Combining Demand Management and Merge Control in an Equilibrium Network Model

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Abstract

Equilibrium models under congested traffic conditions, and especially those addressing blocking back, are very useful to estimate the demand conditions that ITS policies should be able to manage, for instance to maintain congestion within controlled areas and avoiding that they further spillback and cause more serious and/or less controllable congestion states.

The objective of this paper is to supplement the equilibrium model, developed by the authors in recent research, with a more thorough analysis of merge behaviour, especially in cases of blocked nodes. Regulating the merger behaviour together with the demand pattern can lead to certain desired stationary states. It has a great practical significance when congestion is inevitable, while demand management and merge control are able to retain queues and spill-backs within the local area.

Keywords: Equilibrium, Queuing, Merging, Blocking back, Merging control policy.

1. Introduction

This paper integrates and extends recent modelling developments of the authors in the area of quasi-dynamic traffic assignment problems [Smi13], which are recently being proposed as a convenient trade-off between modelling parsimony requirements sought in network equilibrium analysis and the more complex network effects caused by traffic congestion.

By adopting a novel spatial queuing approach, in our previous work we derived equilibrium conditions that explicitly consider buffer spaces occupied by queues on links, which in turn determine the extent to which vehicles move in free driving mode within a link and the capacity restrictions due to blocking back at nodes. This allowed us to study the effect of limited queue storage capacity, and to determine signal time responses to both saturation and buffer space capacities that guarantee an equilibrium to be reached, under certain conditions.

In this study we focus on the impact of different merging policies on queue sizes and on equilibrium. Furthermore, a better understanding of the relationship between node models and equilibrium conditions allows us to assess different control strategies and give recommendations on how to manage (sub-)networks in an efficient way, either by metering competing flows so that they match both equilibrium and merging requirements, or by adopting specific signal setting policies on the merges that would guarantee solution convergence to equilibrium.

The findings in this paper are relevant for ITS as they will contribute to network-wide dynamic traffic management by means of dynamic demand and supply control strategies which will be quickly found and used to control traffic in real time. A clear advantage of these models is their simplicity and their applicability within more complex online control systems.

2. Equilibrium with queuing, block-back and capacity constraints

This section summarizes the main model developments in [Smi13], which represent the basic information used to apply and analyse different merge models.

Under congested conditions, queues emerge at bottlenecks, and under some condition they back-propagate. In line with the philosophy adopted in [Smi13] we are not considering in this study the spatio-temporal propagation of the queue fronts within a link and onto nodes. Instead, we account for the effects of space taken up by traffic queues and spillback onto nodes within an equilibrium model, which requires only the explicit calculation of steady state conditions, and not necessarily its transient evolution. This type of approach is being referred to as quasi-dynamic equilibrium formulation (see e.g. [Dag98], [Bli12]).

2.1 The basic link model

The quasi-dynamic assignment formulation developed in [Smi13] lays its foundations on a special link model, which explicitly takes into account the reduced space in free-driving mode due to the occurrence of a queue in the link.

Let v_i be the flow entering link i, s_i the saturation flow at the exit of link i, and Q_i the queue of vehicles waiting to exit link i; let the maximum possible value of Q_i be $MaxQ_i$ and the time to traverse the entire length of link i (when the queue $Q_i = 0$) is

$c_i(v_i)$. Feasibility conditions are the inequalities $v_i \leq s_i$ and $Q_i \leq MaxQ_i$.

Let also b_i be the delay due to queuing experienced by vehicles, from the moment they joined the queue until they exited link i. Using the classical Little's law we can assume $b_i = Q_i/s_i$.

Consider the case where $0 \leq Q_i \leq MaxQ_i$. In this case, the queue occupies part of the length of link i, so the cost in free-driving mode tends to decrease with increasing Q_i. The time for traversing link i will thus be the sum of a non-queueing component and a queueing component, which are mutually varying. This mutual consistency can be modelled in a rather simple manner with the following relationship, which provides the total cost spent to traverse link i:

$$Sum_i = c_i(v_i) + k_i b_i \tag{1}$$

where k_i is denoted as the 'shrinkage' factor. In [Smi13] it has been shown that in case of no blocking back this shrinkage factor takes the expression

$$k_i = 1 - s_i c_i(s_i)/MaxQ_i \tag{2}$$

Expression (2) prevents the typical overestimation problem when vertical queuing is used; in fact, if $Q_i = 0$ then $Sum_i = c_i(v_i)$, which is rather straightforward as the total costs is consisting of only the driving time to traverse the whole link as function of the flow, while if $Q_i = MaxQ_i$ then $Sum_i = MaxQ_i/s_i = Maxb_i$, so the total cost to traverse the fully congested link consists of only the queuing delay. For all values $0 < Q_i < MaxQ_i$ it can be observed that $k_i > 0$.

