Baumann Henri 16.04.2010

Final report

Absolute gravimeter Intercomparison

EURAMET Project no. 1093

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1. Introduction

From May 19th to May 21st 2008, a comparison between the absolute gravimeters of the European Center for Geodynamics and Seismology (ECGS), the Royal Observatory of Belgium and the Federal Office for Metrology METAS was hosted in the Underground Laboratory for Geodynamics in Walferdange. All three absolute gravimeters have been manufactured by Microg-Lacoste and are working on the same principle [1] even if their construction concepts differ from each other. The aim of the project was to ensure that there is good agreement between these three test facilities.

The participant persons in EURAMET 1093 have been:

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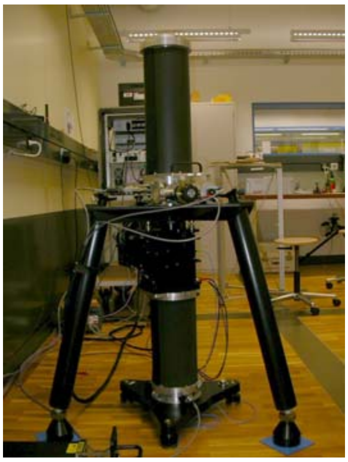
2. Instrument description

The instruments used in the context of this project are ballistic free fall absolute gravimeters provided by Microg-Lacoste. The absolute gravimeter of the Royal Observatory of Belgium, FG5 #202, is in operation since 1996 and the two other one, from the ECGS, FG5 #216, and from METAS FG5 #209, since 2001.

FG5 #202, ROB

FG5 #216, ECGS



FG5 #209, METAS

Figure 2.1: Absolute gravimeters used during this comparison

As it can be seen in Figure 2.1, even if all three gravimeters have the same working principle their design in the construction is slightly different. The main difference lies in the integration of the laser. If for more recent gravimeters, #216, #209, the lasers are mechanically separated from interferometer it is not the case for the gravimeter #202 where the laser is integrated in the interferometer.

3. Test procedure

The test procedure applied during this comparison is quiet simple. From May 19th 2008 to May 21st 2008, each gravimeter was placed alternatively on the reference stations B3 and B4 of the Underground Laboratory for Geodynamics in Walferdange. For each station the absolute value of gravity was measured by each gravimeter for 24 hours.

4. Measurement results

The results obtained during the comparison at the station B3 and B4 by the three gravimeters are summarized in Table 4.1. The values are given in µGal (µGal = 10^{-8} ms^{-2}).

	ROB (#202)		METAS (#209)		UL (#216)	
Stations	**g-g_{offset} (µGal)**	**u (µGal)**	**g-g_{offset} (µGal)**	**u (µGal)**	**g-g_{offset} (µGal)**	**u (µGal)**
B3	67.76	2.6	65.07	2.6	64.07	2.6
B4	62.62	2.6	58.48	2.6	58.11	2.6

Table 4.1: Gravity value measured by the three gravimeters at stations B3 and B4 (g_{offset} = 980964000 µGal)

The uncertainty given in the Table 4.1 corresponds to the square root of the quadratic sum of the typical uncertainty of an FG5, and the site dependant uncertainty [3].

The gravity values measured by each gravimeter at each station are graphically represented in the Figure 4.1.

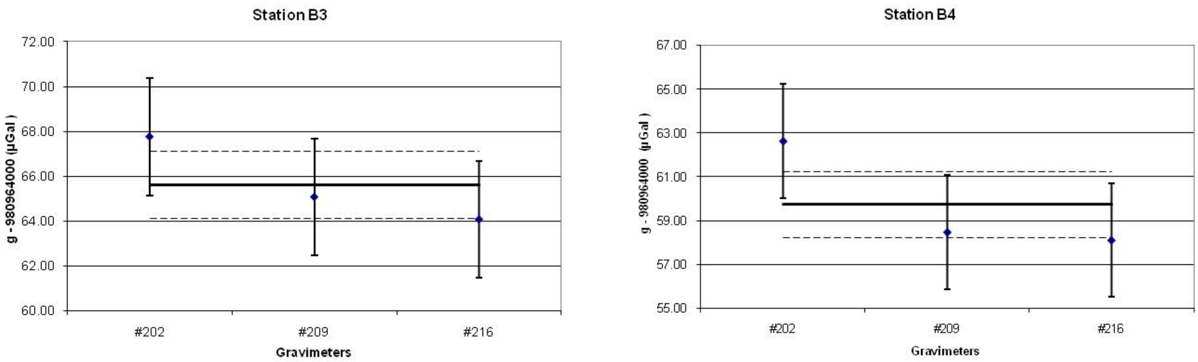


Figure 4.1: Gravity value measured by the three gravimeters at the tow stations B3 and B4. The continues line represents the reference value, and the dot lines its uncertainty.

