Natural extension of median algebras.

The concept of natural extension for algebras in an internally residually finite prevariety $\mathcal{A}$ has been introduced as a generalization of canonical extension of lattice-based algebras. An algebra A embeds as a topological dense subalgebra of its natural extension, which is a topological algebra isomorphic to the profinite completion of A. If $\mathcal{A} = \mathbb{ISP}(M)$ is a dualisable finitely generated prevariety then the natural extension of A can be concretely obtained as the topological algebra of the structure preserving maps from the dual of A to the structure $\underset{\sim}{M}$ that generates the duality.

This tool has already been applied as an alternative way to study canonical extension of finitely generated varieties of lattice-based algebras. Actually, it is clearly well designed for any dualisable finitely generated variety, in which algebras do not necessarily have a lattice reduct.

In this talk we propose to illustrate these constructions for the variety $\mathcal{A}$ of median algebras, *i.e.*, for $\mathcal{A} = \mathbb{ISP}(M)$ where $M =< \{0, 1\}, (\cdot, \cdot, \cdot) >$ where $(\cdot, \cdot, \cdot)$ is the the ternary majority function on $\{0, 1\}$. In the perspective of illustrating the techniques of natural extension, this variety is interesting for at least three reasons: (1) it admits a strong duality, (2) it is not lattice based but (3) has nevertheless some interesting order-theoretic properties. It is indeed known that any median algebra can be regarded as a median semilattice (which are meet-semilattices in which every principal ideal is a distributive lattice and in which any three elements have an upper bound whenever each pair of them is bounded above) and conversely.

Hence, we investigate and illustrate how natural extension behaves with regard to median algebras and their correspondent median semilattices. We also show how to extend maps between median algebras to maps between their canonical extension. This allows us to consider natural extension of expansions of median algebra and the problem of preservation of equations through natural extension.