FACULTY OF LAW, ECONOMICS AND FINANCE

Department of Economics and Management

Discussion Paper

2026-10

Economics

Department of Economics and Management
University of Luxembourg

Is Carbon Capture and Storage Socially Desirable under a Net-Zero Emission Target?

https://www.uni.lu/fdef-en/research-departments/department-of-economics-and-management/publications/

Yiwen Chen, Shandong Agricultural University, Taiwan
Nora Paulus, DF, University of Luxembourg, LU
Xi Wan, DEM, University of Luxembourg, LU
Benteng Zou, DEM, Université du Luxembourg, LU

May 2026

For editorial correspondence, please contact: dem@uni.lu
University of Luxembourg
Faculty of Law, Economics and Finance
4, Rue Alphonse Weicker
L-2721 Luxembourg

UNIVERSITÉ DU LUXEMBOURG

The opinions and results mentioned in this paper do not reflect the position of the Institution

Is Carbon Capture and Storage Socially Desirable under a Net-Zero Emission Target? *

Yiwen Chen † Nora Paulus‡ Xi Wan§, Benteng Zou¶

May 23, 2026

Abstract

Achieving Net-Zero emissions by 2050, in line with the Paris Agreement, has emerged as a binding constraint for climate policy, yet most dynamic models treat carbon neutrality as an endogenous outcome. This paper studies the role of Carbon Capture and Storage (CCS), as a representative case of abatement under terminal constraints, when the deadline for reaching the net-zero target (T) is exogenously fixed. We derive a closed-form welfare criterion that compares CCS-based and renewable-only transition pathways through the difference in inherited pollution damages at T. This criterion delivers policy-relevant results along three dimensions: (i) the net-zero date; (ii) learning-by-doing effects on CCS cost trajectories; and (iii) storage capacity requirements needed for CCS to remain welfare-desirable. Using a linear–quadratic framework calibrated to different target dates, we show that CCS dominates a renewable-only pathway whenever its transition period welfare gains are large enough to offset the higher post-transition climate damages it generates relative to renewables. We therefore derive a general welfare criterion linking transition emissions to terminal carbon stock under an exogenously imposed net-zero constraint. Our results provide decision-relevant thresholds for net-zero planning, complementing recent work on the timing of CCS deployment and the feasibility of negative emissions.

Keywords: CCS, renewable energy, Pollution abatement, Irreversible pollution control, net-zero emission target

JEL classification: Q54, Q58

*Yiwen Chen acknowledges support from Shandong Provincial Natural Science Foundation (ZR2025QC751) and from the *Research Excellent Scheme* while visiting the University of Luxembourg. We benefit from the discussions with Raouf Boucekkine and Weihua Ruan.

†School of Economics and Management, Shandong Agricultural University. 61 Daizong Avenue, Tai'an, China. E-mail: yiwen.chen@sdau.edu.cn.

‡Corresponding author. Department of Finance, University of Luxembourg. 6, rue Richard Coudenhove-Calergi, L-1359, Luxembourg. E-mail: nora.paulus@uni.lu.

§International Joint Audit Institute, Nanjing Audit University, 86 West Yushan Road, 211815 Nanjing, China. E-mail: xiiwan@gmail.com.

¶Department of Economics and Management, University of Luxembourg. 6, rue Richard Coudenhove-Calergi, L-1359, Luxembourg. E-mail: benteng.zou@uni.lu.

1 Introduction

Many countries have adopted net-zero targets with explicit, legally binding deadlines as part of their climate action plans. Net zero refers to the balance between the amount of greenhouse gases emitted into the atmosphere and the amount removed, effectively achieving a state where no new emissions are added to the atmosphere. Sweden and Germany aim for carbon neutrality by 2045; France, Denmark, Spain, Hungary and Luxembourg have committed to 2050; China and Brazil target 2060. Under such binding commitments, a central question is which mitigation strategies can achieve net-zero at the lowest social cost.

Carbon capture and storage (CCS) is frequently proposed alongside renewable energy as a viable mitigation option, especially for hard-to-abate sectors and for the reduction of stranded asset risk (Clark and Herzog 2014; Welsby et al. 2021)[1]. Yet its role remains contested in both the academic literature and policy debates. Proponents argue that CCS can offset residual emissions where direct abatement is prohibitively costly or technically infeasible. Critics, however, contend that its economic viability is undermined by high capital and operating costs, technological uncertainties, and the potential carbon leakage from storage sites (Bacilieri et al. 2023; Van der Zwaan and Gerlagh 2009). A further concern is that CCS investment may crowd out renewables and reinforce fossil fuels dependence, thereby delaying the transition to a low-carbon economy (Vergragt et al. 2011).

Against this background, this paper asks whether, under an exogenously imposed and binding net-zero target, CCS constitutes a more effective pathway to achieving net zero compared to an equally costly renewable energy alternative. Taking the net-zero date as a strict policy constraint, we derive an explicit welfare criterion to compare a fossil-fuel-with-CCS pathway against a renewable-only transition. The comparison depends on the trade-off between production gains during the transition and environmental damages at the net-zero date. We show that CCS can be welfare-enhancing even when it generates higher gross emissions prior to the deadline, and examine how the timing of the target, cost parameters, and storage capacity determine this result.

The key reason is as follows. Welfare consists of production benefits, environmental damage, and abatement (CCS) or transition (renewables) costs, both during and after the transition period. Assuming homogeneous cost structures and post-transition production output for both pathways, post-transition welfare depends solely on environmental damage, which is determined by the inherited carbon stock at Net Zero (carbon stock declines only through natural absorption thereafter). Thus, the welfare ranking reflects a combination of transition-period production benefits (output), transition-period environmental damage, and inherited carbon stock at Net Zero — all depending on net emissions. For the CCS-based pathway, the decision maker chooses both transition-period production (hence gross emission) and abatement, for the renewables pathway, only transition-period production (and gross emission).

Two effects are at play. First, the net emission effect: the accumulated stock of carbon determines environmental damage during the transition period and, upon reaching the net-zero date, is carried over as the inherited carbon stock that continues to drive damage in the post-

[1] According to Welsby et al. (2021), 60% of the oil and gas reserves and 90% of the known coal reserves should not be extracted in order to achieve the net-zero target. In addition, the related infrastructure, as well as workers with technology-specific skills in those industries will be affected (Bos and Gupta 2018; Van der Ploeg and Rezai 2020).

transition period. Second, the economic benefit effect: higher production, thus higher carbon emission as a byproduct, yields greater economic benefits. If production and abatement yield lower transition-period net emissions than under the renewables pathway, overall environmental damage falls. However, the economics benefit effect may reinforce or counteract this positive net emission effect, making the combined effect ambiguous and dependent on the relative strengths of the two effects.

