Behavioral Analysis of Aggregation in Multicriteria Decision Aid

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EXTENDED ABSTRACT

1 Introduction

Let us consider a finite set of *alternatives* $A = \{a, b, c, \ldots\}$ and a finite set of *criteria* $N = \{1, \ldots, n\}$ in a multicriteria decision making problem. Each alternative $a \in A$ is associated with a *profile* $x^a = (x_1^a, \ldots, x_n^a) \in \mathbb{R}^n$, where, for any $i \in N$, x_i^a represents the partial score of a related to criterion i. We assume that all the partial scores are defined according to the same interval scale, that is, they are defined up to the same positive linear transformation. Particularly, this will enable us to embed the scale in the unit interval $[0, 1]$.

From the profile of any alternative $a \in A$, one can compute a global score $M(x^a)$ by means of an aggregation operator $M : \mathbb{R}^n \to \mathbb{R}$ which takes into account the weights of importance of the criteria. Once the global scores are computed, they can be used to rank the alternatives or select an alternative that best satisfies the given criteria.

Until recently, the most often used aggregation operators were the weighted arithmetic means, that is, operators of the form

$$M_\omega(x) = \sum_{i=1}^{n} \omega_i \, x_i,$$

with $\sum_i \omega_i = 1$ and $\omega_i \geq 0$ for all $i \in N$. However, since these operators are not able to model in any understandable way an interaction among criteria, they can be used only in the presence of independent criteria. They are not appropriate for the aggregation of interacting criteria.

In order to have a flexible representation of complex interaction phenomena between criteria (e.g. positive or negative synergy between some criteria), it is useful to substitute to the weight vector ω a non-additive set function on N to define a weight not only on each criterion, but also on each subset of criteria. For this purpose the concept of *fuzzy measure* [16] has been introduced.

Now, a suitable aggregation operator, which generalizes the weighted arithmetic mean, is the discrete Choquet integral, whose use was proposed by many authors (see e.g. [5] and the references there). Of course, the large flexibility of this aggregation operator is due to the use of a fuzzy measure, which makes it possible to model interaction phenomena existing among criteria. However, the meaning of the values of such a fuzzy measure is not always clear for the decision maker. These values, which represent the importance of each combination of criteria, do not give immediately the global importance of the criteria nor the degree of interaction among them.

In fact, from a given fuzzy measure, it is possible to derive some indices or parameters that describe the behavior of the fuzzy measure or, equivalently, that of the Choquet integral that is used to aggregate the criteria. Alternatively, when the fuzzy measure is not completely known, such indices can help the decision maker to assess it. This corresponds to the inverse problem of identifying the weights from parametric specifications on criteria, see [10].

The aim of this paper is to present the following behavioral indices: the global importance of criteria, the interaction among criteria, and the tolerance of the decision maker.

The outline is as follows. In Section 2 we recall the definition of the discrete Choquet integral and some of its particular cases. Sections 3 and 4 are respectively devoted to the importance and interaction indices. Finally, Sections 5 and 6 deal with the tolerance of the decision maker by means of the concepts of conjunction and disjunction degrees as well as the veto and favor indices.

In order to avoid an heavy notation, cardinality of subsets $S, T, \ldots$ will be denoted whenever possible by the corresponding lower case letters $s, t, \ldots$, otherwise by the standard notation $|S|, |T|, \ldots$. Moreover, we will often omit braces for singletons, e.g., writing $N \setminus i$ instead of $N \setminus \{i\}$.

For any subset $S \subseteq N$, e_S will denote the characteristic vector of S in $\{0,1\}^n$, i.e., the vector of $\{0,1\}^n$ whose i-th component is 1 if and only if $i \in S$.

