Conditioned Higher Moment Portfolio Optimisation Using Optimal Control

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Outline

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   - Portfolio optimisation with conditioning information

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2. Empirical study
   - Description
   - Results

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3. Summary

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Problem context

- Discrete-time optimisation
- Minimise portfolio risk for a given expected portfolio return (or, equivalently, maximise expected utility)
- Postulate that there exists some relationship $\mu(s)$ between a signal s and each asset return r observed at the end of the investment interval:

  $$r_t = \mu(s_{t-1}) + \epsilon_t,$$

  with $E[\epsilon_t | s_{t-1}] = 0$.
- How do we optimally use this information in an otherwise classical (unconditional moment) portfolio optimisation process?

Problem history

- Hansen and Richard (1983): functional analysis argument suggesting that unconditional moments should enter the optimisation even when conditioning information is known
- Ferson and Siegel (2001): closed-form solution of unconstrained mean-variance problem using unconditional moments
- Chiang (2008): closed-form solutions to the benchmark tracking variant of the Ferson-Siegel problem
- Basu et al. (2006), Luo et al. (2008): empirical studies covering conditioned mean-variance optima of portfolios of trading strategies

Conditioned optimisation use of unconditional moments

This choice of moments is standard and can be justified in three different ways:

- Hansen and Richard (1983): the unconditional conditioned efficient frontier contains the corresponding conditional conditioned efficient frontier as a proper subset in general
- Investment managers applying conditioned portfolio optimisation will be evaluated by agents who take an unconditioned (Markowitz) view on performance (Ferson and Siegel (2001))
- Empirical studies (Chiang (2008)) provide statistical evidence that the use of conditional moments in conditioned optimisation underperforms with respect to the use of unconditional moments

Optimal control formulation

- Suggested by Boissaux and Schiltz (2010) as a general description for different variants of conditioned portfolio problems.
- This formulation allows for the numerical solution of problem variants (e.g. conditioned MV with weight constraints) for which no closed-form solution is available.
- In the present, we obtain and exercise optimal control formulations of conditioned problems involving higher moments of returns to evaluate the impact of conditioning information on higher moment optimisation for the first time.

Higher-moment optimisation

- Model user preferences with respect to the third and fourth moments of returns (skewness (S) and kurtosis (K)) as well as mean (M) and variance (V).
- Can either work as in MV case, replacing expected return or variance by, respectively, skewness or kurtosis (-> MK efficient frontier) or
- use (polynomial) utility functions to capture investor preferences with respect to more than two moments at the same time (-> MVK and MVSK optimisation).

MK optimisation as an optimal control problem

$$
\begin{array}{ll}
\text{minimise} & J_{I_S}(x(s), u(s)) = \int_{I_S} I_4(u, s) p_s(s) ds \\
\text{subject to} & \dot{x}(s) = I_1(u, s) p_s(s) \ \forall s \in I_S \\
 & x(s^-) = 0, x(s^+) = \mu_P \\
\text{and} & u(s) \in U \ \forall s \in I_S
\end{array}
$$

where $I_1(u, s)$ and $I_4(u, s)$ are integrands chosen such that the signal domain integral of $I_i p_s(s)$ corresponds in either case to unconditional ith moment metrics of expected portfolio returns, μ_P is the expected unconditional portfolio return and $p_s(s)$ is the signal density function.

MVK/MVSK optimisation as an optimal control problem

$$
\begin{aligned}
\text{minimise} \quad & J_{I_S}(x(s), u(s)) = -\int_{I_S} \left(a_1 \frac{dx_1}{ds} + a_2 \frac{dx_2}{ds} + a_3 \frac{dx_3}{ds} + a_4 \frac{dx_4}{ds} \right) ds \\
\text{subject to} \quad & \frac{dx_i}{ds} = I_i(u, s) p_s(s), i \in \{1, 2, 3, 4\}, \\
& x_1(s^-) = x_2(s^-) = x_3(s^-) = x_4(s^-) = 0 \\
\text{and} \quad & u(s) \in U \, \forall s \in I_S
\end{aligned}
$$

where the $I_i(u, s)$ are integrands chosen such that the $x_i(s^+)$ correspond to unconditional *i*th moment metrics of expected portfolio returns.
Optimisation involving the third moment always entails a nonconvex cost function: this is problematic from the numerical point of view.

Outline

Aim

- Carry out backtests executing constrained-weight higher moment conditioned optimisation strategies with different settings.
- In this way, collect additional evidence for the practical applicability of conditioned optimisation and show the flexibility of the optimal control formulation.