Our analysis is related to three, partly complementary strands of the literature. The first examines optimal transition paths to net-zero, asking how carbon neutrality can emerge as a long-run equilibrium outcome (Hepburn et al. 2025; Dolphin et al. 2023; Hoel 2025). The most closely related paper is Hoel (2025), who analyzes the role of carbon dioxide removal (CDR) technologies, including CCS, in achieving net-zero emissions, and shows that negative emissions may be optimal when storage capacity is sufficiently large. Our paper differs from Hoel (2025) in two ways. First, while net-zero emissions arise endogenously as an outcome of the social optimum in his framework, we instead impose a strict and exogenous net-zero deadline T to reflect different legally binding commitments. Net zero enters our model as a terminal condition on net emissions, which directly governs the pollution stock inherited at date T and links transition-period dynamics to post-transition environmental damages. Second, whereas Hoel (2025) focuses on the interaction among fossil fuel extraction, CDR, and storage capacity in shaping long-run outcomes, we provide a welfare comparison across three scenarios: a benchmark without net-zero, net-zero achieved through the CCS pathway, and net-zero achieved through the renewable-only pathway (complete fossil fuel phase-out). Using a tractable linear–quadratic framework, we derive closed-form solutions and a welfare decomposition into transition-period benefits and post-transition damages.

A second strand examines the economics of CCS and carbon removal, focusing on whether and when CCS should be deployed under alternative cost, learning, storage conditions, and market structure (Lafforgue et al. 2008; Ayong Le Kama et al. 2013; Grimaud and Rouge 2014; Moreaux and Withagen 2015; Amigues et al. 2016; Moreaux, Amigues, et al. 2024)[2].

A third strand compares CCS with alternative mitigation strategies, particularly renewable energy pathways. This literature shows that the relative attractiveness of CCS over renewables depends on the policy environment, technology costs, and the structure of the energy transition. CCS and renewables can either act as substitutes or complements, depending on how policy incentives and decarbonization constraints are specified (Fikru et al. 2024; Holz et al. 2021; Moreaux, Amigues, et al. 2024).

Building on this literature, our main contribution is to examine whether the CCS pathway is socially desirable under an exogenously imposed net-zero target and to evaluate its welfare relative to a renewable-only pathway. Instead of focusing on the optimal timing of CCS, we analyze how a fixed transition date changes the economic role of CCS, considering variations in deployment costs, learning effects, and storage constraints. Additionally, we conduct a comparative analysis of a benchmark scenario without a fixed net-zero target, allowing us to explore the implications of transitioning to a low-carbon economy without an exogenously imposed deadline.

Our main results can be summarized as follows. First, we establish an explicit welfare criterion

[2]See Chen et al. (2024) for a detailed review.

for when CCS is socially preferable to a renewable-only path. This criterion compares transition-period welfare gains from continued fossil energy use under CCS to the discounted difference in post-transition environmental damages, which depends solely on the inherited carbon stock at net zero. Second, we show that CCS can be welfare-enhancing even when it raises gross emissions prior to net zero. CCS allows the society to sustain higher fossil-energy use during the transition while reducing net emissions, thereby lowering the carbon stock at net zero. Since post-transition damages depend solely on this stock, which then declines through natural decay, a lower stock implies permanently lower future damages. This gain can outweigh the upfront cost of CCS. Third, we examine three policy-relevant dimensions. The timing of the net-zero deadline governs the trade-off between near-term production and discounted future damages: Later targets favor economies that weight near-term production heavily, while earlier transitions are preferred when climate damages are large or future welfare weighted more. Learning-by-doing lowers CCS costs and expands the range over which it is welfare-desirable. In contrast, binding storage constraints raise effective costs and can eliminate its advantage below a critical capacity threshold. Fourth, numerical results show that a later net-zero date raises the inherited stock but can increase cumulative welfare by extending transition-period production. Lower CCS costs reduce the stock and future damages, strengthening its welfare advantage. Together, these results identify when, given target timing, costs, and storage, CCS outperforms a renewable-only strategy in intertemporal welfare.

The remainder of the paper is organized as follows. Section 2 presents the benchmark economy without a net-zero constraint. Section 3 introduces the net-zero target, analyzes optimal abatement strategies, and compares the impact of CCS and renewable-only pathways on CO_2 accumulation and welfare. Section 4 develops welfare criteria for assessing when the CCS pathway is socially desirable under an exogenously imposed net-zero deadline while incorporating policy-relevant extensions, including timing, learning, and storage limits. Section 5 provides a numerical analysis, illustrating how net-zero timing and CCS cost dynamics influence both the transition period pollution stock and intertemporal social welfare. Section 6 concludes.

2 The economy without a net-zero constraint

2.1 Model setup in the unconstrained regime

We begin by analyzing an economy without a binding net-zero constraint, in which the society faces no exogenously imposed deadline for achieving zero net emissions. Consequently, there is no terminal constraint on either emissions or the pollution stock, and emissions remain a control variable for all $t \geq 0$. We consider two cases: a benchmark economy without CCS and an alternative economy with CCS. Our core research questions are: Does the availability of CCS reduce long-run atmospheric pollution accumulation?[3] What are the welfare implications of CCS relative to a scenario without its deployment? This section provides a reference point against which the implications of an exogenously imposed net-zero constraint will be evaluated in subsequent sections.

[3] Throughout the paper, CCS is representative example of carbon dioxide removal (CDR) technologies. The analysis of CCS in this paper applies equally to other CDR options discussed in Hoel (2025), such as direct air capture, provided that they reduce net emissions without directly enhancing productivity.

The decision maker maximizes social welfare by optimally choosing the emission rate $y(t)$ and the abatement level $x(t)$ at each time t. The objective function can be written as:

$$\begin{aligned} W &= \max_{x,y\geq 0} \int_0^{+\infty} (U(y) - D(z) - B(x))e^{-rt}dt \\ &= \max_{x,y\geq 0} \int_0^{+\infty} \left[\left(a_1 y - \frac{a_2 y^2}{2} \right) - \frac{c\, z^2}{2} - \left(b_1 x + \frac{b_2 x^2}{2} \right) \right] e^{-rt}dt, \end{aligned} \tag{1}$$

where r is the time preference, $U(y) = a_1 y - \frac{a_2 y^2}{2}$ is utility from consuming the final output whose production emits pollution $y(t)$, $D(z) = \frac{c\, z^2}{2}$ is the damage from the accumulated pollution stock z and $B(x) = b_1 x + \frac{b_2 x^2}{2}$ is cost according to abatement $x(t)$. We adopt the standard linear–quadratic approach (Dockner and Van Long 1993; Benchekroun and Van Long 1998; Dockner 2000; Bertinelli et al. 2014; Boucekkine et al. 2023) to ensure analytical tractability and closed-form solutions. All parameters a_1, a_2, b_1, b_2, c are positive constants with a_1 being sufficiently large to ensure positive optimal emissions. The productivity parameters a_1 and a_2 covert emissions into output, b_1 and b_2 denote the unit costs of CCS deployment, and the parameter c defines the unit damage cost, capturing the increased damage costs of higher levels of emissions.

