Infinitary aggregation

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Abstract. In this paper, based on [12, 18], we present infinitary aggregation functions on sequences possessing some a priori given properties. General infinitary aggregation is also discussed, and the connection with integrals, e.g., Lebesgue, Choquet and Sugeno integrals, is given.

Aggregation of finitely many inputs, directly related to many applications, were investigated in many fields [1–3, 5, 7, 12, 15, 24, 26]. Aggregation of infinitely but still countably many inputs is important in several mathematical areas, such as discrete probability theory, but also in non-mathematical areas, such as decision problems with an infinite jury, game theory with infinitely many players, etc. Though these theoretical tasks seem to be far from reality, they enable a better understanding of decision problems with extremely huge juries, game theoretical problems with extremely many players, etc., see [20, 22, 25].

In our contribution, based on [12, 18], we discuss infinitary aggregation functions on sequences possessing some a priori given properties, such as additivity, comonotone additivity, symmetry, etc. Based on these properties, infinitary OWA operators are discussed, among others, see [23]. On the other side we discuss infinitary aggregation functions $\mathsf{A}^{(\infty)}\colon [0,1]^{\mathbb{N}} \to [0,1]$ related to a given extended aggregation function $\mathsf{A}\colon \cup_{n\in\mathbb{N}} [0,1]^n \to [0,1]$, where special attention is paid to t-norms, t-conorms, and weighted arithmetic means, where a connection with Toeplitz matrix (see [4, 11]) was obtained. Note that the discussion of the infinitary arithmetic mean $\mathsf{AM}^{(\infty)}\colon [0,1]^{\mathbb{N}} \to [0,1]$ can be found in [13, 14].

General infinitary aggregation is also discussed (see [12, 19]), thus extending the results concerning aggregation of infinite sequences. Note that in such case, some restrictions on the domain of aggregation functions is usually necessary. For example, to apply Lebesgue, Choquet or Sugeno integrals, see [21], one should require the measurability of the input function to be aggregated.

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