Note that it would be more 'natural' to adopt a shrinkage factor to multiply $c_i(v_i)$, as it seems more intuitive to observe a reduced space in free-driving mode. However, it can be shown that a multiplier $h_i = [MaxQ_i - Q_i]/MaxQ_i$ associated to $c_i(v_i)$ leads to the same expression. In addition, k_i does not depend on Q_i, which is found only solving the equilibrium problem, while expression (2) solely depends on constant and predetermined parameters. A demonstration of the equivalence between shrinkage factors k_i and h_i can be found in [Smi13].

Expressions (1)-(2) assume no blocking back, so that the condition $Q_i > 0$ can occur only if $v_i = s_i$. When link outflow is restricted by downstream queue filling a downstream link and overflowing, the flow along link i can be less than s_i. The shrinkage factor k_i becomes in this case dependent on v_i and is no longer constant:

$$Sum_i = c_i(v_i) + k_i b_i = c_i(v_i) + [1 - v_i c_i(v_i)/MaxQ_i] b_i \tag{3}$$

2.2. Network representation

The network representation in this study follows the multi-level framework used extensively in past studies to model multi-commodity flows. For more details one should look at [Smi13].

2.3. Equilibrium formulation

We use the standard Wardrop [War52] notion that for each OD pair more costly routes are unused. Again, details on equilibrium conditions and the Variational Inequality formulation derived are found in [Smi13]. For sake of keeping the paper to the required size limits we directly deal with the problem analysing a simple network example, which serves as proof of concept.

2.4 A simple numerical example

Consider the network in **Figure 1**; two OD pairs are joined by three routes as follows.

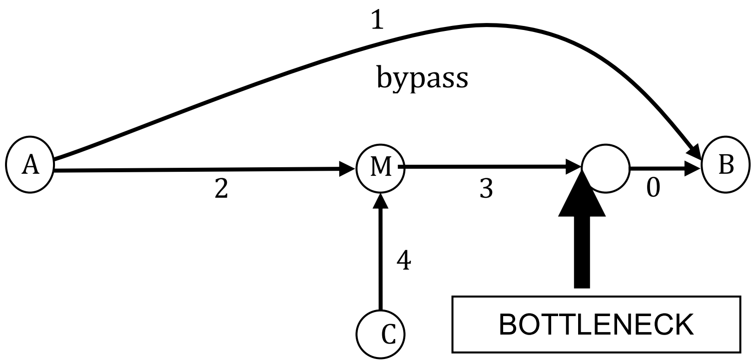

Figure 1: *Simple network with one bottleneck, two origins and one destination*

- Route 1 (the bypass) joins the OD pair [A,B], and has just link 1.
- Route 2 joins OD pair [A,B], and has links 2, 3 and 0.
- Route 3 joins OD pair [C,B], and has links 4, 3 and 0.

Link 2 has a saturation flow at the exit of s_2 vehicles per minute and link 3 has a saturation flow at the exit of s_3 vehicles per minute. All other links will have very large saturation flows and $s_2 > s_3$ so the exit of link 3 is a bottleneck. Links 2 and 4 merge at M. The steady OD flow rate from A to B is fixed at T_{AB} vehicles per minute where $T_{AB} > 0$. The steady OD flow rate from C to B is fixed at T_{CB} vehicles. If v_i is the flow rate along link i, we suppose that with no queueing the time taken to traverse link i (i = 0, 1, 2, 3, 4) will be $c_i(v_i)$. We assume that link 0 has zero travel time as it does not influence the results.

