Signal control strategies in day-to-day dynamic process

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Abstract: *Signal control affects travellers' route choice behaviour and flow patterns in urban traffic networks. The combined traffic assignment and control problem was introduced to investigate the interaction between route choice and signal control. This concept has been developed for decades since the seminal works of Allsop and Gartner. This paper adopts a day-to-day dynamic modelling framework for traffic assignment and proposes a unifying dynamic process model for study on the combined problem. The day-to-day dynamic model allows for more realistic travel behaviour in terms of travellers' learning and route choice adaptation, as well as more flexible control strategies in response to the re-routing of traffic. Control policies regarding different objectives and updating strategies are designed. The proposed dynamic model is tested and verified in an example network. Different control policies are also evaluated and compared through the numerical study.*

Keywords: Combined traffic assignment and control, day-to-day dynamic model, signal control

1. Introduction

Signal control is an effective way to regulate traffic in urban networks by adjusting road capacities to the demand. Many operating signal plans at local intersections are determined assuming given flow pattern, without consideration on the changes in flow pattern due to travellers' re-routing. Regarding the re-routing of traffic and flow assignment, signal control can influence route choice process thus should be chosen so as to induce desirable flow pattern and network performance. Early works of Allsop (1974) and Gartner (1975) both concerned about how to incorporate route choice as part of traffic control setting and contributed to give a novel view on the close interaction between assignment and control. The Combined Traffic Assignment and Control problem (CTAT) was introduced to deal with this interaction and has been an interesting subject of study for decades (Taale and Van Zuylen, 2001); it still attracts much research attention nowadays (Huang et al., 2013).
Most of the studies on CTAC employ equilibrium-based models for traffic assignment and give particular attention to the properties of the final equilibrium state, such as existence, uniqueness and stability. While it is a specific network design method of designing optimal signal control consistent with user equilibrium, it is not able to describe travellers' route choice adjustment based on their (forecasting) perceptions of travel cost (usually referred to as travellers' learning) which evidently occurs in the underlying traffic networks, especially for commuting traffic. With the emergence of day-to-day dynamic models for representing more realistic route choice adjustment processes, the adoption of dynamic models to accommodate more sensible re-routing behaviour becomes an attractive stream of research (Cantarella and Cascetta, 1995; Cantarella, 2010). Due to the re-routing of traffic, signal timings are adapted to the resulting flow pattern, specifying a dynamical system which may be characterized as route choice adjustment and signal setting adjustment from day to day. Within this context it is important to analyse the interaction between day-to-day dynamic route choice and signal

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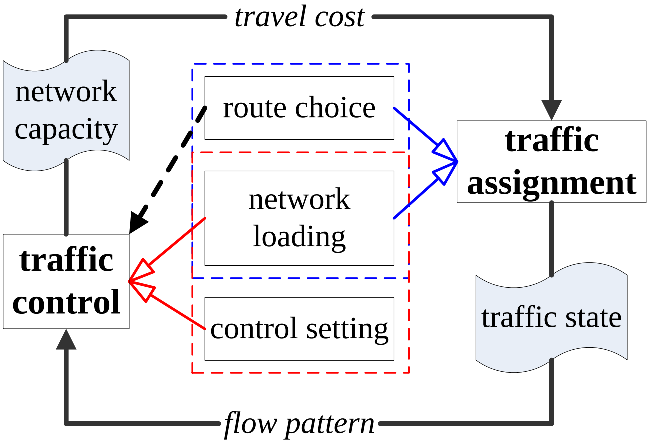
control, and to design effective control strategies. This is the main objective of the present study.
It is worth mentioning that this paper focuses on design of signal control with day-to-day dynamic route choice; it is not designed for optimal control under network equilibrium as in the equilibrium-based approach. Moreover, it is not intended for online traffic control and management, which is concerned more with short term traffic dynamics and optimal network performance assuming flow pattern is not subject to re-routing effect. In this paper, we aim at long term signal design for tracking the temporal evolution of traffic states. It can be regarded as offline signal setting adjustment or fine-tuning of the signal timings which is responsive to travellers' travel time learning and route choice adaptation, on a daily basis or a (more general) pre-specified updating frequency. In short, this study on signal control with day-to-day route choice is to design signal updating/fine-tuning laws which allows for dynamics of route choice behaviour in the long run.
In the rest of the paper, we first illustrate the interaction procedure and propose a unifying dynamic process model for the CTAC problem in section 2. Section 3 presents an analysis on the equilibrium point. Based on the influence of signal control on solutions to the combined problem, different control strategies are designed. The effects of control settings are then tested and compared in a simple network in section 4. Finally some conclusions are drawn and future concerns on developing a flexible learning control system are discussed in section 5.

