Non-destructive Damage Assessment Using Non-linear Vibration

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NOMENCLATURE

M, m	Mass matrix, mass
C, c	Damping matrix, damping value
K, k	Stiffness matrix, stiffness
F	Excitation force
y	Deformation
Y	Response
H	Frequency response function

ABSTRACT

The dynamic properties like natural frequencies and damping values of linear behaving structures can be identified by means of frequency response functions. However, in many cases, structures do not behave strictly linear, e.g. for concrete structures a non-linear material behaviour has to be considered. Furthermore, defects, as for instance cracks in reinforced concrete, lead to additional non-linearities that increase with the level of damage. Exciting the structure harmonically, a force dependency of all the modal parameters and the occurrence of higher harmonics can be observed. Hence, the use of dynamic methods to assess the state of damage has to take care of these nonlinearities to avoid misinterpretation. Moreover, the nonlinearities themselves can be explicitly used as damage indicators, as they are dependent on the damage state. This paper presents the results of an experimental analysis with three reinforced concrete beams at different damage states. Furthermore, the first results of forced excitation tests on a gradually damaged bridge concerning non-linear-behaviour are shown.

1 INTRODUCTION

Assuming linear behaviour of a harmonically excited structure, the differential equation (1) describes the vibration adequately, where *M* is the mass matrix, *C* the damping matrix, *K* the stiffness matrix and *F* the excitation force.

$$M\ddot{y}(t) + C\dot{y}(t) + Ky(t) = F(t) \tag{1}$$

The dynamic properties (natural frequencies, damping, mode shapes) are identified by means of the frequency response function (FRF). The frequency response function according to equation (2) can be established by dividing the vibration response $Y(\omega)$ by the input signal $F(\omega)$ in the frequency domain.

$$H_{lk}(\omega) = \frac{Y_l(\omega)}{F_k(\omega)} \tag{2}$$

For a SDOF system with a mass *m*, a spring *k* and a damper c the FRF is defined:

$$H_{lk}(\omega) = \frac{Y_l(\omega)}{F_k(\omega)} = \frac{1}{(k - \omega^2 m) + i(\omega c)} \tag{3}$$

Exciting the system with its natural frequency $\omega = \omega_0$ the FRF takes maximum value. When the FRF is known or can be measured, the damping value and the natural frequency can directly be extracted.

In case of a linear behaving system, the FRF is independent of the excitation force and vibration amplitude. Disturbances in the structure like cracks lead to non-linear stiffness- and damping matrices. In this case the coefficients in the equation of motion (1) and the dynamic properties are dependent on the vibration amplitude, velocity and thus on the excitation force. Damages in reinforced concrete structures like friction in cracks, change in stiffness due to the alternately opening and closing of cracks under dynamic excitation or amplitude dependent material behaviour lead to particular distinct non-linear-behaviour.

2 EXPERIMENTAL APPROACH

Three reinforced concrete beams with different damage levels are examined concerning their non-linear dynamical behaviour [1],[2]. The 6 m long beams are made of concrete C40/50 and six reinforcement bars of diameter 16 mm are equally distributed over the tension and the compression zone. Figure 1 shows the three beams.

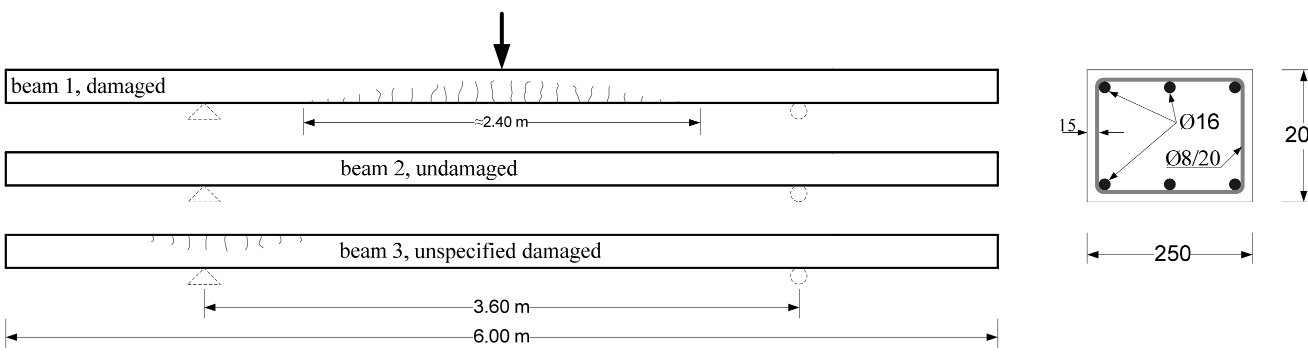

Figure 1. Three examined beams

Beam 1 has been loaded up to 45% of its calculated strength during a symmetrical three point bending test. Beam 2 can be considered as undamaged. The analysis of the dynamical behaviour of beam 3 (unspecified damaged) detects a damage in the area of one support due to unscheduled loading during storage. In order to avoid environmental influences, the beams are suspended to flexible springs during the dynamic test [3]. A shaker excites the beam via a force sensor. The dynamic responses are measured with acceleration sensors at three points on the beam.

Initially the natural frequencies of the three beams are measured using hammer impact. The energy associated with an individual frequency is small and non-linear effects do not appear clearly [4]. So only the quasi-linear dynamic properties of the system can be observed, when using hammer impact method. Table 1 figures the first eight natural frequencies of the three beams. As expected the natural frequencies of the damaged beam 1 are smaller compared to the undamaged beam, due to the reduced stiffness.

