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Compound Model of Inverter Driven Grids

Stability Analysis of Island Power Grids Containing Only Voltage Source Inverters

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I. Model Conception

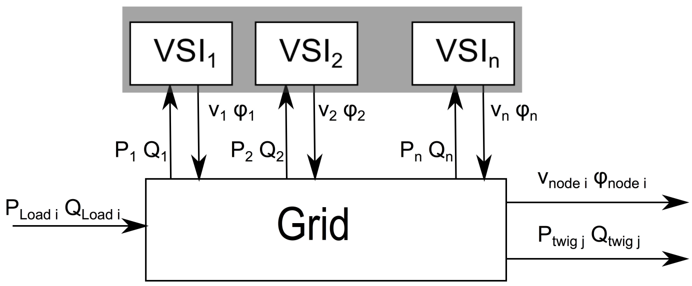

Figure 1. *Separating Grid from Voltage Source Inverters*

In order to derive a compound model of an inverter driven island grid, a semantic separation is made, as illustrated in Figure 1, into the nodes to which a VSI is connected and the remaining nodes of the passive grid with its lines and connection points for loads. Input parameter to the model are the load charges at grid nodes, as active and reactive power $P_{Load,i}$ and $Q_{Load,i}$. Output parameters are the voltage v and angle φ at these load nodes and the power flows through the grid brances (twigs). Parameters exchanged between the grid model and the VSI model are the active power P_j and the reactive power Q_j which the VSI inject based on the grid's power equations and based on the current load situation. Parameters exchanged from the VSI model to the grid model, are voltage v_j and angle φ_j the VSI adjust at their point of connection (PoC).

II. Voltage Source Inverter Modelling

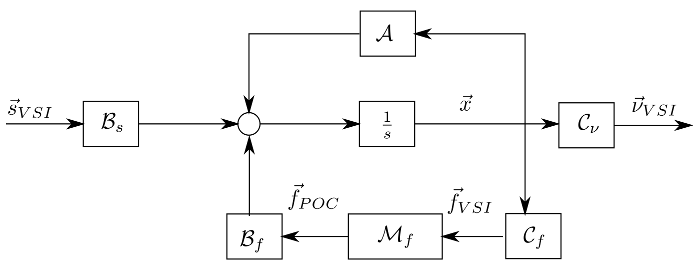

Figure 2. *Multi VSI State Space Model*

A VSI in droop control mode acts as a voltage source and adjusts frequency and voltage amplitude according to the actual power injection. The VSI model input are active power P and reactive power Q, momentarily injected into the grid. Further, the actual grid frequency at the point of connection f_{POC} is provided. The outputs of the VSI model are the voltage amplitude v, the voltage angle φ and the injection frequency f_{VSI}. A state space model of the form dx/dt=Ax+Bu, y=Cx is obtained for a droop controlled voltage source inverter. For any VSI indexed with i, the input vector is split into $u_i = [s_i\ f_{POC,i}]^T$ with $s_i = [P_i\ Q_i]^T$, and the output vector is split into $y_i = [v_i\ f_{VSI}]^T$ with $\nu_i = [\varphi_i\ v_i]^T$. While the system matrix A remains unchanged, the input matrix B and the output matrix C can be separated in two sub-matrices according to the mentioned splits in the input and output vectors. Thus the state space representation of a single VSI becomes $\dot{\vec{x}}_i = \mathcal{A}_i\vec{x}_i + \left[\ \mathcal{B}_{si}\mathcal{B}_{fi}\ \right] \begin{bmatrix} \vec{s} \\ f_{POC} \end{bmatrix}_i$ and $\begin{bmatrix} \vec{\nu} \\ f_{vsi} \end{bmatrix}_i = \begin{bmatrix} \mathcal{C}_{\nu i} \\ \mathcal{C}_{fi} \end{bmatrix} \vec{x}_i$. A multi VSI model can be obtained by stacking all input vectors $\vec{s}$, output $\vec{\nu}$ and state vectors $\vec{x}$ and by assembling all VSI state space matrices in bock diagonal form. Separating the frequency values from the variables $\vec{s}$ and $\vec{\nu}$, which are exchanged with the grid model and defining a matrix $\mathcal{M}_f$, measuring the mutual frequency interference, leads to the overall multi-VSI state space model illustrated in Figure 2. The system matrix becomes $\mathcal{A}' = \mathcal{A} + \mathcal{B}_f\mathcal{M}_f\mathcal{C}_f$.

