Students' Social Origins and Targeted Grade Inflation*

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Abstract

Grade inflation or soft grading is a common feature of the educational systems of many countries. In this paper I analyse grade inflation in a setting where students differ in social background, a firm decides its hiring strategy and the school's grading policy can be targeted according to student type. A targeted grade inflation may exacerbate the job opportunities of disadvantaged students compared to advantaged students. This result emerges since the school has an incentive in inflating grades for a larger proportion of students coming from an advantaged social background.

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- Praise, like gold and diamonds, owes its value only to its scarcity -
Samuel Johnson, 1751

1 Introduction

Grade inflation arises when teachers award students with higher grades than they deserve, leading to a higher concentration of students with top grades.[1] The presence of grade inflation makes it increasingly difficult to distinguish a student's ability, both in evaluating university applications and in job recruitment, and brings about potential distortions. Nowadays, the presence of grade inflation is a common feature in several educational systems. In the United States for example, the evidence of grade inflation has been recently documented by Rojstaczer and Healy (2011), who collected historical data on letter grades awarded by more than 200 four-year colleges and universities. Their results show the drastic rise in the share of A grades awarded over the years.[2] In Canada, Allahar and Côté (2007) show that the 52.6 % of high-school graduates applying to universities in Ontario in 1995 had an A average, and then this rose to 61% in 2004. Also in Ontario in 1995, the 9.4% of high school graduates reported an A+ average and it increased to 14.9% in 2003. In addition, the average grade of university applicants was 80% in 1997, and this percentage has steadily increased since then. In the United Kingdom, graduates who obtain a first-class honours rose from 7.7% of total graduates in year 1996/97 to 14% in year 2008/09. For graduates with an upper-second class honour, the percentage rose from 41.1% of total

[1] Grade inflation literally refers to an increase of average grade of students over time. However, in the related theoretical literature, grade inflation refers to the fact a student obtains a grade that over-estimate her own ability. A better definition for this phenomenon could be "soft grading". In order to be in line with the literature, along the paper I will use the expression "grade inflation", but having in mind "soft grading".

[2] In earlier contributions, Rosovsky and Hartley (2002) and Johnson (2003) survey the empirical literature on grade inflation in the U.S. The emergence of grade inflation can also be observed in Figlio and Lucas (2004), who analyse the impact of standard grades in educational achievement in the Alachua County, Florida.

graduates in year 1996/97 to 48% in year 2008/09 (Higher Education Statistic Agency). At the secondary school level (GCSE), Lodgen *et al.* (2009) compared the results of 3000 fourteen-year-olds sitting a mathematics paper containing questions identical to one set in 1976. He found similar overall levels of attainment between the two cohorts. Thus teenagers' maths skills are no better than 30 years ago, despite soaring GCSE passes. In Italy, the analysis "Stella" reports one third of graduates achieved the highest grade (110/110) in 2004 and 2005 (Modica, 2008). The established presence of grade inflation across countries requires the attention of policy makers. A theoretical understanding of its consequences becomes necessary in order to design an adequate policy intervention.

In this paper I examine the effects of grade inflation when this can be targeted according to a student's social background. Recent evidence shows that students differing in social origins are graded differently. Prenzel *et al.* (2005), Kiss (2010), and Lüdemann and Schwerdt (2010) show that immigrant or lower class children in Germany receive lower grades, and this results holds even if one controls for the individual student's intelligence. Burgess and Greaves (2009), using data from British examination records, compare grades obtained by students in central examinations with teacher assessments of student abilities. They show that some ethnic minorities of students are more likely to be underassessed by the teacher compared to their performance in the central exam than white students. Wikström and Wikström (2005) find that private universities in Sweden, attended mainly by privileged students, are more lenient in their grading policy.[3] All this evidence shows that the grades of students with an advantaged social background are more likely to be inflated compared to students coming from a disadvantaged social background. I will refer to the grade inflation that can be given differently to students with different social origins as "targeted"

[3] An opposite effect is found by Himmler and Schwager (2012), who exploit Dutch school data, finding that schools with a higher proportion of disadvantaged students tend to set lower standards.

grade inflation.

I consider a signalling game in a static setting, with one firm and one school as players, where students differ both in ability (high or low) and social background (advantaged or disadvantaged). In the model, a student's social background represents her family environment, household income, neighbourhood, peer effects and ethnic origins. I assume that students with an advantaged social background are more likely to have high ability, this due to the influences of a more adequate environment to develop skills and by a stronger parental and social pressure about life achievement. The school aims to maximise the job opportunities of its students and appoint them a grade in a school-leaving exam. In the model, a grade is inflated if the school gives a high grade to a low-ability student. The assumption of targeted grade inflation implies that the school can differentiate grading according to a student's social background, so that, for instance, it may inflate, with a certain proportion, the grades of advantaged students and with a different proportion the grades of disadvantaged students. After attending school, students apply for a position at the firm. The firm observes the students' grades and social backgrounds, the grading policy adopted by the school, and wants to hire only high-ability students. Thus the firm forms a belief on a student's ability according to distribution of ability in the advantaged and disadvantaged population. In this sense, the model can be considered of "statistical discrimination", according to which an employer prefers a worker's type since is believed as more productive.[4]

In the analysis, I show the effects of targeted grade inflation by comparing this policy with the alternative case in which the school cannot differentiate the grading according to students social origins ("untargeted" grade inflation) The results suggest that optimally targeted grade inflation may exacerbate

[4]See Arrow (1973), in which discrimination emerges by the relationship between the employers' belief of a worker's productivity and his or her actual productivity. Development of this strand are Coate and Loury (1993), Moro and Norman (2003, 2004), and Norman (2003, *inter alia*).

the differences in job opportunities between advantaged and disadvantaged students, as more advantaged students have inflated grades than in the untargeted case. Intuitively, the fact that advantaged students are more likely to have high ability provides an incentive for the school to inflate grades for a greater proportion of low-ability students with advantaged background. This result is in line with the empirical evidence showing that students coming from an advantaged background are more likely to obtain high grades, irrespective of their ability (Prenzel *et al.*, 2005, Kiss, 2010, Lüdemann and Schwerdt, 2010, Burgess and Greaves, 2009, and Wikström and Wikström, 2005). The results are also consistent with the empirical evidence according to which social background influences students' job opportunities. For instance, Glyn and Salverda (2000) and Berthoud and Blekesaune (2006) show that a disadvantaged social background negatively affects the chance of finding a job in OECD countries and the United Kingdom, respectively.

