Analysis of the salary trajectories in Luxembourg

A finite mixture model approach

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joint work with Jean-Daniel GUIGOU (University of Luxembourg)
& Bruno LOVAT (University Nancy II)

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Outline

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General description of Nagin's model

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This is still an inter-individual model, but unlike other classical models such as standard growth curve models, it allows the existence of subpolulations with completely different behaviors.

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Aim of the analysis: Find r groups of trajectories of a given kind (for instance polynomials of degree 4, $P(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \beta_4 t^4$.

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- mixture : population composed of a mixture of unobserved groups

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$$y^*_{i_t} = \beta^j_0 + \beta^j_1 Age_{i_t} + \beta^j_2 Age^2_{i_t} + \beta^j_3 Age^3_{i_t} + \beta^j_4 Age^4_{i_t} + \varepsilon_{i_t}, \tag{4}$$

where $\varepsilon_{i_t} \sim \mathcal{N}(0, \sigma)$, σ being a constant standard deviation.

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Hence,

$$\begin{array}{lll} y_{i_t} = S_{min} & \text{si} & y^*_{i_t} < S_{min}, \\ y_{i_t} = y^*_{i_t} & \text{si} & S_{min} \leq y^*_{i_t} \leq S_{max}, \\ y_{i_t} = S_{max} & \text{si} & y^*_{i_t} > S_{max}, \end{array}$$

where S_{min} and S_{max} dennote the minimum and maximum of the censored normal distribution.

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$$p^j(y_{i_t} = S_{max}) = 1 - \Phi\left(\frac{S_{max} - \beta^j x_{i_t}}{\sigma}\right). \tag{7}$$

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Software:

SAS-based Proc Traj procedure
by Bobby L. Jones (Carnegie Mellon University).

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Finally,

$$L = \frac{1}{\sigma} \prod_{i=1}^{N} \sum_{j=1}^{r} \frac{e^{\theta_j}}{\sum_{j=1}^{r} e^{\theta_j}} \prod_{t=1}^{T} \phi\left(\frac{y_{i_t} - \beta^j x_{i_t}}{\sigma}\right). \qquad (10)$$

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Mplus package by L.K. Muthén and B.O Muthén.

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3. can create the illusion of non-existing groups.

Model Selection

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Rule:

The bigger the BIC, the better the model!

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To be classified into a small group, an individual really needs to be strongly consistent with it.

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OCC_j should be greater than 5 for all groups.

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The ratio of the two should be close to 1.

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Diagonostic 4: Confidence Intervals for Group Membership Probabilities

The confidence intervals for group membership probabilities estimates should be narrow, i.e. standard deviation of π_j should be small.

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- transforming all the salaries above 7.577 to 7.577
- creation of the time variables necessary for the Proc Traj procedure

