Aging Impact on the pay-as-you-go pension systems: The case of Luxembourg

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Abstract:
Financing of the Luxembourg pension system is based on a pay-as-you-go system and hence on an inter-generational contract. As is the case for most other European countries, this system will be exposed to the effects of demographic aging over the coming decades.
The aim of this paper is to evaluate the impact of this demographic deficit on the long term sustainability of the Luxembourg pension system.

We proceed in two steps.
In a first step, we highlight the evolution of salaries in Luxembourg. To this end, we use the recent statistical group based trajectory model of D. Nagin (Nagin 2005). We estimate model parameters from a single database, provided by the general social security inspection office (IGSS) and containing annual salaries of all wage earners in the Luxembourg private sector. As a result we divide up the population into nine groups, each with its own mean salary trajectory in time and its relative weight in society.

In a second step, we evaluate the pension system by means of a new criterion. This is the sustainability coefficient, which we define as the average amount (in euros) that the labor force has to earn to fund one euro of pension payments, based on current legislation. Our estimations show the high sensitivity of the coefficient to demographic variables and highlight the risks threatening the Luxembourg pay-as-you-go system.
To this aim, we develop a theoretical model consistent with our statistical analysis. This model allows for the determination of both the initial level and the evolution of the pensions for each of the groups identified in the first step of our work. Knowing the weight of each of the groups within the Luxembourg population, we are then able to evaluate the sustainability coefficient of the system. This corresponds to comparing the sum of all incomes of the labor force to that of all the pensions paid to pensioners at a given date while taking into account the growth rate of the population in time.

Finally, we interpret our results in the light of other sustainability criteria, such as the rate of contributions which ensures long-term stability of the pension level under the current legislation or the pension level that current contributions are able to finance in the long run.

1. Introduction

The financial balance of PAYG systems (systems in which the pension of a given period are financed by levies on the earnings of the same period) depends directly on the dependency ratio, that is to say the ratio between the number of pensioners and the number of contributors. Therefore, the aging of the population expected over the next decades (and the arrival at the age of retirement for many generations of baby boomers), because this rate degrades, endangers the long-term sustainability of PAYG systems.

The purpose of this paper is to propose an assessment of the impact of this demographic deficit on the long-term sustainability of the pension system in Luxembourg. We proceed in two steps.
Initially, we highlight the curves of wage developments in Luxembourg. To achieve this, we use a statistical method based on recent developmental trajectories (Nagin 2005). We estimate the parameters of statistical model from a single database, provided by the IGSS, including annualized salaries of all employees in the private sector in Luxembourg. We get nine tracks and averages of wage developments and the weight of each of them into society.

In a second step, we propose an evaluation of the Luxembourg pension system from a new criterion: the coefficient of sustainability. This is the number of Euros that the assets must earn on average to ensure every dollar of retirement with the legislation. Our estimates show the high sensitivity of the coefficient for demographic variables, thus highlighting the risk to the PAYG system in Luxembourg.

To do this, we develop a theoretical model consistent with statistical analysis. This model allows to determine

the initial level and changes in pension for each group identified the first step. Knowing the weight of each of these groups within the population of Luxembourg, then we can assess the sustainability factor of the system, ie comparing the sum of all wages earned by the assets with the sum of all pensions paid to retirees on a given date, based on the rate of population growth over time.

We then interpret our results in light of other criteria of sustainability, as the contribution rates to ensure long-term level of benefits to current law or alternatively, the level of benefits that can be supported long term by the current level of contribution.

Section 2 presents the main characteristics of the general pension scheme in Luxembourg. Section 3 analyzes the trajectories of wages in Luxembourg organized around common patterns. Section 4 develops a model to assess the long-term sustainability of the pension system luxemborgeois according to population change.

2. The pension system in Luxembourg

The statutory pension system in Luxembourg is composed of a general scheme for private sector employees and non employees and a special scheme for civil servants. The system includes old age pensions, invalidity and survivors. In what follows, only the general scheme will be considered and special interest will be paid to old age pensions.

Funding for the general scheme is based on an allocation system for periods of 7 years with mandatory formation of a minimum reserve equal to 1.5 times the amount of annual benefits. The contribution rate is fixed at the beginning of each period of coverage to ensure funding of the scheme throughout the period. The rate is 24% of assessable payroll, split evenly between workers, employers and the state. A new period of coverage was initiated in 2006. The compensation reserve amounts to EUR 8 046.4 million at December 31, 2007 and represents 3.42 times the amount of annual benefits. As a result, it will ignore this subject, which is a compensation reserve and can not be understood as a capital coverage guarantee of future benefits.