The remainder of the paper is organised as follows. Section 2 briefly surveys some of the related literature. The model is presented in Section 3, and Section 4 examines the main results. Section 5 extends the analysis to the case with school competition, with Section 6 providing concluding remarks.

2 Related literature

The economic literature has only recently taken on interest in grade inflation, with some noteworthy contributions. Yang and Yip (2003) present a model where universities have an incentive to inflate grades and they mutually reinforce each other's practise, thus determining a competitive effect in grade inflation. This is due to the fact that each university does not consider the collective reputation of graduates, but is willing to help some of its own low-ability students by inflating their grades, which leads to a free-riding problem. Popov and Bernhardt (2013) develop a similar model to Yang and

Yip (2003) to identify the increase over time in the quantity of good jobs as a driving force of grade inflation. They also extend the analysis by considering students with varying social skills.

Chan *et al.* (2007) develop a signalling model where firms observe the students' grade but are not aware of their ability and the proportion of talented ones in the population of students. This gives rise to an incentive to help some low ability students by giving them good grades. They also show that when the average quality of students among schools is correlated, soft grades are strategic complements, and thus inflating schools mutually reinforce each other's practices. Ehlers and Schwager (2012) modifies the analysis of Chan *et al.* (2007) by introducing a reputational element. They add a second cohort of graduates that arrives on the labor market when the first cohort has already revealed their true ability and therefore the school's grading policy. The reputation effect may shrink the level of grade inflation.

Bar *et al.* (2012) examine the recent policy of "putting grades in context", according to which American colleges can reveal the distribution of grades in different disciplines to employers, in order to prevent the distortion in information caused by grade inflation. Accordingly, they propose a framework where students can choose different courses and the university can vary grading standards according to the course. They show that, when information on grading policies is provided only to students, some of them become more attracted to leniently graded courses. On the other hand, if the information is provided to both students and employers, some students choose the strictly graded courses and some choose the leniently graded courses.

The main difference between these papers and my analysis is that the differences in students' social background and grade inflation is targeted according to a student type.[5] My analysis is mostly related to Schwager (2012),

[5]The paper is also related to the theoretical literature on educational standards, which examines the criteria adopted by schools in evaluating students. Costrell (1994) considers a policymaker who maximises social welfare under the assumption that utility-maximising students choose whether to meet the standard, thus leading to the fact that earnings

who develops a labour matching model with grade inflation and student differing in social background. In his paper, students are matched with firms offering different kinds of jobs, according to the grade and the expected ability. Regardless of social background, it is possible that mediocre students receive a high grade caused by grade inflation. Also, the high-ability students from advantaged backgrounds may benefit from grade inflation since this shields them from the competition on the part of able and disadvantaged students. Compared to this analysis, I share the same assumptions on the distributions of ability with differing social backgrounds, but Schwager (2012) (i) focuses on the matching between workers and firms, whereas I do not consider the matching in the labour market, and (ii) assumes the same degree of grade inflation along different social class, whereas I assume that the school may target its grading policy. More importantly, in Schwager (2012) grade inflation is a parameter, while in the present paper grade inflation is endogenously determined. Given the different framework, in my results disadvantaged students may in fact benefit from the presence of grade inflation.

are an endogenous function of educational achievement. The welfare analysis shows that more egalitarian policymakers set lower standards. Betts (1998) instead argues that an egalitarian policy maker might prefer higher standards than would a policy maker whose goal was to maximize the sum of earnings. The result is based on the assumption of heterogeneous ability among workers. As a consequence, a rise in educational standards will increase the earnings of both the most-able and the least-able workers. The only workers whose earnings fall are those workers who after the increase fail to continue meeting the standard. Himmler and Schwager (2012) extend the Costrell (1994)'s analysis by assuming that, in addition to the standard, also the social origin affects the wage earned by graduates. For a given standard, students from disadvantaged backgrounds obtain a lower wage than students from other social classes. Schools with a disadvantaged student body set lower standards than other schools, even if the abilities of the disadvantaged students are identical to those of others. Standards are inflated in this way because the wage discount experienced by graduates from unfavourable backgrounds depresses the return to learning effort for these students. They are thus less willing to satisfy any given standard than students from an average social background.

3 The framework

For simplicity, I abstract from student effort and from competition across schools, and I focus on the interplay between one school that grades students and one firm interested in hiring students after school.[6] All students attend school and afterward apply for a job in the firm.

3.1 Students

I study an economy with a continuum of students, with measure normalised to one. Students can have high (H) or low (L) ability and an advantaged (a) or disadvantaged (d) social background. Social background is public information, and can be seen as a bivariate measure of family environment, income, neighbourhood, peer effects, ethnic origins and so forth.[7] The public knowledge of social background seems plausible: in the real world, school teachers and personnel managers can tell the student/job candidate's social background through some information such as ethnic origins, name, address, language style, manners, clothing, and so on.

I denote as $\eta \in (0, 1)$ the proportion of advantaged students, and $p_a, p_d \in (0, 1)$ as the probability that an a or d student has high-ability, respectively. I assume $p_a > p_d$, that is students with advantaged social background are more likely to have high ability. This assumption can be justified as follows. Given the same distribution of innate ability within a population with differing social backgrounds, an advantaged environment can foster development via parental and peer pressure so that, on average, the overall "ability" is likely

[6] In Section 5 I illustrate how the baseline model can be developed by introducing competition between schools.

[7] Peer effects arise if students learn better in a group of more able students. Relevant empirical studies are, *inter alia*, Summers and Wolfe (1977), Henderson et al. (1978), Epple et al. (2003) and Zimmer and Toma (2000). From a theoretical point of view, Arnott and Rowse (1987), de Bartolome (1990) and Epple and Romano (1998) consider explicitly the peer group effect. In the present analysis the presence of peer-effects is considered within the background.

to be higher for students with an advantaged background. The assumption is in line with past research documenting that family and environmental factors are major predictors of the individuals' ability (Cunha *et al.*, 2006, Carneiro and Heckman, 2003, Joshi and McCulloc, 2000).