2. Day-to-day dynamic model for combined traffic assignment and signal control

2.1. Interaction between traffic assignment and control

The interaction between travellers' route choice behaviour and controller's signal setting decision is depicted in Figure 1. A mutual procedure assumes the controller to respond to the travellers' route choice; in turn, travellers choose the route based on travel cost which accounts for signal delays; assuming the consistency exists, this cyclic process holds until a mutually consistent point is reached. However, the controller may anticipate the response of travellers when making decisions by considering the route choice mechanism when updating the signals. This results in another type of interaction, in which controller anticipates route choice reactions, showing a leader's role, and the travellers follow the signal timings and make route choice according to the resulting travel cost. This leader-follower architecture highlights the asymmetric impact of the traffic control optimization facet with respect to the user-optimal assignment side, and suggests the use of global optimization schemes, which can take into account this asymmetry.

Figure 1 : Interaction between traffic assignment and traffic control



As introduced, this interaction can be investigated within two kinds of traffic assignment models, i.e. equilibrium-based models and day-to-day dynamic models. In traditional equilibrium-based models, the underlying hypothesis is the very notion of a market in equilibrium: a typical isolated, 'self-consistent' state of the network which, if attained, would persist under certain rational rules of travel behaviour. In the day-to-day approach, the underlying belief is in the behavioural dynamics, namely how the behaviour is affected by behaviour and the state of the network on previous days (Watling and Hazelton, 2003). Comparing to the equilibrium-based approach, the day-to-day modelling approach is more general in that it recognizes the dynamics and non-equilibrium of the route choice process and includes equilibrium state as one particular solution. Some researches start to focus on the study of traffic control and traffic assignment in day-to-day dynamic process, exploring the models and solution properties (such as existence, stable and convergence of equilibrium), and also the performance of solution algorithms (such as convergence speed, global or local optimality) (Bie and Lo, 2010; Cantarella et al., 2012; Smith, 2011).
Here we adopt the day-to-day dynamic modelling framework and present a unifying dynamic process model for route choice adjustment and signal setting adjustment.

2.2. A general unifying process model

We propose the following general model to simulate route choice adjustment and signal setting adjustment.

$$\begin{aligned}
\mathbf{c}^t &= \mathbf{c}^{t-1} + \alpha\boldsymbol{\omega}^t = \mathbf{c}^{t-1} + \alpha(C(\mathbf{f}^{t-1}, \mathbf{g}^{t-1}) - \mathbf{c}^{t-1}) \\
\mathbf{f}^t &= \mathbf{f}^{t-1} + \beta\boldsymbol{\phi}^t = \mathbf{f}^{t-1} + \beta(F(\mathbf{c}^t) - \mathbf{f}^{t-1}) \\
\mathbf{g}^t &= \mathbf{g}^{t-1} + \gamma\boldsymbol{\psi}^t = \mathbf{g}^{t-1} + \gamma(G(\mathbf{f}^t) - \mathbf{g}^{t-1})
\end{aligned}$$

Where $\mathbf{c}^t$ represents the travel cost vector at day t, $\boldsymbol{\omega}$ is cost changing rate with a weight α; $C(.)$ is the link cost function depending on flow $\mathbf{f}$ and green split $\mathbf{g}$; $\boldsymbol{\phi}$ is the flow changing rate with a weight β; $F(.)$ defines the flow function; $\boldsymbol{\psi}$ is the signal changing rate with a weight γ; $G(.)$ can result from a control optimization problem, or represents some specified control policy, which approximates control optimization.
This modelling approach is well fitted for representing travellers' learning and adaptation in a day-to-day updating process, in which travellers forecast travel cost (learning) and make route choice based on the learning. The model allows for describing the transient behaviour and tracking the evolution of system states from different initial conditions and under different instances of the response functions for both the control setting adjustment and the route choice adjustment. Incorporated within the day-to-day modelling framework, control setting can be evaluated and defined as the domain of attraction associated with the equilibrium state.

