Statistical shape analysis

Application to the classification of renal tumors appearing in early childhood

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Outline

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1. Statistical shape analysis

2. Renal tumors in early childhood

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3. Experimental Results

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Every object o_i in a space V of dimension m is thus represented in a space of dimension $k \cdot m$ by a set of landmarks:

$$\forall i = 1 \dots N,\ o_i = \{l_1 \dots l_k\}, l_j \in \mathbb{R}^m. \tag{1}$$

Removing the scale

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1. For every $i, i = 1, ..., N$, the size of each object is determined as the euclidian norm of their landmarks.

$$\|o_i\| = \sqrt{\sum_{j=1}^{k} \|l_j^i\|_m^2}. \tag{2}$$

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$$\|o_i\| = \sqrt{\sum_{j=1}^{k} \|l_j^i\|_m^2}. \qquad (2)$$

2. The landmarks are standardized by dividing them by the size of their object:

$$\tilde{l}_j^i = \frac{l_j^i}{\|o_i\|}. \qquad (3)$$

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2. We center all the landmarks by subtracting this mean:

$$\bar{l}_j^i = l_j^i - z^i \tag{5}$$

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Hence, we are able to work completely in the standard three-dimensional space with the euclidian norm.

We do not need any further procrustes analysis nor any complicated stochastic geometry.

The mean shape

To compare the standardized and centered sets of landmarks, we need to define the mean shape of all the objects and a distance function which allows us to evaluate how ”near” every object is from this mean shape.

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If X demotes a random variable defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with values in a metric space (Ξ, d), an element $m \in \Xi$ is called a mean of $x_1, x_2, ..., x_k \in \Xi$ if

$$\sum_{j=1}^{k} d(x_j, m)^2 = \inf_{\alpha \in \Xi} \sum_{j=1}^{k} d(x_j, \alpha)^2. \qquad (6)$$

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That means that the mean shape is defined as the shape with the smallest variance of all shapes in a group of objects.

The algorithm of Ziezold (1994)

To begin, we fix the mean of all the standardized and centered objects as starting value:

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1. $$\tilde{m} \mapsto w_i(\tilde{m}) = \begin{cases} \frac{\langle \tilde{m}, o_i \rangle}{|\langle \tilde{m}, o_i \rangle|} & \text{if } \langle \tilde{m}, o_i \rangle \neq 0 \\ 1 & \text{if } \langle \tilde{m}, o_i \rangle = 0 \end{cases} \tag{7}$$

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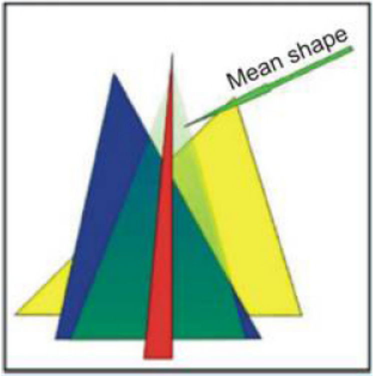
The stopping rule is $\tilde{m} = T(\tilde{m})$.

Example



The green triangle is the mean shape of the group of three triangles (yellow, red and blue).

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Renal tumors in early childhood

- Wilms-tumors (nephroblastoma) growing next to the kidney.

  Genetic cause. There are four types of tissue (a, b, c, d) and three stages of development (I, II, III).

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  Quite frequent (80 per year).
- Clear cell carcinoma growing next to bones.

  Rare (12 per year).

The data

Research sample:

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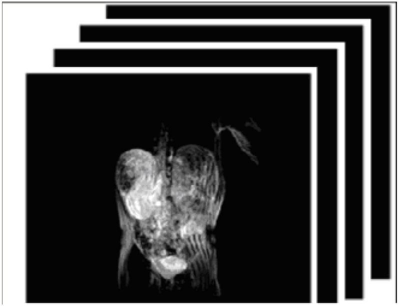
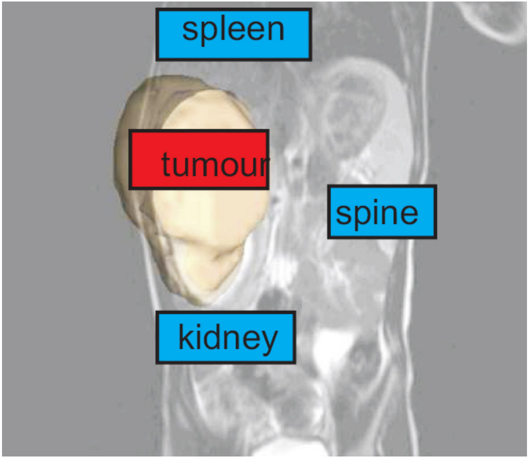


MRI of a renal tumor in frontal view.

The three-dimensional object



Three-dimensional model of a tumor.