Data set

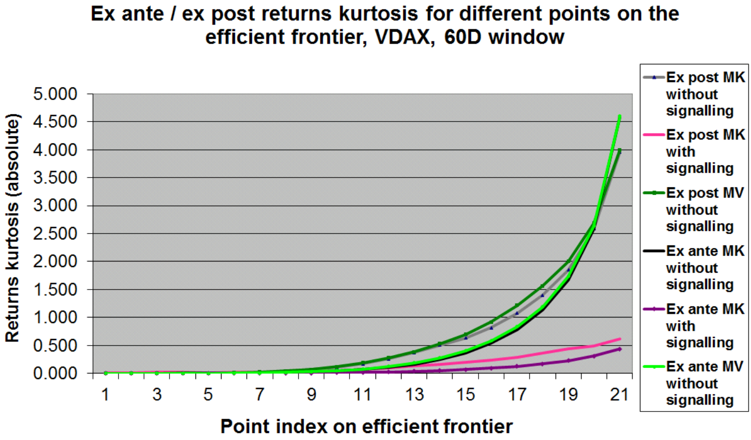
- 11 years of daily data, from January 1999 to February 2010 (2891 samples)
- Risky assets: 10 different EUR-based funds commercialised in Luxembourg chosen across asset categories (equity, fixed income) and across Morningstar style criteria
- Risk-free proxy: EURIBOR with 1 week tenor
- Signal: VDAX - an indicator of pure equity risk (perception)

Backtest logic

- Rebalance classically optimal portfolio alongside conditioned optimal portfolio, in a market of risky assets only, over the 11-year period
- Assume lagged relationship $\mu(s)$ between signal and return can be represented by a linear regression
- Use kernel density estimates for signal densities
- Estimate the above using a rolling window size of 60 points
- Use direct collocation method for numerical problem solutions: see e.g. Betts (2001)
- Obtain discretised arc of efficient mean-kurtosis frontier (MK) or optimise expected quadratic utility value (MVK/MVSK) to obtain strategy weights
- Compare kurtoses, adjusted Sharpe ratios and utilities (both *ex ante* and *ex post*) of both strategies

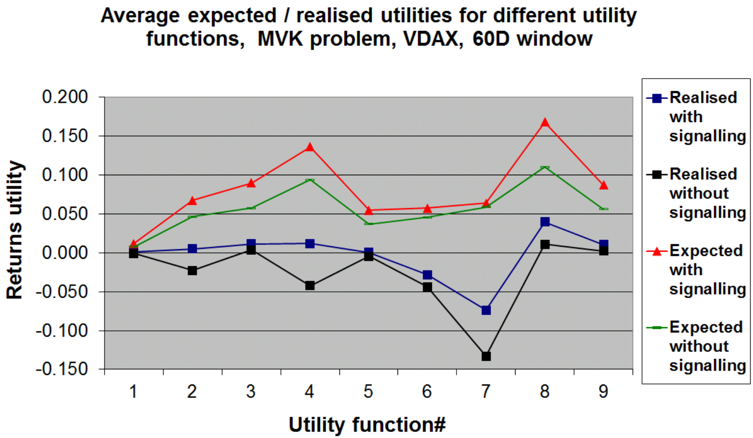
Outline

Ex ante and ex post kurtoses for MK optimisation

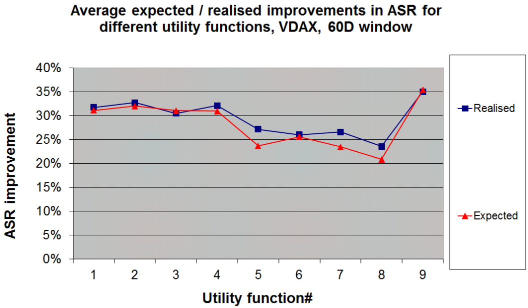


- Large reduction in kurtosis both ex ante and ex post seen when conditioning information is used.

Ex ante and ex post utility values for MVK optimisation



Ex ante and ex post improvements in ASR for MVSK optimisation



- The adjusted Sharpe ratio (ASR, Pézier and White (2006)) takes into account third and fourth moments of expected returns.
- MVSK improvements are consistent with the MVK case: some evidence that skewness preferences may be taken into account in practice.

Summary

- Analysis gives an example of how the optimal control formulation of conditioned problems may be applied to different problem variations not previously accessible.
- Results provide further evidence (in addition to the existing empirical two-moment literature) to suggest that conditioned optimisation increases strategy performance in a universal and robust manner.