The CO_2 accumulation is given by

$$\dot{z}(t) = y - x + \alpha x - \delta z, \quad z(0) = z_0 \ \text{ given,} \tag{2}$$

where δ is the natural absorption rate and $\alpha \in (0, 1)$ measures the additional emissions per unit induced by the energy requirements of CCS. Indeed, CCS operation requires extra energy and therefore leads to additional emissions (IPCC 2022). Whereas the polar case $\alpha = 0$ corresponds to abatement without an energy penalty, real-world applications generally imply $\alpha > 0$.

2.2 The case without CCS abatement

We first consider the case without CCS abatement, i.e., $x \equiv 0$. Using subscript b to denote this benchmark case, the decision maker solves:

$$W_b = \max_{y\geq 0} \int_0^{+\infty} \left[\left(a_1 y - \frac{a_2 y^2}{2} \right) - \frac{cz^2}{2} \right] e^{-rt}dt, \tag{3}$$

subject to

$$\dot{z} = y - \delta z$$

with initial CO_2 stock at time $t = 0$, z_0, given.

For any $t \geq 0$, the accumulated CO_2 stock and optimal emissions are given by:

$$\begin{cases} z_b(t) = (z_0 - z_b^*)e^{\mu_b t} + z_b^*, \\ y_b(t) = \dfrac{a_1 + \lambda_b(t)}{a_2} = \dfrac{c}{a_2(\mu_b - (r + \delta))}(z_0 - z_b^*)e^{\mu_b t} + \delta z_b^*, \end{cases} \tag{4}$$

where $z_b(t)$ denotes the atmospheric carbon stock, while $\lambda_b(t)$ is the associated co-state variable, representing the present discounted sum of future damage increases induced by a marginal increase in the carbon stock at time t. z_b^* represents the long-run steady state of carbon accumulation. This steady state is asymptotically stable, with a rate of convergence given by $|\mu_b|$. Furthermore,

$$z_b^* = \frac{a_1(r+\delta)}{a_2\delta(r+\delta)+c}(>0), \qquad \mu_b = \frac{1}{2}\left[r - \sqrt{(r+2\delta)^2 + 4c/a_2}\right](<0). \tag{5}$$

The steady-state outcome reflects the decision maker's trade-off between the current benefits from production, which are accompanied by emissions, and the future welfare losses generated by carbon accumulation. This trade-off becomes more pronounced when damage costs are higher ($\frac{\partial z_b^*}{\partial a_1} > 0$, $\frac{\partial z_b^*}{\partial c} < 0$). Additionally, the result is influenced by the decision maker's time preference. A more impatient decision maker places greater emphasis on immediate production gains over future environmental damages, leading to a higher long-run pollution stock ($\frac{\partial z_b^*}{\partial r} > 0$). The effect of the natural absorption rate is ambiguous ($\frac{\partial z_b^*}{\partial \delta} \gtrless 0$): when marginal damages are sufficiently high relative to production costs, faster decay allows the economy to sustain higher steady-state emissions ($\frac{\partial z_b^*}{\partial \delta} > 0$), whereas the opposite holds otherwise.

The shadow value $\lambda_b(t)$ is given by

$$\lambda_b(t) = \frac{c}{\mu_b - (r+\delta)}(z_0 - z_b^*)e^{\mu_b t} + \lambda_b^*, \tag{6}$$

where λ_b^* denotes the long-run steady state of the shadow value:

$$\lambda_b^* = -\frac{cz_b^*}{r+\delta} = -\frac{a_1 c}{a_2\delta(r+\delta)+c}(<0). \tag{7}$$

The detailed derivation of these expressions is in Appendix A.1.

We impose the following assumption throughout the benchmark analysis:

Assumption 1 *Suppose* $z_0 < z_b^*$.

Assumption 1 sets the initial CO_2 stock below its steady-state level, ensuring positive accumulation over time. Along the optimal emission path $y(t)$, the stock increases strictly and monotonically over time, i.e., $\dot{z}_b > 0$ for all t, until it reaches the steady-state level z_b^*. At the same time, both the shadow value of pollution and the optimal emission rate strictly and monotonically decrease over time: $\dot{\lambda}_b < 0$ and $\dot{y} < 0$, given that $\delta + \mu_b < 0$.

2.3 The case with CCS abatement

We now consider an alternative case with positive abatement $x(t) > 0$ and solve (1) accordingly. Denoting this case with the subscript c, the optimal emissions and CCS abatement for any $t \geq 0$ are given by:

$$y_c(t) = \frac{a_1}{a_2} + \frac{\lambda_c(t)}{a_2} \quad \text{and} \quad x_c(t) = \begin{cases} 0 & \text{if} \quad \lambda_c(t) \geq -\frac{b_1}{1-\alpha}, \\ -\frac{b_1}{b_2} - \frac{(1-\alpha)\lambda_c(t)}{b_2}(>0) & \text{if} \quad \lambda_c(t) < -\frac{b_1}{1-\alpha}, \end{cases} \tag{8}$$

where $\lambda_c(t)$ is the shadow value of the state variable z_c. CCS is activated only when the shadow cost of atmospheric carbon accumulation is sufficiently high, namely when $\lambda_c(t) < -\frac{b_1}{1-\alpha}$. Otherwise, the optimal policy sets $x_c(t) = 0$.

