# The weighted lattice polynomials as aggregation functions

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In lattice theory, *lattice polynomials* have been defined as well-formed expressions involving variables linked by the lattice operations $\wedge$ and $\vee$ in an arbitrary combination of parentheses. In turn, such expressions naturally define *lattice polynomial functions*. For instance,

$$p(x_1, x_2, x_3) = (x_1 \wedge x_2) \vee x_3$$

is a 3-ary lattice polynomial function.

The concept of lattice polynomial function can be straightforwardly generalized by regarding some variables as "parameters", like in the 2-ary polynomial

$$p(x_1, x_2) = (c \vee x_1) \wedge x_2,$$

where $c$ is a constant.

We investigate those "parameterized" polynomial functions, which we shall call *weighted lattice polynomial* functions. Particularly, we show that, in any bounded distributive lattice, those functions can be expressed in disjunctive and conjunctive normal forms. We also show that they include the discrete Sugeno integral, which has been extensively studied and used in the setting of nonlinear aggregation and integration. Finally, we prove that those functions can be characterized by means of a remarkable median based functional system of equations.

## References

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