Proc Traj procedure

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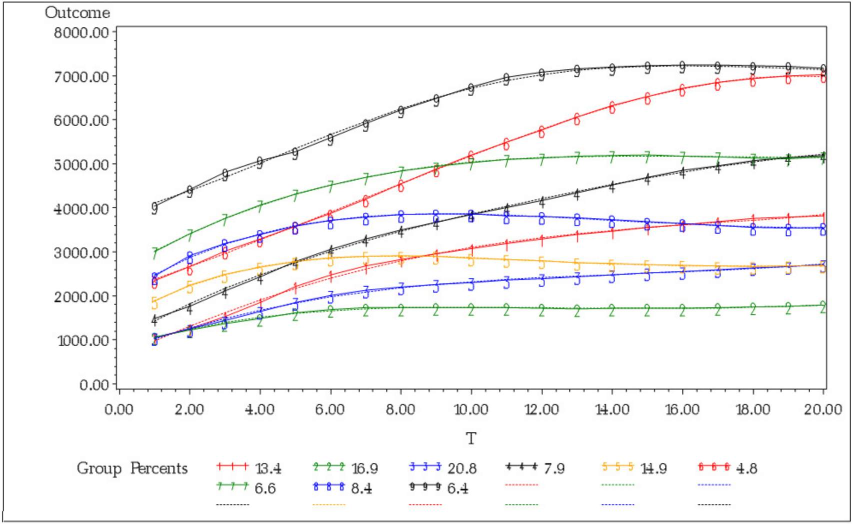
PROC TRAJ DATA=TEST OUTPLOT=OP OUTSTAT=OS OUT=OF
OUTEST=OE ITDETAIL;
      ID ID; VAR O1-O20; INDEP T1-T20;
      MODEL CNORM; MAX 8000; NGROUPS 6; ORDER 4 4 4 4 4 4;
RUN;
```

Results for 9 groups (1)

Results for 9 groups (1)



Results for 9 groups (2)

Maximum Likelihood Estimates
Model: Censored Normal (CNORM)

Group	Parameter	Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
1	Intercept	589.03067	18.46813	31.894	0.0000
	Linear	387.72145	11.31617	34.263	0.0000
	Quadratic	-14.36621	2.12997	-6.745	0.0000
	Cubic	-0.01563	0.15109	-0.103	0.9176
	Quartic	0.00856	0.00358	2.395	0.0166
2	Intercept	784.79156	15.75939	49.798	0.0000
	Linear	277.63602	9.78078	28.386	0.0000
	Quadratic	-28.36731	1.83236	-15.481	0.0000
	Cubic	1.17739	0.12972	9.076	0.0000
	Quartic	-0.01635	0.00307	-5.330	0.0000
3	Intercept	709.28728	15.90545	44.594	0.0000
	Linear	318.88029	8.97949	35.512	0.0000
	Quadratic	-21.54540	1.69611	-12.703	0.0000
	Cubic	0.62010	0.12002	5.167	0.0000
	Quartic	-0.00440	0.00284	-1.554	0.1203

Outline

1^{st} group

13.4 % of the population

1^{st} group

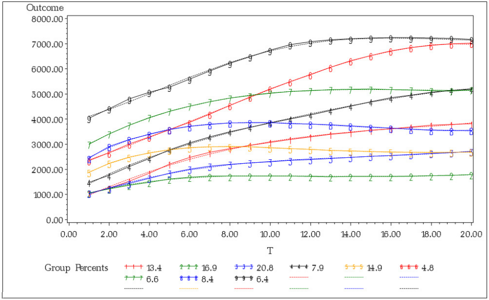
13.4 % of the population

$$P(x) = 590 + 388t - 14t^2 + 0.009t^4$$

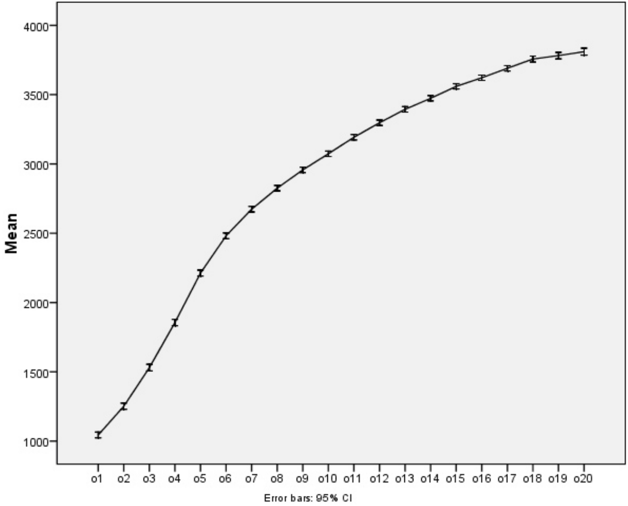
1^{st} group

13.4 % of the population

$$P(x) = 590 + 388t - 14t^2 + 0.009t^4$$



1^{st} group

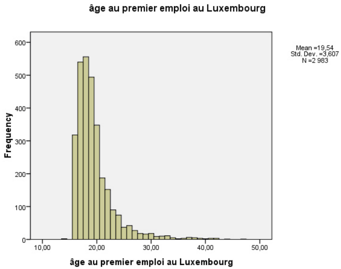


Mean
4000
3500
3000
2500
2000
1500
1000
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20
Error bars: 95% CI

1^{st} group

Age_initial

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	13,00	2	,1	,1	,1
	15,00	318	10,6	10,7	10,7
	16,00	540	18,1	18,1	28,8
	17,00	556	18,6	18,6	47,5
	18,00	494	16,5	16,6	64,0
	19,00	348	11,7	11,7	75,7
	20,00	187	6,3	6,3	82,0
	21,00	152	5,1	5,1	87,1
	22,00	90	3,0	3,0	90,1
	23,00	74	2,5	2,5	92,6
	24,00	37	1,2	1,2	93,8
	25,00	42	1,4	1,4	95,2
	26,00	27	,9	,9	96,1
	27,00	18	,6	,6	96,7
	28,00	16	,5	,5	97,3
	29,00	18	,6	,6	97,9
	30,00	9	,3	,3	98,2
	31,00	10	,3	,3	98,5
	32,00	11	,4	,4	98,9
	33,00	5	,2	,2	99,0
	34,00	2	,1	,1	99,1
	35,00	3	,1	,1	99,2
	36,00	6	,2	,2	99,4
	37,00	5	,2	,2	99,6
	38,00	3	,1	,1	99,7
	39,00	2	,1	,1	99,7
	40,00	3	,1	,1	99,8
	41,00	3	,1	,1	99,9
	43,00	1	,0	,0	100,0
	46,00	1	,0	,0	100,0
	Total	2983	99,9	100,0	
Missing	System	3	,1		
Total		2986	100,0		



1^{st} group

Sexe

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	masculin	2100	70,4	70,4	70,4
	féminin	883	29,6	29,6	100,0
	Total	2983	100,0	100,0	

1^{st} group

Sexe

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	masculin	2100	70,4	70,4	70,4
	féminin	883	29,6	29,6	100,0
	Total	2983	100,0	100,0	

Résidence et nationalité

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	résident de nationalité luxembourgeoise	1979	66,3	66,3	66,3
	résident étranger	543	18,2	18,2	84,5
	frontalier	461	15,5	15,5	100,0
	Total	2983	100,0	100,0	

1^{st} group

Men:

Classe d'employé

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	1213	57,8	57,8	57,8
	employé privé	887	42,2	42,2	100,0
	Total	2100	100,0	100,0	

1^{st} group

Men:

Classe d'employé

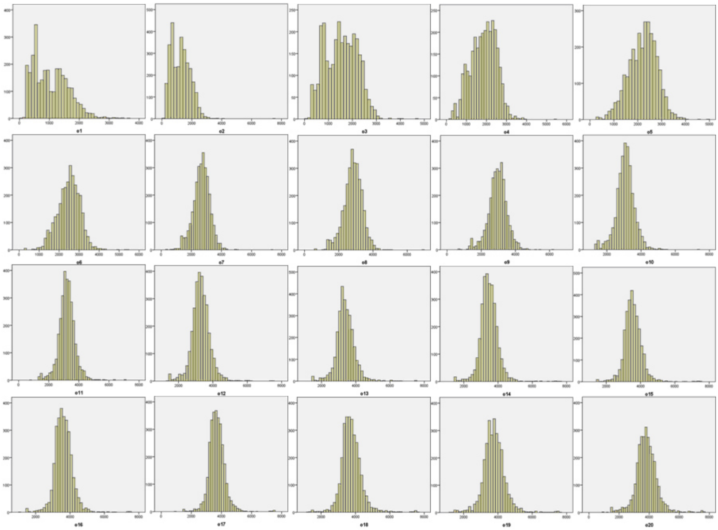
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	1213	57,8	57,8	57,8
	employé privé	887	42,2	42,2	100,0
	Total	2100	100,0	100,0	

Women:

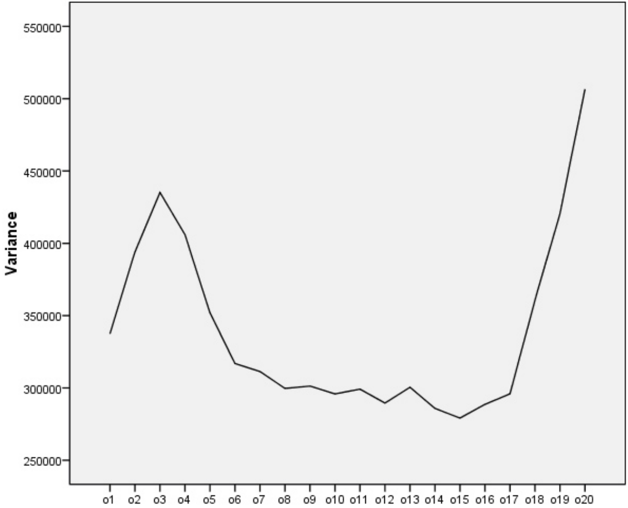
Classe d'employé

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	43	4,9	4,9	4,9
	employé privé	840	95,1	95,1	100,0
	Total	883	100,0	100,0	

1^{st} group

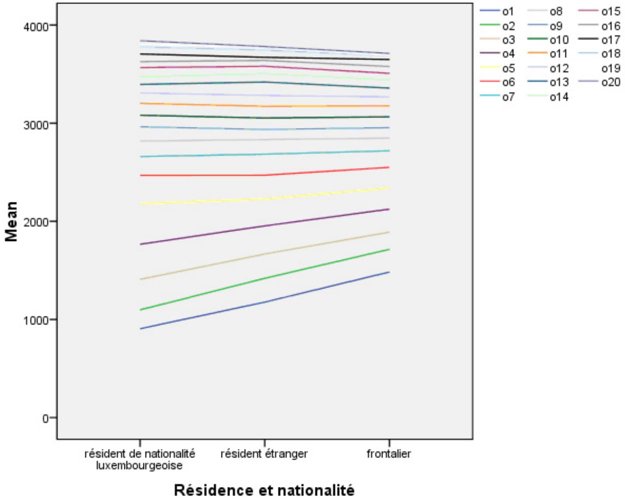


1^{st} group



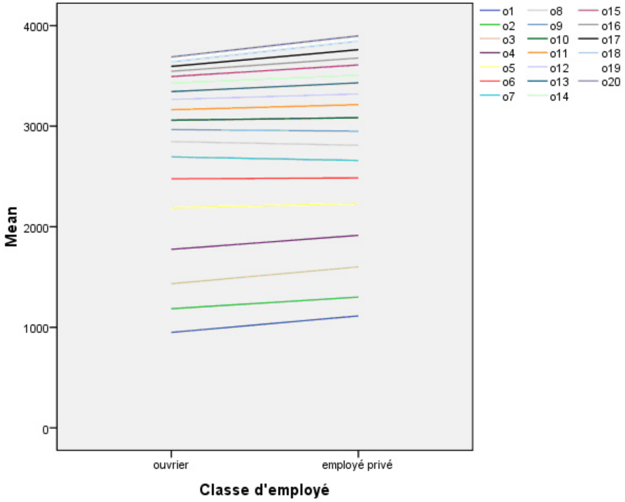
550000
500000
450000
400000
350000
300000
250000
Variance
o1 o2 o3 o4 o5 o6 o7 o8 o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19 o20

1^{st} group



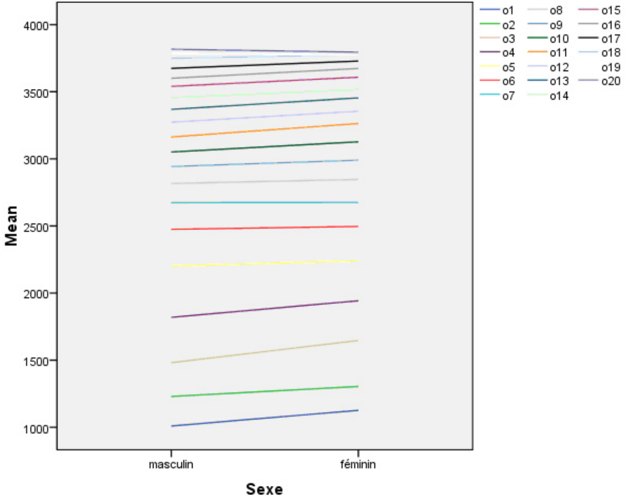
4000
3000
2000
1000
0
Mean
résident de nationalité luxembourgeoise
résident étranger
frontalier
Résidence et nationalité
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

1^{st} group



Mean
4000
3000
2000
1000
0
ouvrier
employé privé
Classe d'employé
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

1^{st} group



Mean
4000
3500
3000
2500
2000
1500
1000
masculin
féminin
Sexe
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

2^{nd} group

16.9 % of the population

2^{nd} group

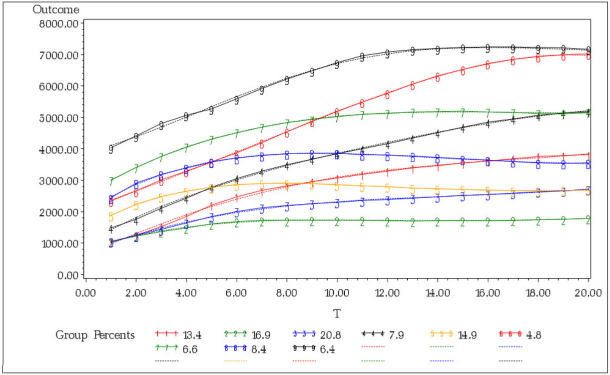
16.9 % of the population

$$P(x) = 785 + 278t - 28t^2 + 1.18t^3 - 0.016t^4$$

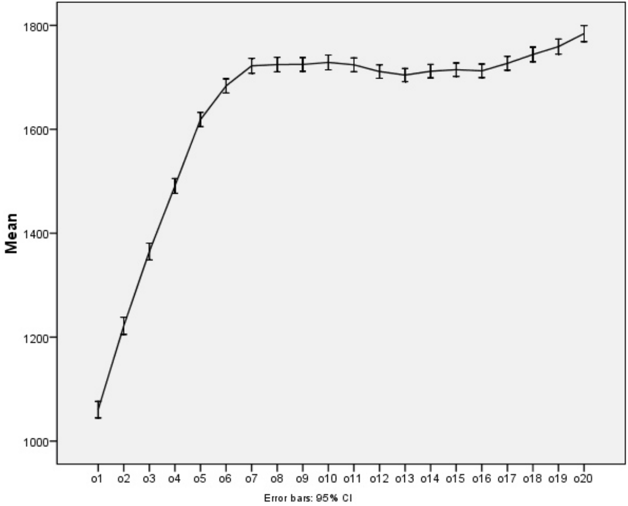
2^{nd} group

16.9 % of the population

$$P(x) = 785 + 278t - 28t^2 + 1.18t^3 - 0.016t^4$$



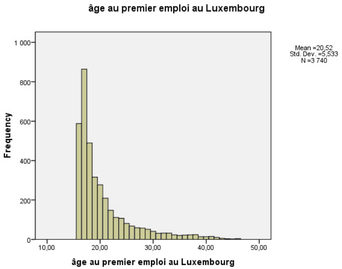
2^{nd} group


Mean
1800
1600
1400
1200
1000
o1 o2 o3 o4 o5 o6 o7 o8 o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19 o20