3.2 School

The school prepares students for a final exam, with equal teaching effort irrespective of the student type, and learns the student's ability during the period spent by a student at school, through their tests and assessments results. The final exam can be interpreted either as a grade for a final test or as the average grades among the school examinations. The possible exam outcomes are a high (A) or a low (B) grade.

The school decides which grade to appoint each student type. I define $g_{ji} \in [0,1]$, where $j \in \{H, L\}$, and $i \in \{a, d\}$, as the probability that the school appoints an A grade to a ji student. I refer to "grade inflation" when the school appoints an L student with an A grade. The fact that the school can differentiate its grading according to a student background can be interpreted in several ways. For instance, the school may track students of different social origins, due to a different initial preparation. Another situation in which a targeted grading policy may take place emerges in those school courses in which students' achievement can be assessed very subjectively, for example in oral tests. In this case a grading policy being targeted according to student types is easy to implement.

There is not a standard way of modeling school behaviour. In the economic models of school (or university) competition, the number of enrolled students or the overall amount of tuition fees enter in the school/university objective function (Epple and Romano, 1998, Del Rey, 2001, De Fraja and Iossa, 2002, Brunello and Rocco, 2008, Maldonado, 2008, Ferreyra, 2012, *inter alia*). Other models propose a school objective function determined by the average qualification (De Fraja and Landeras, 2006, De Fraja *et al.*, 2010, Albornoz *et al.*, 2011), the average and the variance of the qualification (Ritzen *et al.*, 1979), the quality of school (Epple *et al.*, 2003 and 2006) and the quality or the attendance in the case of a public school (Epple *et al.*, 2002).[8] Here I assume that a school wants the maximum number of students to be hired, and obtains a benefit for each hired student. I denote as $z_{\xi i} \in [0,1]$, $\xi \in \{A, B\}$ the probability that the firm hires a student according to grade and social background (see the next paragraph).[9] Therefore the school objective function is as follows:

$$\Pi^U = \eta \left[p_a b_{Ha} + (1 - p_a) b_{La} \right] z_{\xi a} + \tag{1}$$

$$(1 - \eta) \left[p_d b_{Hd} + (1 - p_d) b_{Ld} \right] z_{\xi d},$$

where b_{ji} is the benefit that the school obtains from the hiring of a ji student. I make the following assumption.

Assumption 1 $b_{Ha} = b_{Hd} > b_{La} = b_{Ld} > 0.$

In words, the school wants the maximum number of students to be hired and values more the employment of an H student. Accordingly, (i) each student's employment increases the school reputation as an effective institution for obtaining a job and (ii) the school obtains a higher benefit from the hiring of H students which ensures the school's credibility to the firm. It is important to stress that, according to Assumption 1, the school has no preferences whatsoever about a student's social background.[10]

[8] Some recent contributions (Albornoz *et al.*, 2011, Donze and Gunnes, 2011) rely on the "goal theory" (Covington, 2000), according to which achievement goals influence the quality, timing and appropriateness of the students' engagement in their own learning. This effort together with innate ability affect the student's accomplishments. As a consequence, parents and teachers play a key role in influencing the students' achievement goals and, in turn, their effort.

[9] As will be clear shortly, firms cannot observe a student's ability and use the grade as a signal of it.

[10] In Section 4.3 I consider different assumptions about the school's preferences.

3.3 Firm

The firm evaluates to hire some of the students after the school period by observing their final grade and social background. The public knowledge of social background seems plausible: in the real world, a personnel manager can probably tell the job candidate's social background through some information such as ethnic origins, name, address, language style, manners, clothing, and so on. Also, the firm is fully aware of the school's grading policy. The latter assumption reflects the situations in the real world in which either (i) an educational institution claims its own grading policy, (ii) the grading policy is public information due to reputation effects or (iii) some policy intervention induces them to reveal the distribution of grades, like "putting grades in context" in the United States (see Bar *et al.*, 2012). After observing a grade, the firm has a belief, consistent with Bayes' rule, about a student ability, conditional on all the information it has: the student's grade, the distribution of ability according to the student's social background and the school strategy.

The firm hires with probability $z_{\xi i}$ a student with background $i \in \{a, d\}$ who finished school with grade $\xi \in \{A, B\}$ and offers a single job type. Also, I assume that ability of employees determines the firm's profit entirely. In particular, each high- and low-ability employee yields a net profit of $\mu > 0$ and -1, respectively. The assumption of a negative profit by hiring a L student can be interpreted in many ways: low-ability employees may have a marginal productivity which is lower than salary cost. In addition, the firm may want to lay off an unproductive employee but this action still comes at a cost, e.g. industrial disputes, wasted training costs and time, and so on. Given these assumptions, the expected firm's profit for a student ji is given by:

$$\Pi^F = \mu\pi\left(H\left|g_{ji}; p_i\right.\right) - \pi\left(L\left|g_{ji}; p_i\right.\right),$$

where $\pi\left(j\left|g_{ji}; p_i\right.\right)$ is the belief of the firm about the ability of a student with social background i and whose grade depends on the school grading policy, g_{ji}. According to the Bayes rule, the firm's beliefs about a student's ability are defined as follows.

Definition 1 *The firm's beliefs on the students' ability which are consistent with the Bayes' rule are*

$$\begin{aligned}\pi\left(H\left|g_{ji};p_i\right.\right) &= \frac{p_i g_{Hi}}{p_i g_{Hi}+\left(1-p_i\right)g_{Li}}, \\ \pi\left(L\left|g_{ji};p_i\right.\right) &= \frac{\left(1-p_i\right)g_{Li}}{p_i g_{Hi}+\left(1-p_i\right)g_{Li}},\end{aligned} \tag{2}$$

Finally, I define $J \in [0,1)$ as *labour demand.* The fact that J cannot cover all of the students rules out the unrealistic case where the job market is cleared, and has important consequences on the behaviour of the school. As will be clear below, given the limited amount of job placements and Assumption 1, the school will adopt a grading policy such that none of the L students obtain a job at the expenses of an H student. In turn, in equilibrium $z_{\xi i}$ will equal 0 or 1 for all student types.

3.4 The game

Figure 1 summarises the timing of the game. Nature draws the student types, then the school grades the student in the final exam. Finally, all the students apply for a job in the firm, that decides whether to hire or not each job candidate.