Table 1. Natural frequencies [Hz] using hammer-impact

mode	damaged beam 1	undamaged beam 2	unsp. dam. beam 3
B1	16.6	22.8	21.7
B2	52.2	63.1	57.2
B3	106	123	113
T1	165	194	187
B4	172	201	191
B5	250	295	283
T2	350	391	375
B6	361	407	390

For the investigation of the non-linear-behaviour, the beams are excited harmonically by means of a swept sine excitation starting with an excitation frequency smaller than the first natural frequency and ending with an excitation frequency higher than the third natural frequency of the beam. In order to transfer as much as possible of the oscillation energy into the nonlinear parts of the oscillation, the beams are excited with a swept sine with a small sweep-rate of 0.3Hz/s. To avoid additional damage due to the test the excitation force for the undamaged beam 2 and unspecified damaged beam 3 has been restricted to 80N in the first natural frequency. This results into maximum tensile stress of approximately 2.7N/mm^2, which is smaller than the nominal tensile stress of concrete C40/50.

In case of linear systems, the FRF is independent of the excitation force. However, as already mentioned in the introduction, the stiffness and damping values are dependent on the excitation force for non-linear systems. Therefore, observing nonlinear systems necessitates a comparison between the dynamic parameters of the system under different excitation forces. Figures 2, 3 and 4 show the frequency response functions for the first three modes of the beams for different excitation forces.

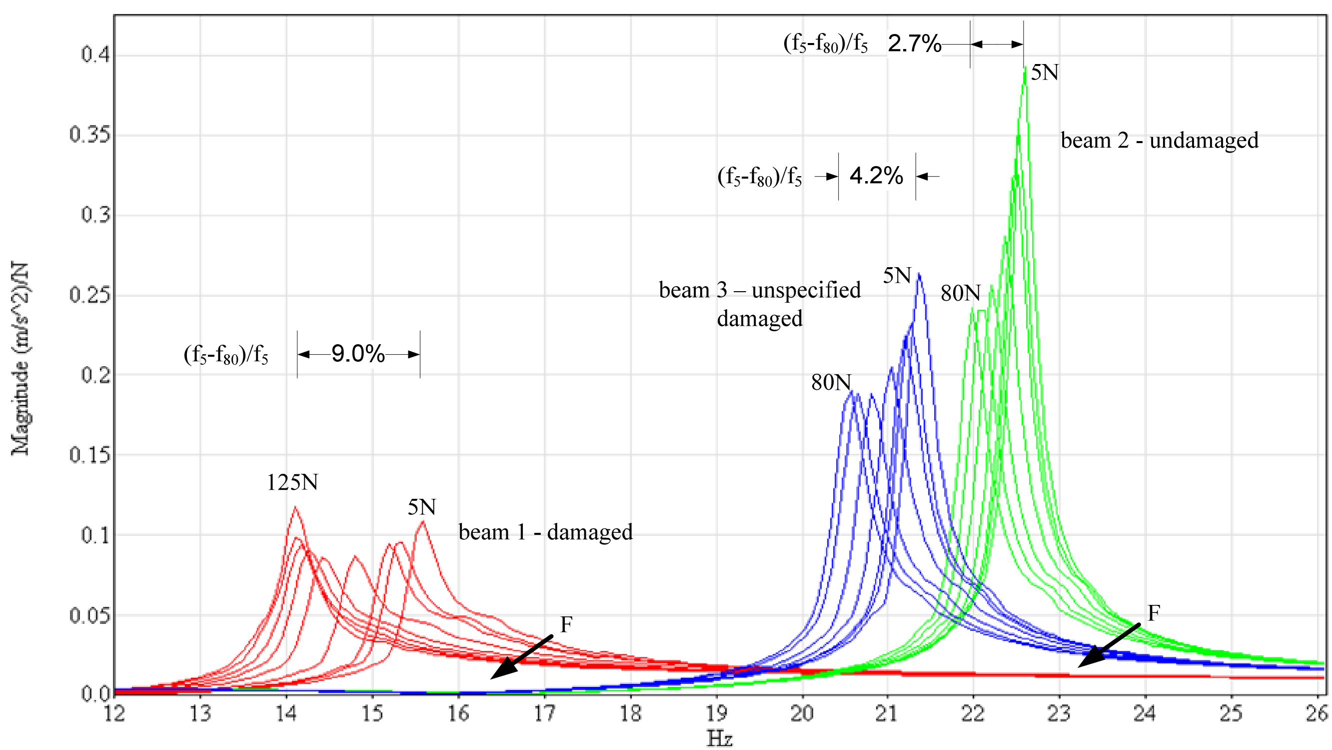

Figure 2. FRF, mode 1, damaged beam 1 (red), undamaged beam 2 (green), unspecified damaged beam 3 (blue)