3. <u>Analysis on different control strategies</u>

3.1. Equilibrium point and solution stability regarding control strategies

In an iterative approach, the equilibrium state can be defined as the following fixed point, by embedding signal control in the cost function.

$$\begin{aligned}
&\mathbf{c}^* = C(F(\mathbf{c}^*), G(F(\mathbf{c}^*))) \quad or \\
&\mathbf{f}^* = F(C(\mathbf{f}^*, G(\mathbf{f}^*)))
\end{aligned}$$

Where $\mathbf{c}^*$ and $\mathbf{f}^*$ denote equilibrium link cost and flow vectors, respectively. According to the theory of non-linear dynamical systems, the solution stability depends on the eigenvalues of the Jacobian of $\mathbf{J}(\mathbf{c}^t)$ and $\mathbf{J}(\mathbf{f}^t)$.

$$\mathbf{J}(\mathbf{c}^t) = \frac{\partial \mathbf{c}^t}{\partial \mathbf{c}^{t-1}} = \alpha\beta \frac{\partial C}{\partial \mathbf{f}^{t-1}} \frac{\partial F}{\partial \mathbf{c}^{t-1}} + \alpha\beta\gamma \frac{\partial C}{\partial \mathbf{g}^{t-1}} \frac{\partial G}{\partial \mathbf{f}^{t-1}} \frac{\partial F}{\partial \mathbf{c}^{t-1}} + (1-\alpha)$$

$$\mathbf{J}(\mathbf{f}^t) = \frac{\partial \mathbf{f}^t}{\partial \mathbf{f}^{t-1}} = \alpha\beta \frac{\partial F}{\partial \mathbf{c}^t} \frac{\partial C}{\partial \mathbf{f}^{t-1}} + \alpha\beta\gamma \frac{\partial F}{\partial \mathbf{c}^t} \frac{\partial C}{\partial \mathbf{g}^{t-1}} \frac{\partial G}{\partial \mathbf{f}^{t-1}} + (1-\beta)$$

The above formulae show the dependence of the solution stability on the control policy *G(.)* and the corresponding signal updating strategy with parameter γ. This is the theoretical basis for designing dynamic control strategies with day-to-day adjustment characteristics. One interesting topic for future research is to elaborate conditions on different parameters for stability analysis.

3.2. Control policies and signal updating strategies

Signal control can be determined in various ways. In this paper we discuss two control policies with considerations on minimum delay and maximum capacity respectively, as well as different signal updating strategies.

1) Control policies regarding different objective functions

Two control policies are employed in this study, equisaturation policy (Webster, 1958) and P_0 policy (Smith, 1979). In equisaturation control policy, the green splits are distributed in a way that the degrees of saturation of all signal phases being considered are equal. This policy has been widely employed for signal setting.

$$Equal\ [\frac{f_I}{g_I s_I}]$$

In which I denotes the signal phases; s is the saturation flow. By equalizing the degrees of saturation, it is well known that it approximates those strategies that minimize total intersection delay and is fitted for low travel demand level.

Another widely used method is P_0 control policy. In this policy, signal pressure of each phase, which is defined as product of saturation flow and delay, is equalized.

$$Equal\ [s_I . d_I(g_I)]$$

Where *d(.)* is the average delay as a function of green split. It has been verified that this policy encourages traffic flow to take the approaches with higher saturation flow by assigning them more green time to yield lower delay. Therefore it makes full use of the network capacity and approximates those strategies that maximize capacity; thus can serve high travel demand level.

2) Fixed-time control and fully responsive control

Different signal updating strategies apply with respect to different traffic conditions. In the situation that traffic conditions are fairly stable and the traveller route choice does not fluctuate significantly, fixed-time control is applied with signal updating parameter γ=0.

$$\mathbf{c}^t = \mathbf{c}^{t-1} + \alpha(C(\mathbf{f}^{t-1}, \mathbf{g}) - \mathbf{c}^{t-1})$$
$$\mathbf{f}^t = \mathbf{f}^{t-1} + \beta(F(\mathbf{c}^t) - \mathbf{f}^{t-1})$$

Here **g** is given and determined under some average demand level.