Error bars: 95% CI

2^{nd} group

Age_initial

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	15,00	588	15,7	15,7	15,7
	16,00	864	23,1	23,1	38,8
	17,00	489	13,1	13,1	51,9
	18,00	316	8,4	8,4	60,3
	19,00	277	7,4	7,4	67,8
	20,00	209	5,6	5,6	73,3
	21,00	148	4,0	4,0	77,3
	22,00	111	3,0	3,0	80,3
	23,00	107	2,9	2,9	83,1
	24,00	81	2,2	2,2	85,3
	25,00	68	1,8	1,8	87,1
	26,00	59	1,6	1,6	88,7
	27,00	57	1,5	1,5	90,2
	28,00	51	1,4	1,4	91,6
	29,00	41	1,1	1,1	92,7
	30,00	31	,8	,8	93,5
	31,00	32	,9	,9	94,4
	32,00	32	,9	,9	95,2
	33,00	23	,6	,6	95,8
	34,00	20	,5	,5	96,4
	35,00	22	,6	,6	97,0
	36,00	23	,6	,6	97,6
	37,00	24	,6	,6	98,2
	38,00	14	,4	,4	98,6
	39,00	14	,4	,4	99,0
	40,00	15	,4	,4	99,4
	41,00	10	,3	,3	99,6
	42,00	5	,1	,1	99,8
	43,00	3	,1	,1	99,8
	44,00	2	,1	,1	99,9
	45,00	4	,1	,1	100,0
	Total	3740	99,9	100,0	
Missing	System	2	,1		
Total		3742	100,0		



2^{nd} group

Sexe

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	masculin	1200	32,1	32,1	32,1
	féminin	2540	67,9	67,9	100,0
	Total	3740	100,0	100,0	

2^{nd} group

Sexe

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	masculin	1200	32,1	32,1	32,1
	féminin	2540	67,9	67,9	100,0
	Total	3740	100,0	100,0	

Résidence et nationalité

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	résident de nationalité luxembourgeoise	1631	43,6	43,6	43,6
	résident étranger	1439	38,5	38,5	82,1
	frontalier	670	17,9	17,9	100,0
	Total	3740	100,0	100,0	

2^{nd} group

Men:

Classe d'employé

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	994	82,8	82,8	82,8
	employé privé	206	17,2	17,2	100,0
	Total	1200	100,0	100,0	

2^{nd} group

Men:

Classe d'employé

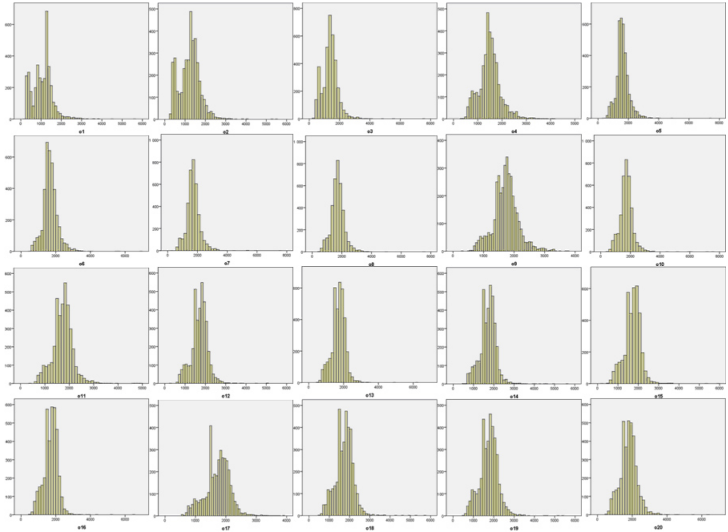
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	994	82,8	82,8	82,8
	employé privé	206	17,2	17,2	100,0
	Total	1200	100,0	100,0	

Women:

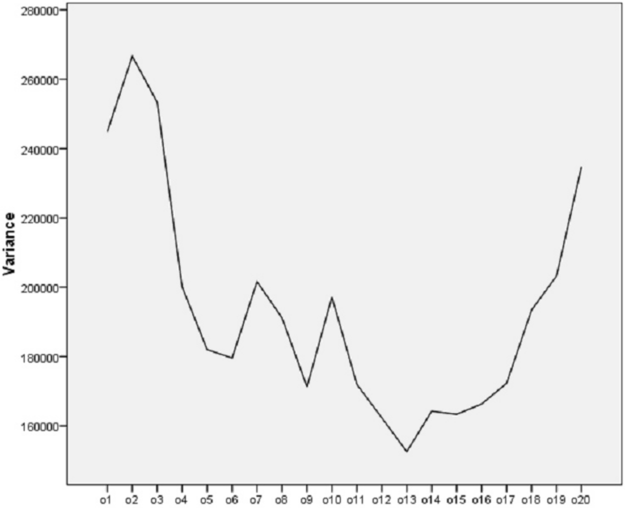
Classe d'employé

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	1453	57,2	57,2	57,2
	employé privé	1087	42,8	42,8	100,0
	Total	2540	100,0	100,0	

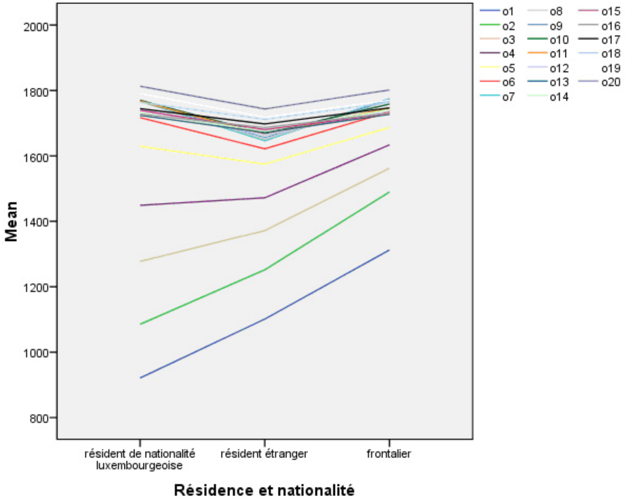
2^{nd} group



2^{nd} group

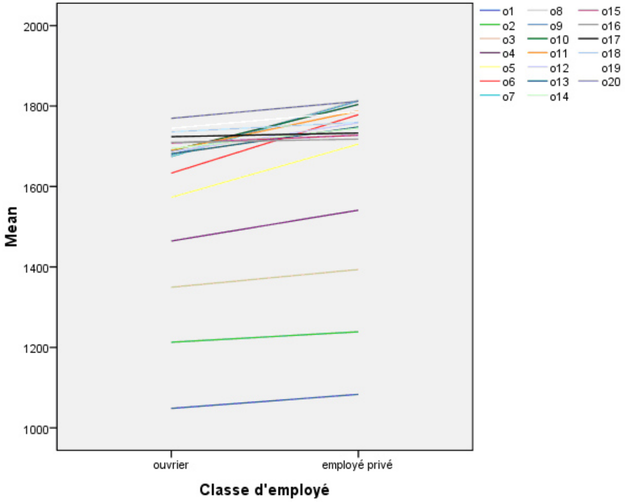

Variance
280000
260000
240000
220000
200000
180000
160000
o1 o2 o3 o4 o5 o6 o7 o8 o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19 o20

2^{nd} group



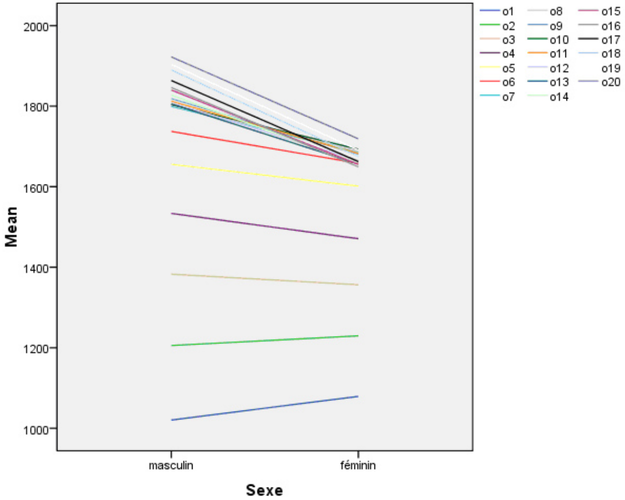
Mean
2000
1800
1600
1400
1200
1000
800
résident de nationalité luxembourgeoise
résident étranger
frontalier
Résidence et nationalité
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

2^{nd} group



Mean
2000
1800
1600
1400
1200
1000
ouvrier
employé privé
Classe d'employé
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

2^{nd} group



Mean
2000
1800
1600
1400
1200
1000
masculin
féminin
Sexe
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

3^{rd} group

20.8 % of the population

3^{rd} group

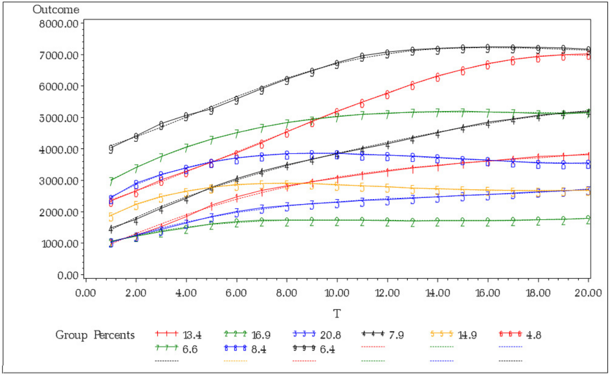
20.8 % of the population

$$P(x) = 709 + 318t - 21.5t^2 + 0.62t^3$$

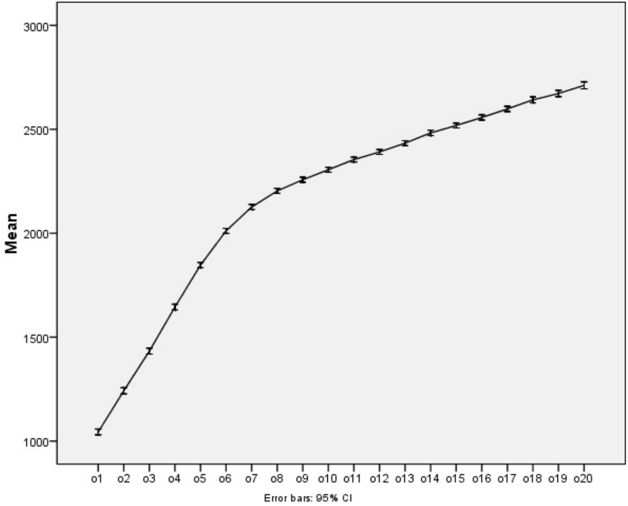
3^{rd} group

20.8 % of the population

$$P(x) = 709 + 318t - 21.5t^2 + 0.62t^3$$



3^{rd} group

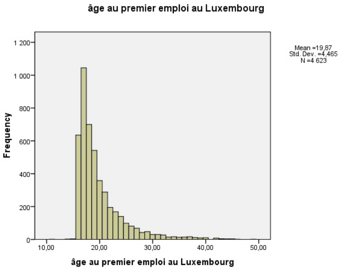


Mean
3000
2500
2000
1500
1000
o1 o2 o3 o4 o5 o6 o7 o8 o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19 o20
Error bars: 95% CI

3^{rd} group

Age_initial

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	10,00	1	,0	,0	,0
	13,00	2	,0	,0	,1
	14,00	3	,1	,1	,1
	15,00	635	13,7	13,7	13,9
	16,00	1045	22,6	22,6	36,5
	17,00	700	15,1	15,1	51,6
	18,00	542	11,7	11,7	63,3
	19,00	358	7,7	7,7	71,1
	20,00	288	6,2	6,2	77,3
	21,00	195	4,2	4,2	81,5
	22,00	168	3,6	3,6	85,2
	23,00	140	3,0	3,0	88,2
	24,00	98	2,1	2,1	90,3
	25,00	81	1,8	1,8	92,1
	26,00	68	1,5	1,5	93,5
	27,00	42	,9	,9	94,4
	28,00	47	1,0	1,0	95,5
	29,00	30	,6	,6	96,1
	30,00	30	,6	,6	96,8
	31,00	27	,6	,6	97,3
	32,00	14	,3	,3	97,6
	33,00	16	,3	,3	98,0
	34,00	13	,3	,3	98,3
	35,00	14	,3	,3	98,6
	36,00	16	,3	,3	98,9
	37,00	11	,2	,2	99,2
	38,00	9	,2	,2	99,4
	39,00	10	,2	,2	99,6
	41,00	8	,2	,2	99,7
	42,00	4	,1	,1	99,8
	43,00	3	,1	,1	99,9
	44,00	3	,1	,1	100,0
	45,00	1	,0	,0	100,0
	48,00	1	,0	,0	100,0
	Total	4623	100,0	100,0	
Missing	System	1	,0		
Total		4624	100,0		



3^{rd} group

Sexe

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	masculin	3136	67,8	67,8	67,8
	féminin	1487	32,2	32,2	100,0
	Total	4623	100,0	100,0	

3^{rd} group

Sexe

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	masculin	3136	67,8	67,8	67,8
	féminin	1487	32,2	32,2	100,0
	Total	4623	100,0	100,0	

Résidence et nationalité

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	résident de nationalité luxembourgeoise	2234	48,3	48,3	48,3
	résident étranger	1399	30,3	30,3	78,6
	frontalier	990	21,4	21,4	100,0
	Total	4623	100,0	100,0	

