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THE USE OF THE DISCRETE SUGENO INTEGRAL IN MULTI-CRITERIA DECISION MAKING

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Abstract

We present a model allowing to aggregate decision criteria when the available information is of qualitative nature. The use of the SugenO integral as an aggregation function is justified by an axiomatic approach.

Assume $A = \{a, b, c, \dots\}$ is a finite set of potential alternatives, among which the decision maker must choose. Consider also a finite set of criteria $N = \{1, \dots, n\}$ to be satisfied. Each criterion $i \in N$ is represented by a mapping g_i from the set of alternatives A to a given finite ordinal scale

$$X_i = \{r_1^{(i)} < \cdots < r_{k_i}^{(i)}\} \subset \mathbb{R},$$

that is, a scale which is unique up to order. For example, a scale of evaluation of importance of scientific papers by referees such as

1=Poor, 2=Below Average, 3=Average, 4=Very Good, 5=Excellent

is a finite ordinal scale. The coding by real numbers is used only to fix an order on the scale.

For each alternative $a \in A$ and each criterion $i \in N$, $g_i(a)$ represents the evaluation of a along criterion i. We assume that all the mappings g_i are given beforehand.

Our central interest is the problem of constructing a single comprehensive criterion from the given criteria. Such a criterion, which is supposed to be a representative of the original criteria, is modeled by a mapping g from A to a given finite ordinal scale

$$X = \{r_1 < \cdots < r_k\} \subset \mathbb{R}.$$

The value $g(a)$ then represents the global evaluation of alternative a expressed in the scale X. Without loss of generality, we can embed this scale in the unit interval $[0, 1]$ and fix the endpoints $r_1 := 0$ and $r_k := 1$.

In order to aggregate properly the partial evaluations of $a \in A$, we will assume that there exist n non-decreasing mappings $U_i : X_i \to X$ ($i \in N$) and an aggregation function $M : X^n \to X$ such that

$$g(a) = M\big[U_1(g_1(a)), \dots, U_n(g_n(a))\big] \qquad (a \in A).$$

The mappings U_i, called ***utility functions***, enable us to express all the partial evaluations in the common scale X, so that the function M aggregates commensurable evaluations. We will also make the assumption that $U_i(r_1^{(i)}) = 0$ and $U_i(r_{k_i}^{(i)}) = 1$ for all $i \in N$.

We present an axiomatic framework for defining a suitable aggregation model. As presented above, this model is determined by the mapping g, which can be constructed in two steps:

1. The aggregation function M can be identified by means of an axiomatic approach. The one we propose here leads to the discrete Sugeno integral, that is, a function of the form

$$\mathcal{S}_v(x) := \bigvee_{i=1}^{n} \Big[x_{(i)} \wedge v(\{(i),\ldots,(n)\})\Big] \qquad (x \in [0,1]^n),$$

   where v is a fuzzy measure on N, that is, a monotone set function $v : 2^N \to [0,1]$ fulfilling $v(\emptyset) = 0$ and $v(N) = 1$. Also, $(\cdot)$ indicates a permutation on N such that $x_{(1)} \leq \cdots \leq x_{(n)}$.

2. Each utility function U_i ($i \in N$) can be identified by asking appropriate questions to the decision maker. We present a procedure to obtain these functions.

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