Practical weight-constrained conditioned portfolio optimisation using risk aversion indicator signals

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Outline

1. Context
   - Portfolio optimisation with conditioning information

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Problem context

- Discrete-time optimisation
- Minimise portfolio variance for a given expected portfolio mean
- Postulate that there exists some relationship $\mu(s)$ between a signal s and each asset return r observed at the end of the investment interval:
  $$r_t = \mu(s_{t-1}) + \epsilon_t,$$
  with $E[\epsilon_t | s_{t-1}] = 0$.
- How do we optimally use this information in an otherwise classical (unconditional mean / unconditional variance) portfolio optimisation process?

Problem history

- Hansen and Richard (1983): functional analysis argument suggesting that unconditional moments should enter the optimisation even when conditioning information is known
- Ferson and Siegel (2001): closed-form solution of unconstrained mean-variance problem using unconditional moments
- Chiang (2008): closed-form solutions to the benchmark tracking variant of the Ferson-Siegel problem
- Basu et al. (2006), Luo et al. (2008): empirical studies covering conditioned optima of portfolios of trading strategies
- Boissaux and Schiltz (2010): optimal control formulation of problem that allows for generic numerical solutions

Possible signals

Taken from a continuous scale ranging from purely macroeconomic indices to investor sentiment indicators. Indicators taking into account investor attitude may be based on some model or calculated in an ad-hoc fashion. Examples include

- short-term treasury bill rates (Fama and Schwert 1977);
- CBOE Market Volatility Index (VIX) (Whaley 1993) or its European equivalents (VDAX etc.);
- risk aversion indices using averaging and normalisation (UBS Investor Sentiment Index 2003) or PCA reduction (Coudert and Gex 2007) of several macroeconomic indicators;

Possible signals (2)

- global risk aversion indices (GRAI) (Kumar and Persaud 2004) based on a measure of rank correlation between current returns and previous risks;
- option-based risk aversion indices (Tarashev et al. 2003);
- sentiment indicators directly obtained from surveys (e.g. University of Michigan Consumer Sentiment Index)

Outline

Aim

- Carry out backtests executing constrained-weight conditioned optimisation strategies with different settings

Data set

- 11 years of daily data, from January 1999 to February 2010 (2891 samples)
- Risky assets: 10 different EUR-based funds commercialised in Luxembourg chosen across asset categories (equity, fixed income) and across Morningstar style criteria
- Risk-free proxy: EURIBOR with 1 week tenor
- Signals: VDAX, volatility of bond index, PCA-based indices built using both 2 and 4 factors and estimation window sizes of 50, 100 and 200 points, Kumar and Persaud currency-based GRAI obtained using 1 month and 3 month forward rates

Individual backtest

- Rebalance Markowitz-optimal portfolio alongside conditioned optimal portfolio, both with and without the availability of a risk-free proxy asset, over the 11-year period
- Assume lagged relationship $\mu(s)$ between signal and return can be represented by a linear regression
- Use kernel density estimates for signal densities
- Estimate the above using a given rolling window size (15 to 120 points)
- Use direct collocation method for numerical problem solutions

Individual backtest(2)

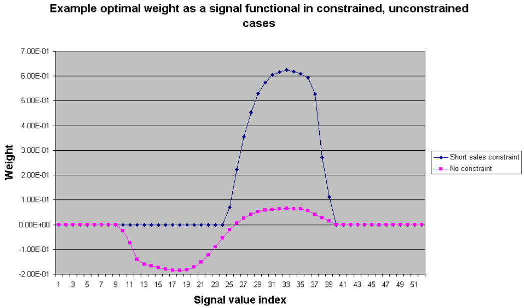
- Obtain efficient frontier for every date and choose portfolio based on quadratic utility functions with risk aversion coefficients between 0 and 10
- Compare Sharpe ratios (ex ante), additive observed returns (ex post), observed standard deviations (ex post), maximum drawdowns / drawdown durations (MD/MDD) of both strategies

Set of backtests

- Decide on one baseline case - VDAX index, 60 point estimation window, weights constrained to allow for long investments only
- Vary these parameters to check both for robustness of strategy results and whether results can be further improved while staying with a linear regression model for the relationship between signal and returns