In case of no merging traffic, T_{CB}=0. We assume that $T_{AB} > s_3$ so that not all the demand can be served in the considered time period (here and for all the analysis considered unitary), and that $c_2(v_2) + c_3(v_3) < c_1(v_1) \leq c_2(v_2) + Maxb_3$ so at equilibrium queuing occurs at link 3, but conditions are such that this queue does not get longer than the maximum buffer space $MaxQ_3$. To determine the bottleneck delay on link 3 at equilibrium we use link performance model (3). Then $c_1(v_1) = c_2(s_3) + c_3(s_3) + k_3 b_3$, and considering that $T_{AB} = v_1 + s_3 + Q_3$ hence

$$b_3 = \frac{c_1(T_{AB} - s_3 - Q_3) - (c_2(s_3) + c_3(s_3))}{[1 - s_3 c_3(s_3)/MaxQ_3]} \tag{4}$$

considering $k_3 = [1 - s_3 c_3(s_3)/MaxQ_3]$ by (2), and $Q_3 = b_3 s_3$ by the Little's law. Thus, the steady-state equilibrium queueing delay (and the equilibrium queue) depends on the uncongested travel times, the outflow capacity of link 3 and the storage-capacity of link 3.

We now consider the case of blocking back occurring at node M; we also assume that there is not yet merging traffic flow. Suppose that$c_2(v_2) + Maxb_3 < c_1(v_1) \leq Maxb_2 + Maxb_3$. The link 3 queue must then spillback onto link 2. The equilibrium queues will be such that the total delay incurred on the two queued links 2 and 3 equals the uncongested travel time difference between two alternative routes joining A and B. So, now $c_1(v_1) = c_2(s_3) + k_2 b_2 + Maxb_3$ using (3). Considering now that $T_{AB} = v_1 + s_3 + Q_2 + MaxQ_3$ in this case

$$b_2 = \frac{[(c_1(T_{AB} - s_3 - Q_2 - MaxQ_3) - (c_2(s_3) + c_3(s_3))) - (MaxQ_3/s_3 - c_3(s_3))]}{[1 - s_3 c_2(s_3)/MaxQ_2]} \tag{5}$$

considering $k_2 = [1 - s_3 c_2(s_3)/MaxQ_2]$ and $Q_2 = b_2 s_3$. Queuing delay becomes this time a function of the link travel times, the link saturation flow of link 3 and the storage-capacity of both links 2 and 3. These equations show the dependence of the steady-state queue on a link upstream of the block back node M and on the queue storage-capacity of the blocked link 3. One may easily verify that equilibrium cannot be achieved in this network if $c_1(v_1) > Maxb_3 + Maxb_2$, i.e. route 1 is not appealing even with fully saturated links 2 and 3. Any queue in this case would spillback onto origin node A, and thus outside the analysed network.

We have so far analysed situations where no merging flow enters the system from node M coming from origin C. If $T_{CB} > 0$ some portion of the flow and the queue observed at link 3 is taken by this demand, and reduces the opportunity for link 2.

Given therefore a fixed demand, T_{CB}, we should specify a realistic merge operation. To add this extra complexity, we need to specify a merge model. In the following section we introduce different merge models proposed in literature, and analyse how the choice of a specific model form affects the existence of equilibrium.

3. Merge models

To analyse the impact of different merge models we refer again to the example network of **Figure 1**, therefore we deal with the problem in this study of only two merging flows onto one capacitated link. We will elaborate a general formulation with multiple merging and diverging links in future research.

It is straightforward to observe that, to have equilibrium, and if conservation of vehicles principle holds, there must be no distinction between sending and receiving flows, as it is instead done in dynamic network loading models. No stationariety would otherwise be observed in the system and queues would be time dependent. This can be formulated as:

$$r_2 = \frac{v_2}{v_2 + v_4}, r_4 = \frac{v_4}{v_2 + v_4} \Rightarrow p = \frac{r_2}{r_4} = \frac{v_2}{v_4} \tag{6}$$

Therefore, the type of priority observed at the merge determines the proportion between flows v_2 and v_4 to be met at equilibrium. This holds for any arbitrary time period, and therefore also for the analysed unitary time period.

At equilibrium, four possible states can occur, involving two merging links: 1) No queue is observed at any of the entering links as demand is low; 2) Only link 2 is queued; 3) Only link 4 is queued; and 4) Both links are queued.

Various merge ratios are suitable for different layouts, different junction geometries and different "controls". A variety of factors in real life determine how these output flows and delays are distributed among the two approaching traffic streams. Here we distinguish and discuss some basic rule:

- ***Fixed merge models***

Merge ratios can be assumed constant, and the ratio p can be fixed according to observed behaviour in real merging situations. Examples of these models in literature are the Daganzo's fixed merge model [Dag98], where p can take any arbitrary value. An instance of Daganzo's fixed merge model is the "zipper" rule, which assumes that drivers give way to the other approach in such a way that vehicles from the two approaches alternate with equal frequency. The model is obtained by letting the distribution fractions in the previous model be ½.