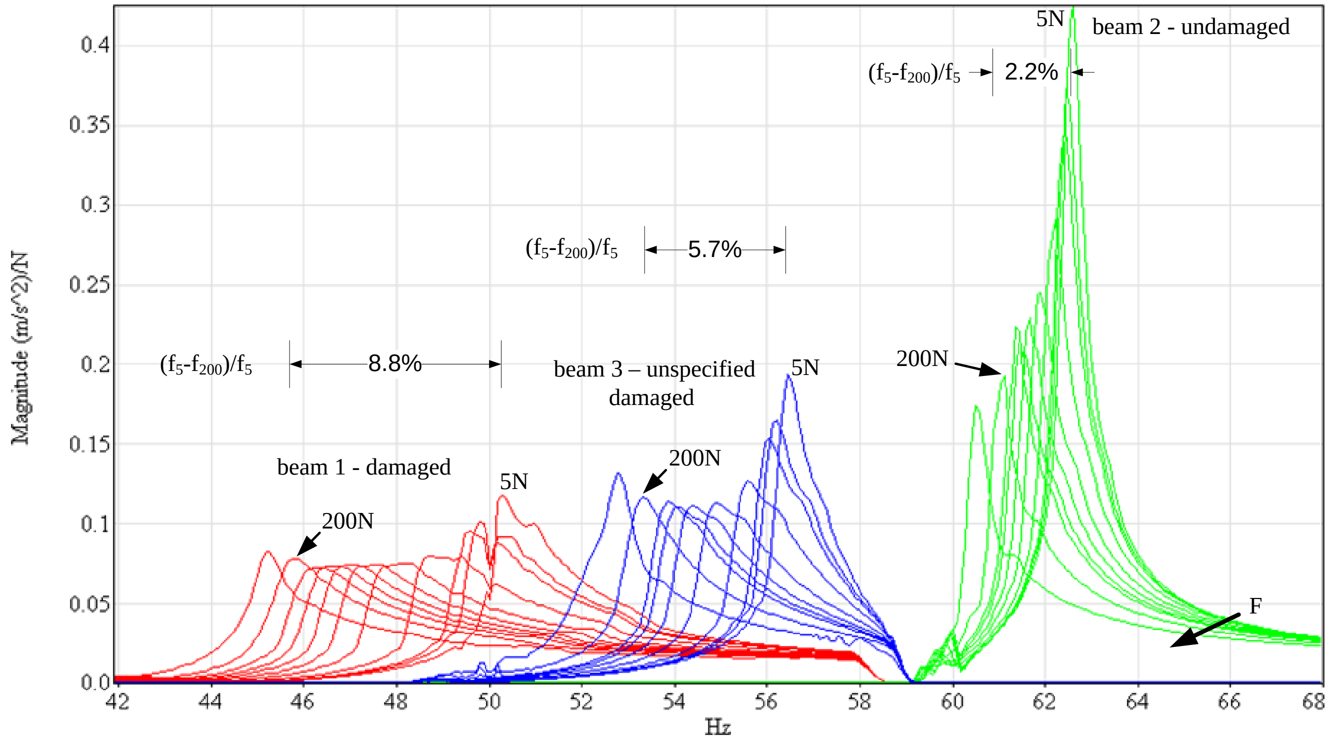

Figure 3. FRF, mode 2, damaged beam 1 (red), undamaged beam 2 (green), unspecified damaged beam 3 (blue)