3^{rd} group

Men:

Classe d'employé

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	2452	78,2	78,2	78,2
	employé privé	684	21,8	21,8	100,0
	Total	3136	100,0	100,0	

3^{rd} group

Men:

Classe d'employé

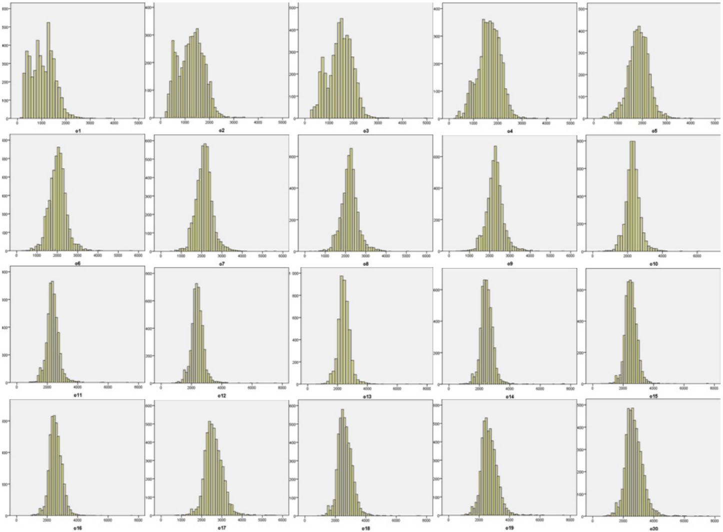
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	2452	78,2	78,2	78,2
	employé privé	684	21,8	21,8	100,0
	Total	3136	100,0	100,0	

Women:

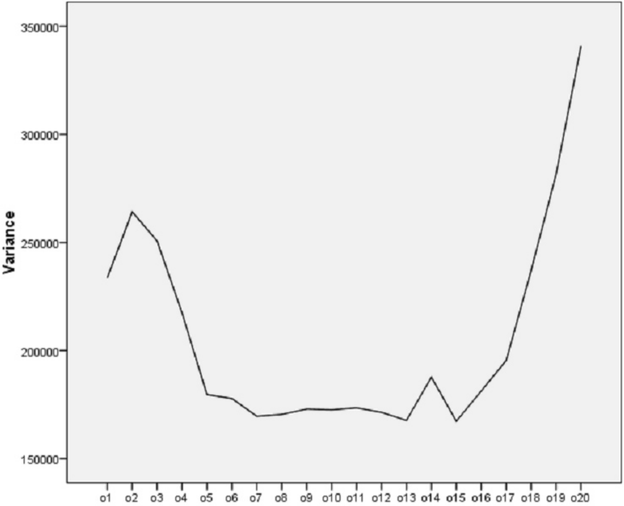
Classe d'employé

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	539	36,2	36,2	36,2
	employé privé	948	63,8	63,8	100,0
	Total	1487	100,0	100,0	

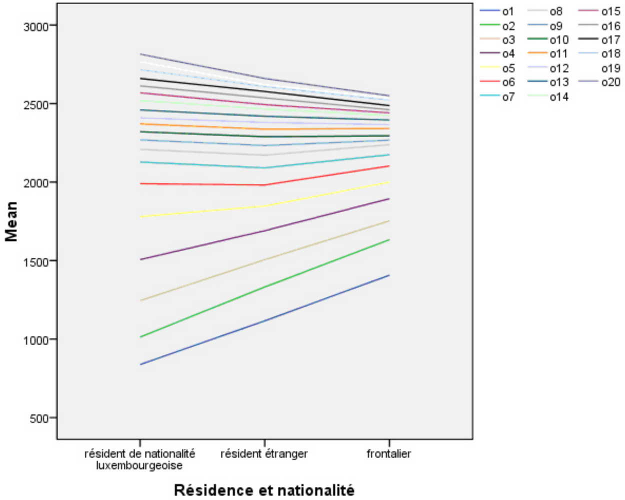
3^{rd} group



3^{rd} group

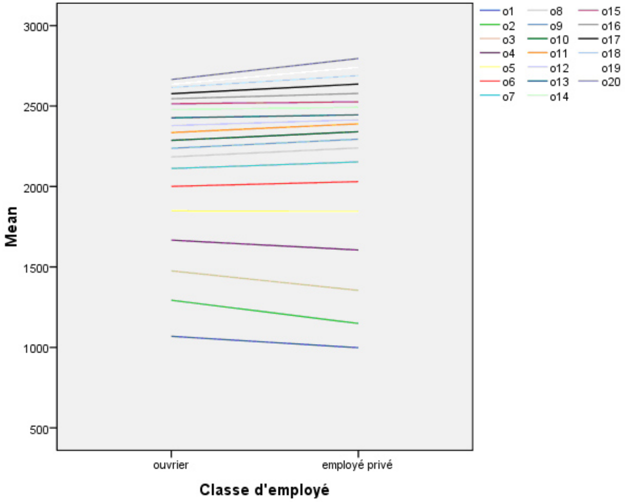

Variance
350000
300000
250000
200000
150000
o1 o2 o3 o4 o5 o6 o7 o8 o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19 o20

3^{rd} group

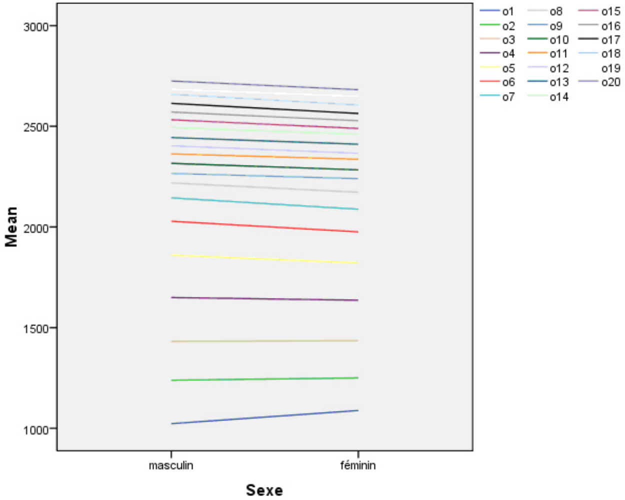


Mean
3000
2500
2000
1500
1000
500
résident de nationalité luxembourgeoise
résident étranger
frontalier
Résidence et nationalité
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

3rd group


3000
2500
2000
1500
1000
500
Mean
ouvrier
employé privé
Classe d'employé
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

3^{rd} group



Mean
3000
2500
2000
1500
1000
masculin
féminin
Sexe
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

4^{th} group

7.9 % of the population

4^{th} group

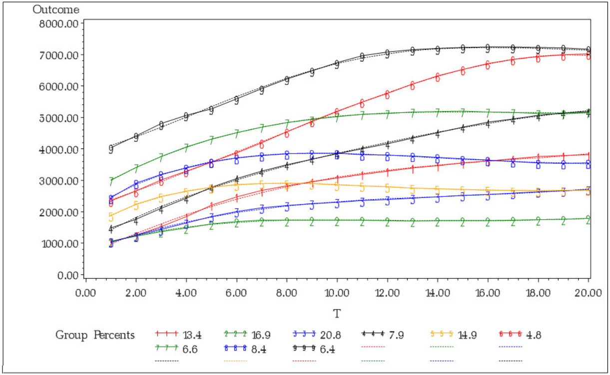
7.9 % of the population

$P(x) = 976 + 474t - 29.6t^2 - 0.029t^4$

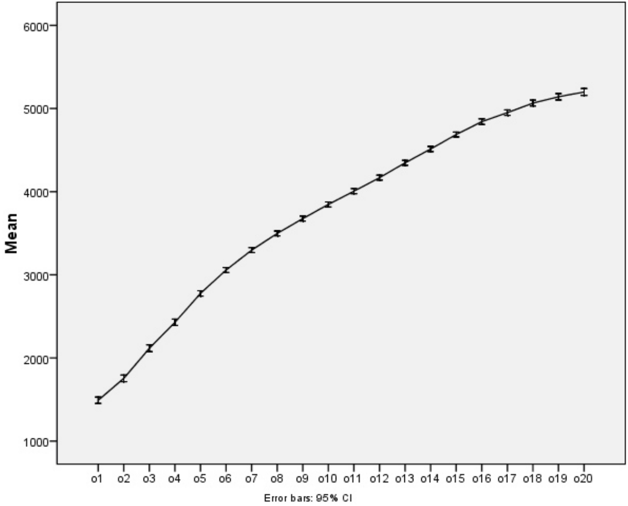
4^{th} group

7.9 % of the population

$$P(x) = 976 + 474t - 29.6t^2 - 0.029t^4$$



4^{th} group

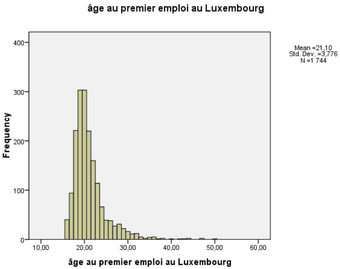


6000
5000
4000
3000
2000
1000
Mean
o1 o2 o3 o4 o5 o6 o7 o8 o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19 o20
Error bars: 95% CI

4^{th} group

Age_initial

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	15,00	40	2,3	2,3	2,3
	16,00	94	5,4	5,4	7,7
	17,00	221	12,7	12,7	20,4
	18,00	303	17,4	17,4	37,7
	19,00	303	17,4	17,4	55,1
	20,00	220	12,6	12,6	67,7
	21,00	160	9,2	9,2	76,9
	22,00	114	6,5	6,5	83,4
	23,00	66	3,8	3,8	87,2
	24,00	39	2,2	2,2	89,4
	25,00	38	2,2	2,2	91,6
	26,00	27	1,5	1,5	93,2
	27,00	31	1,8	1,8	95,0
	28,00	22	1,3	1,3	96,2
	29,00	16	,9	,9	97,1
	30,00	11	,6	,6	97,8
	31,00	12	,7	,7	98,5
	32,00	5	,3	,3	98,7
	33,00	2	,1	,1	98,9
	34,00	4	,2	,2	99,1
	35,00	5	,3	,3	99,4
	36,00	1	,1	,1	99,4
	37,00	2	,1	,1	99,5
	39,00	1	,1	,1	99,6
	41,00	1	,1	,1	99,7
	42,00	1	,1	,1	99,7
	43,00	2	,1	,1	99,8
	46,00	2	,1	,1	99,9
	49,00	1	,1	,1	100,0
	Total	1744	99,9	100,0	
Missing	System	2	,1		
Total		1746	100,0		



4^{th} group

Sexe

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	masculin	1100	63,1	63,1	63,1
	féminin	644	36,9	36,9	100,0
	Total	1744	100,0	100,0	

4^{th} group

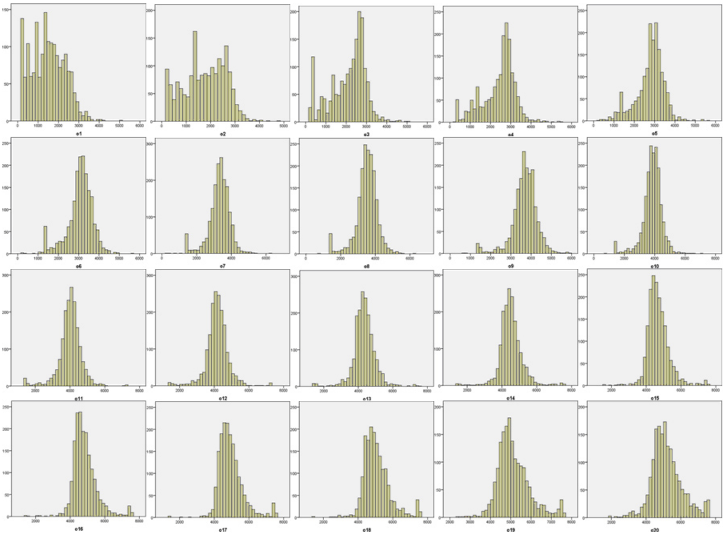
Résidence et nationalité

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	résident de nationalité luxembourgeoise	1211	69,4	69,4	69,4
	résident étranger	260	14,9	14,9	84,3
	frontalier	273	15,6	15,7	100,0
	Total	1744	99,9	100,0	
Missing	System	2	,1		
Total		1746	100,0		

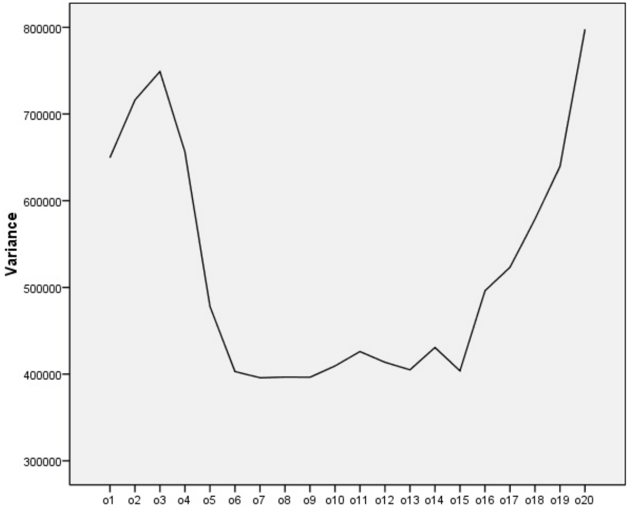
Classe d'employé

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	117	6,7	6,7	6,7
	employé privé	1627	93,2	93,3	100,0
	Total	1744	99,9	100,0	
Missing	System	2	,1		
Total		1746	100,0		

4^{th} group

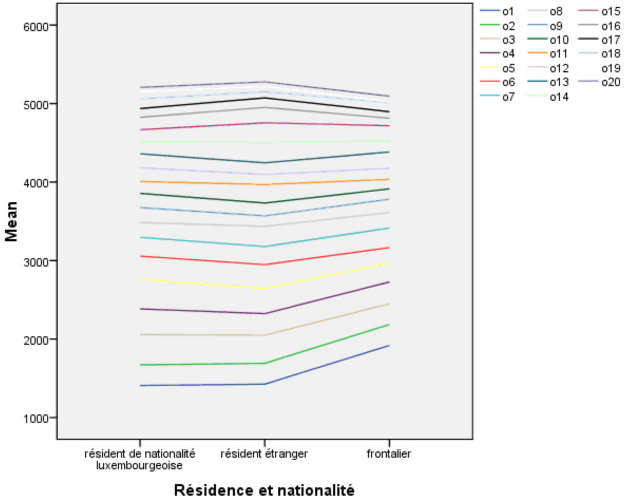


4^{th} group



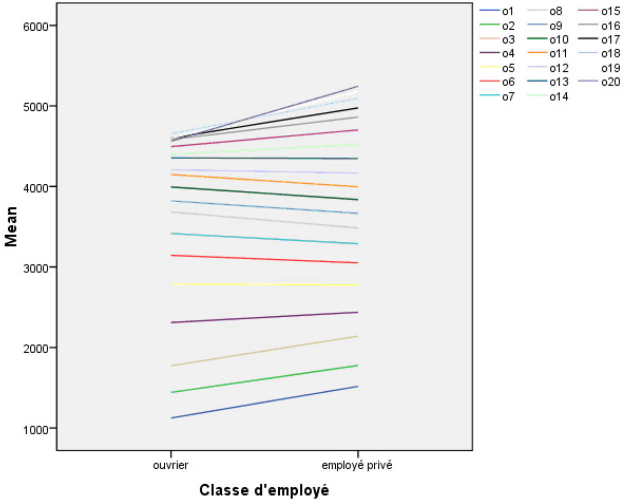
Variance
800000
700000
600000
500000
400000
300000
o1 o2 o3 o4 o5 o6 o7 o8 o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19 o20

4^{th} group



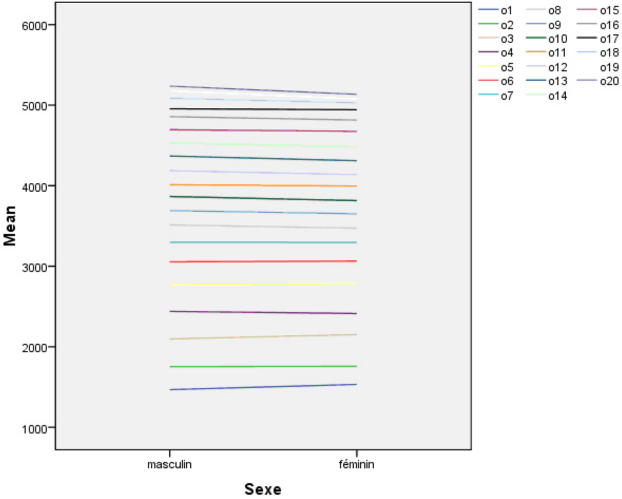
Mean
6000
5000
4000
3000
2000
1000
résident de nationalité luxembourgeoise
résident étranger
frontalier
Résidence et nationalité
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

4^{th} group



Mean
6000
5000
4000
3000
2000
1000
ouvrier
employé privé
Classe d'employé
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

4^{th} group



Mean
6000
5000
4000
3000
2000
1000
masculin
féminin
Sexe
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

5^{th} group

14.9 % of the population

5^{th} group

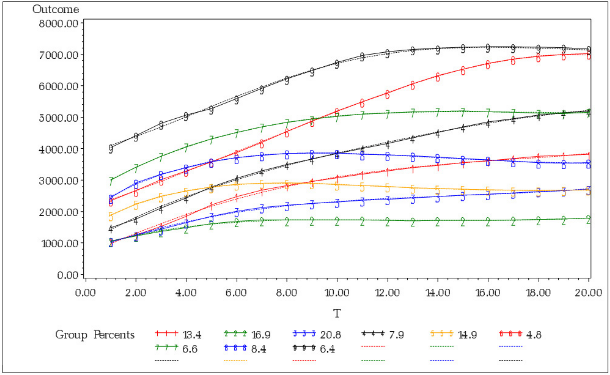
14.9 % of the population

$$P(x) = 1452 + 490t - 29.6t^2 + 1.38t^3 - 0.028t^4$$

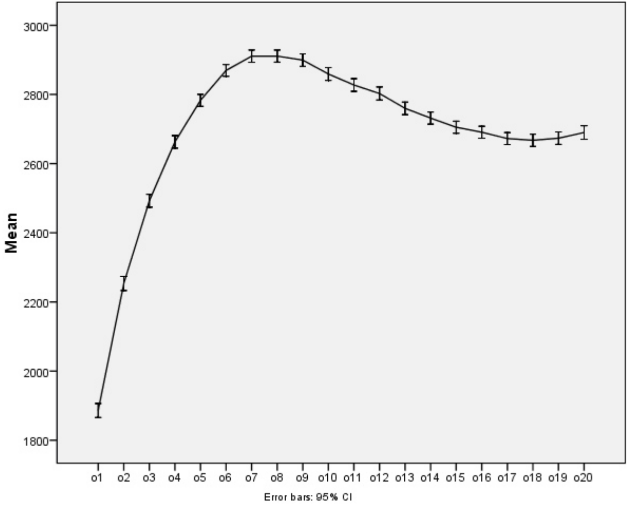
5^{th} group

14.9 % of the population

$$P(x) = 1452 + 490t - 29.6t^2 + 1.38t^3 - 0.028t^4$$



5^{th} group

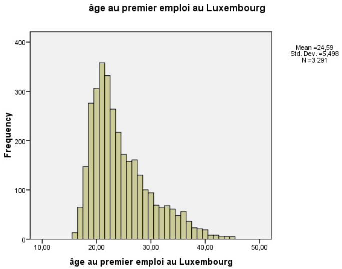


Mean
3000
2800
2600
2400
2200
2000
1800
o1 o2 o3 o4 o5 o6 o7 o8 o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19 o20
Error bars: 95% CI

5^{th} group

Age_initial

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	15,00	13	,4	,4	,4
	16,00	65	2,0	2,0	2,4