In other cases, traffic fluctuates due to the re-routing behaviour. Signal setting should also be adapted to the changed condition. Thus responsive control is taken with γ=1.

$$\begin{aligned}
\mathbf{c}^t &= \mathbf{c}^{t-1} + \alpha(C(\mathbf{f}^{t-1}, \mathbf{g}^{t-1}) - \mathbf{c}^{t-1}) \\
\mathbf{f}^t &= \mathbf{f}^{t-1} + \beta(F(\mathbf{c}^t) - \mathbf{f}^{t-1}) \\
\mathbf{g}^t &= G(\mathbf{f}^t)
\end{aligned}$$

3) Daily response and periodical response

Besides responding daily, the signal response can also be made periodically. This is understandable in the situation that the control setting keeps until certain flow pattern forms. It can also be interpreted as control settings reacting to some long-term flow patterns. We formulate a *T*-day period (denoted by *p*) signal update as follows.

$$\begin{aligned}
\mathbf{c}^t &= \mathbf{c}^{t-1} + \alpha(C(\mathbf{f}^{t-1}, \mathbf{g}^{p-1}) - \mathbf{c}^{t-1}) \\
\mathbf{f}^t &= \mathbf{f}^{t-1} + \beta(F(\mathbf{c}^t) - \mathbf{f}^{t-1}) \\
\mathbf{g}^p &= \mathbf{g}^{p-1} + \gamma(G(\mathbf{f}^T) - \mathbf{g}^{p-1})
\end{aligned}$$

Actually, the signal updating parameter γ makes a flexible signal control structure and leads to different types of response. The daily and periodical response both refer to adaptation to historical information. Other response mechanisms can be also adopted with reaction to current information, which is represented in *G(.)*. The application of combined assignment and control in perturbed network, where controller may respond adaptively to perturbations requires further elaboration on these two response mechanisms.

Similar to describing travellers' learning in the dynamic process models, the signal updating parameter shows an idea that the controller also takes learning from day to day based on previous operations. This motivates future development of a learning control approach for designing control strategies.

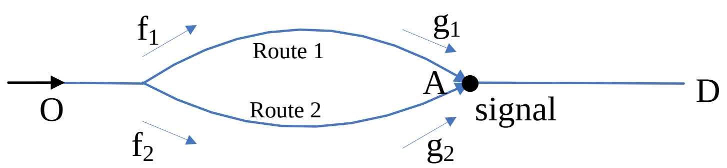
4. Numerical examples

4.1. The example network

A simple two-route network is employed to test the unifying process model and illustrate the effects of different control strategies.

Consider the network in Figure 2, one OD pair is joined by two routes and demand is fixed at d=1000veh/h. A signal light is placed at node A. Travel cost is simply calculated as summation of running time and signal delay. Logit route choice model is employed and the dispersion parameter is denoted by Θ.

Figure 2 : A two-route example network



$$C(f, g) = k + Af + B\frac{f}{gs}$$

Here k is free flow travel time; A is a flow coefficient and B is a signal coefficient; k, A and B are constants.

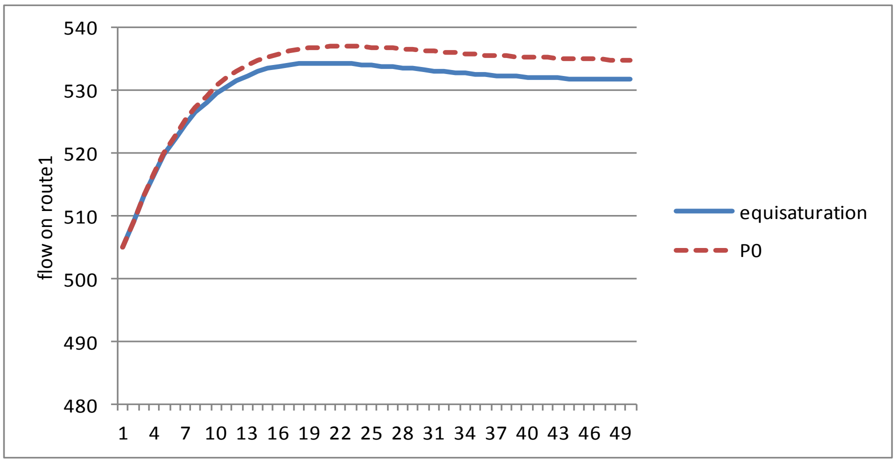
4.2. Testing on different control strategies