Figure 1. The game timing

Stage 1. Nature		**Stage 2. School**		**Stage 3. Firm**
4 student types: H or L with a or d background.	$\rightarrow$	chooses the grade to give to each student type.	$\rightarrow$	chooses whether hiring or not each student.

The equilibrium concept is perfect Bayesian equilibrium, which is a combination of students, school and firm strategies and firm beliefs where all the agents maximise their payoff. For each grade, the firm must maximise its expected profit, given its belief and the school strategy. Labour demand requires that the number of hired students is at most J. In turn the school chooses its grading strategy in order to maximise its expected payoff, given the set of students, the firm's strategy and labour demand J.[11]

4 Results

4.1 The baseline problem

In this section I show the results of the baseline model. First notice that, since a student who scored B has low ability with probability 1, then the firm will never hire one of them, so that $z_{Bi} = 0$ for $i \in \{a, d\}$. Indeed while the school may want to inflate the grade of a low-ability student in order to increase the number of students who obtain a job, it would never appoint a B to a high-ability student. This simplifies the exposition of the results and allows me to focus on A students. By the same token, it is always better for the school to confer a grade A to an H student, as this unambiguously raises the payoff of both the school and the firm. Therefore, in all the possible scenario the probability of an H student to obtain an A is $g_{Hi} = 1$ for $i \in \{a, d\}$.

The firm's expected payoff of hiring an A student with social background i according to the beliefs (2) is

$$\Pi^F(g_{Li}; A, p_i) = \frac{p_i \mu - (1 - p_i) g_{Li}}{p_i + (1 - p_i) g_{Li}}. \tag{3}$$

The firm hires from a population of students if its expected payoff is nonnegative, $\Pi^F(g_{Li}; A, p_i) \geq 0$. Assume for a moment that labour demand is $J = 1$.

[11] Notice that the school has complete information about the firm's behaviour, therefore it is not necessary to determine its beliefs.

For every $i \in \{a, d\}$ and $p_i \geq \frac{1}{1+\mu}$, the firm's expected payoff (3) is positive for $g_{Li} = 1$. For $p_i < \frac{1}{1+\mu}$, the firm's expected payoff (3) is nonnegative for $g_{Li} \leq \frac{p_i \mu}{(1-p_i)}$, so that the school strategy is $g_{Li} = \frac{p_i \mu}{(1-p_i)}$. Hence the threshold point $\frac{1}{1+\mu}$ is a function of the firm's benefit from hiring an H student, μ. The higher the benefit, the higher the firm's expected payoff by hiring an A student. Therefore the firm tends to hire more A students when μ is high, and in turn, the school more likely inflates the students' grades.

Consider now the case in which job positions are limited, $J < 1$. Since the school prefers that an H rather than an L student obtains a job, it will inflate grades at most for the amount of labour demand net to the share of H students, denoted as

$$\Phi \equiv p_a \eta + p_d (1 - \eta), \tag{4}$$

so that the remainder of labour demand is $J - \Phi$. This ensures that none of the L students would obtain a job opportunity at the expenses of an H student. Finally, if the expected profits are positive from hiring both an a and a d students, then the firm can compare the two expected profit:

$$\Pi^F (g_{La}; A, p_a) = \frac{p_a \mu - g_{La} (1 - p_a)}{p_a + g_{La} (1 - p_a)} \lesseqgtr \frac{p_d \mu - g_{Ld} (1 - p_d)}{p_d + g_{Ld} (1 - p_d)} = \Pi^F (g_{Ld}; A, p_d). \tag{5}$$

Given Assumption 1, the school strategy ensures that the maximum number of students would be hired and that all the students who scored A are going to obtain a job. This is due to the following reason. Since the school favours H over L students, it will never choose a strategy such that an L student obtains a job at the expenses of an H student. In other words, grade inflation cannot go against the job opportunities of H students. This may happen for instance if the number of students who scored A is higher than J. In this case some of the H students may not be hired, implying that some of the A students did not obtain a job. Therefore the school's strategy to give

A at most to $J - \Phi$ students ensures that all the A students will receive a job offer. Of course, if the number of open positions is lower than the number of H students, then multiple equilibria arise, as the school would be indifferent to give A to all H students only, or to give A to a number of H students that covers J (irrespective of a student social background).[12] In both cases, not all H students will obtain a job, but only a number J. The discussion can be summarised in the following lemma.

Lemma 1 *For $J \in [\Phi, 1)$, all the students who scored A will obtain a job.*

Lemma 1 implies that the firm's strategy is $z_{Ai} = \min\{J, 1\}$ always. This is due to the fact that the school inflates grades in such a way that the firm's profit is always nonnegative.

A situation where $J < \Phi$ is not very interesting as does not represent what happens in the real world. Thus from now on I will focus on the case in which labour demand is larger than the number of high ability students, according to the following assumption.

Assumption 2 $\Phi < J < 1$.

4.2 Untargeted vs targeted grade inflation

As a benchmark, first I will show the situation in which grade inflation cannot be targeted, then I will compare this with the case with targeted grade inflation. With untargeted grade inflation, the grading policy is the same for both populations. The characteristics of the equilibria can be summarised as follows.

Proposition 1 *Suppose Assumption 1 and 2 hold, and grade inflation is untargeted.*

[12] The last situation may be defined as "grade deflation equilibrium", since not all the H students obtain a high grade.

- ***Pooling equilibrium.*** *For* $p_d \geq \frac{1}{\mu+1}$*, grade inflation is given with the same proportion to a and d students, and the probability of obtaining a job is the same for a and d students.*

- ***Separating equilibrium.*** *For* $p_d < \frac{1}{\mu+1}$*, grade inflation is given in proportion to more a than d students, and the probability of obtaining a job is greater for a rather than d students.*