2 The Choquet integral

The use of the Choquet integral has been proposed by many authors as an adequate substitute to the weighted arithmetic mean to aggregate interacting criteria, see e.g. [5, 9]. In the weighted arithmetic mean model, each criterion $i \in N$ is given a weight $\omega_i \in [0,1]$ representing the importance of this criterion in the decision. In the Choquet integral model, where criteria can be dependent, a fuzzy measure [16] is used to define a weight on each combination of criteria, thus making it possible to model the interaction existing among criteria.

Definition 2.1 *A fuzzy measure on N is a set function $v : 2^N \to [0,1]$ satisfying the following conditions:*

i) $v(\emptyset) = 0, v(N) = 1,$

ii) $S \subseteq T \;\Rightarrow\; v(S) \leq v(T).$

The set of all fuzzy measures on N will be denoted by $\mathcal{F}_N$ as we continue. Moreover, for any fuzzy measure v on N and any permutation π on N, πv will denote the fuzzy measure on N defined by $\pi v(\pi(S)) = v(S)$ for all $S \subseteq N$, where $\pi(S) = \{\pi(i) \,|\, i \in S\}$.

For any $S \subseteq N$, $v(S)$ can be interpreted as the weight of importance of the combination S of criteria, or better, its importance or power to make the decision alone (without the remaining criteria).

The concept of Choquet integral was first introduced in capacity theory [1]. Its use as a (fuzzy) integral with respect to a fuzzy measure was then proposed by Murofushi and Sugeno [13, 14].

Definition 2.2 *Let $v \in \mathcal{F}_N$. The Choquet integral of $x : N \to \mathbb{R}$ with respect to v is defined by*

$$\mathcal{C}_v(x) := \sum_{i=1}^{n} x_{(i)}\,[v(A_{(i)}) - v(A_{(i+1)})],$$

where $(\cdot)$ indicates a permutation on N such that $x_{(1)} \leq \ldots \leq x_{(n)}$. Also $A_{(i)} = \{(i), \ldots, (n)\}$, and $A_{(n+1)} = \emptyset$.

Thus defined, the Choquet integral has very good properties for aggregation (see e.g. Grabisch [5]). For instance, it is continuous, non decreasing, comprised between min and max, stable under the same transformations of interval scales in the sense of the theory of measurement, and coincides with the weighted arithmetic mean (discrete Lebesgue integral) as soon as the fuzzy measure is additive. Moreover, in [8, 9] the author proposed an axiomatic characterization of the class of all the Choquet integrals with n arguments. The statement is the following.

Theorem 2.1 *The operators $M_v : \mathbb{R}^n \to \mathbb{R}$ $(v \in \mathcal{F}_N)$ are*

- *linear w.r.t. the fuzzy measure, that is, there exist 2^n functions $f_T : \mathbb{R}^n \to \mathbb{R}$ $(T \subseteq N)$, such that*

$$M_v = \sum_{T \subseteq N} v(T)\, f_T, \qquad v \in F_N.$$

- *non decreasing in each argument,*
- *stable for the admissible positive linear transformations, that is,*

$$M_v(r\,x + s\,e_N) = r\,M_v(x) + s$$

  for all $x \in \mathbb{R}^n$, $r > 0$, $s \in \mathbb{R}$.

- *properly weighted by v, that is,*

$$M_v(e_S) = v(S), \qquad S \subseteq N,\ v \in F_N,$$

if and only if $M_v = \mathcal{C}_v$ for all $v \in \mathcal{F}_N$.

The axioms presented in the previous characterization are natural enough in the context of multicriteria decision making. The first one is proposed to keep the aggregation model as simple as possible. The second axiom says that increasing a partial score along any criterion cannot decrease the global score. The third axiom only demands that the aggregated value is stable with respect to any change of scale. Finally, assuming that the partial score scale is embedded in $[0, 1]$, the fourth axiom suggests that the weight of importance of any subset S of criteria is defined as the global evaluation of the alternative that completely satisfies criteria S and totally fails to satisfy the others.

The fourth axiom is fundamental. It gives an appropriate definition of the weights of subsets of criteria, interpreting them as global evaluation of particular profiles.