In this case study, we take $(k_1,k_2)=(0.04,0.03)$, $(A_1,A_2)=(0.0008,0.0012)$, $(B_1,B_2)=(0.05,0.04)$, $(s_1,s_2)=(1200,800)$ and solve the problem via an heuristic iterative approach. Equisaturation and P_0 are compared. As shown in Table 1 and Figure 3, P_0 assigns more green times and attracts more flows to route 1, which has higher saturation flow. In this toy network, Equisaturation introduces slightly lower delay while P_0 reaches higher capacity.

	demand=1000veh/h		demand=2000veh/h	
	Equisaturation	P_0	Equisaturation	P_0
Green split for route 1	0.49	0.59	0.50	0.61
Green split for route 2	0.51	0.41	0.50	0.39
Flow on route 1 (veh/h)	532	535	1097	1107
Flow on route 2 (veh/h)	468	465	903	893
Total delay (veh.h)	46	47	182	186
Capacity (veh/h)	994	1036	1001	1043

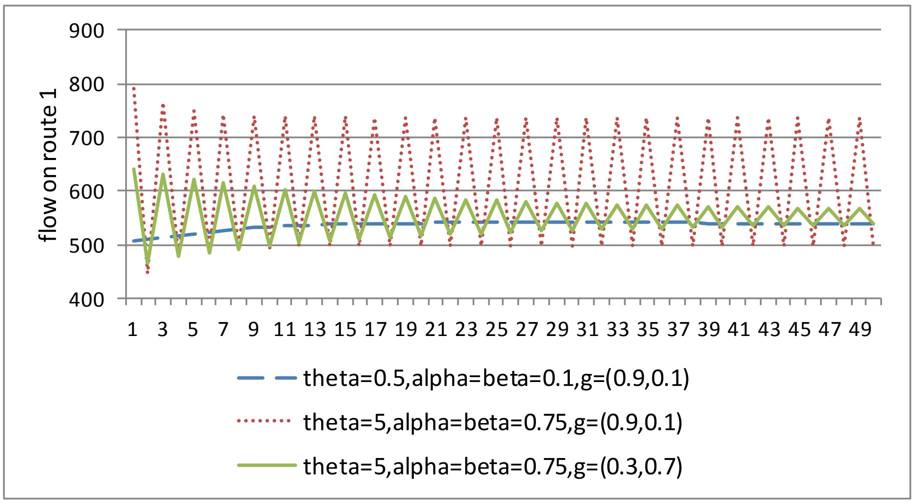
Table 1 : Comparison between equisaturation and P0 control

Figure 3 : Flow trajectory under equisaturation control and P_0 control



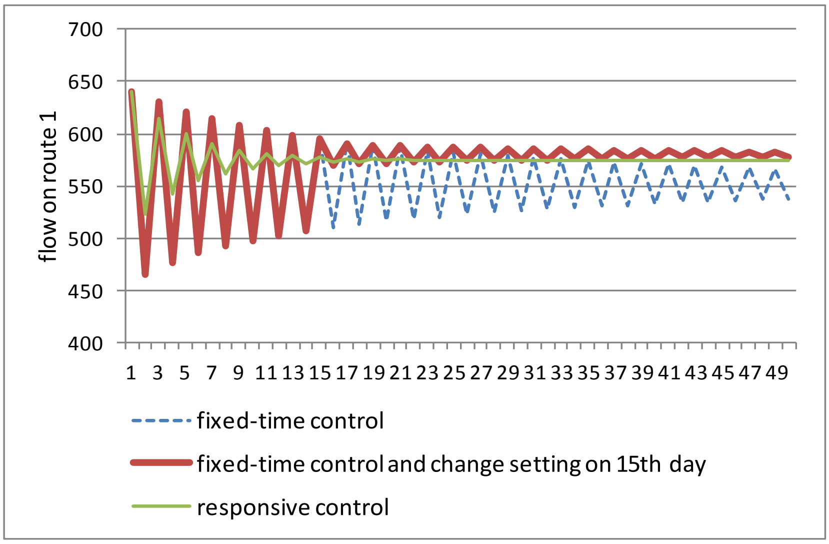
A fixed-time control $(g_1,g_2)=(0.9,0.1)$ is implemented under a relative stable condition, with a small Θ (Θ=0.5) and small α and β ($\alpha=\beta$=0.1) showing more conservative choice behaviour. As shown in Figure 4 flow converges within 50 days. While in a network which fluctuates due to rash traveller behaviour and re-routing (Θ=5, $\alpha=\beta$=0.75), the equilibrium flow pattern does not hold any more. Different control setting $(g_1,g_2)=(0.3,0.7)$ in the same network could change the convergence trajectory of the process.