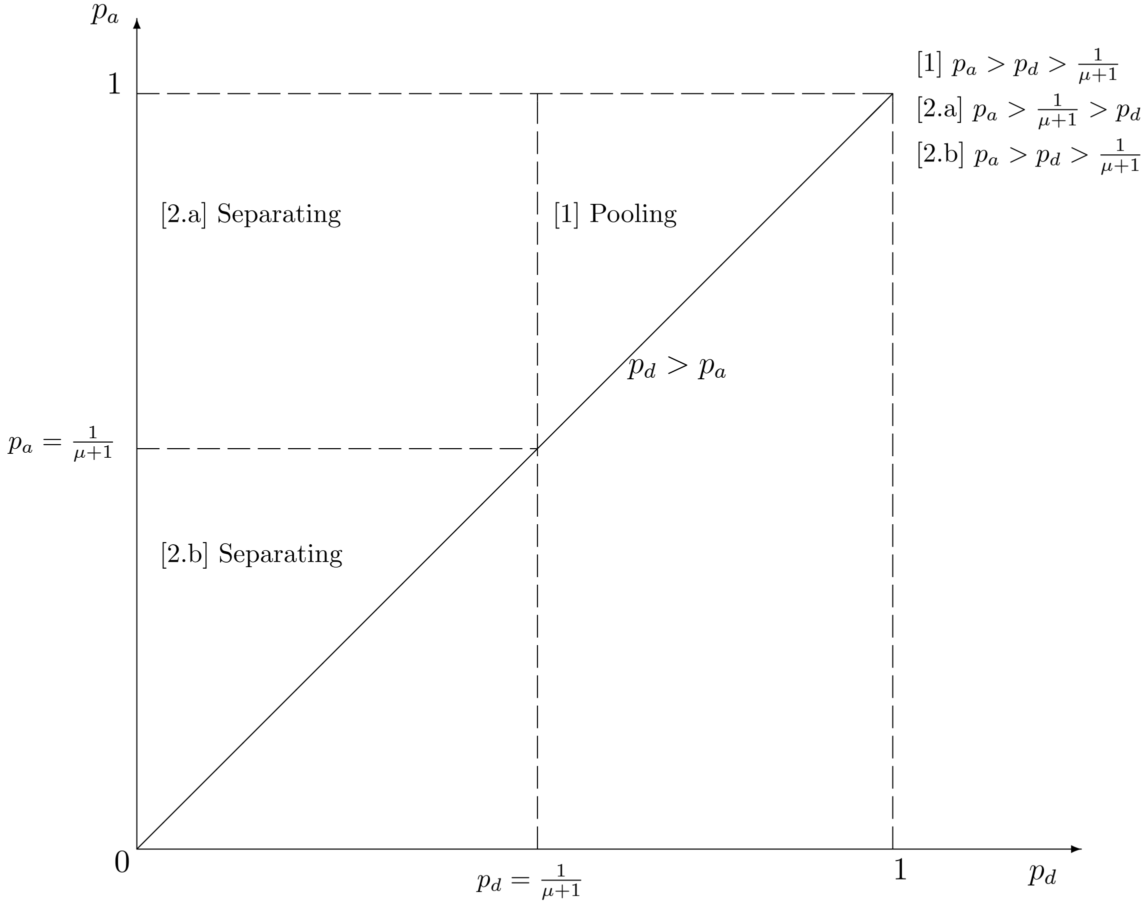
Proof. See the appendix. ■

Figure 2 shows the equilibria in the parameter space (p_d, p_a). If the distribution of ability is very high in both population $\left(p_d > \frac{1}{\mu+1}\right)$, the firm believes that it is very likely that the ability both of advantaged and disadvantaged students is high. The school then inflates their grades as much as it can according to J, in order to maximise the chance of a job for H rather than L students. The result is a pooling equilibrium, in which social background does not matter in order to obtain a job.

Things change when the distribution of ability is low in the disadvantaged population $\left(p_d < \frac{1}{\mu+1}\right)$. Here, there are two subcases, according to which the advantaged population has a high number of H students $\left(p_a \geq \frac{1}{\mu+1} > p_d\right)$ or not $\left(\frac{1}{\mu+1} > p_a > p_d\right)$, and where the results are quantitatively similar. Given the lower expectations about the ability of d students, the school inflates grades to a lower proportion of them. This leads to a separating equilibrium, in which a students are more likely to obtain a job than d students. The results obtained in Proposition 1 are qualitatively similar to Schwager (2012), who considers untargeted grade inflation, differences in students' social background and matching in the labour market.

Suppose now that grade inflation can be targeted. This Assumption is in line with some recent evidence showing different behaviour of teachers according to students social background (Burgess and Greaves, 2009, in the UK, and Prenzel *et al.*, 2005, Kiss, 2010 and Lüdemann and Schwerdt, 2010 in Germany). The following proposition summarises the features of the equilibria.

Figure 2. Equilibria with positive grade inflation



Proposition 2 *Suppose Assumption 1 and 2 hold and the school can target grade inflation:*

- ***Pooling equilibrium.*** *For* $p_d > \frac{1}{\mu+1}$*, the results are the same as in Proposition 1.*

- ***Separating equilibrium.*** *For* $p_d < \frac{1}{\mu+1}$*, the probability of obtaining a job is higher for a than d students. Compared to the targeted case,* *an d student has a relatively lower probability of obtaining a job than an a student.*

Proof. See the appendix. ■

The results are qualitatively similar to the case with untargeted grade inflation, and the intuitive interpretation is much the same. The main difference is due to the fact that, in the separating equilibrium, the job opportunities of a students will increase compared to the untargeted case, with the effect of exacerbating class differences. In other words, a student with disadvantaged background has relatively less job opportunities compared to an advantaged student than in the untargeted case. Indeed, it is optimal for the school to inflate grades for a higher proportion of a than d students, given their difference in the ability distribution. The result is consistent with the evidence showing that students coming from a disadvantaged background have less job opportunities in the job market (Glyn and Salverda, 2000, and Berthoud and Blekesaune, 2006, *inter alia*).

4.3 Different school objectives

So far, the analysis has been carried out with the assumption that the school did not have any redistributive aim whatsoever in the students' job opportunities. Nonetheless, it is noteworthy to consider whether these results are robust when the school has different objectives. In what follows I analyse a school with preferences over social background. In the two scenarios considered, the school still favours H over L students to keep a good reputation towards the firm, hence Lemma 1 still holds.

First I examine a situation in which the school is interested in helping the job opportunities of disadvantaged students. I will call it a "redistributive" school. Accordingly, I assume:

Assumption 3 $b_{Hd} > b_{Ha} > b_{Ld} > b_{La}$.

In the attempt to increase the job opportunities of disadvantaged students, the redistributive school can design its grading policy by keeping a sufficiently high grade inflation for a students that makes d students to be preferred in the job market, i.e., hiring a d students ensures a higher expected payoff. However, a lower grade inflation implies that a lower proportion of d students receive an A and in turn a job. Therefore the results obtained in Proposition 2 hold even when a school has redistributive intentions. This discussion is summarised in the following proposition.