The net replacement rate after a full career in this regime is one of the highest in Europe[1]: The net replacement rate withholds amounts of pensions and wages net of social contributions. It reached nearly 100% (exactly 96%) of average pre-retirement for a worker on average earnings who have contributed for 40 years (IGSS 2009).[2]

3. Analysis of the salary trajectories

In order to evaluate Luxembourgish pension system, we need the average trajectories of the salaries in this country. To this aim, we apply a recent method based on trajectories of development (3.1). We use a SAS procedure called Proc Traj to estimate model parameters from a database containing the annual salaries of all the employees of the private sector in Luxembourg (3.2). We thus obtain a regouping of the salary trajectories around common patterns (3.3).

3.1 A statistical method based on clustering

Longitudinal data are the empirical basis of research on various subjects in the social sciences and in medicine. The common statistical aim of these various application fields is the modelisation of the evolution of an age or time based phenomenon (Nagin 2002). In the 1990s, the generalized mixed model assuming a normal distribution of unobserved heterogeneity (Bryk and Raudenbush 1992), latent growth curves modeling (Muthén 1989, Willett and Sayer 1994) and the nonparametric mixture model, based on a discrete distribution of heterogeneity (Jones, Nagin and Roeder 2001) have emerged. We choose this variation of the generalized mixed model because of the growing interest in this approach to answer questions about atypical subpopulations (see Eggleston, Laub and Sampson 2004).

[1] The net replacement rate withholds amounts of pensions and wages net of social contributions.

[2] According to simulations made by AXA Luxembourg (2008), it appears that the pension level is comfortable (higher replacement rate to 75% of last wage) for workers with full careers and having a normal salary in - below the contribution limit. In contrast, employees with high income (above the contribution limit and those without full career in Luxembourg (mixed career abroad and Luxembourg or women who have interrupted their careers for family reasons) the amount of retirement may be insufficient to maintain living standards achieved.

The nonparametric mixed model is also called semiparametric mixture model (Nagin 1999) or latent class analysis for growth curves (Muthén 2001). It is designed specifically to detect the presence of distinct subgroups among a set of trajectories and represents an interesting compromise between analysis around a single mean trajectory and case studies (Von Eye and Bergman 2003).

Like many time-varying economic data, incomes and pensions can be collected in longitudinal research designs if they are measured in fixed timpepoints. They can be part of hierarchical models where individuals belong to sub-groups themselves included in one or several groups. There are hence to aspects to pensions. On the one hand, the pension of an individual depends on his income trajectory (intra-group variability), but on the other hand it also depends on his profession, because his profession links him to a part of society (inter-group variability).

Compared to subjective classification methods, the nonparametric mixed model has the advantage of providing a formal framework for testing the existence of distinct groups of trajectories. This method does not assume a priori that there is necessarily more than one group in the population. Rather, an adjustment index is used to determine the number of sub-optimal groups. This is a significant advance over other categorical methods which determine the number of groups only subjectively (Von Eye and Bergman 2003). Moreover, this method allows to evaluate the accuracy of the assignement of the individuals to the different sub-groups and to consider the variation of this accuracy in subsequent analyses (Dupere et al. 2007).

The SAS procedure Proc Traj, programmed by Daniel Nagin and Bobby Jones, allows to estimate the parameters of a semiparametric mixture model for longitudinal data that are either normal (censored) distributed or follow a Poisson or Bernoulli distribution. The subgroup tracetories can be modelled by polynomials up to the fourth degree. The procedure enables to calculate the posterior probability of group membership in terms of risk factors that are stable in time. Moreover, time-dependent covariates can influence the trajectories and cause different effects in different subgroups.

Nagin's nonparametric mixed model starts from a set of individual trajectories and tries to divide the population into a number of homogeneous sub-populations and to estimate a mean trajectory for each of these sub-populations.

Consider a statistical variable Y defined on a population of size N. Let $Y_i = y_{i1},\ y_{i2},...,\ y_{iT}$ denote a longitudinal sequence of measurements on individual i over T periods.

Let $P(Y_i)$ dennote the probability of Y_i. For count data $P(Y_i)$ is specified as the Poisson distribution, for censored data it is specified as the censored normal distribution, and for binary data it is specified as the binary logit distribution. In our application, we assume that it follows a censored normal distribution.

The purpose of the analysis is to find r trajectories of a given type, in general polynomials of degree 4, $P(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \beta_4 t^4$.

Let $P^j(Y_i)$ denote the probability of obtaining the observed data for individual i given membership in group j and π_j the probability of an individual chosen at random to belong to the group number j. Hence π_j is the size of group j.

We try to estimate a set of parameters $\Omega = \{\beta_0^j,\ \beta_1^j,\ \beta_2^j,\ \beta_3^j,\ \beta_4^j,\ \pi_j;\quad j = 1,...,\ r\}$ which maximises the probability of Y_i is maximal. The ideal number of groups r is also an outcome of the analysis. For a given group, conditional independence is assumed for the sequential realisations of the elements of Y_i, y_{it}, over the T periods of measurment. The likelihood L of the sample is then given by

$$L = \frac{1}{\sigma}\prod_{i=1}^{N}\sum_{j=1}^{r}\pi_j\prod_{t=1}^{T}\phi\left(\frac{y_{it} - \beta^j x_{it}}{\sigma}\right),$$

where ϕ denotes the density function of the standard normal distribution. These equations are too complicated to hope to obtain an algebraic solution. Bobby L. Jones (Carnegie Mellon University) has programmed a SAS

procedure based on a quasi-Newtonian maximum search method (Dennis, Gay & Welsch 1981). The estimated standard deviations are obtained by inverting the observed information matrix.