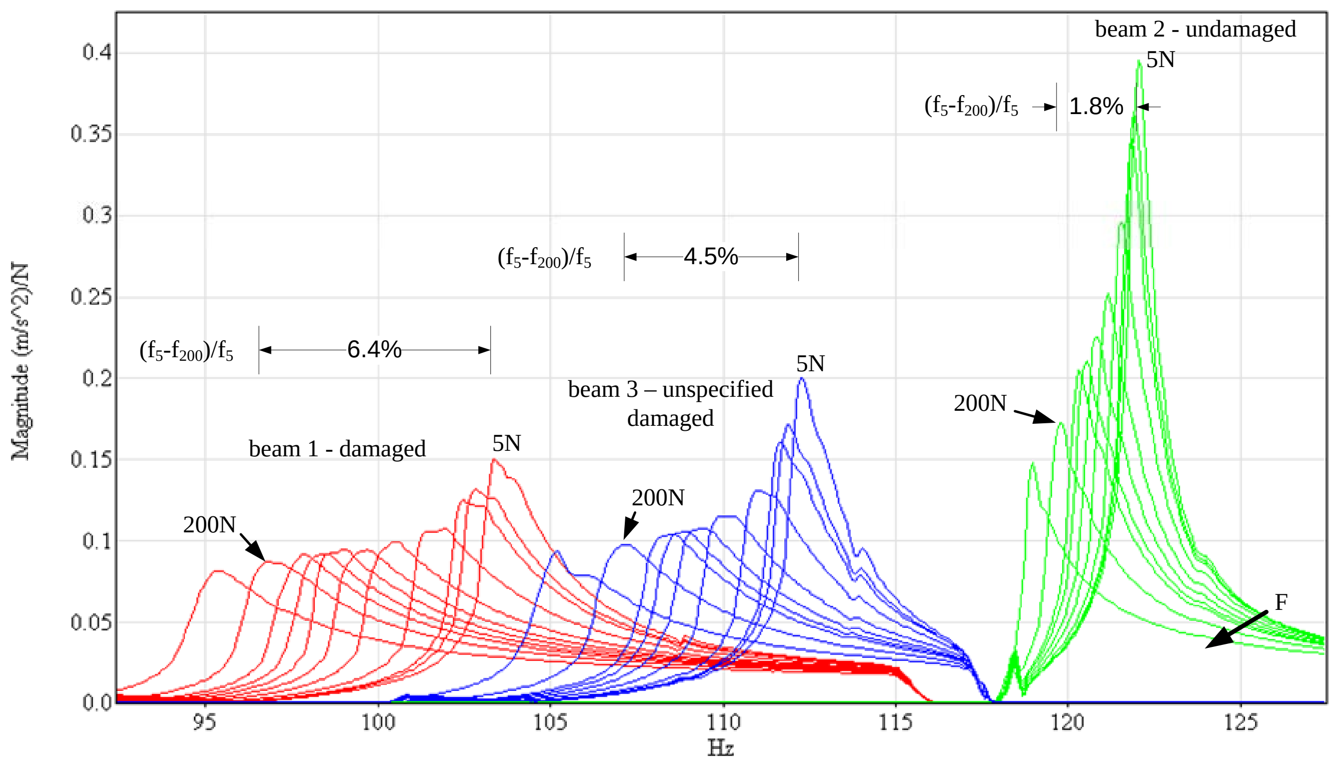

Figure 4. FRF, mode 3, damaged beam 1 (red), undamaged beam 2 (green), unspecified damaged beam 3 (blue)

For the undamaged beam 2 both the natural frequencies and the amplitude of the FRF decrease with increasing excitation force. This property can be explained by means of the strongly dependent hysteretic material behaviour of concrete, probably due to inevitable microcracks. The decrease is most evident for mode 1 compared to modes 2 and 3. The deformation in the first mode is greater than in the higher modes and thus the vibration contains more non-linear behaviour concerning material.

Comparing the behaviour of the three beams in the first mode, it has to be noted, that beam 3 and beam 2 indicate qualitative analogue behaviour. Differences in the amplitudes of the FRFs are due to varying material behaviour. The properties of the damaged beam 1 show a clear difference. It is obvious that the frequency decreases up to a certain excitation force more than the frequencies of the undamaged beam 2. At higher excitation forces there is a remarkable increase of amplitudes. These observations can be explained by means of the behaviour of cracked reinforced concrete under forced vibration. According to the excitation force, the cracks are opened and thus the stiffness is reduced. This results in an intense decrease of the natural frequencies with increasing excitation force. Due to the bond between concrete and reinforcement (a hysteretic behaviour), the damping ratio increases and according to this the amplitude of the FRF decreases. In case of low excitation forces there is only shear bond (static friction, stick-condition); in case of higher force values bond changes between static and dynamic friction (permanent change between stick and slip condition) and thus increasing damping. In this state the resonance frequency is still decreased. Exciting the beam with higher excitation forces, the bond is just ensured by dynamic friction (slip-condition), because the duration of the stick-condition is very short, due to the increased strain-rate. In this state the stiffness, so the natural frequency is no longer influenced and keeps a constant value. As the friction force still slightly decreases with increasing excitation force, the damping ratio is also decreasing. Therefore, the FRF amplitude increases again. The described behaviour can be regarded as an indicator for a typical damage in RC structures.

The described effect can also been observed when investigating the FRF of the second mode of the unspecified damaged beam 3. This is an indicator for a damage concerning mode 2.

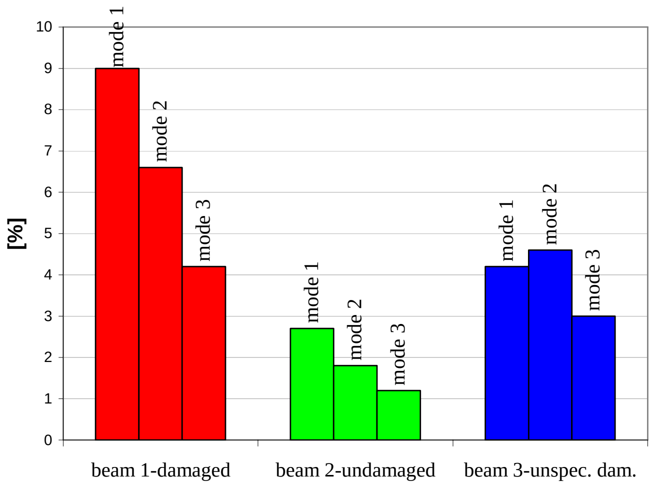

Figure 5. Decrease of the first three natural frequencies from 5N to 80N

Figure 5 contains the percentage decrease of the resonance frequencies from an excitation force from 5N to 80N. The most significant decrease can be denoted for the first mode of the damaged beam 1. The natural frequencies of the undamaged beam 2 show comparatively small changes. Likewise, the first mode is influenced the most. This is due to the distortion dependent non-linear material behaviour. As the deflection of mode 1 is always greater than the deflections of mode 2 and 3, the influence on the first mode is more significant. This is different when inspecting the unspecified damaged beam 3. Here the decrease of the second natural frequency (f_2) is dominant. The decrease of f_1 is similar to f_1 of the undamaged beam 2. f_3 of beam 3 tends to the behaviour of f_3 of the damaged beam 1.

Fitting the above-mentioned results into the big picture, the state of the three beams can be assessed. The investigation of the dependence of the natural frequencies on the excitation force has been very useful, because it showed a strong non-linear-behaviour especially for the damaged beam 1 and for mode 2 of the unspecified damaged beam 3. By means of these most affected modes (figure 6), it is possible to confirm the damage location described at the beginning of this chapter. Beam 1 is damaged in the area of great modal curvature of mode 1, beam 3 in the area of great modal curvature of mode 2 (near support during storage). There is nonlinear behaviour of beam 2 (undamaged) as well, but this is due to hysteretic material behaviour. For more detailed information compare [1] and [2].