Proposition 3 *Suppose Assumption 2 and 3 hold. Then the results are the same as in the case in which the school has not redistributive intentions (Proposition 2).*

Proof. See the appendix. ■

Consider next a school (an "elitist" school) willing to favour advantaged rather than disadvantaged students. A school may prefer hiring advantaged students for pecuniary reasons. For instance it may happen in those countries, like the United States, in which wealthy alumni would give donations to their college. Indeed evidence shows that students who receive loans or financial aid are less likely in the future to make a gift in the future (Meer and Rosen, 2011). The behaviour of an elitist school can be implemented as follows:

Assumption 4 $b_{Ha} > b_{Hd} > b_{La} > b_{Ld}$.

Even in this case the results do not change compared to the case with no redistributive aims. The reason is the following. In order to favour advantaged students, the elitist school may grade in such a way to keep a sufficiently high grade inflation for d students that makes a students be preferred in the job market, i.e., hiring a a students ensures a higher expected payoff. However, inflating less the grades of a students (i) reduces the proportion of a students who obtain an A, and (ii) it is not really necessary given the assumption $p_a > p_d$.

Proposition 4 *Suppose Assumption 2 and 4 hold. Then the results do not change compared to the case in which the school has no preferences for a student's social background (Proposition 2).*

Proof. See the appendix. ■

The results of Proposition 3 and 4 are due to the fact that the school is limited in their use of grade inflation by the risk of damaging the H students of the less favoured population.

5 School competition

In this section, I consider the effects of competition among schools on my findings about grade inflation. In this setting, I assume that students can decide in which school to be enrolled, and they would prefer to attend the school that increases their chance of obtaining a job. I show that the essential results carry over to settings with competition.

5.1 Symmetric oligopoly

First assume a competitive school market in which $n \geq 2$ schools operate with no entry of other competitors. A school has a payoff according to Assumption 1, i.e., it prefers that an H rather than an L student obtains a job, and it is indifferent to student social background. In a competitive setting, Assumption 1 implies that a school prefers to enroll H rather than L students, as they will give to the school a higher payoff in the case they are employed.

Each school can admit a number of students of at most $h_u \in (0, 1)$, identical for each school, i.e.

Assumption 5 $\sum_u h_u = 1$, $h_u = h_s > 0$, *for all* $u, s = 1, 2, .., n$, $u \neq s$.

This is an important assumption, since a school grading strategy cannot increase its own capacity but only helps to fulfill the maximum number of admitted students. In other words, the maximum supply of enrollment is fixed. Each school decides whether or not to inflate grades and, if so, the amount of grade inflation to provide.

Begin with the case in which labour demand is lower than the number of H students, $J < \Phi$. Given the limited number of expected positions and the fact that a school prefers to enroll H rather than L students, the school will favour the former over the latter by not inflating grades.

Lemma 2 *Suppose Assumption 1 and 5 hold and $J \leq \Phi$. Then each school never inflates grades and the firm is indifferent to a student's social background.*

Consider next $J > \Phi$. In this case a school may consider inflating grades in order to increase the number of its students by enrolling some of the L students. There are two possible cases. In the first, assume $\Phi < h_u$. If all but one schools inflate grades, then the non-inflating school will attract all the H students, and the firm would hire those students only. If $n-1$ schools do not inflate grades while one school deviates, then all H students would attend one of the $n-1$ schools, in which there is always room for them since $\Phi < h_u$, so that the payoff of the school that inflates grades will be zero. Therefore even in this case, none of the school inflates grades.

Lemma 3 *Suppose Assumption 1 holds and $\Phi < h_u$. Then each school never inflates grades and the firm is indifferent to a student's social background.*

In the second case, assume $h_u < \Phi < J$. If $n-1$ schools inflate grades and one school deviates, it will attract the H students according to its own size h_u, and since a school prefers to enroll H over L students, it will increase its own payoff. Hence even in this case a situation in which all the schools inflate grades is never an equilibrium.

Similarly, if $n-1$ schools do not inflate grades and a school deviates, but $\Phi \leq (n-1)\, h_u$, then all the H students find a placement in the other schools, so that the inflating-grades school obtains a zero payoff. Therefore this is not an equilibrium, since none of the schools are willing to inflate grades.

Lemma 4 *Suppose Assumption 1 holds and $h_u < \Phi \leq (n-1)\, h_u$. Then each school never inflates grades and the firm is indifferent to a student's social background.*

On the other hand, if some of these H students cannot find a placement in the other schools, i.e., $\Phi > (n-1)\, h_u$, they attend the school who inflated grades. Indeed, an H student will score A with certainty for Lemma 1, since even with competition, a school who inflates grades still will do that in such a way to favour her over an L student. In this case, the school compares the payoff of deviation with the payoff of no grade inflation:

$$\frac{\Phi}{n} b_H > \left[\Phi - (n-1)\, h_u\right] b_H + \left[g_{La}\eta\,(1-p_a) + g_{Ld}\,(1-\eta)\,(1-p_d)\right] b_L, \quad (6)$$

so that two situations may occur:

Proposition 5 *Suppose Assumption 1 holds and $(n-1)\, h_u < \Phi < J$.*

1. *If inequality (6) holds, in equilibrium all schools will not inflate grades;*
2. *If inequality (6) does not hold, in equilibrium one school inflates grades and the $n-1$ schools will not inflate grades.*

Note that, in point 2 of Proposition 5, the deviation strategy would increase the payoff of all the other non-inflating-grades schools, since they would fill their students' capacity with H students. Therefore none of the other schools has an incentive in deviating, which ensures stability to the equilibrium. More important, for the case analysed in point 2 of Proposition 5, the equilibria depicted above in the analysis with no school competition hold for the population of students net of the H students attending the school that gives grade inflation. Hence the results analysed in the non-competitive case may apply as a benchmark for some market with symmetric competition.

5.2 Asymmetric duopoly

The result depicted in the previous section strictly depends on the assumption of equal size of all schools. To understand why, consider this second competitive model. Suppose for simplicity a school duopoly in which one school (denoted by 1) is larger and the number of H students is higher than the size of the smaller school (denoted by 2), i.e.:

Assumption 6 $h_1 > h_2 > 0$, $h_1 + h_2 = 1$.