III. Grid Structure Modelling

For stability analysis, a linear, time-invariant (LTI) model is needed. Under the condition that the angle difference $\delta = \varphi_i - \varphi_j$ between two nodes i and j remains small and the voltages remain in the range of $v_i \approx v_j \approx 1.0\,\mathrm{p.u.}$ the power flow can be linearized and be calculated by $s_k \approx \mathbf{y}_k \cdot (\nu_i - \nu_j)$ with the 2×2 admittance matrix $\mathbf{y}_k = \begin{bmatrix} -B_k & G_k \\ -G_k & -B_k \end{bmatrix}$, based on the admittance G_k and susceptance B_k of branch k.

All branch admittances are set in the block diagonal matrix $\mathcal{Y} = \mathrm{diag}(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_n)$. The node incidence matrix $\mathcal{K}$ contains the topological information of the grid. $\mathcal{Y}$ and $\mathcal{K}$ together with the stacked vector s of all branch powers, the stacked vector ν of all node voltages (angle φ and amplitude v), allow the formulation of the power flow equation $s = -\mathcal{Y}\mathcal{K}^T \cdot \nu$. In $\mathcal{K}$ the nodes with VSI connected are sorted upwards, so it can be split in two sub-matrices $\mathcal{K} = \begin{bmatrix} \mathcal{K}_{VSI} \\ \mathcal{K}_N \end{bmatrix}$. Accordingly the vector ν of the node voltags is split into the top part ν_{VSI} of the voltages at the nodes with VSI contected and the bottom part ν_{N} of the voltages at the remaining grid nodes. Thus the power equation becomes

$$s = \begin{bmatrix} -\mathcal{Y}\mathcal{K}_{VSI}^T & -\mathcal{Y}\mathcal{K}_N^T \end{bmatrix} \cdot \begin{bmatrix} \nu_{VSI} \\ \nu_N \end{bmatrix}.$$

The VSIs are the only power sources in the grid, injecting the power vector s_{VSI}. Only at the non-VSI nodes load power s_{Load} can be consumed. Thus the sum of power flows in all twigs of the grid must always equal the injected or consumed power at the grid boundary. This is mathematically described by $-\begin{bmatrix} s_{VSI} \\ s_{Load} \end{bmatrix} = \begin{bmatrix} \mathcal{K}_{VSI} \\ \mathcal{K}_N \end{bmatrix} \cdot s$. Combining the previous two equations leads to

$$\begin{bmatrix} s_{\mathrm{VSI}} \\ s_{\mathrm{Load}} \end{bmatrix} = \begin{bmatrix} \mathcal{K}_{\mathrm{VSI}}\mathcal{Y}\mathcal{K}_{\mathrm{VSI}}^T & \mathcal{K}_{\mathrm{VSI}}\mathcal{Y}\mathcal{K}_{\mathrm{N}}^T \\ \mathcal{K}_{\mathrm{N}}\mathcal{Y}\mathcal{K}_{\mathrm{VSI}}^T & \mathcal{K}_{\mathrm{N}}\mathcal{Y}\mathcal{K}_{\mathrm{N}}^T \end{bmatrix} \cdot \nu = \begin{bmatrix} \mathcal{K}_{\mathrm{YA}} & \mathcal{K}_{\mathrm{YB}} \\ \mathcal{K}_{\mathrm{YC}} & \mathcal{K}_{\mathrm{YD}} \end{bmatrix} \cdot \begin{bmatrix} \nu_{\mathrm{VSI}} \\ \nu_{\mathrm{N}} \end{bmatrix}.$$