One problem is that numerical estimates of probabilities of group membership π_j must be a number between 0 and 1. This is a difficult constraint when using iterative approximation procedures. Hence the problem is circumverted by setting

$$\pi_j = \frac{e^{\theta_j}}{\sum_{j=1}^{r} e^{\theta_j}},$$

where θ_j are real parameters. As the sum of all the π_j is equal to 1, it suffices to estimate *r-1* parameters θ_j. By convention, we set $\theta_1 = 0$.

The final form of the likelihood is hence equal to

$$L = \frac{1}{\sigma} \prod_{i=1}^{N} \sum_{j=1}^{r} \frac{e^{\theta_j}}{\sum_{j=1}^{r} e^{\theta_j}} \prod_{t=1}^{T} \phi\left(\frac{y_{it} - \beta^j x_{it}}{\sigma} \right).$$

A hard part of the problem is selecting the right model, which means primarily the selection of the optimal number of trajectory groups. One possibility to do is to use the likelihood ratio test. Unfortunately, the null hypothesis of this test is on the border of the parameter space which invalidates the asymptotic results (Ghosh & Sen 1985). Because of this, the mathematical criterion generally used is the Bayesian Information Criterion BIC defined by

$$\mathrm{BIC} = \log(L) - 0{,}5k \log(N),$$

where *k* denotes the number of parameters in the model. The difference of the BIC between two models can then be used as an approximation of the logarithm of the Bayes factor (Kass & Wasserman 1995). Keribin (1997) shows that, under certain conditions this approximation is valid to test the number of components in a mixture model. Finally, we chose the model with the highest BIC meaning the one with the BIC closest to zero.

Nagin's model also allows to determine to wich group a given individual belongs. The posterior probability $P(j / Y_i)$ for an individual *i* to belong to group number *j* is indeed given by the Bayes theorem:

$$P(j / Y_i) = \frac{P(Y_i / j)\hat{\pi}_j}{\sum_{j=1}^{r} P(Y_i / j)\hat{\pi}_j}.$$

A large posterior probability estimate for a small group requires that Y_i be so strongly consistent with the small group that $P(Y_i / j)$ for that group is very large in comparison to its companion probabilities for the big groups (Nagin 2005).

3.2 The IGSS database

The analysis relies on a file containing the salaries of all employees of the private sector in Luxembourg. The data cover the period from 1940 to 2006. Since the file contains the careers of those started to work from the beginning of the forties onwards, it is not complete during the first years, but becomes so gradually. In particular it includes all the employees of the private sector in Luxembourg from the beginning of the seventies till 2006.

This file originates from the General Inspectorate of Social Security (Inspection générale de la sécurité sociale - IGSS). The main variables are the net annual taxable salary, measured in constant euros (2006 euros), sex, age at first employment, residence and nationality (Luxembourg national living in Luxembourg, foreigner living in Luxembourg and forntalier) and the type of employment contract (blue or white collar worker). Initially, the file consisted of about 7 000 000 lines showing the salaries of some 718 054 workers. Each line gave the annualized salary of a worker. We converted the initial file into a file with 718 054 lines with the wages from 1940 to 2006 as variables. Most lines contained many zeros since nobody has worked all the time from 1940 to 2006. In addition, many careers are incomplete for many reasons. Moreover, for immigrant workers, we know only the part of their careers made in Luxembourg and know nothing about what they have done in their country of origin. Finally, the percentage of employees who quit prematurely with a disability pension or take pre-retirement or quit early for family reasons (women stopping work or interrupting their work to look after their children for example) is around 50 per cent.

The domestic employment (which includes the frontaliers étrangers working in Luxembourg) has experienced strong growth since the mid-eighties, with an average increase of 3.5% annually and an increase of more than 110 000 jobs between 1986 and 2001 (compared to 20 000 jobs in the period 1975-1985) (Source: STATEC). The development of the financial and the growing needs of the public sector are key drivers of this evolution. Today, the services sector represents more than three-quarters of total employment. These changes are not without consequences in terms of professional status, so that changes in careers before the 80s are necessarily significantly different from those of twenty-five years.
We have therefore decided to pay interest careers of individuals who began working in Luxembourg between 1982 and 1986 and who work for at least 20 years. Using macros programmed in Mathematica, we selected persons who meet these criteria. In our file were 487 052 persons who started to work in Luxembourg at the earliest in 1982. The final file used for our analysis includes data from 22 203 employees and private workers. That all persons who started work after 1982 and who have worked for at least 20 years in the private sector. Noted that in Luxembourg, the maximum contribution ceiling on pension insurance is 5 times the minimum wage, or 7 577 € (Euro 2006) per month. Wages in our data are also limited by that number.