In cases $J < \Phi$, $J > \Phi > h_1$, and $J > h_2 > \Phi$, both schools never inflate grades and the firm is indifferent to a student's social background. The argumentations of these results are the same described in the symmetric market.

Consider now $J > h_1 > \Phi > h_2$. If school 1 does not inflate grades, the school 2's dominant strategy is not to inflate grades. Indeed school 1 may enroll all the H students and thus leaving school 2 with L students only. However, if school 2 does not inflate grades, then school 1 compares the payoff of inflating grades with the payoff of no grade inflation, i.e., inequality (6) when $n = 2$:

$$\frac{\Phi}{2} b_H > (\Phi - h_2)\, b_H + \left[g_{La}\eta\,(1 - p_a) + g_{Ld}\,(1 - \eta)\,(1 - p_d)\right] b_L. \tag{7}$$

Again, two equilibria may occur:

Proposition 6 *Suppose Assumption 1 and 6 hold and $J > h_1 > \Phi > h_2$.*

1. *If inequality (7) holds, in equilibrium both schools will not inflate grades;*

2. *If inequality (7) does not hold, in equilibrium school* 1 *inflates grades and the school* 2 *does not inflate grades.*

In particular for the case illustrated in point 2 of Proposition 6, the equilibria depicted above in the analysis with no school competition hold for the population of students net of the H students attending school 2. Similarly to point 2 of Proposition (5), the results analysed in the non-competitive case may apply as a benchmark in some market with asymmetric school competition.

6 Concluding remarks

This paper has examined the effects of grade inflation when it can be targeted according to students' social background. In line with the empirical evidence, I have shown that grade inflation is given to a higher proportion of advantaged students. This exacerbates the job opportunities of disadvantaged students compared to advantaged students. I assume that the school has not preferences on a student social background, but the results hold also by relaxing this assumption. Also, the analysis is robust to some extent in markets with school competition.

A possible development may be to evaluate the relationship between grade inflation and school reputation (see Ehlers and Schwager, 2012) when students differ in social background. In this direction, it would be necessary to modify the framework in a either repeated or dynamic setting, and to relax the assumption of perfect information about the school strategy. An interesting follow up paper could analyse how grade inflation affects students' effort incentives. For instance, does more grade inflation create perverse incentives for the students to put forth effort? The relation between grade inflation and students' effort is left for future work.

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Appendix

Proof of Proposition 1

Consider $J = 1$. For $p_a > p_d > \frac{1}{\mu+1}$, the equilibrium would be

$$g_{La} = g_{Ld} = 1, \tag{8}$$

thus all the students obtain a job. Plugging (8) into (5), hiring an a student always gives a higher expected profit. When $p_a\eta + p_d(1-\eta) < J < 1$, this strategy may lead L students to obtain a job at the expense of H students. Therefore the school inflates grades to a number of students that covers labour demand net to the share of H, i.e.:

$$g_{La} = g_{Ld} = \frac{J - \Phi}{\eta(1-p_a) + (1-p_d)(1-\eta)}, \tag{9}$$

so that an L and a student has the same chance of obtaining a job than an L and d student. Plugging (9) into (5), hiring an a student gives a higher expected payoff always.

Consider next $p_a > \frac{1}{\mu+1} > p_d$ and $J = 1$. The optimal school strategy is

$$g_{La} = 1,\ g_{Ld} = \frac{p_d\mu}{1-p_d}. \tag{10}$$

This strategy cannot be implemented with the assumption of untargeted grade inflation, as in equilibrium the proportion of grade inflation differ across students populations. In this case, the best grading policy is determined according to the distribution of ability in the d population. Otherwise, if the university applies the optimal level of grade inflation for a students, none of the d students would be hired, as the firm's expected profit from hiring them would be negative, given the excessive grade inflation. Thus by adopting this policy, a and L students would obtain a job over d and H students, going against Assumption 1. Thus the school strategy is

$$g_{La} = g_{Ld} = \frac{p_d \mu}{1 - p_d}. \tag{11}$$

When $\Phi < J < 1$, this strategy may lead L students to obtain a job at the expense of H students. Thus the school provides grade inflation for the proportion of the expected available positions for low ability students, i.e.:

$$\frac{J - \Phi}{(1 - p_a)\,\eta + (1 - p_d)\,(1 - \eta)}. \tag{12}$$

The strategy in equilibrium is

$$g_{La} = g_{Ld} = \frac{p_d \mu}{1 - p_d} \frac{J - \Phi}{(1 - p_a)\,\eta + (1 - p_d)\,(1 - \eta)}. \tag{13}$$

Consider next $\frac{1}{\mu+1} > p_a > p_d$ and $J = 1$. The optimal school strategy is

$$g_{La} = \frac{p_a \mu}{1 - p_a},\ g_{Ld} = \frac{p_d \mu}{1 - p_d}. \tag{14}$$

As in the previous case, the school strategy is based on the distribution of ability in the d population, hence its strategy is

$$g_{La} = g_{Ld} = \frac{p_d \mu}{1 - p_d}. \tag{15}$$

When $\Phi < J < 1$, the school strategy becomes:

$$g_{La} = g_{Ld} = \frac{p_d \mu}{1 - p_d} \frac{J - \Phi}{(1 - p_a)\,\eta + (1 - p_d)\,(1 - \eta)}. \tag{16}$$

Plugging (16) into (5), hiring an a student gives a higher expected profit always.