	17,00	147	4,5	4,5	6,8
	18,00	276	8,4	8,4	15,2
	19,00	306	9,3	9,3	24,5
	20,00	358	10,9	10,9	35,4
	21,00	332	10,1	10,1	45,5
	22,00	264	8,0	8,0	53,5
	23,00	217	6,6	6,6	60,1
	24,00	172	5,2	5,2	65,3
	25,00	158	4,8	4,8	70,1
	26,00	161	4,9	4,9	75,0
	27,00	130	3,9	4,0	79,0
	28,00	100	3,0	3,0	82,0
	29,00	94	2,9	2,9	84,9
	30,00	69	2,1	2,1	87,0
	31,00	65	2,0	2,0	88,9
	32,00	68	2,1	2,1	91,0
	33,00	61	1,9	1,9	92,9
	34,00	48	1,5	1,5	94,3
	35,00	56	1,7	1,7	96,0
	36,00	36	1,1	1,1	97,1
	37,00	23	,7	,7	97,8
	38,00	21	,6	,6	98,5
	39,00	19	,6	,6	99,0
	40,00	8	,2	,2	99,3
	41,00	8	,2	,2	99,5
	42,00	6	,2	,2	99,7
	43,00	5	,2	,2	99,8
	44,00	5	,2	,2	100,0
	Total	3291	99,9	100,0	
Missing	System	2	,1		
Total		3293	100,0		



5^{th} group

Sexe

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	masculin	2359	71,7	71,7	71,7
	féminin	932	28,3	28,3	100,0
	Total	3291	100,0	100,0	

5^{th} group

Sexe

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	masculin	2359	71,7	71,7	71,7
	féminin	932	28,3	28,3	100,0
	Total	3291	100,0	100,0	

Résidence et nationalité

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	résident de nationalité luxembourgeoise	862	26,2	26,2	26,2
	résident étranger	768	23,3	23,3	49,5
	frontalier	1661	50,5	50,5	100,0
	Total	3291	100,0	100,0	

5^{th} group

Men:

Classe d'employé

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	1910	81,0	81,0	81,0
	employé privé	449	19,0	19,0	100,0
	Total	2359	100,0	100,0	

5^{th} group

Men:

Classe d'employé

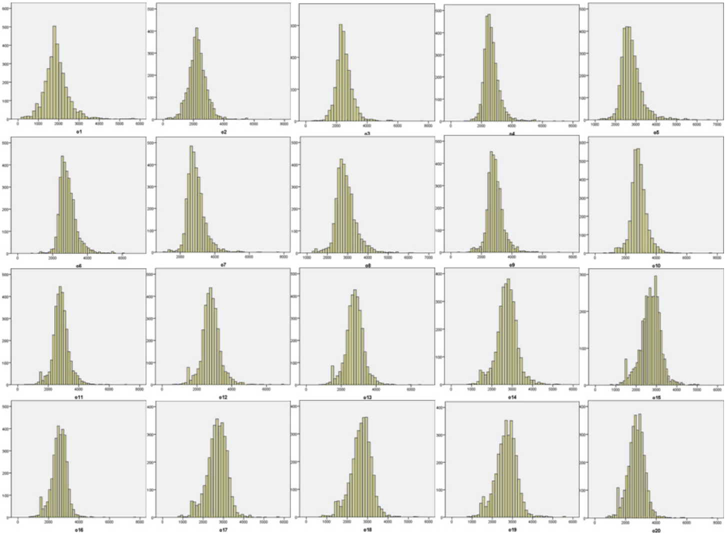
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	1910	81,0	81,0	81,0
	employé privé	449	19,0	19,0	100,0
	Total	2359	100,0	100,0	

Women:

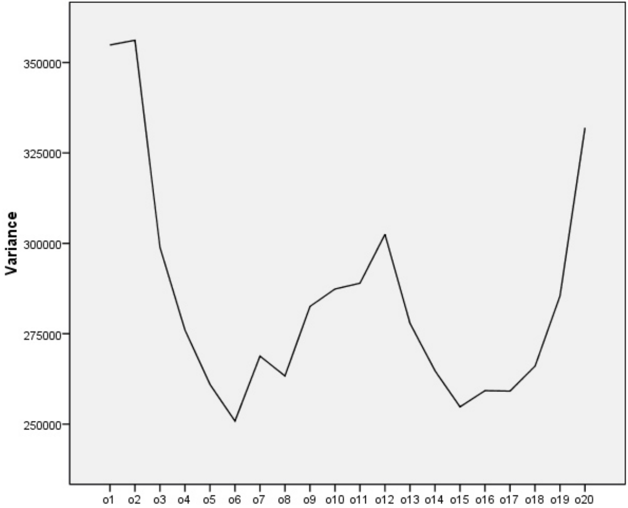
Classe d'employé

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	71	7,6	7,6	7,6
	employé privé	861	92,4	92,4	100,0
	Total	932	100,0	100,0	

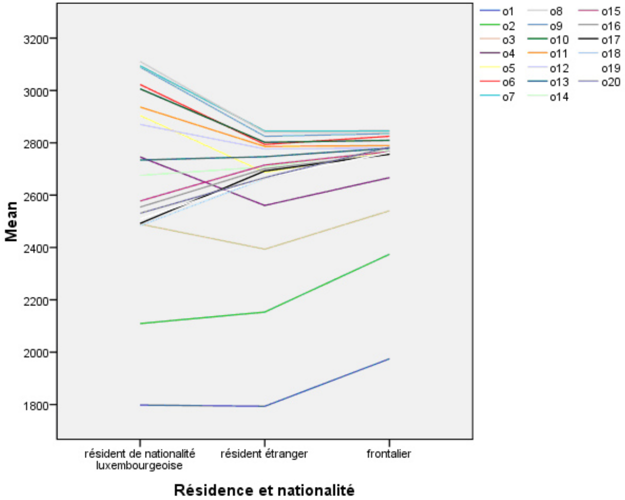
5^{th} group



5^{th} group

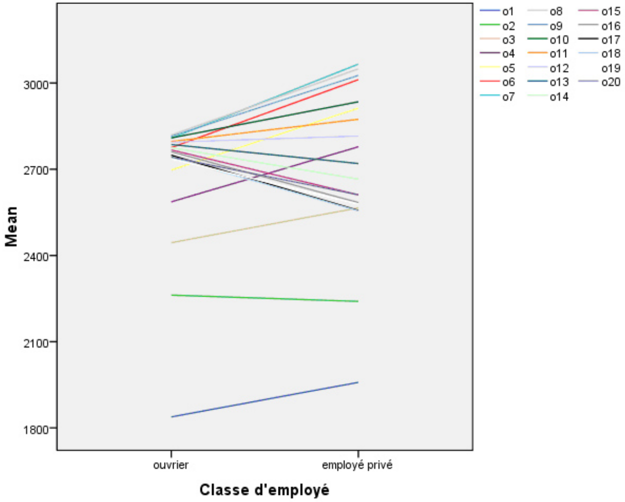

Variance
350000
325000
300000
275000
250000
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

5^{th} group



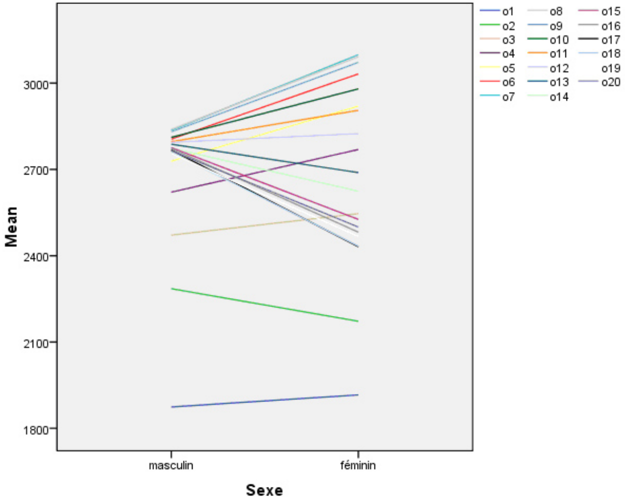
Mean
3200
3000
2800
2600
2400
2200
2000
1800
résident de nationalité luxembourgeoise
résident étranger
frontalier
Résidence et nationalité
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

5^{th} group



Mean
3000
2700
2400
2100
1800
ouvrier
employé privé
Classe d'employé
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

5^{th} group



o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20
Mean
3000
2700
2400
2100
1800
masculin
féminin
Sexe

6^{th} group

4.8 % of the population

6^{th} group

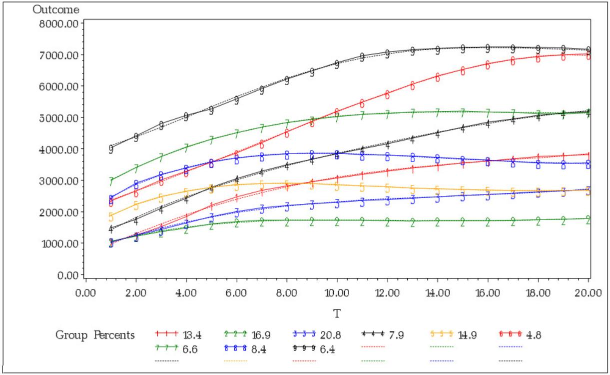
4.8 % of the population

$P(x) = 2089 - 0.017t^4$

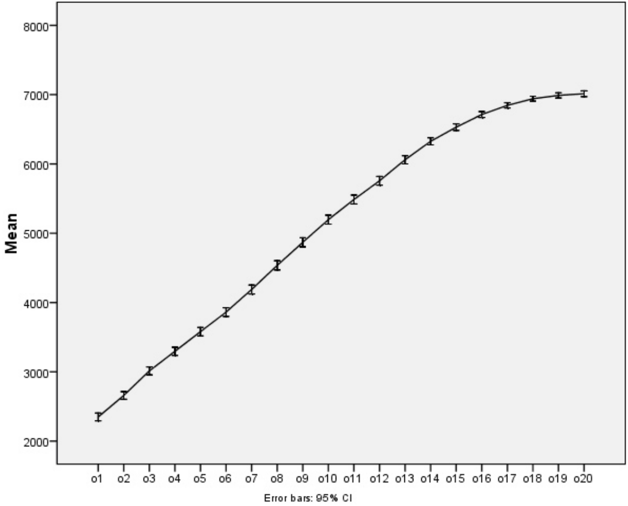
6^{th} group

4.8 % of the population

$P(x) = 2089 - 0.017t^4$



6^{th} group

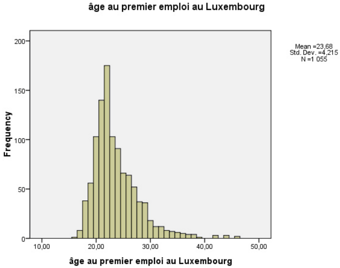


Mean
8000
7000
6000
5000
4000
3000
2000
o1 o2 o3 o4 o5 o6 o7 o8 o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19 o20
Error bars: 95% CI

6^{th} group

Age_initial

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	15,00	1	,1	,1	,1
	16,00	8	,8	,8	,9
	17,00	38	3,6	3,6	4,5
	18,00	56	5,3	5,3	9,8
	19,00	103	9,7	9,8	19,5
	20,00	140	13,2	13,3	32,8
	21,00	175	16,5	16,6	49,4
	22,00	103	9,7	9,8	59,1
	23,00	91	8,6	8,6	67,8
	24,00	66	6,2	6,3	74,0
	25,00	64	6,0	6,1	80,1
	26,00	52	4,9	4,9	85,0
	27,00	37	3,5	3,5	88,5
	28,00	36	3,4	3,4	91,9
	29,00	18	1,7	1,7	93,6
	30,00	12	1,1	1,1	94,8
	31,00	12	1,1	1,1	95,9
	32,00	8	,8	,8	96,7
	33,00	7	,7	,7	97,3
	34,00	6	,6	,6	97,9
	35,00	5	,5	,5	98,4
	36,00	4	,4	,4	98,8
	37,00	4	,4	,4	99,1
	38,00	1	,1	,1	99,2
	41,00	3	,3	,3	99,5
	43,00	3	,3	,3	99,8
	45,00	2	,2	,2	100,0
	Total	1055	99,2	100,0	
Missing	System	8	,8		
Total		1063	100,0		



6^{th} group

Sexe

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	masculin	784	74,3	74,3	74,3
	féminin	271	25,7	25,7	100,0
	Total	1055	100,0	100,0	

6^{th} group

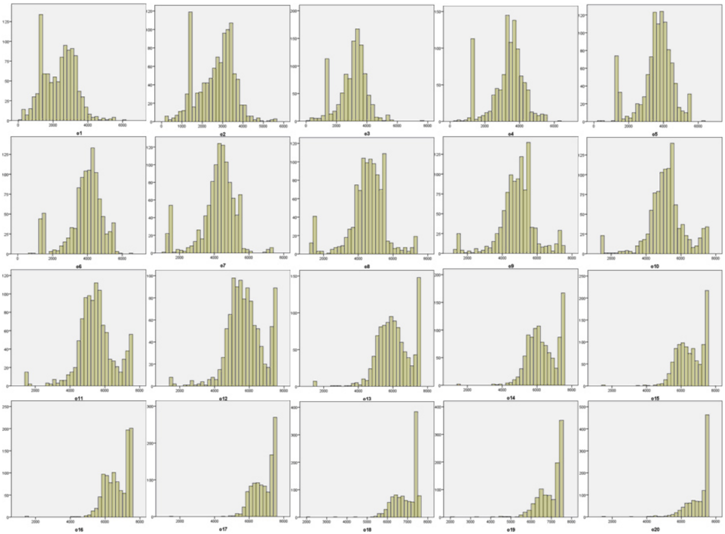
Résidence et nationalité

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	résident de nationalité luxembourgeoise	651	61,2	61,7	61,7
	résident étranger	184	17,3	17,4	79,1
	frontalier	220	20,7	20,9	100,0
	Total	1055	99,2	100,0	
Missing	System	8	,8		
Total		1063	100,0		

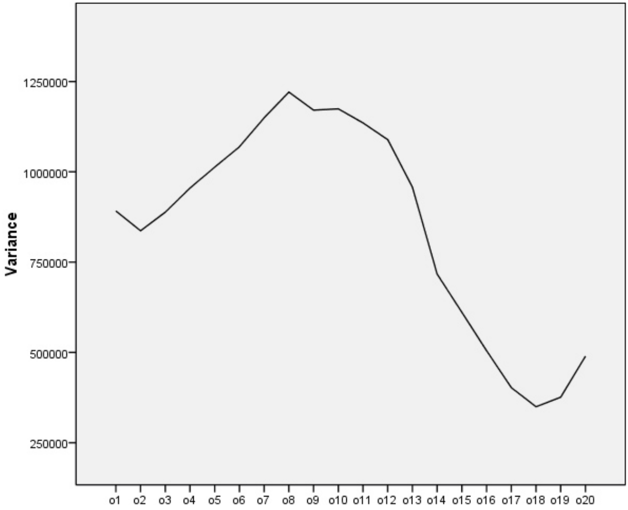
Classe d'employé

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	3	,3	,3	,3
	employé privé	1052	99,0	99,7	100,0
	Total	1055	99,2	100,0	
Missing	System	8	,8		
Total		1063	100,0		

6^{th} group

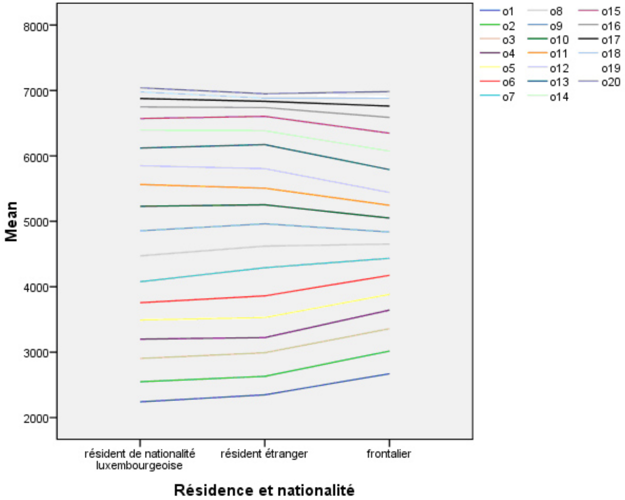


6^{th} group



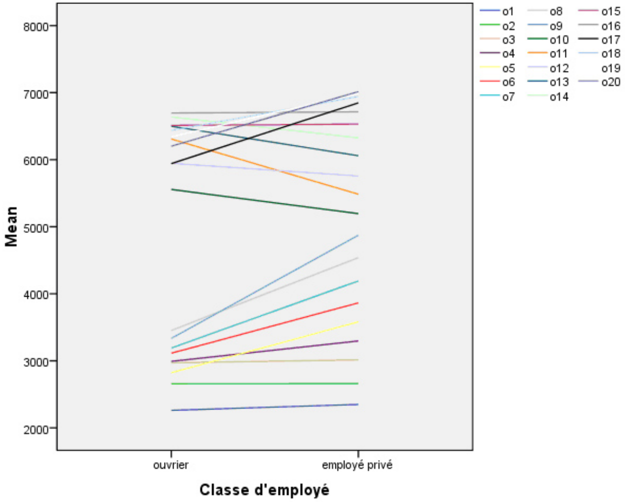
Variance
1250000
1000000
750000
500000
250000
o1 o2 o3 o4 o5 o6 o7 o8 o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19 o20

6^{th} group