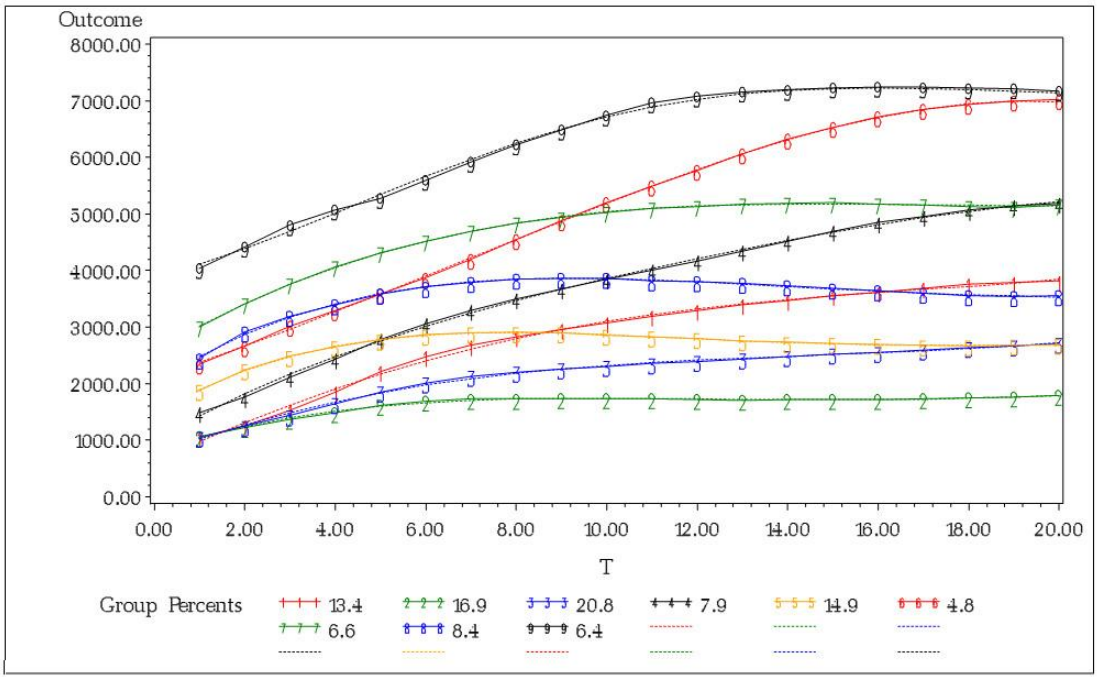
3.3 The mean salary trajectories in Luxembourg

We used the SAS procedure Proc Traj, programmed by Daniel Nagin and Bobby Jones, to determine the mean salary trajectories of 22 203 people who began working between 1982 and 1987 in the private sector in Luxembourg and who worked for at least twenty years.

We established the trajectories for models with between 4 and 20 groups. As the salary trajectories form more or less a continuum in the continuous functions from [1000, 4000] with values in the interval [1200, 7577], the BIC adjustment criterion for determining the ideal number of groups is not well suited. Indeed, BIC increases with the number of groups. This is quite normal, since it just shows that if one assumes more groups, one can necessarily represent reality with more details. On the other hand, one creates smaller groups and an explanatory model more complicated to use. After discussion with the IGSS, we decided to retain a 9 groups solution, for it gives a good representation of the career development in Luxembourg. Solutions with more groups add essentially parallel paths to those present in our model.

To test the stability of trajectories in time, we also established the trajectories for the first 15 years of the careers (careers starting between 1985 and 1992) and for full career of 40 years (careers starting between 1960 and 1967). The trajectories of the first 15 years are very close to the first 15 years of trajectories of 20-year careers. The sizes of the groups vary between 2 and 6% compared to the ones we found and the changes are due to gains or losses to groups with similar salaries. Since moreover the macroeconomic situation has not changed dramatically during the last twenty years, we are fairly confident that the trajectories remain valid, except in cases of severe economic shock that could certainly change the situation completely. The only thing that might change over the coming years is the percentage of frontier workers in the different groups. Since the total number of employees increased faster than the population can do this percentage will continue to grow in all groups. The trajectories of the complete careers are also quite similar to ours, except that they show a clear decline of the wages during the steel crisis for the trajectories representing the high wages.

The chart below shows the average salary trajectories in our 9 groups.