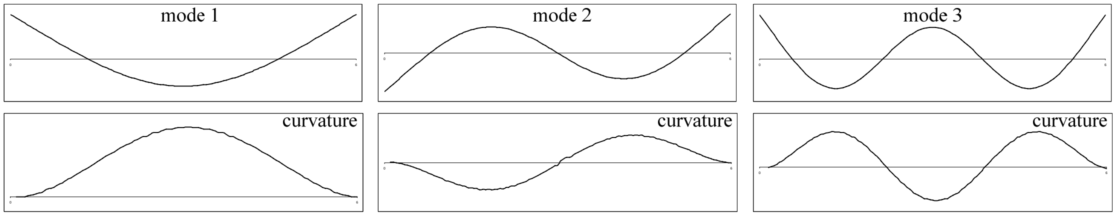

Figure 6. Mode shapes and corresponding modal curvature

3 FORCED EXCITATION TESTS ON A GRADUALLY DAMAGED BRIDGE

For the confirmation of the mentioned results and the investigation whether the observation of non-linear-behaviour is useful in practice, forced excitation tests on a real bridge are performed. The three span bridge was slightly skewed, curved with a radius of 300m and had a total length of 51m. The mid span had a length of 23m, the side spans were 13m and 15m long. The prestressed slab was supported on 16 elastomer bearings (figure 7, axis A, B, D, E). The 29 tendon wires were arranged according to the bending moment, i.e. in the middle of the mid span the tendon wires were arranged in the lower part of the cross-section, above the column the tendon wires were arranged in the upper part of the cross-section. Figure 7 shows the dimensions and the cross-section of the bridge.

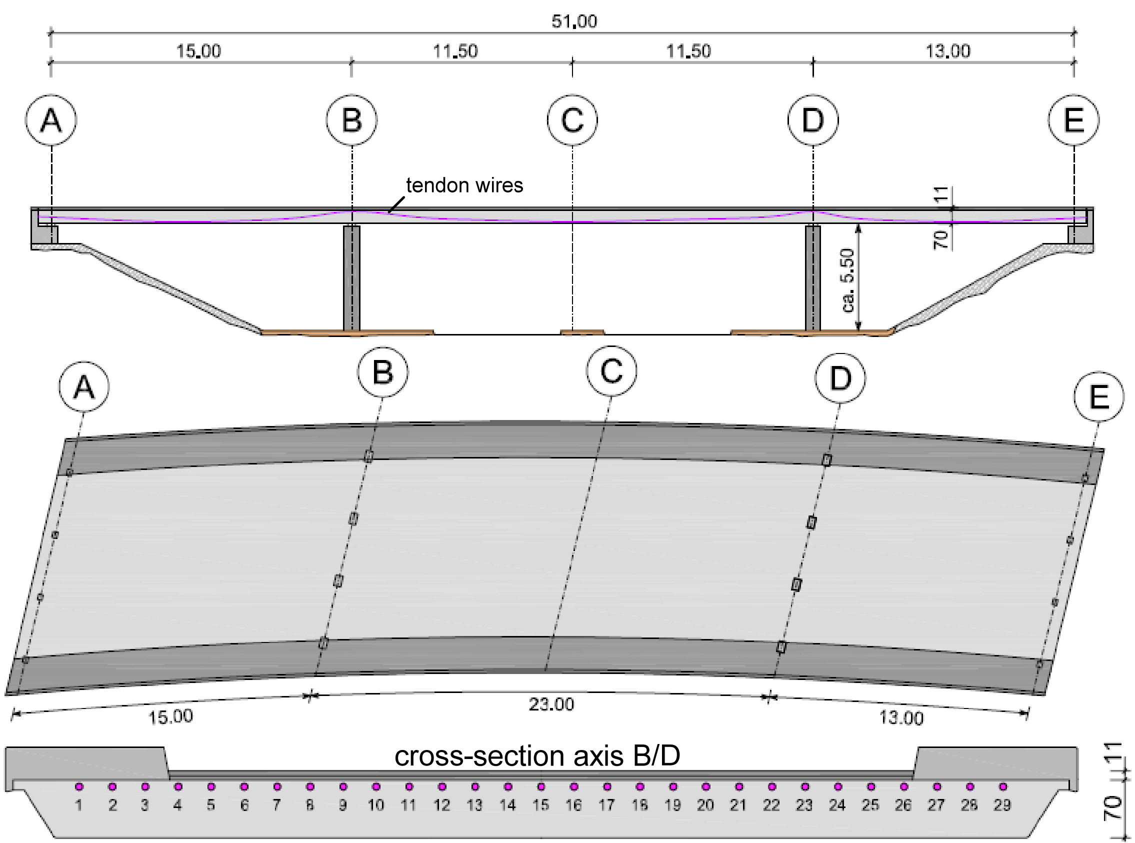

Figure 7. The investigated bridge, cross-section

The bridge was investigated in six different states. After the examination of the undamaged initial state, the roadbed was removed. In the next steps, it was possible to damage the bridge by cutting tendon wires. Table 2 lists the different scenarios.

Table 2. Damage scenarios

ID	scenario	location
#1	undamaged	
#2	undamaged, removed roadbed	
#3	failure of tendon wire 15	C
#4	failure of tendon wires 7, 13, 15, 17, 23	C
#5	failure of tendon wires 5, 7, 9, 13, 15, 17, 21, 23, 25	C
#6	failure of tendon wires 5, 7, 9, 13, 15, 17, 21, 23, 25	B, C, D

For the investigation of the non-linear-behaviour, the bridge was excited with an eccentric mass shaker harmonically by means of a swept sine excitation with a frequency range including the first four natural frequencies. Figure 8 shows the first four mode shapes of the bridge. The sweep-rate was 0.02Hz/s. The tests were performed with excitation forces of 0.9kN, 1.8kN, 2.7kN, 5.4kN, 8.1kN and 9.9kN.