The cost parameter b_1 plays a role analogous to Tobin's Q: The deployment of CCS occurs only if the effective shadow value $(1-\alpha)\lambda$ is sufficiently small, which is determined by the solution to the following equation:

$$\dot{\lambda}_c(t) = (r+\delta)\lambda_c + cz \tag{9}$$

with transversality condition

$$\lim_{t\to\infty} e^{-rt}\lambda(t)z(t) = 0.$$

Substituting the optimal controls (8) into the state equation (2), yields the following piecewise law of motion of the pollution stock:

$$\dot{z} = \begin{cases} -\delta z + \left(\frac{1}{a_2} + \frac{(1-\alpha)^2}{b_2}\right)\lambda + \frac{a_1}{a_2} + \frac{b_1(1-\alpha)}{b_2} & \text{if} \quad \lambda_c(t) < -\frac{b_1}{1-\alpha}, \\ -\delta z + \frac{\lambda}{a_2} + \frac{a_1}{a_2} \quad \text{if} \quad \lambda_c(t) \geq -\frac{b_1}{1-\alpha}, \end{cases} \tag{10}$$

If $\lambda_c(t) \geq -\frac{b_1}{1-\alpha}$, CCS will not be deployed, and the system coincides with the case without CCS in Section 2.1. Consequently, if in the long-run, the shadow value satisfies

$$\lambda_c^* > -\frac{b_1}{1-\alpha}, \quad \text{i.e.,} \quad b_1 > \frac{a_1 c(1-\alpha)}{a_2\delta(r+\delta)+c},$$

then CCS is never adopted, given that $\lambda_c(t)$ decreases monotonically over time and never crosses the activation threshold. To ensure that CCS deployment is triggered within finite time, we impose the following condition.

Assumption 2 *Suppose that* $b_1 \leq \dfrac{a_1 c(1-\alpha)}{a_2\delta(r+\delta)+c}$.

Intuitively, CCS investment cost must not be too high for deployment to occur along the optimal path.[4] In line with the objective of the present study, we assume that there exists a time $T_c \geq 0$ at which CCS is triggered. The law of motion for the pollution stock z_c is then:

$$\dot{z} = \begin{cases} -\delta z + \frac{\lambda}{a_2} + \frac{a_1}{a_2} \quad \text{if} \quad 0 \leq t \leq T_c, \\ -\delta z + \left(\frac{1}{a_2} + \frac{(1-\alpha)^2}{b_2}\right)\lambda + \frac{a_1}{a_2} + \frac{b_1(1-\alpha)}{b_2} \quad t \geq T_c. \end{cases} \tag{11}$$

Clearly, the special case $T_c = 0$ corresponds to immediate CCS deployment from the outset of the policy maker's decision problem. Moreover, for any $t \geq T_c$, the resulting linear dynamic system defined by (9) and (11) admits a closed-form solution. Since the shadow value decreases monotonically over time, once CCS is initiated, its deployment continues indefinitely.

Since the primary objective of this paper is not to characterize the optimal timing of CCS adoption, which has been examined in Chen et al. (2024) and Chen et al. (2025), but rather to evaluate the contribution of CCS to pollution dynamics and welfare, we restrict our attention

[4] Chen et al. (2024) and Chen et al. (2025) additionally discuss the possibility that CCS may remain inactive and characterize the conditions under which it is activated under different market structures.

to the post-deployment regime. We thus redefine the game where $T_c = 0$, focusing only on the period after CCS deployment, in which the solution to the system (9)–(11) is reduced to:[5]

$$\begin{cases} z_c(t) = (z_0 - z_c^*)e^{\mu_c t} + z_c^*, \\ \lambda_c(t) = \dfrac{c}{\mu_c - (r+\delta)}(z_0 - z_c^*)e^{\mu_c t} + \lambda_c^*, \end{cases} \tag{12}$$

by taking into account the transversality condition, where

$$\mu_c = \frac{1}{2}\left[r - \sqrt{(r+2\delta)^2 + 4c\left(\frac{1}{a_2} + \frac{(1-\alpha)^2}{b_2}\right)}\right] (< 0);$$

and z_c^* and λ_c^* denote the asymptotically stable long-run steady state values of the state and co-state variables, respectively, given by

$$\begin{cases} z_c^* = \dfrac{\frac{a_1}{a_2} + \frac{b_1(1-\alpha)}{b_2}}{\delta + \frac{c}{r+\delta}\left(\frac{1}{a_2} + \frac{(1-\alpha)^2}{b_2}\right)} = \dfrac{a_1(r+\delta) + \frac{b_1(1-\alpha)}{b_2}a_2(r+\delta)}{a_2\delta(r+\delta) + c + \frac{c(1-\alpha)^2 a_2}{b_2}}, \\ \lambda_c^* = -\dfrac{cz_c^*}{r+\delta}. \end{cases} \tag{13}$$

Thus CCS deployment starts immediately, i.e. $T_c = 0$, if and only if

$$\lambda_c(0) = \frac{c}{\mu_c - (r+\delta)}(z_0 - z_c^*) - \frac{cz_c^*}{r+\delta} \leq -\frac{b_1}{1-\alpha}, \tag{14}$$

which characterizes a lower bound on the cost parameter b_1 under which CCS is optimally deployed from the outset of the planning horizon. We summarize the above analysis in the following proposition.

Proposition 1 *Suppose that Assumption 1 and 2 hold, along with condition (14). Then,*

(a) The optimal emission and CCS abatement are given by

$$y_c(t) = \frac{a_1}{a_2} + \frac{\lambda_c(t)}{a_2} \quad \text{and} \quad x_c(t) = -\frac{b_1}{b_2} - \frac{(1-\alpha)\lambda_c(t)}{b_2}, \tag{15}$$

where the shadow value $\lambda_c(t)$ is given by (12).

(b) Optimal emission, $y_c(t)$, is strictly decreasing, while CCS deployment, $x_c(t)$, is strictly increasing over time.

(c) Furthermore, in the long run

$$z_c^* < z_b^*, \quad y_c^* - y_b^* = \frac{c}{a_2(r+\delta)}(z_b^* - z_c^*) > 0. \tag{16}$$

Proposition 1 shows that CCS can effectively reduce CO_2 accumulation. Despite higher gross emissions, net emissions, $y_c(t) - (1-\alpha)x_c(t)$, are lower, implying a lower long-run pollution

[5]Detailed derivations are provided in Appendix A.2.

stock. These environmental gains, however, come at a cost. While optimal welfare can be derived explicitly, a direct analytical comparison is intractable, so we rely on the numerical results below.

Figure 1 yields several implications. Panel (a) shows that the deployment of CCS raises gross emissions at all dates, while panel (c) shows that net emissions are lower at all dates once abatement is accounted for. As a consequence, the carbon stock accumulates more slowly and converges to a substantially lower steady-state, as illustrated in panel (b), consistent with Proposition 1. This reflects that abatement relaxes the constraint on emissions-generating activity, allowing higher gross emissions and production. Panel (d) shows welfare is initially lower due to abatement costs but turns positive and rises over time, as long-run gains from lower net emissions and accumulation dominate. Overall, without a binding net-zero constraint, CCS allows the society to sustain higher gross emissions while simultaneously reducing net emissions, lowering long-run pollution, and improving welfare.