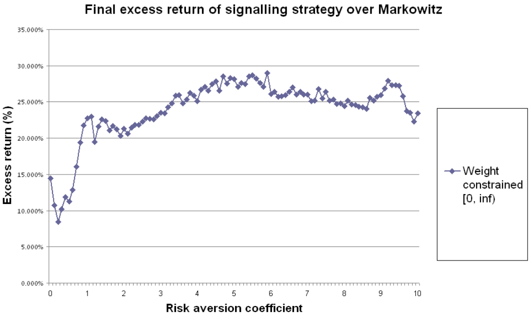
Outline

Typical optimal weight functionals with and without weights constraints



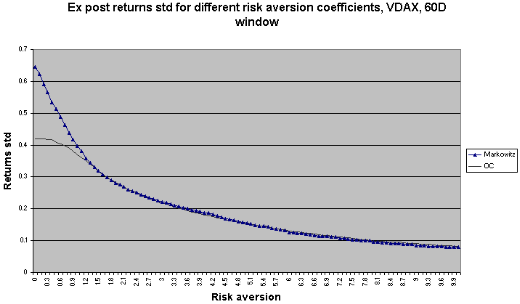
- Constrained optimal weights are not simply a truncated version of the unconstrained optimal (Ferson-Siegel) weights
- Note reduction of leverage for extreme signal values

Base case (with risk free asset): ex post observed relative excess additive returns



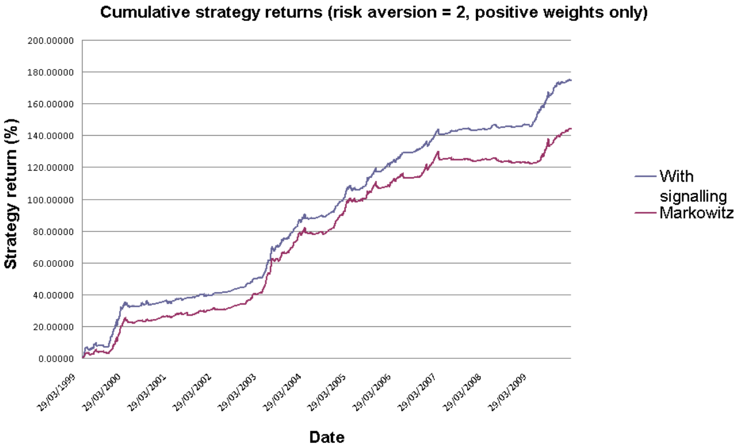
- (Ex ante) average Sharpe ratios: Markowitz - 0.325, using signal - 0.466 (using business daily returns and volatilities)
- General observation: higher risk for Markowitz at low levels of risk aversion, otherwise stable outperformance by conditioned strategies

Base case (with risk free asset): ex post observed standard deviation ratios

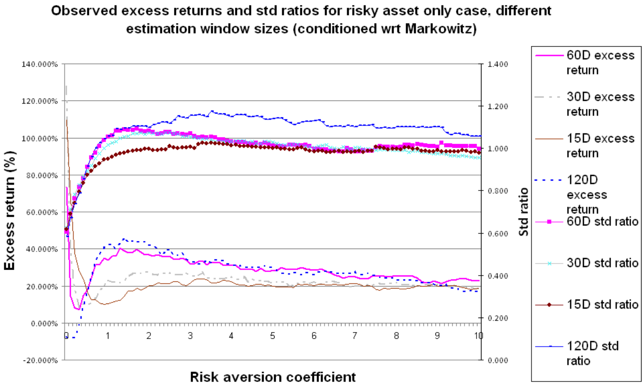


- General observation: Conditioned strategy ex post risk is significantly lower for low levels of risk aversion and slightly higher over the remainder of the range
- Plausible given the typical shape of the conditioned solution

Base case (with risk free asset): Time path of additive strategy returns for $\lambda = 2$