- ***"Fair shares" merge models***

These models assume that merge behaviour is in dependent on the traffic load of each merging link, or has some relationship with the importance of the roads, normally represented by their capacity. In this class of models distribution fractions are proportional to the flows along the incoming links, i.e. the higher flow tends to get more priority. This means, in mathematical terms, that $p = v_1/(v_1 + v_2)$, or they can be proportional to the saturation flows of the incoming links, i.e. more capacitated links get more priority. This means, in mathematical terms, that $p = s_1/(s_1 + s_2)$.

- ***Equal delay merge models***

A third and perhaps more sophisticated merge rule is that merge priority is in some way proportional to the queue lengths or the delay incurred at each merging link. This rule mimics the natural behaviour of drivers who tend to get more risk prone and less inclined to give way to if they had to wait longer for their turn. In mathematical terms, this means that a new equilibrium condition is imposed in the system. In this paper we do not distinguish the two equilibrium conditions (equal queue length or equal delay) as they are equivalent due to the adoption of Little's law.

Equilibrium and merging constraints

We assume that link 1 is used and link 3 is fully saturated. We assume initially that no queues are observed at links 2 and 4, but only, eventually, at link 3. It holds straightforwardly:

$$
\begin{gathered}
c_1(v_1) = c_2(v_2) + c_3(s_3) + k_3 b_3 \\
T_{AB} = v_1 + r_2(s_3 + Q_3) = v_1 + \frac{p}{p+1}(s_3 + Q_3) \\
T_{CB} = r_4(s_3 + Q_3) = \frac{1}{p+1}(s_3 + Q_3)
\end{gathered}
\tag{7}
$$

Equations (7) are necessary conditions to observe equilibrium on the simple network in **Figure 1**, which satisfies the merging fraction p introduced in eqn (6) while no queue emerges on both merging links. Considering $k_3 = [1 - s_3 c_3(s_3)/MaxQ_3]$ by (2), and $Q_3 = b_3 s_3$ by the Little's law, we obtain

$$
b_3 = \frac{c_1(T_{AB} - \frac{p}{p+1}(s_3 + Q_3)) - (c_2(s_3) + c_3(s_3))}{1 - s_3 c_3(s_3)/MaxQ_3}
\tag{8}
$$

If instead maximum queue is reached at link 3, $MaxQ_3$, while link 2 is the only one queued (point **C**), using eqn (2) to account for the relation between free-driving and queuing delay onto the link, we obtain:

$$\begin{gathered} c_1(v_1) = c_2(v_2) + k_2 b_2 + Maxb_3 \\ T_{AB} = v_1 + Q_2 + r_2(s_3 + MaxQ_3) = v_1 + Q_2 + \frac{p}{p+1}(s_3 + MaxQ_3) \\ T_{CB} = r_4(s_3 + MaxQ_3) = \frac{1}{p+1}(s_3 + MaxQ_3) \end{gathered} \tag{9}$$

resulting in

$$b_2 = \frac{c_1\left(T_{AB} - Q_2 - \frac{p}{p+1}(s_3 + MaxQ_3)\right) - \left(c_2(s_3) + c_3(s_3)\right)}{1 - s_3 c_3(s_3)/MaxQ_3} \tag{10}$$

The case of a queuing delay observed only at link 4 results in

$$\begin{gathered} c_1(v_1) = c_2(v_2) + Maxb_3 \\ T_{AB} = v_1 + v_2 + r_2(s_3 + MaxQ_3) = v_1 + \frac{p}{p+1}(s_3 + MaxQ_3) \\ T_{CB} = Q_4 + r_4(s_3 + MaxQ_3) = Q_4 + \frac{1}{p+1}(s_3 + MaxQ_3) \end{gathered} \tag{11}$$

Finally if at both merging links stationary queues are observed it holds:

$$\begin{gathered} c_1(v_1) = c_2(v_2) + k_2 b_2 + Maxb_3 \\ T_{AB} = v_1 + Q_2 + r_2(s_3 + MaxQ_3) = v_1 + Q_2 + \frac{p}{p+1}(s_3 + MaxQ_3) \\ T_{CB} = Q_4 + r_4(s_3 + MaxQ_3) = Q_4 + \frac{1}{p+1}(s_3 + MaxQ_3) \end{gathered} \tag{12}$$

resulting in a similar formulation for b_2 as (10).