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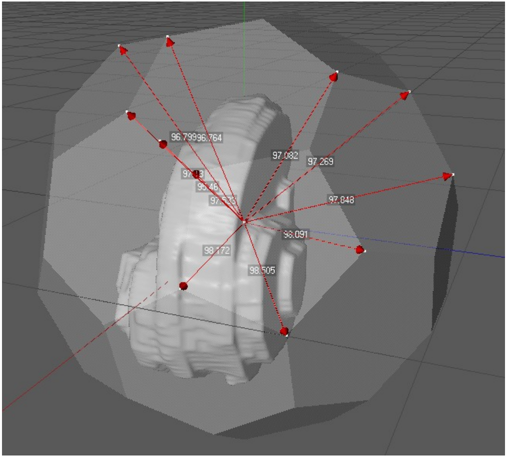
The platonic body C60



For every object, we consider the platonic body C60 whose center lies in the center of the object. This platonic body has 60 edges which give us 60 three-dimensional landmarks for every object.

The landmarks

We take as landmarks the 60 points on the border of each object closest to the edges of the platonic body.

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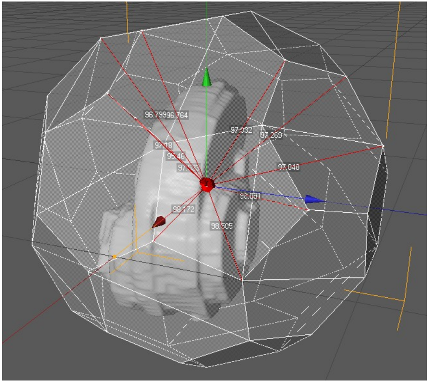

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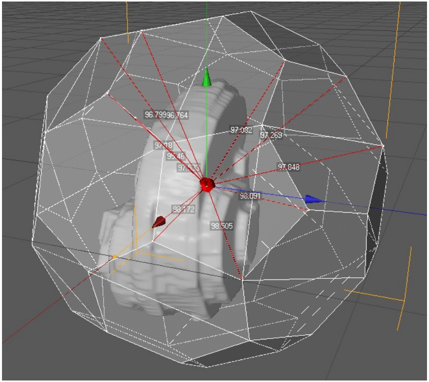
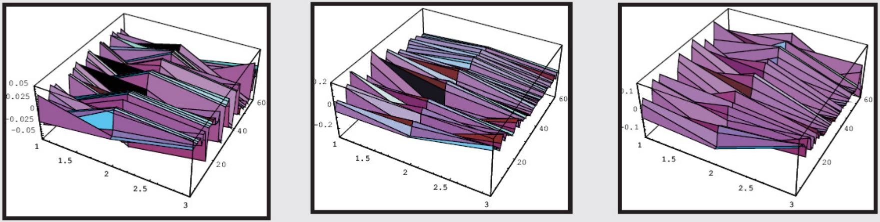


Only real measured points on the border of the tumor are taken, the approximated part of the three-dimensional object is not used.

Examples of tumor shapes



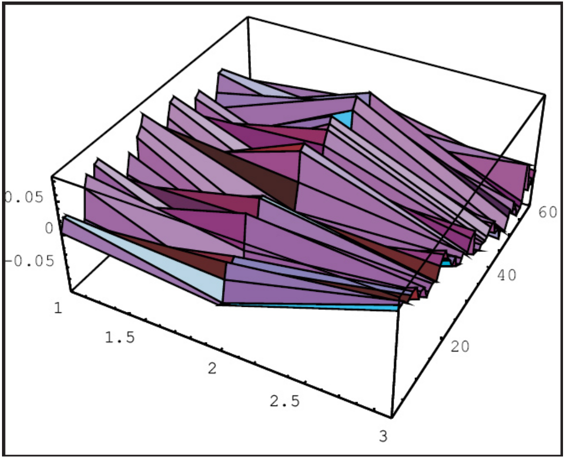
Patient No. 1
d=0.359869

Patient No. 9
d=0,563754

Patient No. 15
d=0,535196

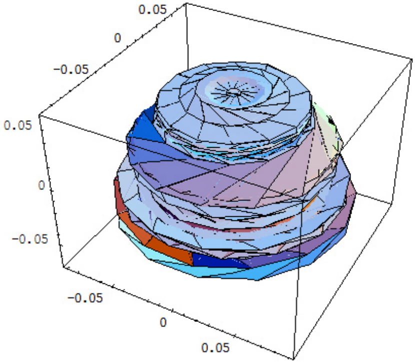
Examples of the mathematical views of the shapes of three tumors.