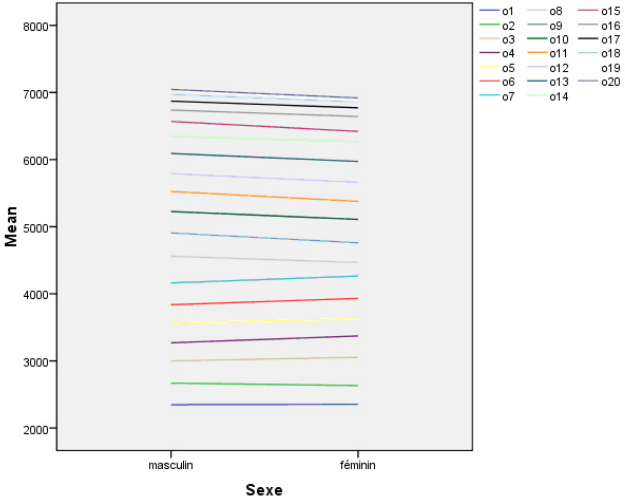
Mean
8000
7000
6000
5000
4000
3000
2000
résident de nationalité luxembourgeoise
résident étranger
frontalier
Résidence et nationalité
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

6^{th} group



Mean
8000
7000
6000
5000
4000
3000
2000
ouvrier
employé privé
Classe d'employé
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

6^{th} group



Mean
8000
7000
6000
5000
4000
3000
2000
masculin
féminin
Sexe
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

7^{th} group

6.6 % of the population

7^{th} group

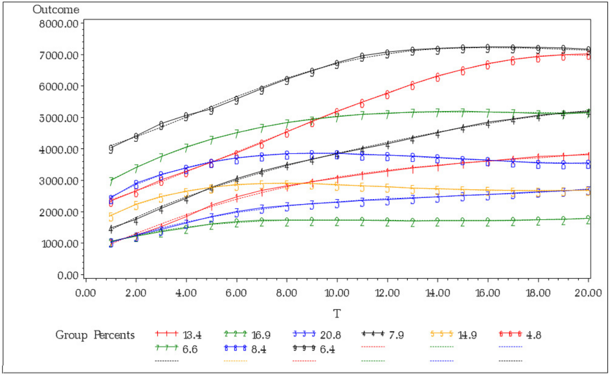
6.6 % of the population

$P(x) = 2556 + 484t - 29.9t^2 + 0.66t^3$

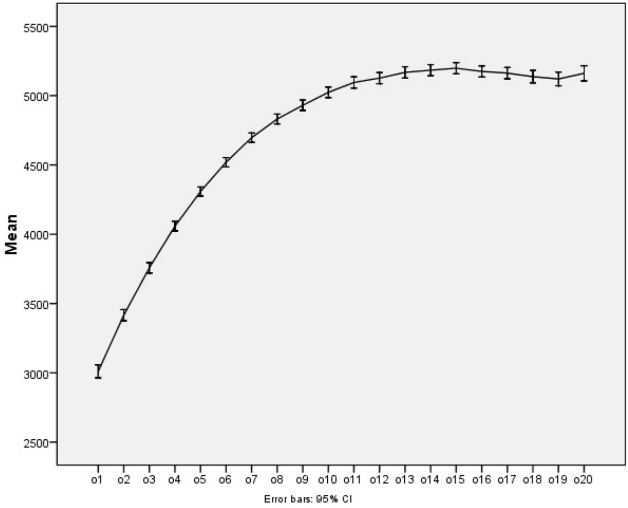
7^{th} group

6.6 % of the population

$$P(x) = 2556 + 484t - 29.9t^2 + 0.66t^3$$



7^{th} group

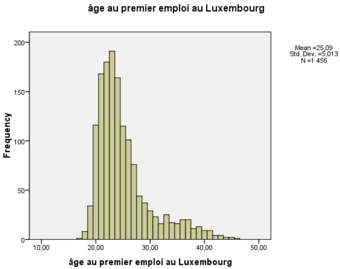


Mean
5500
5000
4500
4000
3500
3000
2500
o1 o2 o3 o4 o5 o6 o7 o8 o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19 o20
Error bars: 95% CI

7^{th} group

Age_initial

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	16,00	1	,1	,1	,1
	17,00	8	,5	,5	,6
	18,00	34	2,3	2,3	3,0
	19,00	116	7,9	8,0	10,9
	20,00	168	11,5	11,5	22,5
	21,00	180	12,3	12,4	34,8
	22,00	191	13,1	13,1	47,9
	23,00	164	11,2	11,3	59,2
	24,00	115	7,9	7,9	67,1
	25,00	101	6,9	6,9	74,0
	26,00	76	5,2	5,2	79,3
	27,00	44	3,0	3,0	82,3
	28,00	37	2,5	2,5	84,8
	29,00	29	2,0	2,0	86,8
	30,00	23	1,6	1,6	88,4
	31,00	16	1,1	1,1	89,5
	32,00	25	1,7	1,7	91,2
	33,00	17	1,2	1,2	92,4
	34,00	16	1,1	1,1	93,5
	35,00	20	1,4	1,4	94,8
	36,00	20	1,4	1,4	96,2
	37,00	11	,8	,8	97,0
	38,00	13	,9	,9	97,9
	39,00	9	,6	,6	98,5
	40,00	9	,6	,6	99,1
	41,00	4	,3	,3	99,4
	42,00	4	,3	,3	99,7
	43,00	2	,1	,1	99,8
	44,00	2	,1	,1	99,9
	45,00	1	,1	,1	100,0
	Total	1456	99,6	100,0	
Missing	System	6	,4		
Total		1462	100,0		



7^{th} group

Sexe

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	masculin	874	60,0	60,0	60,0
	féminin	582	40,0	40,0	100,0
	Total	1456	100,0	100,0	

7^{th} group

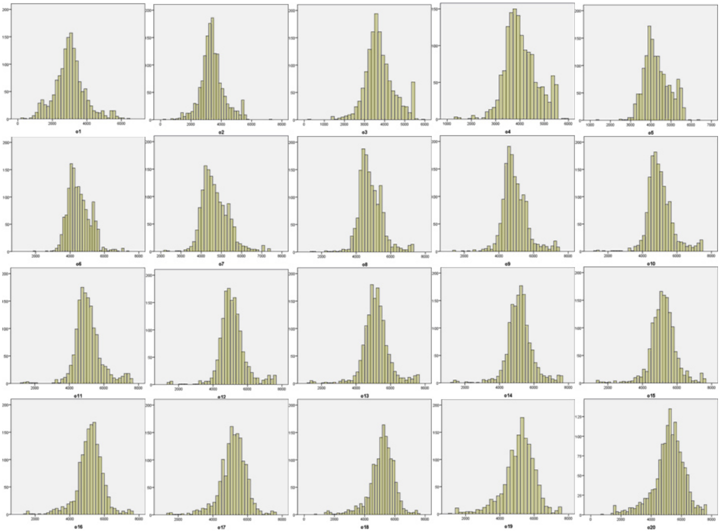
Résidence et nationalité

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	résident de nationalité luxembourgeoise	632	43,2	43,4	43,4
	résident étranger	273	18,7	18,8	62,2
	frontalier	551	37,7	37,8	100,0
	Total	1456	99,6	100,0	
Missing	System	6	,4		
Total		1462	100,0		

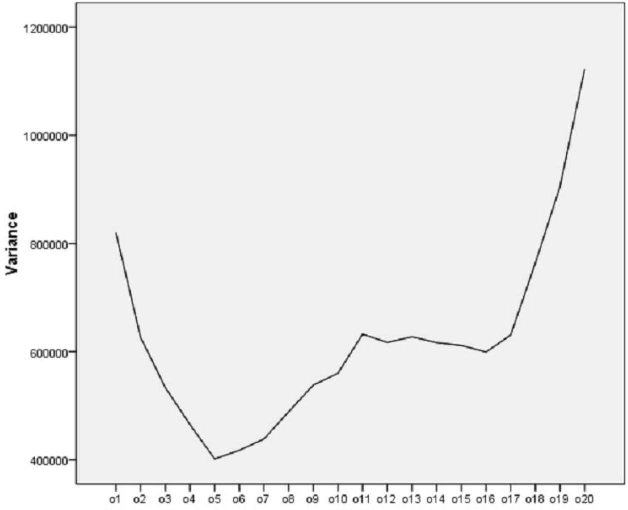
Classe d'employé

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	19	1,3	1,3	1,3
	employé privé	1437	98,3	98,7	100,0
	Total	1456	99,6	100,0	
Missing	System	6	,4		
Total		1462	100,0		

7^{th} group

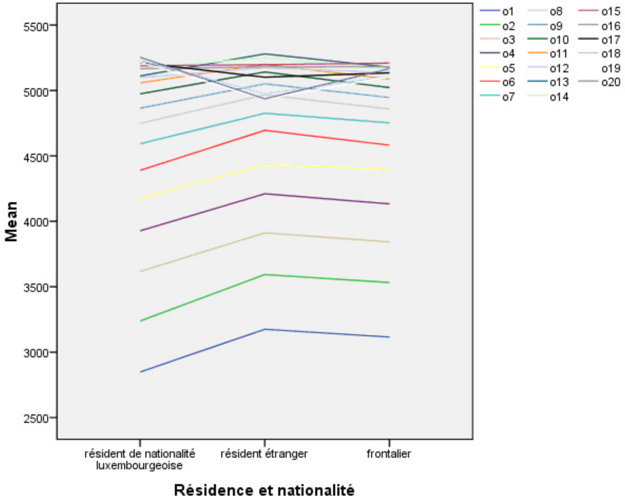


7^{th} group



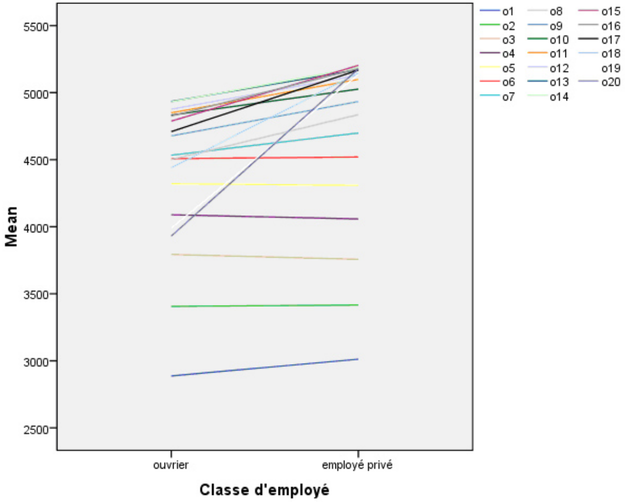
Variance
1200000
1000000
800000
600000
400000
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

7^{th} group



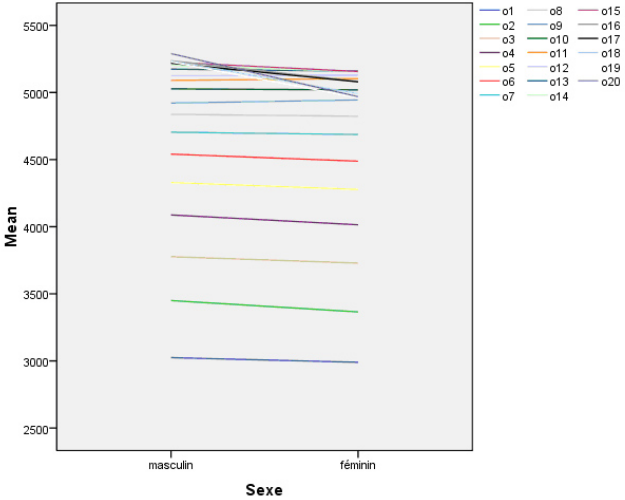
Mean
5500
5000
4500
4000
3500
3000
2500
résident de nationalité luxembourgeoise
résident étranger
frontalier
Résidence et nationalité
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

7^{th} group



Mean
5500
5000
4500
4000
3500
3000
2500
ouvrier
employé privé
Classe d'employé
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

7^{th} group



Mean
5500
5000
4500
4000
3500
3000
2500
masculin
féminin
Sexe
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

8^{th} group

8.4 % of the population

8^{th} group

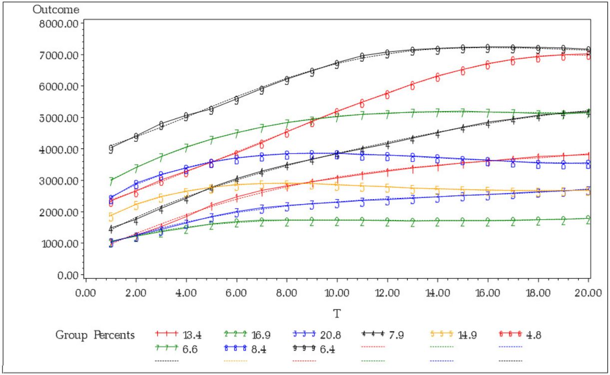
8.4 % of the population

$$P(x) = 1987 + 537t - 52.7t^2 + 2.06t^3 - 0.028t^4$$

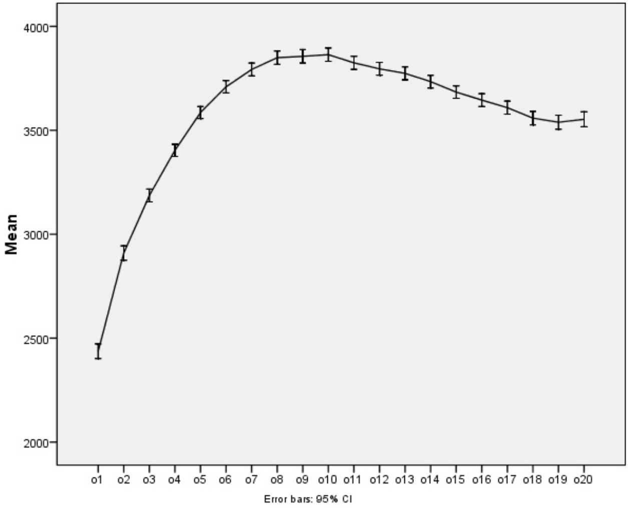
8^{th} group

8.4 % of the population

$$P(x) = 1987 + 537t - 52.7t^2 + 2.06t^3 - 0.028t^4$$



8^{th} group

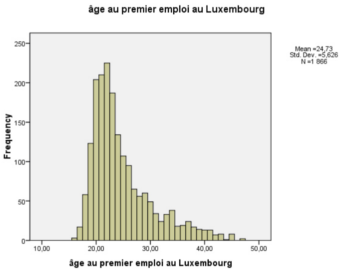


4000
3500
3000
2500
2000
Mean
o1 o2 o3 o4 o5 o6 o7 o8 o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19 o20
Error bars: 95% CI

8^{th} group

Age_initial

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	15,00	3	,2	,2	,2
	16,00	17	,9	,9	1,1
	17,00	58	3,1	3,1	4,2
	18,00	123	6,6	6,6	10,8
	19,00	204	10,9	10,9	21,7
	20,00	210	11,2	11,3	33,0
	21,00	225	12,0	12,1	45,0
	22,00	187	10,0	10,0	55,0
	23,00	134	7,2	7,2	62,2
	24,00	107	5,7	5,7	68,0
	25,00	95	5,1	5,1	73,0
	26,00	65	3,5	3,5	76,5
	27,00	56	3,0	3,0	79,5
	28,00	60	3,2	3,2	82,7
	29,00	49	2,6	2,6	85,4
	30,00	34	1,8	1,8	87,2
	31,00	24	1,3	1,3	88,5
	32,00	33	1,8	1,8	90,2
	33,00	38	2,0	2,0	92,3
	34,00	18	1,0	1,0	93,2
	35,00	19	1,0	1,0	94,3
	36,00	24	1,3	1,3	95,6
	37,00	17	,9	,9	96,5
	38,00	14	,7	,8	97,2
	39,00	13	,7	,7	97,9
	40,00	13	,7	,7	98,6
	41,00	7	,4	,4	99,0
	42,00	8	,4	,4	99,4
	43,00	1	,1	,1	99,5
	44,00	8	,4	,4	99,9
	46,00	2	,1	,1	100,0
	Total	1866	99,8	100,0	
Missing	System	3	,2		
Total		1869	100,0		



8^{th} group

Sexe

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	masculin	1129	60,5	60,5	60,5