4. The pension model

In a general context where both the number of contributors and pensioners varies significantly, it is important to develop a model to estimate the urgency and effectiveness of reforms of the Luxembourg pension system. This model must allow for the identification of both roles and dependencies of all system parameters: duration and rates of contribution, life expectancy, demographic changes, replacement rate regime, saving rates, etc.. At the same time, it must also allow for a faithful representation of these variables and of the links between them. Finally, it must be as simple and realistic as possible.

To meet these requirements, we have calibrated wage trajectories using the Proc Traj procedure in SAS (4.1). Then we formulate assumptions concerning the evolution of relevant variables of the model (4.2).

4.1 Using the salary trajectories

By highlighting nine paths of wages and being able to assign each weight into society that it deserves, we have a stable tool in time for a file based on twenty years of Wages and freed of any individual approach, because the trajectories are collective. We can both get the results on average, have a measure of dispersion, dynamically analyze situations while having the opportunity to incorporate new parameters.

The sizes of the different groups as a percentage of the total population are:

Group 1	Group 2	Group 3	Group 4	Group 5
$p_1 = 13.4\%$	$p_2 = 16.9\%$	$p_3 = 20.8\%$	$p_4 = 7.9\%$	$p_5 = 14.9\%$
Group 6	Group 7	Group 8	Group 9	
$p_6 = 4.8\%$	$p_7 = 6.6\%$	$p_8 = 8.4\%$	$p_9 = 6.4\%$	

The nine curves highlight nine average rates of wage growth λ_i, $i = 1,...,9$: Each λ_i is determined from the trajectory $y_i = P_i(t)$. If β_i is the slope of the least squares line through the cloud of points $(t, \ln P_i(t))$ then $\lambda_i = \exp(\beta_i) - 1$.

Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
$\lambda_i = 3.07\%$	$\lambda_2 = 0.96\%$	$\lambda_3 = 1.45\%$	$\lambda_4 = 2.82\%$	$\lambda_5 = 0.19\%$
Curve 6	Curve 7	Curve 8	Curve 9	
$\lambda_6 = 2.58\%$	$\lambda_7 = 1.28\%$	$\lambda_8 = 0.48\%$	$\lambda_9 = 1.09\%$	

4.2 Model assumptions

Each salary curve models the evolution of a proportion p_i of the workforce. This development follows a growth rate λ_i of its own and leds to a curve of pensions for a proportion $p_i^{'}$ retirees. Assume that $p_i = p_i^{'}$ and that all pension curves grow *r* % every year (the government decides the adjustment every two years).

The population size (active and retired) changes each year to keep pace with demographic rate of intergenerational income (for details on this rate, see Box 1 below). It consists of individuals who worked after *T* years have a life expectancy of *S* years to retirement.

We will assume that *T = 40*, *S = 20* and *d* follows a uniform law on the interval [0%, 4%].

Box 1. Evolution of the demographic intergenerational rate

At the moment *t*, there are n_i individuals who have started working *i* years ago. They form a cloud of points $(i, \ln(n_i))$ which is approached by the regression line $\ln(n_i) = a + bi$.
Writing

$$N_0 = \exp(a)$$

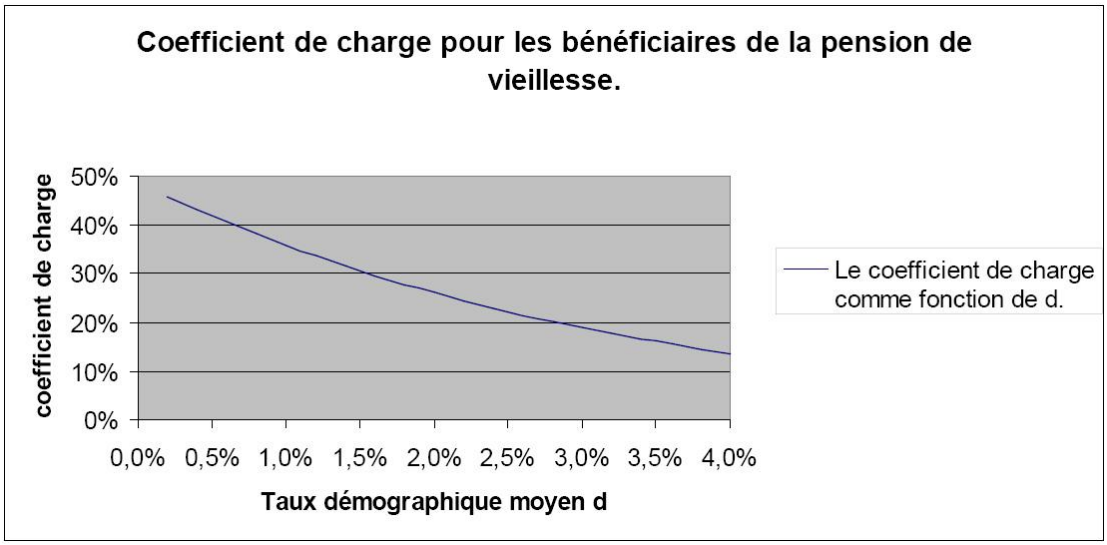
$$d = \exp(-b) - 1$$

gives the best approximation of the real population $(n_0, ..., n_{T+S})$ by the modelised population $(N_0, ..., N_{T+S})$.
We define the number *d by*