Proof of Proposition 2

Consider $J = 1$. For $p_a > p_d > \frac{1}{\mu+1}$, the equilibrium would be the same as in Proposition 1. Indeed in this case, it is optimal to provide the same proportion of grade inflation to each population, even if the school may target it differently. For $p_a > \frac{1}{\mu+1} > p_d$ and $J = 1$, the school strategy is

$$g_{La} = 1, \; g_{Ld} = \frac{p_d \mu}{1 - p_d}. \tag{17}$$

When $\Phi < J < 1$, this strategy may lead L students to obtain a job at the expense of H students. To avoid this, the school provides grade inflation for the proportion of the expected available positions for low ability students. Thus the result would be

$$\begin{aligned} g_{La} &= \frac{J - \Phi}{(1 - p_a)\,\eta + (1 - p_d)\,(1 - \eta)}, \\ g_{Ld} &= \frac{p_d \mu}{1 - p_d} \frac{J - \Phi}{(1 - p_a)\,\eta + (1 - p_d)\,(1 - \eta)}. \end{aligned} \tag{18}$$

Then the firm compares the expected profits from hiring a d or an a student. Plugging (17) into (5) yields:

$$\frac{p_a \mu - (1 - p_a)}{p_a + (1 - p_a)} > 0. \tag{19}$$

For $\frac{1}{\mu+1} > p_a > p_d$ and $J = 1$, the school strategy is

$$g_{La} = \frac{p_a \mu}{1 - p_a}, \; g_{Ld} = \frac{p_d \mu}{1 - p_d}. \tag{20}$$

For $\Phi < J < 1$, this strategy may lead L students to obtain a job at the expense of H students. Hence the school provides grade inflation in the proportion of the available position for low ability students, i.e.:

$$\frac{J-\Phi}{(1-p_a)\,\eta+(1-p_d)\,(1-\eta)}.$$

Thus strategy in equilibrium is

$$\begin{aligned} g_{La} &= \frac{p_a\mu}{1-p_a}\frac{J-\Phi}{(1-p_a)\,\eta+(1-p_d)\,(1-\eta)}, \\ g_{Ld} &= \frac{p_d\mu}{1-p_d}\frac{J-\Phi}{(1-p_a)\,\eta+(1-p_d)\,(1-\eta)}. \end{aligned} \tag{21}$$

Finally, the firm compares the expected profits from hiring a d or an a student. Plugging (20) into (5) yields:

$$\Pi^F\left(g_{Ld};A,p_d\right)=\Pi^F\left(g_{La};A,p_a\right)=0. \tag{22}$$

Proof of Proposition 3

Consider first $J=1$. For $p_a>p_d\geq\frac{1}{\mu+1}$, the school may try to help d students by fully inflating grades to a students ($g_{La}=1$) and keeping the level of grade inflation of d students in such a way than the firm's expected profit is higher by hiring one of them:

$$\frac{p_d\mu-g_{Ld}\,(1-p_d)}{p_d+g_{Ld}\,(1-p_d)}\geq p_a\mu-(1-p_a)\,. \tag{23}$$

Solving for g_{Ld} yields

$$g_{Ld}=\frac{p_d\,(1-p_a)}{p_a\,(1-p_d)}<1. \tag{24}$$

However, given the distribution of ability in the disadvantaged population, the firm would hire all the A and d students, irrespective of the school strategy. Hence the school best strategy is $g_{Ld}=1$, since this raises the number of d students who obtain a job. For $p_a>\frac{1}{\mu+1}>p_d$, the firm's expected profit is still higher by hiring a d student for $g_{Ld} = \frac{p_d(1-p_a)}{p_a(1-p_d)}$. However, comparing this level with the optimal level in Proposition 2, it emerges that

$$\frac{p_d\,(1-p_a)}{p_a\,(1-p_d)} \leq \frac{p_d\mu}{1-p_d}, \text{ for } p_a \geq \frac{1}{\mu+1} \tag{25}$$

Therefore it is optimal for the school to adopt $g_{Ld} = \dfrac{p_d\mu}{1-p_d}$. For $\frac{1}{\mu+1} > p_a > p_d$, the firm's expected profit is higher by hiring a d student for:

$$\frac{p_d\mu - g_{Ld}\,(1-p_d)}{p_d + g_{Ld}\,(1-p_d)} \geq \frac{p_a\mu - g_{La}\,(1-p_a)}{p_a + g_{La}\,(1-p_a)}. \tag{26}$$

Solving for g_{Ld} yields

$$g_{Ld} = \frac{g_{La}p_d\,(1-p_a)}{p_a\,(1-p_d)}. \tag{27}$$

Substituting $g_{La} = \frac{p_a\mu}{1-p_a}$ (the optimal level of grade inflation for a students) yields $g_{Ld} = \frac{p_d\mu}{1-p_d}$. The results for $\Phi < J < 1$ follow as in Proposition 2.

Proof of Proposition 4

Consider first $J = 1$. For $p_a > p_d \geq \frac{1}{\mu+1}$, the school may try to help a students by fully inflating grades to d students ($g_{Ld} = 1$) and keeping the level of grade inflation of a students in such a way than the firm's expected profit is higher by hiring one of them:

$$\frac{p_a\mu - g_{La}\,(1-p_a)}{p_a + g_{La}\,(1-p_a)} \geq p_d\mu - (1-p_d)\,. \tag{28}$$

Solving for g_{La} yields

$$g_{La} = \frac{p_a\,(1-p_d)}{p_d\,(1-p_a)} > 1. \tag{29}$$

Hence in the case the school best strategy is $g_{La} = 1$. For $p_a > \frac{1}{\mu+1} > p_d$, the school may try to help a students by keeping the level of grade inflation of a students in such a way that the firm's expected profit is higher by hiring one of them:

$$\frac{p_a\mu - g_{La}\left(1-p_a\right)}{p_a + g_{La}\left(1-p_a\right)} \geq \frac{p_d\mu - g_{Ld}\left(1-p_d\right)}{p_d + g_{Ld}\left(1-p_d\right)}. \tag{30}$$

Solving for g_{La} yields

$$g_{La} = \frac{g_{Ld}p_a\left(1-p_d\right)}{p_d\left(1-p_a\right)}. \tag{31}$$

Substituting $g_{Ld} = \frac{p_d\mu}{1-p_d}$ (the optimal level of grade inflation for d students) yields $g_{La} = \frac{p_a\mu}{1-p_a}$. However, given the distribution of ability in the advantaged population, the firm would hire all the A and a students, irrespective of the school strategy. Therefore the optimal school strategy is $g_{La} = 1$. For $p_a > p_d > \frac{1}{\mu+1}$, the firm's expected profit is still higher by hiring an a student for $g_{La} = \frac{p_a g_{Ld}(1-p_d)}{p_d(1-p_a)}$. Substituting $g_{Ld} = \frac{p_d\mu}{1-p_d}$ (the optimal level of grade inflation for d students) yields $g_{La} = \frac{p_a\mu}{1-p_a}$. The results for $\Phi < J < 1$ follow as in Proposition 2.