IV. Compound System Model

After a sequence of matrix operations the following set of equations describes interaction of the VSIs with the grid nodes and can be inserted into the compound grid model of Figure 3, where $\mathcal{N}_{Yx}$ indicates the inverse of the corresponding node incidence sub-matrix $\mathcal{K}_{Yx}$ and $\mathcal{I}_l$ is the unity matrix of size l.

$$\mathcal{M}_A = \mathcal{K}_{YA} + \mathcal{K}_{YB}\left(\mathcal{I}_l - \mathcal{N}_{YC}\mathcal{K}_{YB}\right)^{-1}\mathcal{N}_{YC}\mathcal{K}_{YA}$$

$$\mathcal{M}_B = \mathcal{K}_{YB}\left(\mathcal{I}_l - \mathcal{N}_{YC}\mathcal{K}_{YB}\right)^{-1}\mathcal{N}_{YD}$$

$$\mathcal{M}_C = \left(\mathcal{I}_l - \mathcal{N}_{YC}\mathcal{K}_{YB}\right)^{-1}\mathcal{N}_{YC}\mathcal{K}_{YA}$$

$$\mathcal{M}_D = \left(\mathcal{I}_l - \mathcal{N}_{YC}\mathcal{K}_{YB}\right)^{-1}\mathcal{N}_{YD}$$

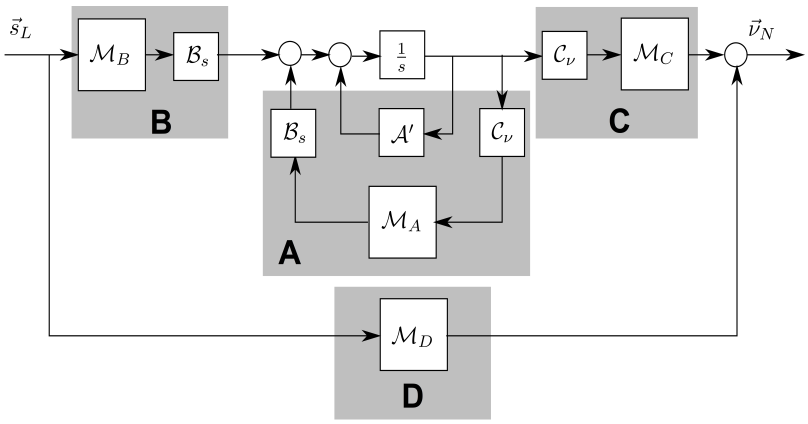

Figure 3. *Compound State Space Model of Inverter Driven Island Grid*

The matrices $\mathcal{B}_s$ and $\mathcal{C}_\nu$ can be moved out of the boundaries of the multi-VSI model across the summation points. The final system state space matrices are

- $\mathcal{A} = \mathcal{A}' + \mathcal{B}_s\mathcal{M}_A\mathcal{C}_\nu$,
- $\mathcal{B} = \mathcal{B}_s\mathcal{M}_B$,
- $\mathcal{C} = \mathcal{M}_C\mathcal{C}_\nu$,
- $\mathcal{D} = \mathcal{M}_D$

and can be used to calculate the Laplace domain transfer function $G(s) = \mathcal{C}\left(s\mathcal{I}_n - \mathcal{A}\right)^{-1}\mathcal{B} + \mathcal{D}$.

Based on this compound model of an inverter driven island grid, stability analysis is performed. Specifically interesting are the instable pole regions and their dependency on inverter parameters like lumped time constant, droop factors for frequency and voltage control or the inverter's rated power. Investigations show instable pole constellations for particular VSI configurations. The model also and particularly allows to investigate the influence of the grid structure (encoded in the node incidence matrix $\mathcal{K}$) on grid stability. Notably in the low voltage grid the line characteristics are no longer of purely inductive but rather inductive-resistive or purely resistive nature, which in turn, influences the inverter's droop control, as it creates a mutual influence of frequency and voltage control.

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