Our mean shape



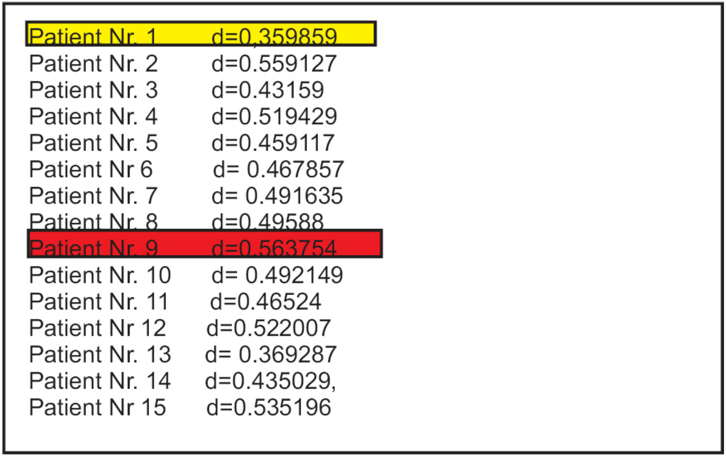
The "mathematical" mean shape of our sample.

The mean shape in 3D



The mean shape of our sample.

Distance from the mean shape for the Wilms tumors



Patient Nr. 1	d=0,359859
Patient Nr. 2	d=0.559127
Patient Nr. 3	d=0.43159
Patient Nr. 4	d=0.519429
Patient Nr. 5	d=0.459117
Patient Nr 6	d= 0.467857
Patient Nr. 7	d= 0.491635
Patient Nr. 8	d=0.49588
Patient Nr. 9	d=0.563754
Patient Nr. 10	d= 0.492149
Patient Nr. 11	d=0.46524
Patient Nr 12	d=0.522007
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The test hypotheses are:

$$\begin{array}{ll} \textit{Hypothesis:} & H_0 : P = Q \\ \textit{Alternative:} & H_1 : P \neq Q \end{array}$$

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4. Comparing the u_0-value to all possible u-values. Computing the rank (small u-value mean a small rank).
5. Calculate the p-value for H_0. $p_{r=i} = \frac{1}{\binom{N}{n}}$ for $i = 1, \ldots, \binom{N}{n}$, where r is the rank for which we assume a uniform distribution.

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  $u = 257 \qquad \text{rank} = 1 - 2$

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  $u = 257 \qquad \text{rank} = 15 - 40$

  Random sample: $n = 1000 \qquad p = 0.040.$

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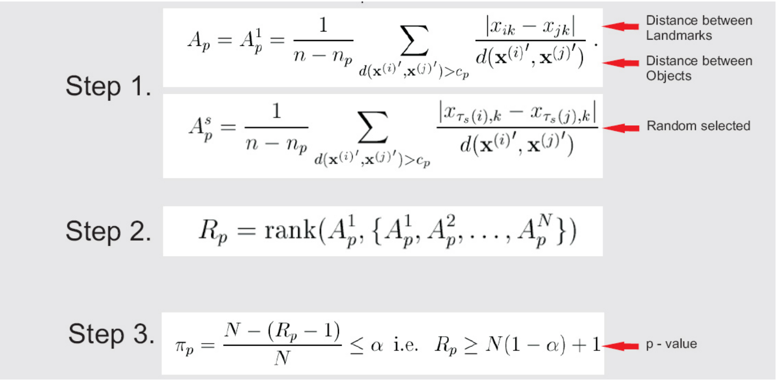

Step 1.

$$A_p = A_p^1 = \frac{1}{n - n_p} \sum_{d(\mathbf{x}^{(i)\prime},\mathbf{x}^{(j)\prime})>c_p} \frac{|x_{ik} - x_{jk}|}{d(\mathbf{x}^{(i)\prime}, \mathbf{x}^{(j)\prime})} .$$

Distance between Landmarks

Distance between Objects

$$A_p^s = \frac{1}{n - n_p} \sum_{d(\mathbf{x}^{(i)\prime},\mathbf{x}^{(j)\prime})>c_p} \frac{|x_{\tau_s(i),k} - x_{\tau_s(j),k}|}{d(\mathbf{x}^{(i)\prime}, \mathbf{x}^{(j)\prime})}$$

Random selected

Step 2.

$$R_p = \mathrm{rank}(A_p^1, \{A_p^1, A_p^2, \ldots, A_p^N\})$$

Step 3.

$$\pi_p = \frac{N - (R_p - 1)}{N} \leq \alpha \text{ i.e. } R_p \geq N(1 - \alpha) + 1$$

p - value

Influence of the different landmarks for Wilms tumors

Landmark	A_p	R_p	p-value
No.1	0.00049721	65	0.65
No.2	0.000498159	61	0.61
No.3	0.00049902	60	0.60
No.4	0.000496514	73	0.73
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All the landmarks have the same importance.

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- It is possible to differentiate the whole set of non-Wilms tumors from the mean shape of Wilms tumors.
- But we cannot use statistical shape analysis to say if a given general tumor is not a Wilms tumor.
- For the Wilms tumors, all the landmarks have the same importance.