We want to stress out that the above components are summed considering a unitary time dimension. For example looking at eqn (15), one should read for the demand equations of T_{AB} and T_{CB} that they are sum of an amount of vehicles flowing ($v_1 + s_3$ and s_3, respectively), and an amount holding in queue ($Q_2 + \frac{p}{p+1}MaxQ_3$ and $Q_4 + \frac{1}{p+1}MaxQ_3$, respectively) during a certain unitary period.

Looking at eqns (7)-(12), a desirable condition in managing (sub-)networks such as the one depicted in **Figure 1**, would be to coordinate the inflow into such systems and the merging priorities in such a way that equilibrium could be met. This could be a very basic area traffic control strategy, which may prevent congestion to back-propagate outside of the controlled area, so that the “damage” of blocking back could be contained to a maximum acceptable extent. It could be even more desirable if an automatic and local control policy would be able to adapt the priority p parameter such that a range of feasible demand conditions could be handled and equilibrated. In the following section we aim at deriving a very simple analytical model with this goal.

A combined demand management and merge control strategy

Taking again the example network of **Figure 1**, a combined inflow and merge control can be designed regarding specific management objectives. For example, if higher priority is given to link 2 in order to make sure that queues stay within downstream links, the merge distribution fraction is determined to guarantee higher merge fractions to flow 2.

Here we analyse the case where we do not put priority to any of the two merging flows, and we derive analytically the conditions for the inflows and the priority p in case of fixed merge models. Considering only the case of a queue blocking node M and back-propagating onto links 2 and 4 (thus equilibrium conditions (12)), if we assume desired queue states $Q_2 \leq MaxQ_2$ to avoid spillback onto node A, and no restriction is imposed on queue Q_4 we have to simply add the constraint

$$T_{AB} - v_1 - \frac{p}{p+1}(s_3 + MaxQ_3) \leq MaxQ_2 \tag{13}$$

which sets a specific range of possible priority fractions p, given T_{AB}. Vice-versa, T_{AB} could be limited in such a way that a certain merge priority p is allowed.

If the equal delay merge model replaces the fixed merge as the merge constraint, an additional constraint determines the extent to which Q_4 can vary. Considering the concurrent use of link 3 determined by the assumed merge priority p we obtain

$$\frac{Q_2}{r_2 s_3} = \frac{Q_4}{r_4 s_3} \tag{14}$$

which, considering that $r_2 = p/(1+p)$ and $r_4 = 1/(1+p)$ it makes the simple constraint $Q_2 = pQ_4$.

More generally, from the manager's point of view, it is more desirable to control the merge behaviour in a way that by adjusting the merge distribution fraction, equilibrium results can be obtained with conditions on feasible demand sets. The P_0 control policy of [Smi79] is analysed here. The main motivation to use this classical local control policy is to complement the spillback-avoiding strategy, guaranteed by inequality (13), with a control policy aimed at maximising the total network throughput, instead of being fair towards each merging link, as guaranteed by the equal delay condition (14).

A P_0-like control policy aimed at managing the system while keeping congestion within the controlled system should take into account the different pressure coming from the merging links (represented by the link saturation flows), but in the same time guarantee that a maximum number of vehicles is sent to the downstream link, which means in **Figure 1**, to make sure that $s_3 + MaxQ_3$ is sent. This is achieved by adding the following extra constraint to the network equilibrium condition:

$$s_2.[r_2(s_3 + MaxQ_3)/r_2 s_3] + Q_2 = s_4.[r_4(s_3 + MaxQ_3)/r_4 s_3] + Q_4 \qquad (15)$$

where the first component of each side controls the flowing part of the system, while the second depends on the queued part.

In future papers we will discuss the properties of this control strategy (especially in terms of stability) and test it onto different networks, and we will compare it with other policies such as the equal delay policy.

Conclusions

This paper has extended an equilibrium model for congested networks, previously developed by the authors, by analysing the impact of different merge models.

Adding merge priorities imposes additional constraints to the existence of an equilibrium. Inversely, using these constraints in combination with Wardrop conditions enables one to identify desirable control states, for which if queues emerge and eventually back-propagate onto the nodes internal to the controllable network, they are likely to stabilize and stay within the controlled area.

Future steps will be to make a more thorough analysis of uniqueness and stability of these control strategies, and to test different management objectives, which could integrate the introduced basic constraints, for instance throughput maximization.

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