We now present some subclasses of Choquet integrals. Any vector $\omega \in [0, 1]^n$ such that $\sum_i \omega_i = 1$ will be called a *weight vector* as we continue.

2.1 The weighted arithmetic mean

Definition 2.3 *For any weight vector $\omega \in [0,1]^n$, the weighted arithmetic mean operator* WAM_ω *associated to ω is defined by*

$$\mathrm{WAM}_\omega(x) = \sum_{i=1}^{n} \omega_i \, x_i.$$

We can easily see that WAM_ω is a Choquet integral $\mathcal{C}_v$ with respect to an additive fuzzy measure:

$$v(S) = \sum_{i \in S} \omega_i, \qquad S \subseteq N.$$

2.2 The ordered weighted averaging operator

Yager [17] defined in 1988 the ordered weighted averaging operators (OWA) as follows.

Definition 2.4 *For any weight vector $\omega \in [0,1]^n$, the ordered weighted averaging operator* OWA_ω *associated to ω is defined by*

$$\mathrm{OWA}_\omega(x) = \sum_{i=1}^{n} \omega_i \, x_{(i)}$$

with the usual convention that $x_{(1)} \leq \cdots \leq x_{(n)}$.

The following result, due to Grabisch [4], shows that any OWA operator is a Choquet integral w.r.t. a fuzzy measure that depends only on the cardinality of subsets.

Proposition 2.1 *Let $v \in \mathcal{F}_N$. Then the following assertions are equivalent.*

i) For any $S, S' \subseteq N$ such that $|S| = |S'|$, we have $v(S) = v(S')$.

ii) There exists a weight vector ω such that $\mathcal{C}_v = \mathrm{OWA}_\omega$.

iii) $\mathcal{C}_v$ is a symmetric function.

The fuzzy measure v associated to OWA_ω is given by

$$v(S) = \sum_{i=n-s+1}^{n} \omega_i, \qquad S \subseteq N, \;\; S \neq \emptyset.$$

2.3 The partial minimum and maximum operators

Definition 2.5 *For any non-empty subset $T \subseteq N$, the partial minimum operator* $\min_T$ *and the partial maximum operator* $\max_T$*, associated to T, are respectively defined by*

$$\begin{aligned} \min\nolimits_T(x) &= \min_{i \in T} x_i, \\ \max\nolimits_T(x) &= \min_{i \in T} x_i. \end{aligned}$$

For the operator $\min_T$, we have

$$v(S) = \begin{cases} 1, & \text{if } S \supseteq T, \\ 0, & \text{else.} \end{cases}$$

For the operator $\max_T$, we have

$$v(S) = \begin{cases} 1, & \text{if } S \cap T \neq \emptyset, \\ 0, & \text{else.} \end{cases}$$

3 Importance indices

The overall importance of a criterion $i \in N$ into a decision problem is not solely determined by the number $v(i)$, but also by all $v(T)$ such that $T \ni i$. Indeed, we may have $v(i) = 0$, suggesting that element i is unimportant, but it may happen that for many subsets $T \subseteq N \backslash i$, $v(T \cup i)$ is much greater than $v(T)$, suggesting that i is actually an important element in the decision.

Shapley [15] has proposed in 1953 a definition of a coefficient of importance, based on a set of reasonable axioms. The *importance index* or *Shapley value* of criterion i with respect to v is defined by:

$$\phi(v, i) := \sum_{T \subseteq N \backslash i} \frac{(n-t-1)!\, t!}{n!} [v(T \cup i) - v(T)]. \tag{1}$$

The Shapley value is a fundamental concept in game theory expressing a power index. Its use in multicriteria decision making was proposed in 1992 by Murofushi [11].

Thus defined, it can be interpreted as a weighted average value of the marginal contribution $v(T \cup i) - v(T)$ of element i alone in all combinations.