Figure 4 : Different convergence properties of fixed-time and responsive control



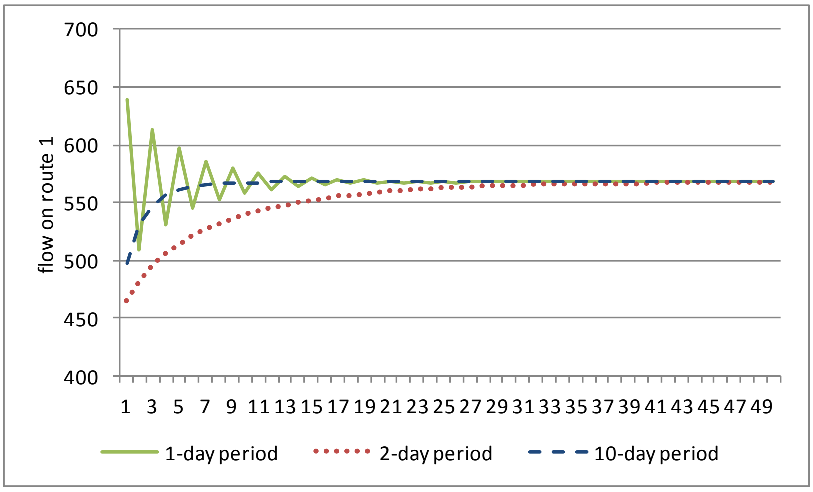
While the fixed-time plan $(g_1,g_2)=(0.3,0.7)$ still cannot equilibrate the network in 50 days, alteration of signal in some day could make it. The following Figure 5 indicates that modifying just the signal setting on the 15th day as $(g_1,g_2)=(0.5,0.5)$ leads to convergence. Besides, a daily responsive control plan implemented in this network also results in equilibrium.

Figure 5 : Effect of a fixed-day signal setting on convergence



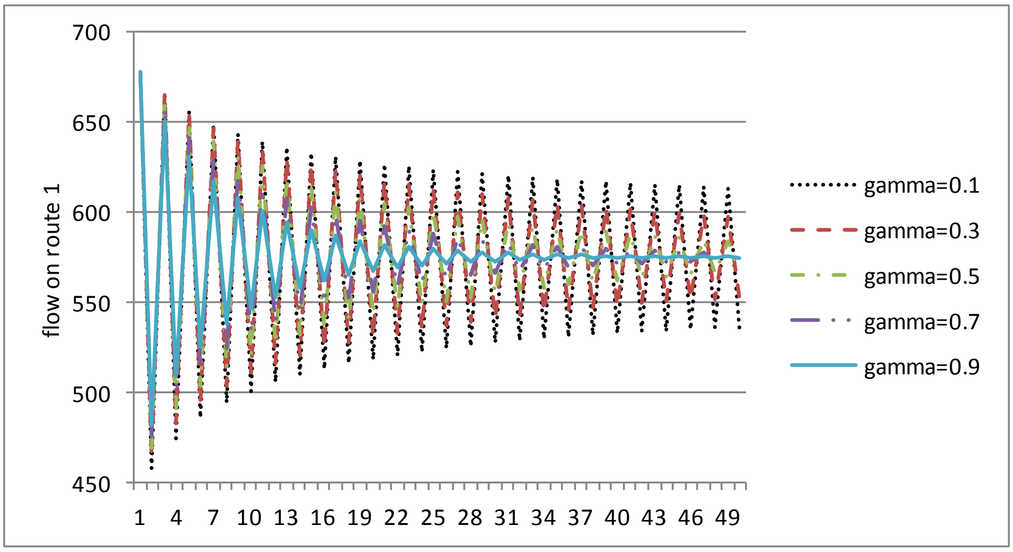
Periodical response of control setting shows a monotonic convergence property in Figure 6. Obviously periodical responsive control with 1-day period is equivalent to the daily responsive control.