	féminin	737	39,5	39,5	100,0
	Total	1866	100,0	100,0	

8^{th} group

Sexe

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	masculin	1129	60,5	60,5	60,5
	féminin	737	39,5	39,5	100,0
	Total	1866	100,0	100,0	

Résidence et nationalité

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	résident de nationalité luxembourgeoise	692	37,1	37,1	37,1
	résident étranger	290	15,5	15,5	52,6
	frontalier	884	47,4	47,4	100,0
	Total	1866	100,0	100,0	

8^{th} group

Men:

Classe d'employé

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	594	52,6	52,6	52,6
	employé privé	535	47,4	47,4	100,0
	Total	1129	100,0	100,0	

8^{th} group

Men:

Classe d'employé

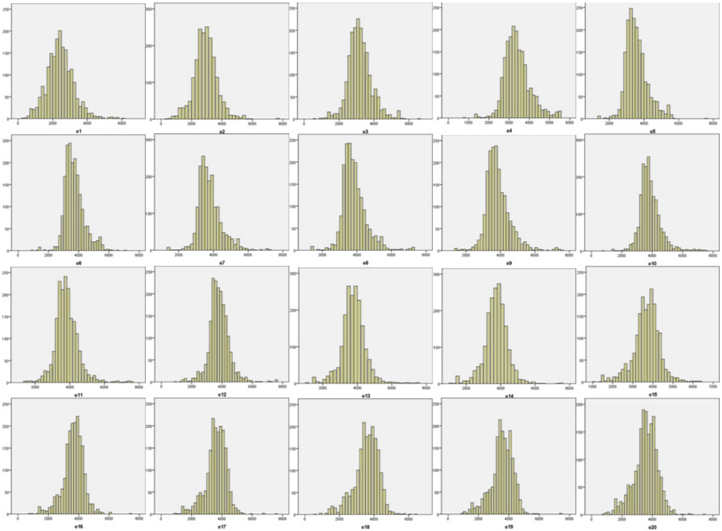
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	594	52,6	52,6	52,6
	employé privé	535	47,4	47,4	100,0
	Total	1129	100,0	100,0	

Women:

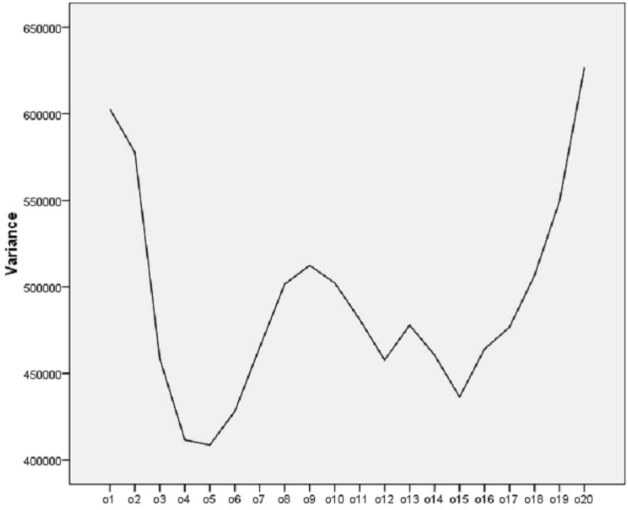
Classe d'employé

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	6	,8	,8	,8
	employé privé	731	99,2	99,2	100,0
	Total	737	100,0	100,0	

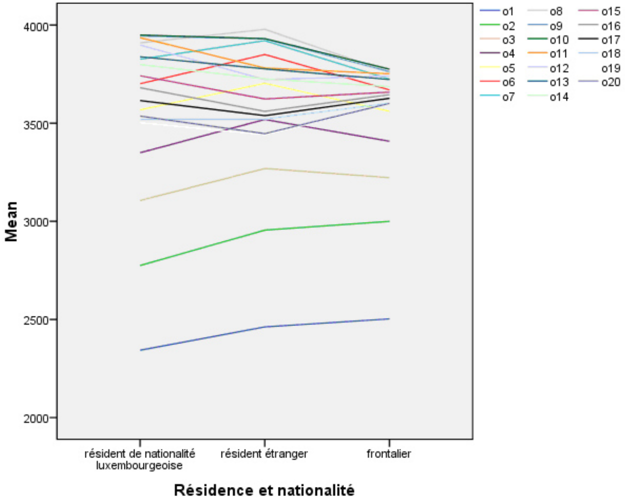
8^{th} group



8^{th} group

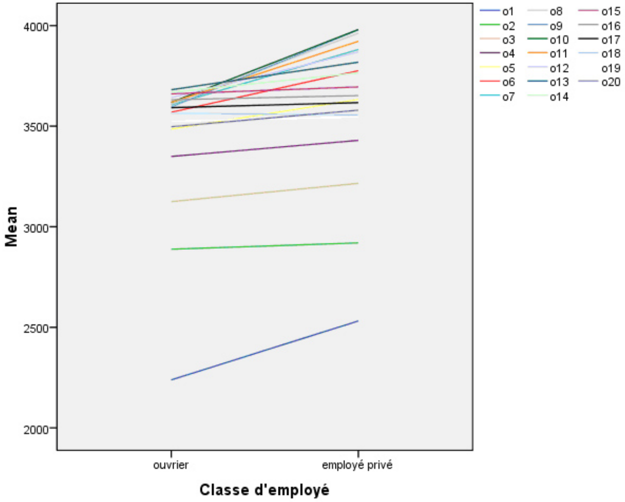

Variance
650000
600000
550000
500000
450000
400000
o1 o2 o3 o4 o5 o6 o7 o8 o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19 o20

8^{th} group



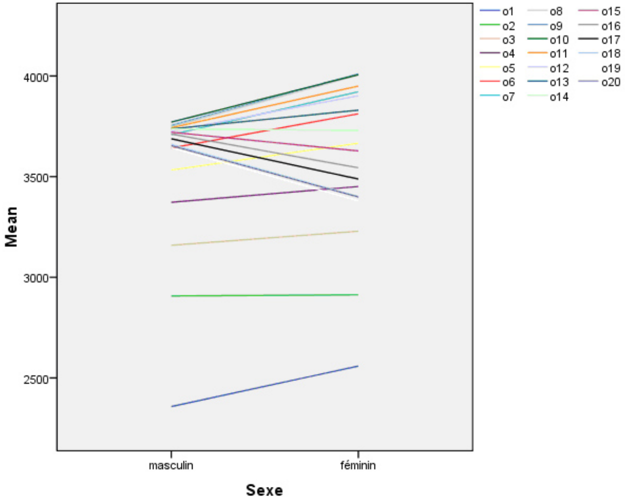
Mean
4000
3500
3000
2500
2000
résident de nationalité luxembourgeoise
résident étranger
frontalier
Résidence et nationalité
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

8^{th} group



Mean
4000
3500
3000
2500
2000
ouvrier
employé privé
Classe d'employé
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

8^{th} group



Mean
4000
3500
3000
2500
masculin
féminin
Sexe
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

9^{th} group

6.4 % of the population

9^{th} group

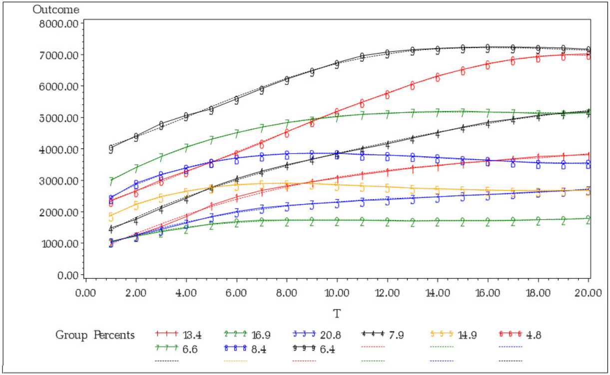
6.4 % of the population

$P(x) = 3873 + 206t + 30t2 - 2.89t^3 + 0.06t^4$

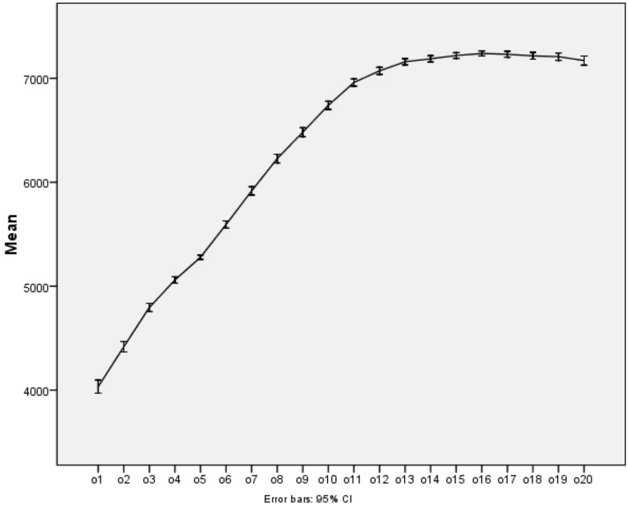
9^{th} group

6.4 % of the population

$$P(x) = 3873 + 206t + 30t2 - 2.89t^3 + 0.06t^4$$



9^{th} group

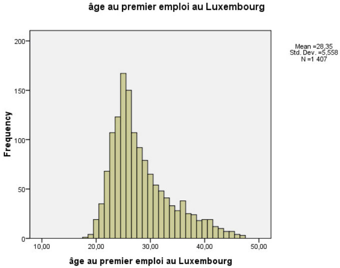


Mean
7000
6000
5000
4000
o1 o2 o3 o4 o5 o6 o7 o8 o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19 o20
Error bars: 95% CI

9^{th} group

Age_initial

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	17,00	1	,1	,1	,1
	18,00	4	,3	,3	,4
	19,00	19	1,3	1,4	1,7
	20,00	35	2,5	2,5	4,2
	21,00	68	4,8	4,8	9,0
	22,00	107	7,5	7,6	16,6
	23,00	123	8,7	8,7	25,4
	24,00	167	11,8	11,9	37,2
	25,00	150	10,6	10,7	47,9
	26,00	107	7,5	7,6	55,5
	27,00	92	6,5	6,5	62,0
	28,00	79	5,6	5,6	67,7
	29,00	65	4,6	4,6	72,3
	30,00	54	3,8	3,8	76,1
	31,00	48	3,4	3,4	79,5
	32,00	41	2,9	2,9	82,4
	33,00	33	2,3	2,3	84,8
	34,00	28	2,0	2,0	86,8
	35,00	38	2,7	2,7	89,5
	36,00	25	1,8	1,8	91,3
	37,00	24	1,7	1,7	93,0
	38,00	18	1,3	1,3	94,2
	39,00	19	1,3	1,4	95,6
	40,00	19	1,3	1,4	96,9
	41,00	12	,8	,9	97,8
	42,00	10	,7	,7	98,5
	43,00	7	,5	,5	99,0
	44,00	7	,5	,5	99,5
	45,00	4	,3	,3	99,8
	46,00	3	,2	,2	100,0
	Total	1407	99,2	100,0	
Missing	System	11	,8		
Total		1418	100,0		



9^{th} group

Sexe

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	masculin	1200	85,3	85,3	85,3
	féminin	207	14,7	14,7	100,0
	Total	1407	100,0	100,0	

9^{th} group

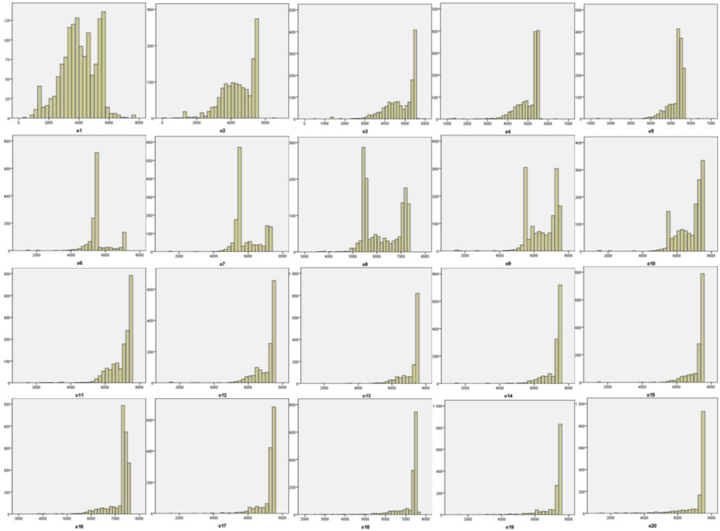
Résidence et nationalité

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	résident de nationalité luxembourgeoise	468	33,0	33,3	33,3
	résident étranger	475	33,5	33,8	67,0
	frontalier	464	32,7	33,0	100,0
	Total	1407	99,2	100,0	
Missing	System	11	,8		
Total		1418	100,0		

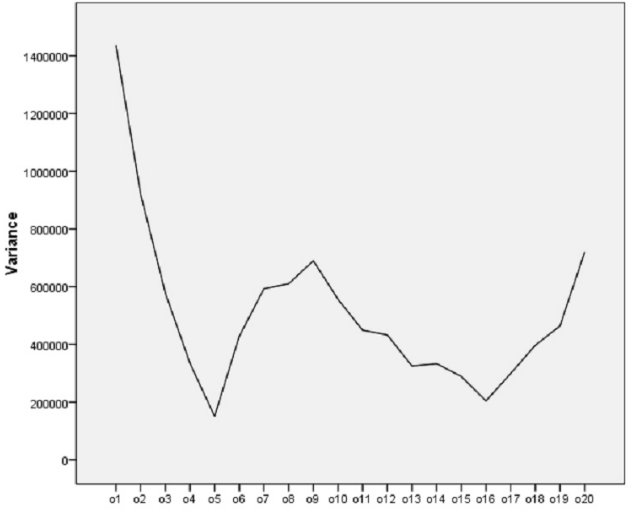
Classe d'employé

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	ouvrier	1	,1	,1	,1
	employé privé	1406	99,2	99,9	100,0
	Total	1407	99,2	100,0	
Missing	System	11	,8		
Total		1418	100,0		

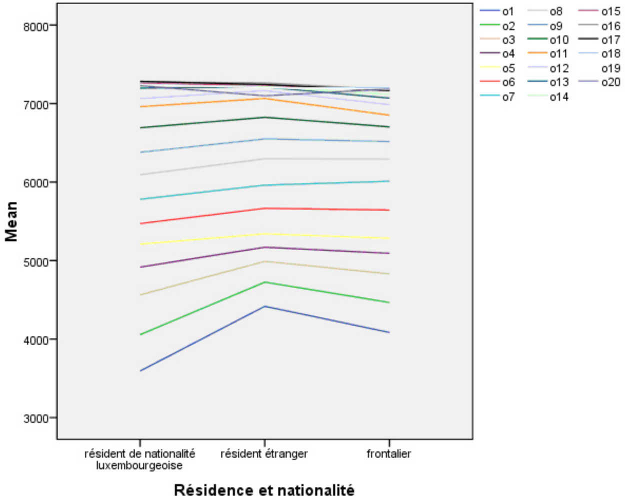
9^{th} group



9^{th} group

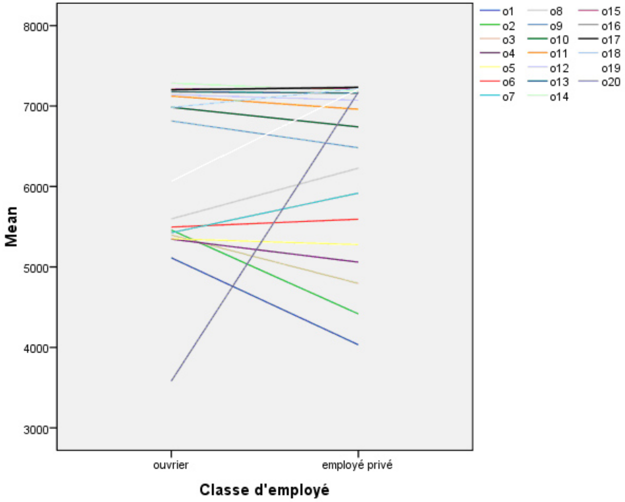

Variance
1400000
1200000
1000000
800000
600000
400000
200000
0
o1 o2 o3 o4 o5 o6 o7 o8 o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19 o20

9^{th} group



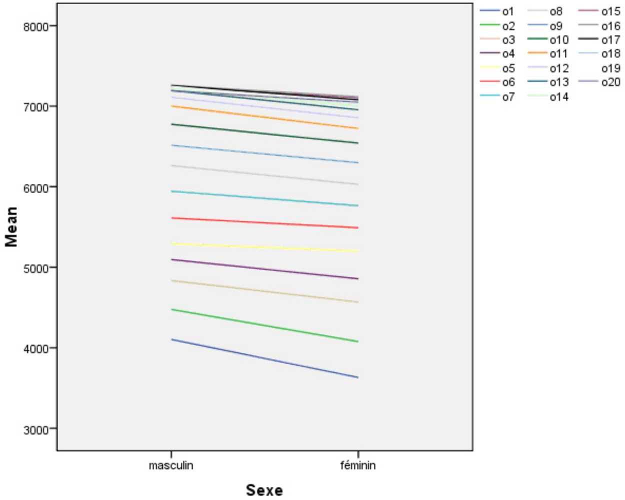
Mean
8000
7000
6000
5000
4000
3000
résident de nationalité luxembourgeoise
résident étranger
frontalier
Résidence et nationalité
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

9^{th} group



Mean
8000
7000
6000
5000
4000
3000
ouvrier
employé privé
Classe d'employé
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

9^{th} group



Mean
8000
7000
6000
5000
4000
3000
masculin
féminin
Sexe
o1
o2
o3
o4
o5
o6
o7
o8
o9
o10
o11
o12
o13
o14
o15
o16
o17
o18
o19
o20

Salary Dynamics

Mean annual salary growth:

Group 1	Group 2	Group 3	Group 4	Group 5
$\lambda_i = 3.07\%$	$\lambda_2 = 0.96\%$	$\lambda_3 = 1.45\%$	$\lambda_4 = 2.82\%$	$\lambda_5 = 0.19\%$