$$N_i = \frac{N_0}{(1+d)^i}.$$

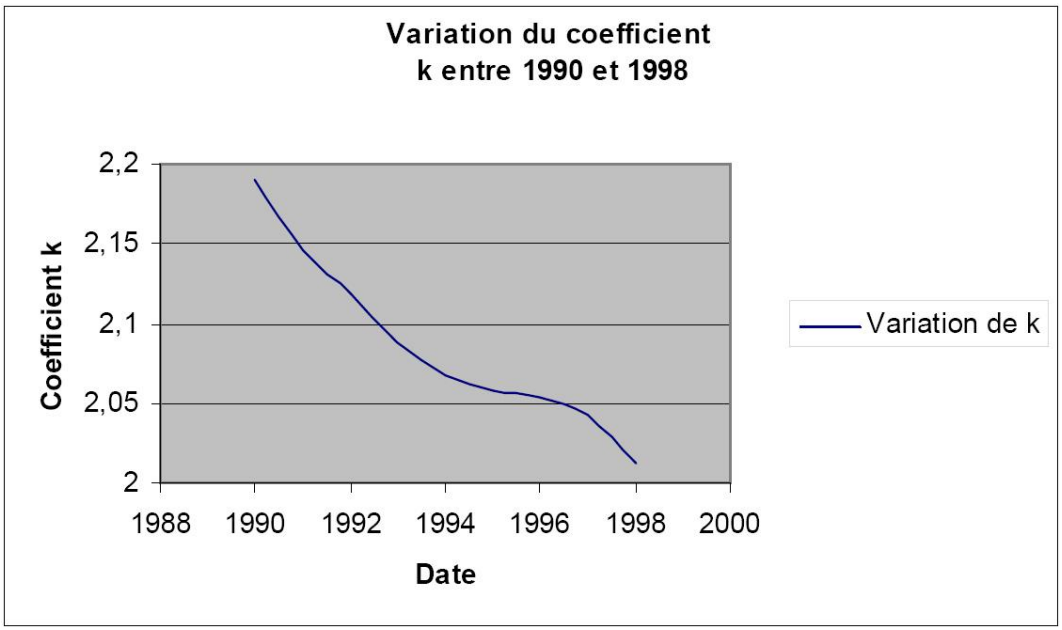
d is then directly linked to the active structure of the country and seems to be strongly correlated to economic indicators like GDP.

Under these assumptions, the number of pensioners per 100 contributors, called the load factor depends explicitly on *d* as shown in the figure below.



If *d* is very close to 0, there are about two times more active than retirees, so that if *d* is the order of 4%, there are seven times more. Note however that the variations of *d* can be only gradual. Indeed, each estimate of *d* is a photograph of the society in which sixty years separate the new from old.

Each year, the number of active volume changes but the structure of the fifty-nine years remains the same: the incoming calls are now one year of age, who had one year of age have two, etc.. After twenty years, two-thirds of the workforce is always that of the start ... unless one accepts the idea of a great variability from one year to another (major changes this year mortality, return to the country ....).



Study the method of financing a pension plan is tantamount to connect within the same balance the inflow of contributions and the outflow of benefits. Historically, Luxembourg is in a logic of division. This is not a pure their spread as a reservation has been made in recent years. However, we do not take into account in our reasoning, because all studies (International Labor Office in Geneva, Banque centrale du Luxembourg, General Inspectorate of Social Security) claim that the money invested will be insufficient to ensure the sustainability of system. They provide, under the assumptions of conventional economic growth, in thirty years, the system does have more reserves.

4.3 Analysis of the pay-as-you-go system

As part of our work, we developed a new indicator of sustainability of the pension division: the sustainability coefficient τ_1 (4.3.1). Then, we evaluated τ_1 for the part of the plan Luxembourgish based on distribution, taking into account a mean intergenerational population growth between 0 and 4% (4.3.2).

4.3.1 Definition of the sustainability coefficient

The principle of distribution prevailing in the Grand Duchy is a principle of solidarity between generations. The pensions paid to retirees are directly financed by contributions levied on earnings assets. In this approach, it is essential to assess the regime's ability to meet the payment of future pensioners. For this, we must at all times compare cash flows generated by the contributors and those for retirees. The sustainability coefficient is a criterion for this comparison.

By definition, the sustainability coefficient of a pay-as-you-go system τ_1 is the sum of all salaries of assets divided by the sum of all pensions paid to retirees at time *t*. For every dollar of pension τ_1 euros are earned in salary. Moreover, τ_1 is a random variable whose values depend on the demographic structure of the country at time *t*.

