Contribution on some construction methods for aggregation functions

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Abstract. In this paper, based on [14], we present some well established construction methods for aggregation functions as well as some new ones.

There is a well-known demand for an ample variety of aggregation functions having predictable and tailored properties to be used in modelling processes. The need for bigger flexibility and ability of fitting more accurate aggregation functions requires the extension of aggregation functions buffer, and one of approaches how to reach this is just based on construction methods. Several construction methods have been introduced and developed for extending the known classes of aggregation functions (defined either on $[0,1]$ or, possibly, on some other domains). There are several construction methods, introduced in many fields [1–5, 7, 9, 14, 15, 24, 25]. Obviously, new construction methods should be a central issue in the rapidly developing field of aggregation functions. In this paper we present some well established construction methods as well as some new ones.

The first group of construction methods can be characterized "from simple to complex". They are based on standard arithmetical operations on the real line and fixed real functions. The second group of construction methods starts from given aggregation functions to construct new ones. Here we can start either from aggregation functions with a fixed number of inputs (e.g., from binary functions only) or from extended aggregation functions. Observe that some methods presented are applicable to all aggregation functions (for example, transformation), while some of them can be applied only to specific cases. Finally, there are construction methods allowing us to find aggregation functions when only some partial knowledge about them is available. For more details on this topic we recommend [14], Chapter 6. In our presentation we will discuss these items:

- transformation of aggregation functions (recall the classical transformation of the sum into the product),
- composed aggregation (recall recursive aggregation functions, convex sums, etc.),
- weighted aggregation functions (quantitative and qualitative approaches),
- aggregation based on optimalisation (mixture operators, for example),
- ordinal sums of aggregation functions (covering in one formula well-known ordinal sums of t-norms and t-conorms).

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