It is worth noting that the Shapley value fulfills the following properties:

$$\phi(v, i) \geq 0, \qquad i \in N,$$

and

$$\sum_{i=1}^{n} \phi(v, i) = 1.$$

Note also that, when v is additive, we clearly have $v(T \cup i) - v(T) = v(i)$ for all $i \in N$ and all $T \subseteq N \setminus i$, and hence

$$\phi(v, i) = v(i), \qquad i \in N. \tag{2}$$

If v is non-additive then some criteria are dependent and (2) generally does not hold anymore. This shows that it is reasonable to search for a coefficient of overall importance for each criterion.

4 Interaction indices

Another interesting concept is that of *interaction* among criteria. We have seen that when the fuzzy measure is not additive then some criteria interact. Of course, it would be interesting to appraise the degree of interaction among any subset of criteria.

Consider first a pair $\{i, j\} \subseteq N$ of criteria. It may happen that $v(i)$ and $v(j)$ are small and at the same time $v(ij)$ is large. Clearly, the number $\phi(v, i)$ merely measures the average contribution that criterion i brings to all possible combinations, but it gives no information on the interaction phenomena existing among criteria.

Suppose that i and j are positively correlated or competitive (resp. negatively correlated or complementary). Then the marginal contribution of j to every combination of criteria that contains i should be strictly less than (resp. greater than) the marginal contribution of j to the same combination when i is excluded. Thus, depending on whether the correlation between i and j is positive or negative, the expression

$$v(T \cup ij) - v(T \cup i) - v(T \cup j) + v(T)$$

is ≤ 0 or ≥ 0 for all $T \subseteq N \setminus ij$, respectively. We call this expression the marginal interaction between i and j, conditioned to the presence of elements of the combination $T \subseteq N \setminus ij$. Now, an interaction index for $\{i, j\}$ is given by an average value of this marginal interaction. Murofushi and Soneda [12] proposed in 1993 to calculate this average value as for the Shapley value. Setting

$$(\Delta_{ij}\, v)(T) := v(T \cup ij) - v(T \cup i) - v(T \cup j) + v(T),$$

the *interaction index* of criteria i and j related to v is then defined by

$$I(v, ij) := \sum_{T \subseteq N \setminus ij} \frac{(n-t-2)!\, t!}{(n-1)!} (\Delta_{ij}\, v)(T).$$

We immediately see that this index is negative as soon as i and j are positively correlated or competitive. Similarly, it is positive when i and j are negatively correlated or complementary. Moreover, it has been shown in [6] that $I(v, ij) \in [-1, 1]$ for all $i, j \in N$.

The interaction index among a combination S of criteria was introduced by Grabisch [6] as a natural extension of the case $s = 2$. It was also axiomatized very recently by Grabisch and Roubens [7]. The *interaction index* of S ($s \geq 2$) related to v, is defined by

$$I(v, S) := \sum_{T \subseteq N \setminus S} \frac{(n-t-s)!\, t!}{(n-s+1)!} (\Delta_S\, v)(T),$$

where

$$(\Delta_S\, v)(T) := \sum_{L \subseteq S} (-1)^{s-l} v(L \cup T).$$

Viewed as a set function, it coincides on singletons with the Shapley value (1).

5 Conjunction and disjunction degrees

Consider the cube $[0, 1]^n$ as a probability space with uniform distribution. Then the expected value of $\mathcal{C}_v(x)$, that is,

$$\mathrm{E}(\mathcal{C}_v) = \int_{[0,1]^n} \mathcal{C}_v(x)\, dx, \tag{3}$$

represents the *average value* of the Choquet integral $\mathcal{C}_v$ over $[0, 1]^n$. This expression gives the average position of $\mathcal{C}_v$ within the interval $[0, 1]$.