4.3.2 Evaluation of τ_1 as a function of *d*

Under our assumptions, the workforce is changing year after year length of service at the rate of population growth for intergenerational income of between 0 and 4%. The SAS procedure Proc Traj allows us to estimate the average wage S_i an active individual with *i* years of age and the average pension P_j of a retiree who began working *j* years ago. We infer the sum of all wages paid at the time *T* in the country:

$$N_a = N_0 S_0 + ... + \frac{N_0 S_T}{(1+d)^T}.$$

The calculation of *d* for the year 1998, for example, leads to *d* = 3.4%.

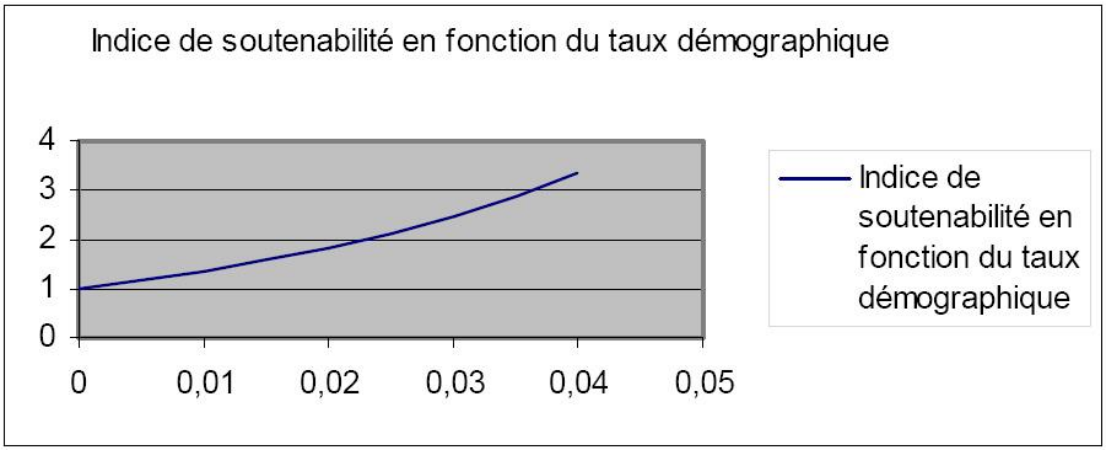
In general, these contributors that we have followed over time through the file IGSS benefit of pension. It is therefore assumed in the insurance old age pension that population growth is that intergenerational assets. To take account of other pension plans, we bring a multiplier *k* for calculating the sum of all pensions granted on the date *T*:

$$N_p = \frac{kN_0}{(1+d)^{T+1}} P_{T+1} + ... + \frac{kN_0}{(1+d)^{T+S}} P_{T+S}.$$

As we said before, k is a new parameter of the problem we assume constant for simplicity. Its estimate from data in our possession led to the average value k = 2.

We can then see that τ_1 is an increasing function of d. More rate demographic structure the country is more powerful the system is sustainable.

$$\tau_1 = \frac{N_a}{N_p} = \frac{S_0 + ... + \frac{S_T}{(1+d)^T}}{\frac{k}{(1+d)^{T+1}} P_{T+1} + ... + \frac{k}{(1+d)^{T+S}} P_{T+S}}$$



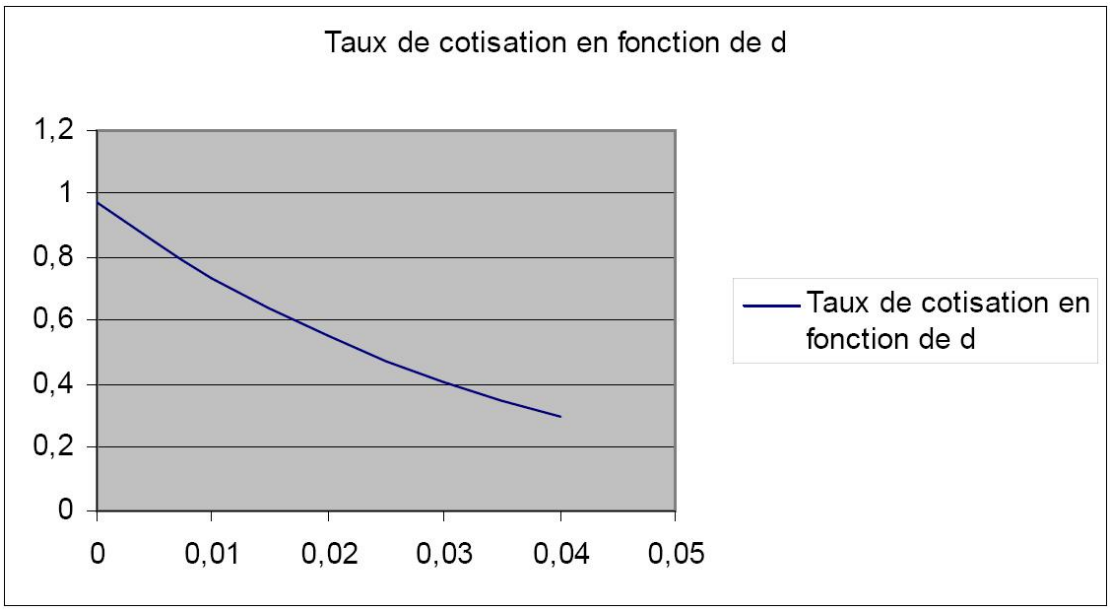
It is easy to see that approximately $E(\tau_1) = 2$ and $\sigma(\tau_1) = 0.8$. On average, an employee wins 2€ for each euro of pension paid.
The coefficient of variation is the ratio of standard deviation to the mean