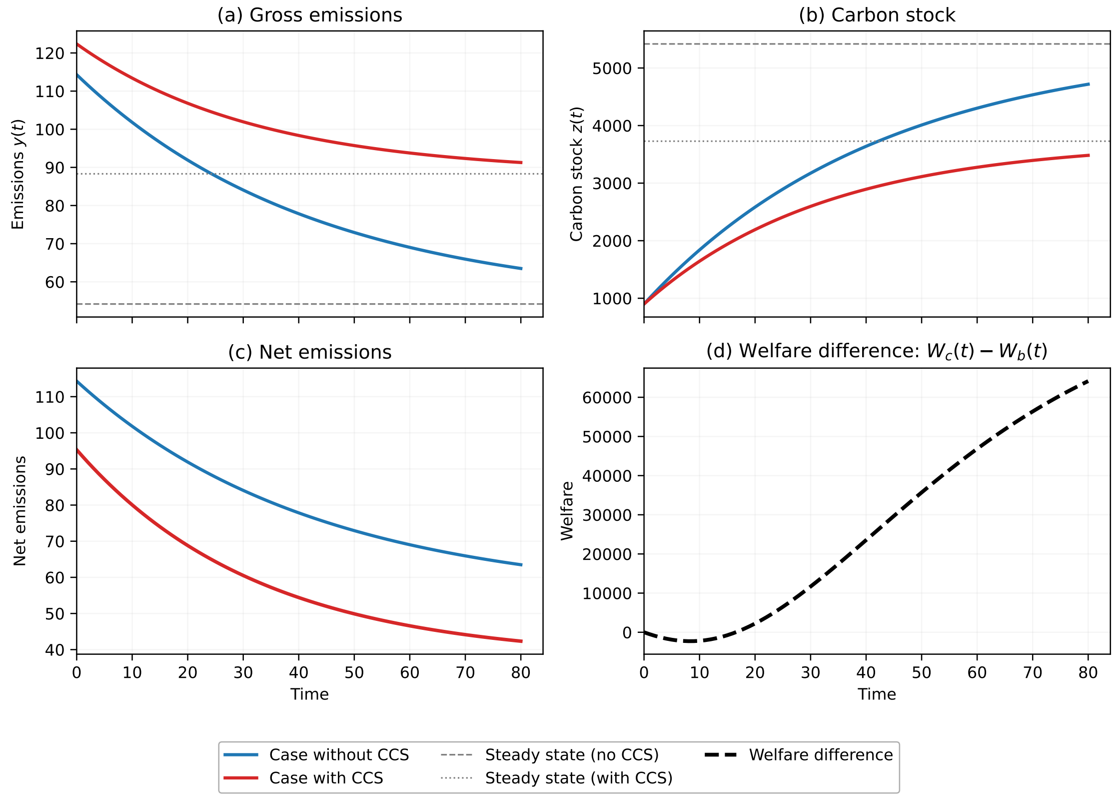

Figure 1: Dynamics under the unconstrained regime: with vs. without CCS.

Notes: Blue line represents no CCS and red line represents CCS without net-zero constraint. Panel (a) shows gross emissions $y(t)$, (b) the carbon stock $z(t)$, (c) net emissions $y(t) - (1 - \alpha)x(t)$, and (d) the welfare difference $\Delta W(t) = W_c(t) - W_b(t)$. Horizontal dashed lines indicate steady-states. Parameters are given in Table 1 (Section 5).

3 The economy under a binding net-zero constraint

We next analyze the economy under a binding net-zero constraint and consider two alternative strategies of achieving the target: one based on CCS and another based on a transition to renewable energy.

Let T denote the exogenously fixed deadline by which net emissions must reach zero. The planning problem naturally splits into two periods: a transition period, $0 \leq t < T$, during which the economy adjusts toward the net-zero requirement, and a post transition period, $t \geq T$, during which net emissions must remain at zero. Under the CCS scenario, fossil fuels are used throughout, with CCS deployed both before and after T to satisfy and sustain the constraint. In contrast, under the renewables scenario, fossil fuels are fully phased out by date T, and production thereafter relies exclusively on zero-emission energy sources. We compare how these two strategies shape the transition path and welfare outcomes.

To ensure comparability across scenarios, we impose two simplifying assumptions. First, we assume that the renewable-energy scenario yields the same post-transition economic benefit as the CCS scenario. Second, the cost of transitioning to renewables equals that of CCS deployment, so the same resources are allocated as would otherwise be used for CCS. These assumptions isolate differences in transition paths rather than post-transition benefits or investment scale, as further discussed in section 3.2.

3.1 Achieving net-zero with CCS

In the fossil-fuel-with-CCS scenario (hereafter referred to as the *with-CCS scenario* for simplicity), the economy continues to use fossil fuels after net-zero date T. Variables are indexed by the subscript zc. Let $y_{zc1}(t)$ and $x_{zc1}(t)$ denote gross emissions and CCS deployment during the transition period $0 \leq t < T$, and $y_{zc2}(t)$ and $x_{zc2}(t)$ for $t \geq T$. During the transition, both emissions and CCS are choice variables of the decision maker, and current emissions affect the evolution of the carbon stock in the usual dynamic way. Once the net-zero date is reached, the constraint becomes binding and requires

$$y_{zc2}(t) = (1-\alpha)x_{zc2}(t), \qquad t \geq T.$$

Hence, in the post-transition regime, emissions and CCS are no longer independent controls.[6] Since net emissions are zero after T, current decisions no longer affect additional carbon accumulation, and the carbon stock thereafter evolves only through natural decay. The resulting optimal control problem is

$$\begin{aligned} W_{zc} &= \max_{y \geq 0,\, x \geq 0} \left[\int_0^T \left[a_1 y - \frac{a_2 y^2}{2} - \frac{cz^2}{2} - \left(b_1 x + \frac{b_2 x^2}{2} \right) \right] e^{-rt} \, \mathrm{d}t \right] \\ &\quad + \max_{x \geq 0} \left[\int_T^\infty \left[(a_1(1-\alpha) - b_1)x - \frac{(a_2(1-\alpha)^2 + b_2)x^2}{2} - \frac{cz^2}{2} \right] e^{-rt} \, \mathrm{d}t \right], \end{aligned} \tag{17}$$

subject to

$$\dot{z} = \begin{cases} y - (1-\alpha)x - \delta z, & \forall t \in [0, T), \\ -\delta z, \qquad y(t) = (1-\alpha)x(t), & \forall t \geq T, \end{cases} \tag{18}$$

with z_0 given.