Figure 6 : Responsive control with periodical response



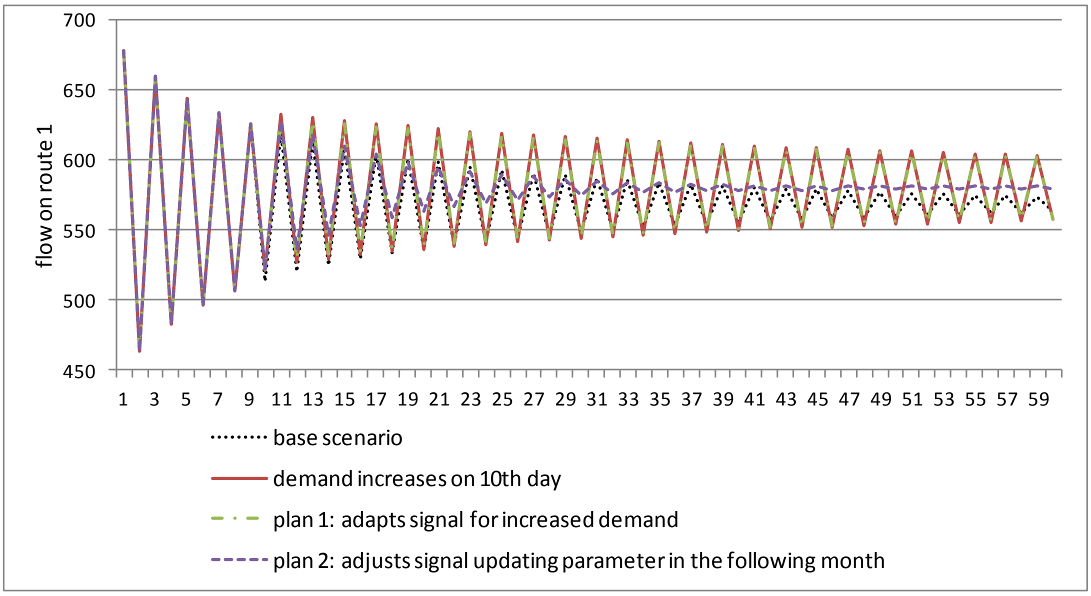
The traffic state evolution also depends on signal updating parameter γ. As shown in Figure 7, in this fluctuating network, higher degree of response leads to quicker convergence.

Figure 7 : Flow evolutions regarding different signal updating strategies



The following result in Figure 8 shows the control effect on network perturbation. OD demand increases on day 10, as d=1020veh/h. The perturbation due to additional demand induces network state fluctuation. Two control plans are implemented; plan 1 just adapts the signal for the higher demand on day 10 while plan 2 also adjusts the signal updating parameter in the following month, reducing the negative effect of control setting on day 10. As shown, the adjusting of updating parameter could re-equilibrate the network state.

Figure 8 : Signal plans for network perturbation



5. Conclusion

This paper proposes a unifying dynamic process model for study on the combined traffic assignment and control problem. The interaction between day-to-day dynamic route choice and signal control is analysed. Control strategies, including two control policies regarding different objectives and different signal updating strategies are designed and evaluated in a toy network. The results verify that signal settings affect the day-to-day interaction. Traffic state and its evolution from day to day depend on the control policies and signal updating strategies. The learning property of control motivates future works on developing a flexible learning control framework. This learning control approach could be employed to systematically design and tune the control settings, and to design robust control for perturbed networks, which should respond to both traveller route choice and network perturbations.

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