$$CV(\tau_1) = \frac{0.8}{2} = 40\%$$

shows that the dispersion around this mean is large.

This is confirmed by the graph above. Between a situation in which the rate of population is low and where there is high, the number of euros earned by contributors to ensure every dollar of retirement can be multiplied by three. This large variability in τ_1 is evidence of danger to the pension system. Not providing a euro pension is easy when d is large so this quickly becomes untenable when d is low. In particular, when d becomes less than 1%, τ_1 approaches the value unity and balance is almost more viable.

The rate of pension contributions currently stands in Luxembourg to 24% of the contributory basis, this charge is divided into three equal parts of 8% between state employees and employers.



The chart above, drawn with the value k = 2, shows that for a fixed horizon to 2040 (we pay for the 80 that extends about 20 +20 = 40 for an individual to be certain of 'be paid even at the end of retirement) and an average growth rate of intergenerational 3% (which is our baseline figure calculated in 1998), the balance between wages earned and pensions requires a rate of contribution of 40%!

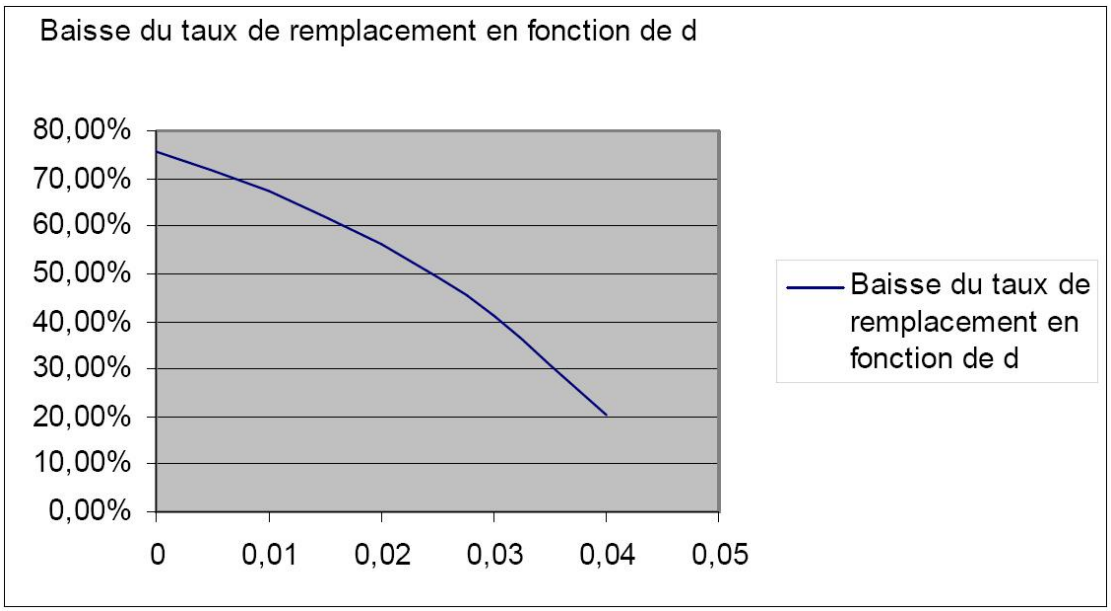
Another way to see the problem is that if the contribution rate c_1 necessary balance is greater than that required by law ($c_0 = 24\%$), then we must reduce benefits by a factor of γ how to have a ratio of pension flows wages as

$$P_1 = \gamma P_0 \text{ et } \frac{P_1}{S} = c_0.$$

Thus we find:

$$\frac{\gamma P_0}{S} = c_0 <=> \gamma c_1 = c_0 <=> \gamma = \frac{c_0}{c_1}.$$

With the same assumptions (k = 2, d = 3%, ₀ c = 24%) the reduction γ of the pensions that allows an equilibrium of the flows is 40%.



Thus, although the system is well managed, he runs a significant risk. With an uncertain number of contributors and an influx of new rights-holders, the pay-as-you-go system could be less favorable than today. Faced with this situation, what to say to people tempted to improve their pension? While it is possible to save individually, but then how to decide how much to capitalize?

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