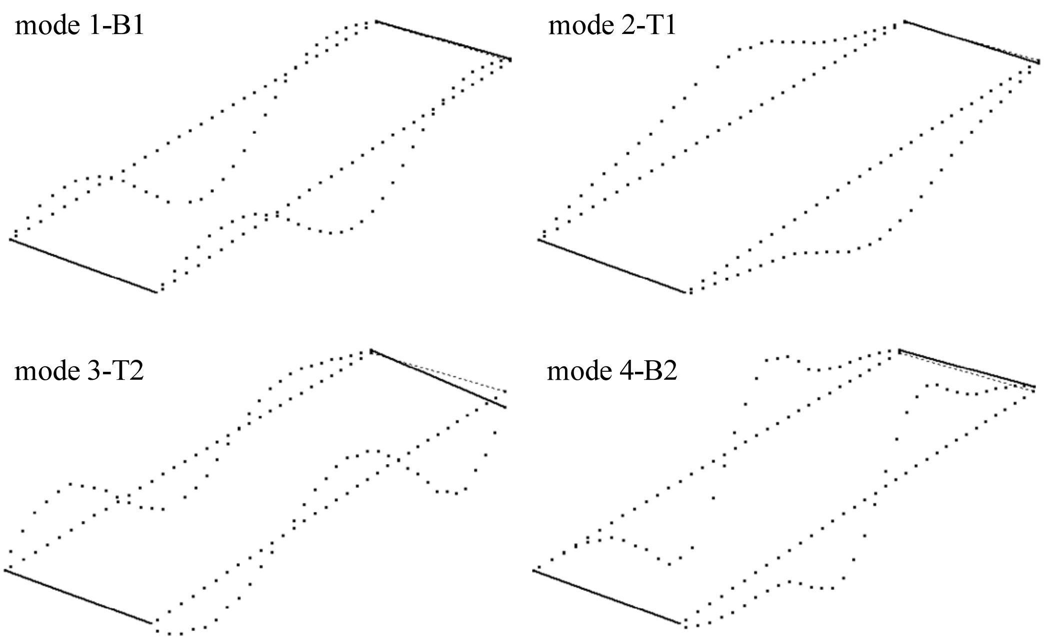

Figure 8. First four mode shapes, B-bending mode, T-torsional mode

Figure 9 shows the FRFs of the bridge in undamaged state #1 for the first two modes, measured on a point in the mid span. The amplitude dependency of the natural frequencies is much smaller compared to the undamaged beam in chapter 2. This is supposed to be due to different material behaviour of reinforced- and prestressed concrete. However, the amplitude dependency is not negligible. It is obvious that also damping is amplitude dependent. Especially when observing mode 1.