Since the Choquet integral is always internal to the set of its arguments, that is

$$\min x_i \leq \mathcal{C}_v(x) \leq \max x_i, \qquad x \in [0, 1]^n,$$

from (3) it follows that

$$\mathrm{E}(\min) \leq \mathrm{E}(\mathcal{C}_v) \leq \mathrm{E}(\max), \qquad v \in \mathcal{F}_N.$$

The relative position of $\mathrm{E}(\mathcal{C}_v)$ with respect to the lower bound of the interval $[\mathrm{E}(\min), \mathrm{E}(\max)]$ is called the conjunction degree or the degree of andness of $\mathcal{C}_v$. It represents the degree to which the average value of $\mathcal{C}_v$ is close to that of min.

Definition 5.1 *The degree of andness of $\mathcal{C}_v$ is defined by*

$$\text{andness}(\mathcal{C}_v) := \frac{\text{E}(\max) - \text{E}(\mathcal{C}_v)}{\text{E}(\max) - \text{E}(\min)}.$$

Similarly, the relative position of $\text{E}(\mathcal{C}_v)$ with respect to $\text{E}(\max)$ is called the disjunction degree or the degree of orness of $\mathcal{C}_v$.

Definition 5.2 *The degree of orness of $\mathcal{C}_v$, is defined by*

$$\text{orness}(\mathcal{C}_v) := \frac{\text{E}(\mathcal{C}_v) - \text{E}(\min)}{\text{E}(\max) - \text{E}(\min)}.$$

An immediate consequence of these definitions is that

$$\text{andness}(\mathcal{C}_v) + \text{orness}(\mathcal{C}_v) = 1.$$

Moreover, we have $\text{andness}(\mathcal{C}_v), \text{orness}(\mathcal{C}_v) \in [0,1]$.

These two concepts have been introduced as early as 1974 by Dujmović [2, 3] in the particular case of power means. Here we have adapted his definitions to the Choquet integral.

The degree of orness is a measure of the tolerance of the decision maker. Indeed, tolerant decision makers can accept that only *some* criteria are satisfied. This corresponds to a disjunctive behavior ($\text{orness}(\mathcal{C}_v) > 0.5$), whose extreme example is max. On the other hand, intolerant decision makers demand that *most* criteria are satisfied. This corresponds to a conjunctive behavior ($\text{orness}(\mathcal{C}_v) < 0.5$), whose extreme example is min. When $\text{orness}(\mathcal{C}_v) = 0.5$ the decision maker is medium (neither tolerant nor intolerant).

In terms of the fuzzy measure v the degree of orness takes the following form, see [8] :

$$\text{orness}(\mathcal{C}_v) = \frac{1}{n-1} \sum_{T \subsetneq N} \frac{(n-t)!\,t!}{n!}\, v(T).$$

The concept of orness was defined independently by Yager [17] in the particular case of OWA operators, see also Yager [18]. His definition is based on the use of the so-called regular increasing monotone quantifiers, that is, increasing functions $Q : [0,1] \to [0,1]$, with $Q(0) = 0$ and $Q(1) = 1$, which represent linguistic quantifiers such as *all*, *most*, *many*, *at least k*.

6 Veto and favor effects

An interesting behavioral phenomenon in aggregation is the veto effect, and its counterpart, the favor effect. A criterion $k \in N$ is said to be a *veto* or a *blocker* for $\mathcal{C}_v$ if its non satisfaction entails necessarily a low global score. Formally, k is a veto for $\mathcal{C}_v$ if

$$\mathcal{C}_v(x) \leq x_k, \qquad x \in [0,1]^n. \tag{4}$$

Similarly, the criterion k is a *favor* or a *pusher* for $\mathcal{C}_v$ if its satisfaction entails necessarily a high global score:

$$\mathcal{C}_v(x) \geq x_k, \qquad x \in [0,1]^n. \tag{5}$$

Note that if the decision maker considers that a given criterion must absolutely be satisfied (veto criterion), then he/she is conjunctive oriented. Indeed, by (4) we have $\text{orness}(\mathcal{C}_v) \leq 0.5$, which is sufficient. Similarly, if the decision maker considers that a given criterion is sufficient to be satisfied (favor criteria) then he/she is disjunctive oriented. By (5) we have $\text{orness}(\mathcal{C}_v) \geq 0.5$.