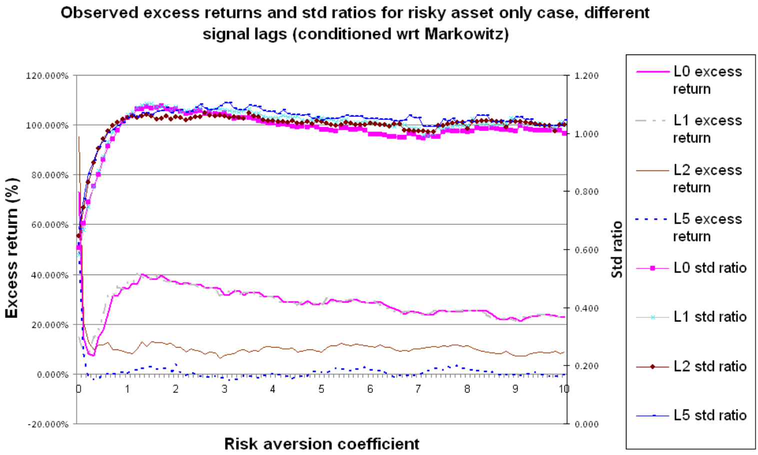


Ex post results for different estimation window sizes



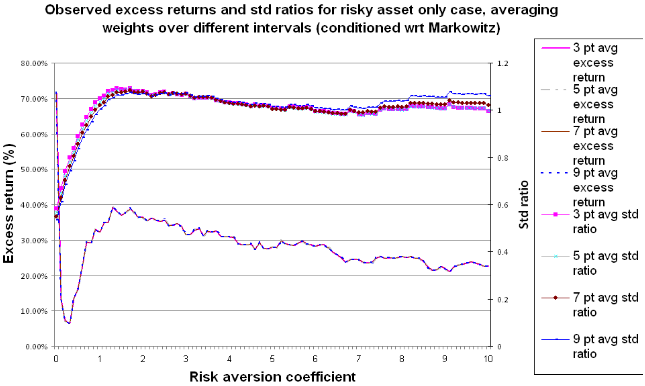
- Excess returns (and standard deviations) larger as window sizes increase
- Tradeoff between statistical quality of estimates and impact of nonstationarity

Ex post results for different signal lags



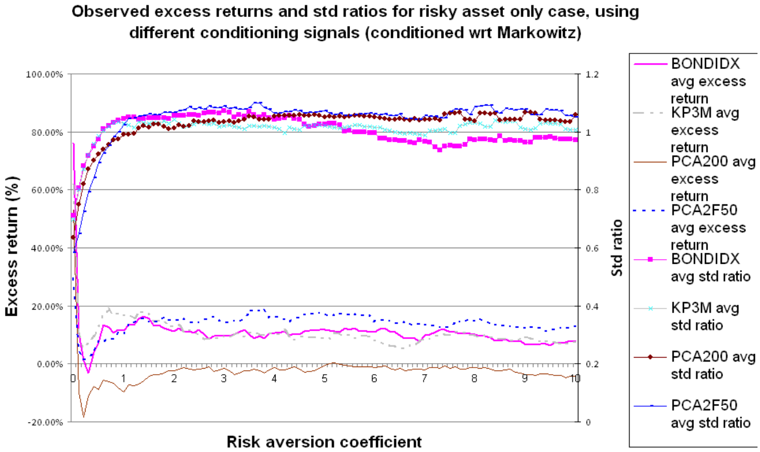
- Signal adds value for at least two additional lags

Ex post results for weight averages over different numbers of signal points



- Negligible changes in excess returns, slight chages in standard deviations: little risk attached to signal observations

Ex post results for different signals



- Best results seen for baseline VDAX signal, averaging seems to detract from signal power

Summary

- Backtesting using a number of different settings shows robust outperformance of the Markowitz strategy given a useful signal is exploited
- In any case,the present strategy shares the characteristics of the Markowitz approach and, as such, the consistently observed improvements reported make it interesting to practitioners investing within the mean-variance framework
- The gap between ex ante and (less good) ex post results as well as the results for lagged signals suggest that use of a signal-return relationship model that captures both autocorrelation and heteroscedasticity is likely to lead to further improvements