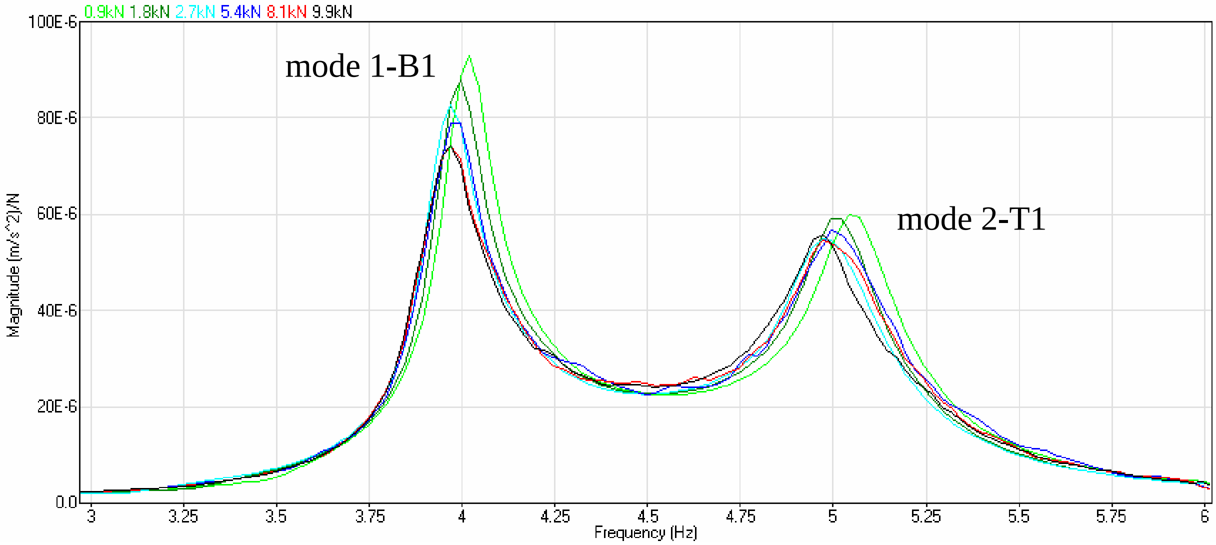

Figure 9. FRF, modes 1 and 2, undamaged state #1, different excitation forces

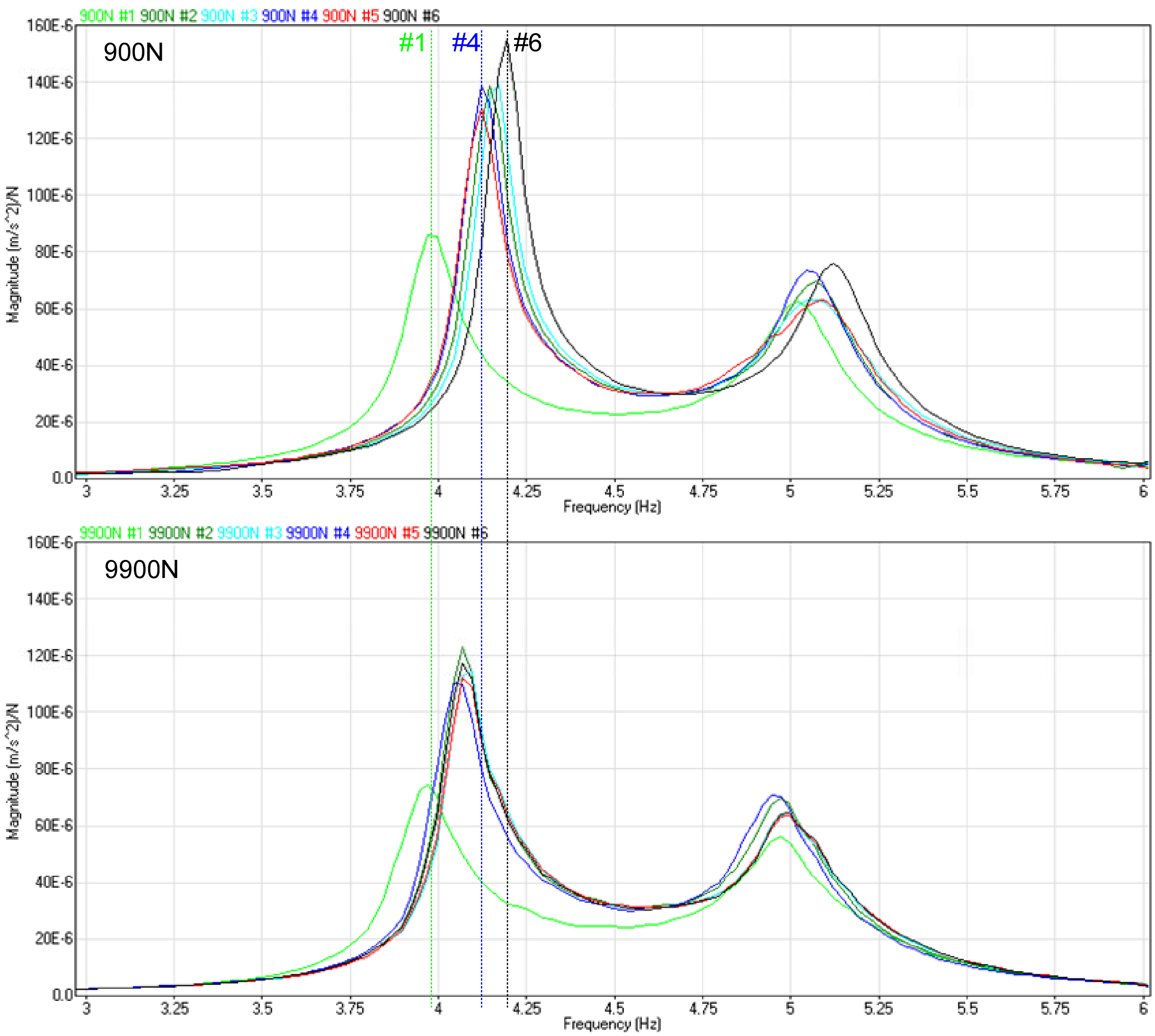

Figure 10. FRF, modes 1 and 2, scenarios #1, #2, #3, #4, #5, #6, excitation forces 900N and 9900N

Figure 10 shows the FRFs in the frequency range of the first two modes for the six scenarios, measured with excitation forces of 900N and 9900N. The first natural frequency in the initial state #1 (light green) has a value of approx. 4.0Hz. As expected, the first natural frequency increases when removing the roadbed. It is also obvious that damping decreases without the roadbed and thus, the amplitude of the FRF increases.

The change of the first two natural frequencies due to damage from scenario #2 to #5 is small. The reason for this is probably due to the behaviour of prestressed concrete under dynamic excitation. Experiments with prestressed concrete beams showed only a very small dependency of the natural frequencies on the pretensioning forces (for pretensioning forces > 7.6N/mm^2) [5]. For scenario #6 only, the first natural frequency increases lightly (the systems changes from a three-span system to a one-span system).

For all scenarios, there is a small dependency of the natural frequencies on the vibration amplitude visible. Table 3 shows the first four natural frequencies for the six scenarios. Figure 11 contains the percentage decrease of the natural frequencies from an excitation force from 900N to 9900N. For scenario #1 the change is small. It has to be noted that the vibration amplitudes for this scenario are smaller compared to the other scenarios, due to the higher damping value. Therefore, a direct comparison is difficult. For the other scenarios, the dependency of the natural frequencies is more evident. Observing scenarios #2 to #5 the dependency of the first natural frequencies varies. This is supposed to be due to the change of the state of stress in the cross-section of the bridge when cutting the tendon wires. The dependency of the second natural frequency is approx. constant. For scenario #6 the dependency is most evident.