5. Evaluation of the measurement results

5.1 Compatibility index En

The compatibility index E_n being defined by,

$$E_n = \frac{x_i - x_j}{\sqrt{U^2(x_i) + U^2(x_j)}} \quad (8)$$

defines the ratio between the difference of two estimated values and the expanded uncertainty (k=2) of the difference. An E_n factor larger than one means, that the difference between the values can not be covered by the uncertainty. With a perfectly repeatable transfer standard, this implies that either at least one of the two values is corrupted, or the claimed uncertainties are too small.
For the measured values at the two station B3 and B4, the E_n factors are listed in Table 5.1.

B3				B4			
	#202	**#209**	**#216**		**#202**	**#209**	**#216**
#202	-	0.37	0.50	**#202**	-	0.56	0.61
#209	-	-	0.14	**#209**	-	-	0.05
#216	-	-	-	**#216**	-	-	-

Table 5.1: E_n factors at the gravity stations B3 and B4 evaluated with the extended uncertainty.

The values of the E_n factors, given in table 5.1 shows that all measures are in equivalence.

5.2 Evaluation of the reference values

As described by M. G. Cox [2], the reference values at each gravity station are determined by the weighted mean y_i.

$$y_i = \frac{x_{i1}/u^2(x_{i1}) + \cdots + x_{iN}/u^2(x_{iN})}{1/u^2(x_{i1}) + \cdots + 1/u^2(x_{iN})} \quad (9)$$

$$\frac{1}{u^2(y_i)} = \frac{1}{u^2(x_{i1})} + \ldots + \frac{1}{u^2(x_{iN})} \quad (10)$$

where:
y_i : weighted mean value for the gravity value at the station i
$u(y_i)$: uncertainty of the weighted mean value for the gravity value at the station i

x_{ij} : estimated gravity value by the gravimeter j at the station i
u_{ij} : uncertainty of the gravimeter j at the station i

The evaluated reference values for each station are given in Table 5.2.

	Weighted mean	
Stations	**g_{ref}-g_{offset} uGal**	**u uGal**
B3	65.63	1.50
B4	59.74	1.50

Table 5.2: Reference gravity value at the stations B3 and B4 (g_{offset} = 980964000 µGal)

To be able to identify systematic errors, it is useful to carry out a statistical test like a χ^2 as described by M. G. Cox [2].

$$\chi^2_{i_{obs}} = \frac{(x_{i1} - y_i)^2}{u^2(x_{i1})} + \cdots + \frac{(x_{iN} - y_i)^2}{u^2(x_{iN})} \qquad (11)$$

where:

$\chi^2_{i_{obs}}$: chi-squared value for the station i
y_i : weighted mean value for the station i

x_{ij} : estimated gravity value by the gravimeter j at the station i
u_{ij} : uncertainty of the gravimeter j at the station i

To evaluate whether the data are consistent the chi-square test has been performed to demonstrate consistency of the data.
The consistency check passes when : $\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$
With v = N - 1 and N is the number of participating labs for the chi-square test.

The chi-squared test is summarized in Table 5.3

	#202	#209	#216		
Stations	**Chi^2**	**Chi^2**	**Chi^2**	**Chi^2**	**Chi^2_{95} from table**
B3	2.0	0.1	1.1	3.2	5.99
B4	3.7	0.7	1.2	5.6	5.99

Table 5.3: Analysis overview of the chi-squared test

The chi-squared test (Table 5.3) is satisfied for both stations B3 and B4.

5.3 Degree of equivalence

For all three gravimeters, the degree of equivalence of the measured gravity values has been evaluated.

Equivalence between institute i and the reference value

The degree of equivalence between the institute and the reference value is given by:

$$d_i = x_i - x_{ref} \qquad (12)$$

$$U(d_i) = 2u(d_i) \qquad u^2(d_i) = u^2(x_i) - u^2(x_{ref}) \qquad (13)$$

The obtained values are given in table 5.4:

ORB		METAS		UL	
d_{202} µGal	**$U(d_{202})$ µGal**	**d_{209} µGal**	**$U(d_{209})$ µGal**	**d_{216} µGal**	**$U(d_{216})$ µGal**
2.09	4.25	-0.60	4.25	-1.60	4.25
2.80	4.25	-1.34	4.25	-1.71	4.25

Table 5.4: Degree of equivalence between the different intuits and the reference value

