Agents in the shadow

(cooperative games with non-cooperating players)

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The aim of this talk is to report on recent investigations about lattice derivatives of Boolean and pseudo-Boolean functions and their interpretations in game theory.

The partial lattice derivatives of a (pseudo-) Boolean function are analogues of the classical partial derivatives, with the difference operation replaced by the minimum or the maximum operation. We focus on commutation properties of these lattice differential operators and relate them to local monotonicity properties. The least and greatest functions that can be obtained from a given function f, by forming lattice derivatives with respect to all variables, are called the lower and the upper shadows of f, respectively. It turns out that the lower and upper shadows of f coincide with the α- and β-effectivity functions of the cooperative game corresponding to f. Thus a function f has a unique shadow if and only if the two effectivity functions coincide, which is equivalent to the fact that certain two-player zero-sum games associated with (the cooperative game corresponding to) f are strictly determined. We formulate a conjecture about Boolean functions (i.e., simple games) with a unique shadow, and present proofs in some special cases as well as results of computer experiments that support our conjecture.

Keywords: cooperative games, non-cooperative games, α- and β-effectivity, simple games, strictly determined games