[6] Equivalently, one may view the decision maker as choosing either $x_{zc2}(t)$, implying $y_{zc2}(t) = (1-\alpha)x_{zc2}(t)$, or $y_{zc2}(t)$, implying $x_{zc2}(t) = y_{zc2}(t)/(1-\alpha)$.

For $0 \leq t < T$, the first-order conditions imply

$$y_{zc1}(t) = \frac{a_1}{a_2} + \frac{\lambda_{zc1}(t)}{a_2}, \qquad x_{zc1}(t) = -\frac{b_1}{b_2} - \frac{(1-\alpha)\lambda_{zc1}(t)}{b_2}, \tag{19}$$

which yield the dynamic system

$$\begin{cases} \dot{z} = -\delta z + \left(\frac{1}{a_2} + \frac{(1-\alpha)^2}{b_2}\right)\lambda + \frac{a_1}{a_2} + \frac{b_1(1-\alpha)}{b_2}, \\ \dot{\lambda} = cz + (r+\delta)\lambda, \end{cases} \qquad \forall t \in [0, T). \tag{20}$$

The corresponding solution is

$$\begin{cases} z_{zc1}(t) = c_1 e^{\mu_{1,zc} t} + c_2 e^{\mu_{2,zc} t} + z_c^*, \\ \lambda_{zc1}(t) = \dfrac{c}{\mu_{1,zc} - (r+\delta)} c_1 e^{\mu_{1,zc} t} + \dfrac{c}{\mu_{2,zc} - (r+\delta)} c_2 e^{\mu_{2,zc} t} + \lambda_c^*, \end{cases} \qquad \forall t \in [0, T), \tag{21}$$

where c_1 and c_2 are constants to be determined, z_c^* and λ_c^* are given by (13), $\mu_{1,zc} = \mu_c$, and

$$\mu_{2,zc} = \frac{1}{2}\left[r + \sqrt{(r+2\delta)^2 + 4c\left(\frac{1}{a_2} + \frac{(1-\alpha)^2}{b_2}\right)}\right].$$

For $t \geq T$, the net-zero constraint implies that emissions and CCS must move one-for-one. The optimal post-transition choice is therefore given by

$$y_{zc2}(t) = (1-\alpha)x_{zc2}(t), \qquad x_{zc2}(t) = \frac{a_1(1-\alpha) - b_1}{a_2(1-\alpha)^2 + b_2}, \tag{22}$$

and the corresponding dynamic system becomes

$$\begin{cases} \dot{z} = -\delta z, \\ \dot{\lambda} = cz + (r+\delta)\lambda, \end{cases} \qquad \forall t \in [T, \infty), \tag{23}$$

with transversality condition

$$\lim_{t\to\infty} e^{-rt}\lambda(t)z(t) = 0.$$

Thus, the above dynamic system yields the following:

$$\begin{cases} z_{zc2}(t) = z_{zc}(T)\, e^{-\delta(t-T)}, \\ \lambda_{zc2}(t) = -\dfrac{c z_{zc}(T)}{r+2\delta}\, e^{-\delta(t-T)}, \end{cases} \qquad \forall t \geq T, \tag{24}$$

where $z_{zc}(T)$ is the pollution stock at the net-zero date, which is undetermined. In fact, CCS reduces the inherited stock $z_{zc}(T)$. Post-T environmental damages are therefore given by

$$D_j^{post} = \int_T^\infty \frac{c z_j^2}{2} e^{-rt} dt = \frac{c z_j^2(T) e^{2\delta T}}{2} \int_T^\infty e^{-(r+2\delta)t} dt = \frac{c z_j^2(T) e^{-rT}}{2(r+2\delta)}, \qquad j = \{zc, zr\}, \tag{25}$$

with $j = zc$ for with-CCS and $j = zr$ for renewable-only. For a given T, the higher the inherited pollution stock, $z(T)$, the larger the damage.

To fix the undetermined coefficient, c_1, c_2, and the transition stock, $z_{zc}(T)$, we rely on the initial condition at $t = 0$, and the transversality condition at $t = T$:

$$\begin{cases} c_1 + c_2 = z_0 - z_c^*, \\ c_1 e^{\mu_{1,zc}T} + c_2 e^{\mu_{2,zc}T} = z_{zc}(T) - z_c^*, \\ \dfrac{c}{\mu_{1,zc} - (r+\delta)} c_1 e^{\mu_{1,zc}T} + \dfrac{c}{\mu_{2,zc} - (r+\delta)} c_2 e^{\mu_{2,zc}T} = -\dfrac{c z_{zc}(T)}{r + 2\delta} - \lambda_c^*. \end{cases} \tag{26}$$

From the last two equations, we have

$$\begin{cases} c_1 = \frac{(\mu_{1,zc} - (r+\delta))(\mu_{2,zc} - (r+\delta))}{e^{\mu_{1,zc}T}(\mu_{1,zc} - \mu_{2,zc})} \left[z_{zc}(T) \left(\frac{1}{r+2\delta} + \frac{1}{\mu_{2,zc} - (r+\delta)} \right) - z_c^* \left(\frac{1}{\mu_{2,zc} - (r+\delta)} + \frac{1}{r+\delta} \right) \right], \\ c_2 = \frac{(\mu_{1,zc} - (r+\delta))(\mu_{2,zc} - (r+\delta))}{e^{\mu_{2,zc}T}(\mu_{2,zc} - \mu_{1,zc})} \left[z_{zc}(T) \left(\frac{1}{r+2\delta} + \frac{1}{\mu_{1,zc} - (r+\delta)} \right) - z_c^* \left(\frac{1}{\mu_{1,zc} - (r+\delta)} + \frac{1}{r+\delta} \right) \right]. \end{cases}$$

Substituting c_1, c_2 into the first equation in (26), we can explicitly solve for $z_{zc}(T)$.