It seems reasonable to define indices that measure the degree of veto or favor of a given criterion. In [8] the author introduced the following indices :

$$\begin{aligned} \text{veto}(\mathcal{C}_v, i) &:= 1 - \frac{1}{n-1} \sum_{T \subseteq N \setminus i} \frac{(n-t-1)!\, t!}{(n-1)!} v(T), \qquad i \in N, \\ \text{favor}(\mathcal{C}_v, i) &:= \frac{1}{n-1} \sum_{T \subseteq N \setminus i} \frac{(n-t-1)!\, t!}{(n-1)!} v(T \cup i) - \frac{1}{n-1}, \qquad i \in N. \end{aligned}$$

The axiomatic that supports these indices is given in the following result.

Theorem 6.1 *The numbers $\psi(\mathcal{C}_v, i)$ $(i \in N, v \in \mathcal{F}_N)$:*

- *are linear w.r.t. the fuzzy measure, that is, there exist real constants p_T^i $(T \subseteq N)$ such that*

$$\psi(\mathcal{C}_v, i) = \sum_{T \subseteq N} p_T^i\, v(T), \qquad i \in N, v \in \mathcal{F}_N.$$

- *are symmetric, that is, for any permutation π on N, we have*

$$\psi(\mathcal{C}_v, i) = \psi(\mathcal{C}_{\pi v}, \pi(i)), \qquad i \in N, v \in \mathcal{F}_N.$$

- *fulfill the "boundary" axiom, that is, for any $T \subseteq N$, $T \neq \emptyset$, and any $i \in T$, we have*

$$\psi(\min_T, i) = 1, \quad (\textit{resp. } \psi(\max_T, i) = 1)$$

- *fulfill the "normalization" axiom, that is, for any $v \in \mathcal{F}_N$,*

$$\begin{gathered} \psi(\mathcal{C}_v, i) = \psi(\mathcal{C}_v, j) \quad \forall i, j \in N \\ \Downarrow \\ \psi(\mathcal{C}_v, i) = \text{andness}(\mathcal{C}_v) \quad (\textit{resp. } \text{orness}(\mathcal{C}_v)) \quad \forall i \in N. \end{gathered}$$

if and only if $\psi(\mathcal{C}_v, i) = \text{veto}(\mathcal{C}_v, i)$ (resp. $\text{favor}(\mathcal{C}_v, i)$) for all $i \in N$ and all $v \in \mathcal{F}_N$.

Let us comment on the axioms presented in this characterization. As for the importance indices, we ask the veto and favor indices to be linear w.r.t. the fuzzy measure and symmetric. Next, the boundary axiom is motivated by the observation that any $i \in T$ is a veto (resp. favor) criterion for $\min_T$ (resp. $\max_T$). Finally, the normalization axiom says that if the degree of veto (resp. favor) does not depend on criteria, then it identifies with the degree of intolerance (resp. tolerance) of the decision maker.

It is easy to observe that $\text{veto}(\mathcal{C}_v, i), \text{favor}(\mathcal{C}_v, i) \in [0, 1]$. Furthermore, we have, for any $v \in \mathcal{F}_N$,

$$\begin{aligned} \frac{1}{n} \sum_{i=1}^{n} \text{veto}(\mathcal{C}_v, i) &= \text{andness}(\mathcal{C}_v), \\ \frac{1}{n} \sum_{i=1}^{n} \text{favor}(\mathcal{C}_v, i) &= \text{orness}(\mathcal{C}_v). \end{aligned}$$

Thus defined, we see that veto$(\mathcal{C}_v, i)$ is more or less the degree to which the decision maker demands that criterion i is satisfied. Similarly, favor$(\mathcal{C}_v, i)$ is the degree to which the decision maker considers that a good score along criterion i is sufficient to be satisfied.

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