Equivalence between institute i and institute j

The degree of equivalence between the institute and the reference value is given by:

$$d_{ij} = x_i - x_j \qquad (14)$$

$$U(d_{ij}) = 2u(d_{ij}) \qquad u^2(d_{ij}) = u^2(x_i) + u^2(x_j) \qquad (15)$$

The obtained values are given in table 5.5 for the station B3 and in table 5.6 for the station B4:

B3							
		#202		**#209**		**#216**	
		d_{i202}	$U(d_{i202})$	d_{i209}	$U(d_{i209})$	d_{i216}	$U(d_{i216})$
#202	d_{202j}	-	-	2.69	7.35	3.69	7.35
#209	d_{209j}	-	-	-	-	1	7.35
#216	d_{216j}	-	-	-	-	-	-

Table 5.5: Degree of equivalence between the gravimeter at station B3

B4							
		#202		**#209**		**#216**	
		d_{i202}	$U(d_{i202})$	d_{i209}	$U(d_{i209})$	d_{i216}	$U(d_{i216})$
#202	d_{202j}	-	-	4.14	7.35	4.51	7.35
#209	d_{209j}	-	-	-	-	0.37	7.35
#216	d_{216j}	-	-	-	-	-	-

Table 5.6: Degree of equivalence between the gravimeter at station B4

6. Link to Walferdange 2007 (Euramet 1030)

To link the comparison to the Euramet project 1030 the mean value of the difference to the respective reference values has been estimated for all three gravimeters. The obtained mean differences have then been compared with the results estimated during the comparison 1030 for which, due to geophysical changes of the local gravity field, another reference value was estimated. These results are graphically represented in Figure 6.1.

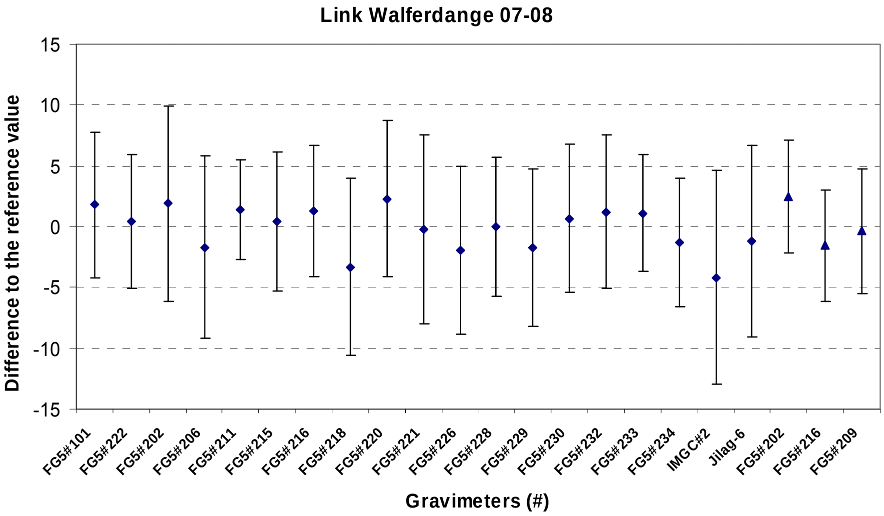


Figure 6.1: Difference to the reference value. ◆: difference to the reference value 07, ▲: difference to the reference value 2008.

It demonstrates that the difference to the reference values estimated in 2007 and 2008 are coherent. Based on this, we can consider that the link between these two comparisons is acceptable.

(The results of the comparison 1030 have been published and presented in 2008 [5]. The draft B from the comparison 1030 will be published)

7. Conclusion

The analysis done in section 5 indicates that the gravity values measured by the three gravimeters, #202, #209 and #216 are in agreement. In Figure 6.1, we reported the present results with the ones obtained during the European Comparison of 19 absolute gravimeters held in Walferdange in November 2007.

8. Literature

[1] Niebauer T M, Sasagawa G S, Faller J E, Hilt R and Klopping F 1995 *Metrologia* **32** 159-180.

[2] Cox, M.G., The evaluation of key comparison data, Metrologia, 2002, 39, 589-595.

[3] Results of the Seventh International Comparison of Absolute Gravimeters ICAG-2005 at the Bureau International des Poids et Mesures, Sèvres, GGEO-2008.

[4] Pearson, E. A., Hartley, H. O., Biometrika Tables for Statisticians, Vol.1 1966, table 8, 137-138.

[5] Francis O *et al* 2008, *Results of the European Comparison of Absolute Gravimeters in Walferdange (Luxembourg) of November 2007.* IAG International Symposium Gravity, Geoid and Earth Observation 2008, Chania, Greece, 23-27 June 2008.