Remark 1 *The presence of a net-zero constraint may alter the optimal timing of CCS deployment relative to the unconstrained benchmark. Since the timing problem is not the focus of the present study, we restrict our attention to cases in which CCS is active in both settings and concentrate on comparing emissions paths and welfare outcomes across regimes.*

3.2 Achieving net zero with renewable energy

In the renewable-only scenario, fossil fuels are completely phased out after date T and replaced by zero-emission renewable energy. Variables associated with this case are indexed by the subscript zr. Since CCS is not used, the transition period $0 \leq t < T$ resembles the setting in Section 2.1, except that the society optimally adjusts fossil-fuel use in anticipation of its elimination at T. After T, the economy switches to an exogenously imposed renewable-only regime, with adjustment costs summarized by $I(T)$. As noted above, to make this scenario comparable to with-CCS scenario, we assume that renewable energy delivers the same post-transition utility flow. Accordingly, for $t \geq T$, utility is given by $a_1 y_{c2} - a_2 y_{c2}^2$. The decision maker's optimal control problem then reduces to:

$$W_{zr} = \max_{y \geq 0} \int_0^T \left(a_1 y - \frac{a_2 y^2}{2} - \frac{c z^2}{2} \right) e^{-rt} dt - I(T) + \int_T^\infty \left(a_1 y_{zc2} - \frac{a_2 y_{zc2}^2}{2} - \frac{c z^2}{2} \right) e^{-rt} dt, \tag{27}$$

subject to

$$\dot{z} = \begin{cases} y - \delta z, & 0 \leq t \leq T, \\ -\delta z, \quad y(t) = 0, & t \geq T, \end{cases} \tag{28}$$

with z_0 given. Note that the last term of the objective function does not contain a choice variable, however, it still contributes to social welfare and is therefore relevant for the comparison analysis.

The term $I(T)$ represents the aggregate cost of transitioning from fossil fuels to renewable energy. This may include capital cost, renewables R&D cost, infrastructure investment, and other expenditures incurred before date T, which can be written as $I(T) = \int_0^T i(s)ds$, where $i(s)$ denotes investment at time s. We analyze two alternative assumptions regarding $I(T)$.

First, transition cost $I(T)$ equals CCS deployment cost $B(x)$ incurred over the period $0 \le t \le T$:

$$I(T) = \int_0^T e^{-rt} \left(b_1 x_{zc1} + \frac{b_2 x_{zc1}^2}{2} \right) dt. \tag{29}$$

Second, transition cost $I(T)$ equals CCS deployment cost $B(x)$ over the entire horizon:

$$I(T) = \int_0^T e^{-rt} \left(b_1 x_{zc1} + \frac{b_2 x_{zc1}^2}{2} \right) dt + \int_T^\infty e^{-rt} \left(b_1 x_{zc2} + \frac{b_2 x_{zc2}^2}{2} \right) dt. \tag{30}$$

Regardless of the specification of $I(T)$, the optimal level of emissions before T is given by

$$y_{zr}(t) = \frac{a_1 + \lambda_{zr}(t)}{a_2}, \quad 0 \le t \le T. \tag{31}$$

The dynamic system is

$$\begin{cases} \dot{z} = -\delta z + \frac{1}{a_2} + \frac{a_1}{a_2}, \\ \dot{\lambda} = cz + (r+\delta)\lambda, \end{cases} \quad \forall t \in [0, T], \tag{32}$$

with explicit solution

$$\begin{cases} z_{1,zr}(t) = k_1 e^{\mu_{1,zr} t} + k_2 e^{\mu_{2,zr} t} + z_b^*, \\ \lambda_{zr1}(t) = \dfrac{c}{\mu_{1,zr} - (r+\delta)} k_1 e^{\mu_{1,zr} t} + \dfrac{c}{\mu_{2,zr} - (r+\delta)} k_2 e^{\mu_{2,zr} t} + \lambda_b^*, \end{cases} \quad \forall t \in [0, T], \tag{33}$$

where k_1, k_2 are undetermined constants, $\mu_{1,zr} = \mu_b$ in (5) and

$$\mu_{2,zr} = \frac{1}{2} \left[r + \sqrt{(r+2\delta)^2 + \frac{4c}{a_2}} \right].$$

Similarly to (24), for $t \ge T$, the state and costate are given by

$$\begin{cases} z_{zr2}(t) = z_{zr}(T)\, e^{-\delta(t-T)}, \\ \lambda_{zr2}(t) = -\dfrac{c z_{zr}(T)}{r+2\delta}\, e^{-\delta(t-T)}, \end{cases} \quad \forall t \ge T, \tag{34}$$

where $z_{zr}(T)$ denotes pollution level at net-zero date and is undetermined. Again, using the initial condition and the transversality condition at T, the following must hold to fix the undetermined coefficients k_1, k_2 and the transition stock $z_{zr}(T)$:

$$\begin{cases} k_1 + k_2 = z_0 - z_b^*, \\ k_1 e^{\mu_{1,zr} T} + k_2 e^{\mu_{2,zr} T} = z_{zr}(T) - z_b^*, \\ \dfrac{c}{\mu_{1,zr} - (r+\delta)} k_1 e^{\mu_{1,zr} T} + \dfrac{c}{\mu_{2,zr} - (r+\delta)} k_2 e^{\mu_{2,zr} T} = -\dfrac{c z_{zr}(T)}{r+2\delta} - \lambda_b^*. \end{cases} \tag{35}$$

Using the same calculations as in the previous subsection, we can obtain $z_{zr}(T)$ and k_1, k_2.

3.3 The contribution of CCS under the net-zero target

Building on the analytical results above, this section presents a numerical illustration of the transition dynamics under an exogenously imposed binding net-zero constraint, comparing CCS and renewables scenarios.

Figure 2 compares the transitional dynamics under the two scenarios when the net-zero date T is exogenous. By construction, both scenarios share the same post-transition economic payoff structure and the same aggregate transition expenditure $I(t)$. The figure therefore isolates differences in the pre-T adjustment path and in the carbon stock at date T.