Table 3. Natural frequencies

mode	#1		#2		#3		#4		#5		#6	
	900N	9900N	900N	9900N	900N	9900N	900N	9900N	900N	9900N	900N	9900N
B1 [Hz]	3.98	3.96	4.15	4.07	4.17	4.07	4.13	4.06	4.12	4.08	4.20	4.08
T1 [Hz]	5.02	4.98	5.08	4.96	5.08	4.99	5.06	4.94	5.11	4.99	5.13	5.00
T2 [Hz]	10.1	10.1	10.2	10.1	10.2	10.1	10.2	10.0	10.2	10.1	10.3	10.1
B2 [Hz]	11.6	11.7	11.9	11.9	12.0	12.0	12.0	11.8	12.0	12.0	12.0	11.8

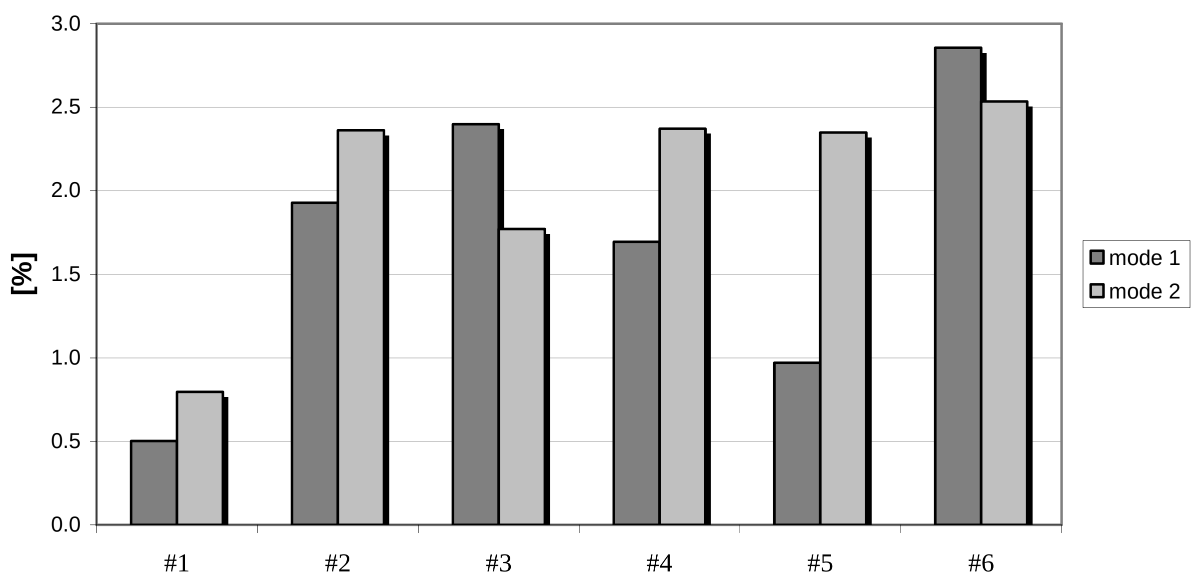

Figure 11. Decrease of the first two natural frequencies from 900N to 9900N

4 CONCLUSION

Using non-linear vibration analysis for detecting damage in civil engineering structures is still in the early stages. The presented experimental investigation showed promising results. There is an obvious relationship between excitation force and natural frequencies. For the undamaged beam, the decrease of the natural frequencies with increasing excitation force is small but not negligible. This and the change of damping is due to non-linear material behaviour. The damaged beam showed a very noticeable relationship between modal parameters and excitation force. The described stick-slip effect can be used as an indicator for damaged reinforced concrete. By means of the most affected modes, first appraisal of damage location is possible.

The results for the investigated bridge are more difficult to interpret. The system of the bridge is more complex than a simple beam; furthermore, the system is prestressed. However, non-linear effects like amplitude dependent natural frequencies and damping are visible. Other tests on bridges are necessary, ideally starting with a simple system like an untensioned reinforced, single span bridge, in order to have more possibility of comparison with the experimental approach. It will be further investigated if this approach can be implemented in the regular inspection program of bridges to document their actual force dependent behaviour and thus, their actual structural state.

REFERENCES

[1] Waltering, M.; Waldmann, D.; Maas, S.; Zuerbes, A.: Untersuchung nichtlinearer Schwingungseigenschaften zur zerstörungsfreien Zustandsprüfung am Beispiel von Stahlbetonbalken; Beton- und Stahlbetonbau 09/2007, Ernst & Sohn, Verlag für Architektur und technische Wissenschaften GmbH & Co.KG, Berlin, 2007

[2] Waltering, M.; Bungard, V.; Waldmann, D.; Zuerbes, A.; Maas, S.; De Roeck, G.: Damage assessment of civil engineering structures by the observation of non-linear dynamic behaviour; International Conference on Experimental Vibration Analysis for Civil Engineering Structures, EVACES 2007, Porto, Portugal, 2007

[3] Maeck, J., De Roeck, G.: Damage Assessment of Civil Engineering Structures by Vibration Monitoring, Department of Civil Engineering, K.U. Leuven, Belgium, 2003

[4] Wordon, K.; Tomlinson, G.R.: Nonlinearity in Structural Dynamics. IOP Publishing Ltd., London, 2001

[5] Büttner, A. Beitrag zur Beschreibung des Dämpfungsverhaltens von Stahlbetonbalken, Fakultät Bauingenieurwesen, Universität Weimar, 1992