Group 6	Group 7	Group 8	Group 9	
$\lambda_6 = 2.58\%$	$\lambda_7 = 1.28\%$	$\lambda_8 = 0.48\%$	$\lambda_9 = 1.09\%$	

Salary Dynamics

Mean annual salary growth:

Group 1	Group 2	Group 3	Group 4	Group 5
$\lambda_i = 3.07\%$	$\lambda_2 = 0.96\%$	$\lambda_3 = 1.45\%$	$\lambda_4 = 2.82\%$	$\lambda_5 = 0.19\%$
Group 6	Group 7	Group 8	Group 9	
$\lambda_6 = 2.58\%$	$\lambda_7 = 1.28\%$	$\lambda_8 = 0.48\%$	$\lambda_9 = 1.09\%$	

Flat curves : groups 2,5 and 8.

Salary Dynamics

Mean annual salary growth:

Group 1	Group 2	Group 3	Group 4	Group 5
$\lambda_i = 3.07\%$	$\lambda_2 = 0.96\%$	$\lambda_3 = 1.45\%$	$\lambda_4 = 2.82\%$	$\lambda_5 = 0.19\%$
Group 6	Group 7	Group 8	Group 9	
$\lambda_6 = 2.58\%$	$\lambda_7 = 1.28\%$	$\lambda_8 = 0.48\%$	$\lambda_9 = 1.09\%$	

Flat curves : groups 2,5 and 8.

Normal salary growth: groups 3,7 and 9.

Salary Dynamics

Mean annual salary growth:

Group 1	Group 2	Group 3	Group 4	Group 5
$\lambda_i = 3.07\%$	$\lambda_2 = 0.96\%$	$\lambda_3 = 1.45\%$	$\lambda_4 = 2.82\%$	$\lambda_5 = 0.19\%$
Group 6	Group 7	Group 8	Group 9	
$\lambda_6 = 2.58\%$	$\lambda_7 = 1.28\%$	$\lambda_8 = 0.48\%$	$\lambda_9 = 1.09\%$	

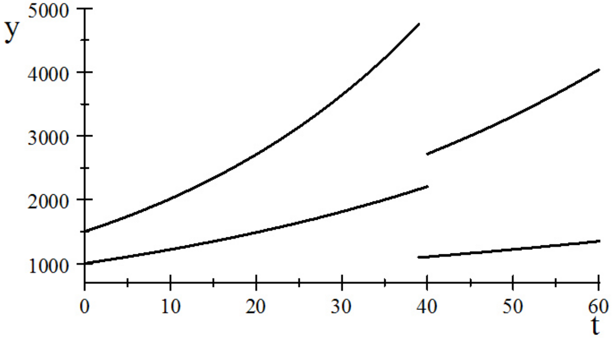
Flat curves : groups 2,5 and 8.

Normal salary growth: groups 3,7 and 9.

Dynamic trajectories: groups 1,4 and 6.

Outline

Dummy example



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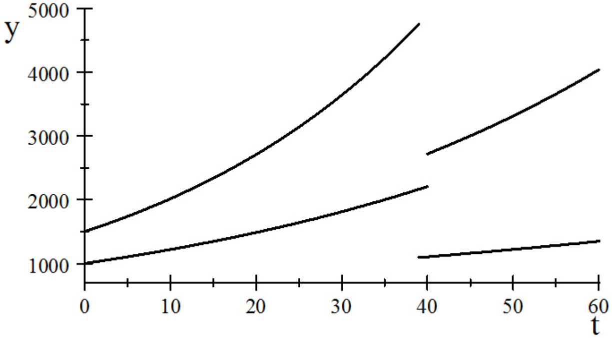

2 trajectories S^1 and S^2 with group size 60% and 40% of the population.

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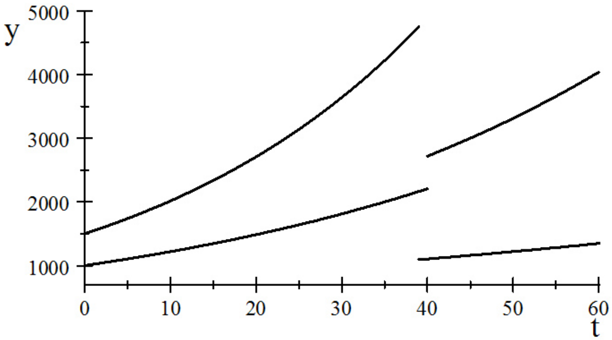

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Length of the professional life: $T = 40$ years.

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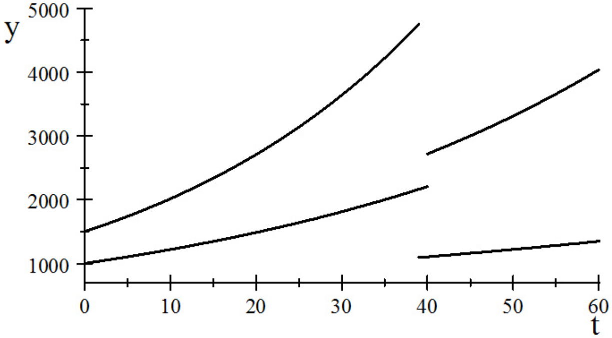

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Length of the professional life: $T = 40$ years.

Additional life expectancy: $T^* = 20$ years.

Hypotheses

Salaries grow linearly, S^1 with a starting value of 1500 and a growth coefficient of 3 %, S^2 with a starting value of 1000 and a growth coefficient of 2 %.

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Luxembourg adopts a repartition model, which means that the current pensions are paid with the tax incomes from the current workers. Each generation hence pays the pension for the generation before it.

Replacement rate in the repartition model

Replacement rate = first pension / last salary

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A worker who's trajectory is S^1 with a probability of 75 % and S^2 with a probability of 25 % has a replacement rate of

$$t_{rep} = \frac{0.75 \times 2718 + 0.25 \times 1104}{0.75 \times 1500(1+0.03)^{39} + 0.25 \times 1000(1+0.02)^{39}} \simeq 56\%.$$

Coverage potential in a repartition & capitalization model

We want to know the sum a that we have to put every year in a saving account to get a desired replacement rate t_{aim}.

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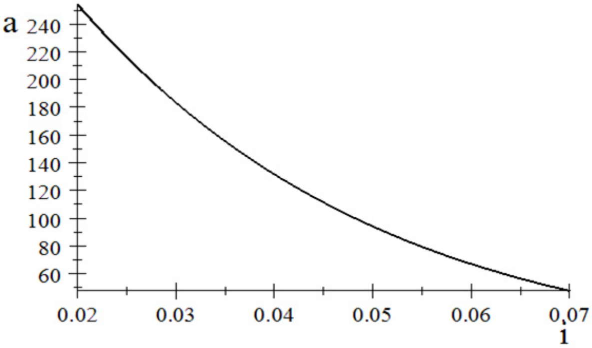

If $i \sim U(2\%; 7\%)$, a varies between 46 euros and 252 euros with a mean of 124 euros.

Capitalization effort coefficient

$\tau_2 =$ Sum of the salaries on the salary trajectory / sum of the investment returns $= 16.5$ on average.

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$$\tau_2 = \frac{S_j}{a_j(i - \lambda_j)} i \frac{(1+i)^T - (1+\lambda_j)^T}{(1+i)^T - 1}.$$

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We need a compromise between a high replacement rate and a small capitalization effort coefficient.

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$$\tau_1 = \frac{\frac{k}{(1+d)^{T+1}} P_{T+1} + \ldots + \frac{k}{(1+d)^{T+T^*}} P_{T+T^*}}{S_0 + \ldots + \frac{S_T}{(1+d)^T}}.$$

τ_1 depends on the demographic rate d.

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τ_1 depends on the demographic rate d. In fact, if $d \sim U(0\%; 5\%)$, τ_1 varies between 6.7 euros and 1.6 euros.

Systemic risk

	Market risk	Demographic risk
Repartition	Negligeable	Extreme
Capitalization	Extreme	Negligeable

Global effort coefficient

$$\tau = x\tau_1 + (1 - x)\tau_2$$

is the number of euros necessary to pay 1 euro for the pension.

Here x euros come from repartition and $1 - x$ euros from capitalization.

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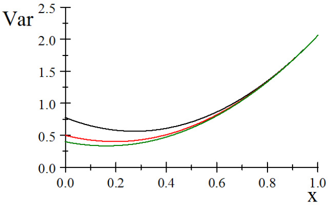
We want to limit the risk of the hybrid system without reducing the pension and in the same time minimize the capitalization effort.



Aim and solution

Aim : volatility of $\tau =$ (volatility of τ_1)$/m$.

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Solution:

$$x = \frac{1}{m^2}.$$