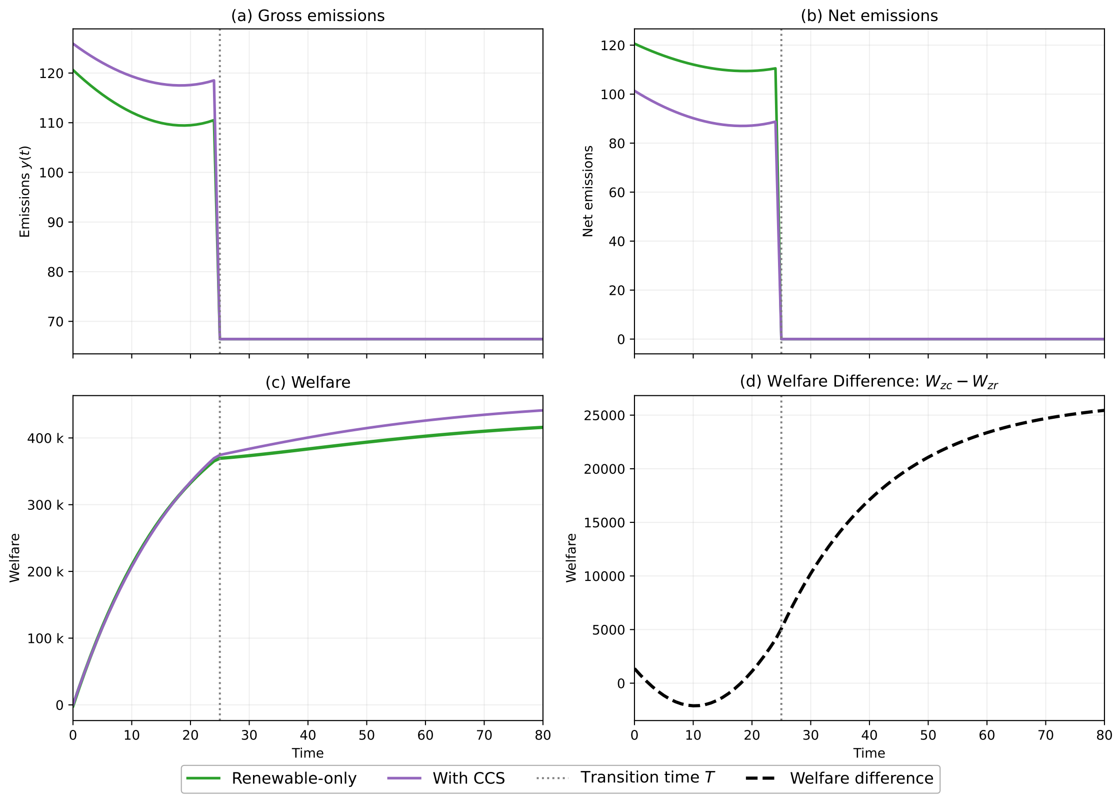

Figure 2: Comparison of Emissions and Welfare under a net-zero constraint.

Notes: The green line denotes the renewable-only scenario and the purple line the with-CCS scenario under the net-zero constraint. Panel (a) shows gross emissions $y(t)$; in the renewable-only scenario, they drop to zero at T. Panel (b) reports net emissions $y(t) - (1 - \alpha)x(t)$, panel (c) shows welfare, and panel (d) the welfare difference $\Delta W(t) = W_{zc}(t) - W_{zr}(t)$. Vertical dashed lines mark T. Parameter values are given in Table 1 (Section 5).

Panels (a) and (b) of Figure 2 together illustrate how the CCS alters the adjustment during transition. Although gross emissions are higher under CCS prior to T, as shown in panel (a), net emissions are lower throughout the same period, as shown in panel (b). CCS therefore provides an additional adjustment instrument, allowing higher fossil fuel use while delivering a lower net-emissions path before the net-zero date. By contrast, in the renewable-only scenario, fossil-fuel use is optimally chosen in anticipation of its complete phase-out at T, leading to a sharper decline in gross emissions during the transition and a discrete drop to zero at T. The sharp fall in net emissions at T in both scenarios reflects a hard regime switch rather than a smooth convergence to net zero.

Panel (c) shows how these differences translate into welfare. CCS entails an initial cost, so welfare is slightly lower at the outset. This gap closes quickly, and welfare under CCS exceeds that of the renewable pathway for most of the transition and remains higher thereafter. The persistent advantage of CCS has two sources. First, lower net emissions before T allow higher fossil-fuel use with lower environmental costs during the transition. Second, this implies a smaller carbon stock at the deadline. Since both scenarios deliver the same post-transition benefit, post-T welfare differences arise solely from this inherited stock: lower carbon under CCS reduces future damages and increases the continuation value.

The welfare difference in panel (d) is negative only initially, reflecting the short-run cost of CCS, but turns positive well before T and increases over time as gains from lower net emissions and carbon stock at T dominate. The welfare advantage of CCS thus stems from both higher welfare during most of the transition and a more favorable environmental state at the post-transition phase. This ranking is, however, parameter-dependent and may reverse under alternative calibrations. Section 4.1 derives formal conditions under which CCS is welfare-improving.

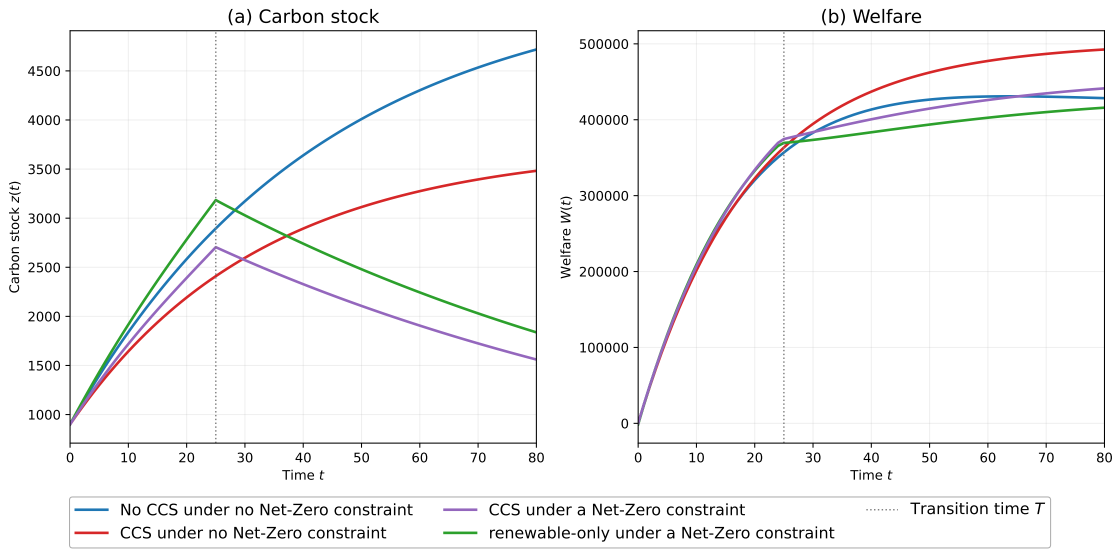

Figure 3: Dynamics of carbon stock and welfare under all scenarios.

Notes: The parameter values used in the figure are reported in Table 1 in Section 5.

Figure 3 extends the comparison to the unconstrained cases. Panel (a) of Figure 3 shows that, before T, the renewable net-zero path lies above the no-CCS path, while CCS under the net-zero constraint lies above unconstrained CCS. This indicates that a binding net-zero requirement shifts emissions toward the transition period, a green-paradox-type effect in the sense that anticipated future constraints accelerate current emissions (Sinn 2008; Ploeg 2013; Van der Ploeg and Withagen 2015). At the same time, constrained CCS remains below the renewable-only net-zero path, reflecting the role of abatement in lowering net emissions during the transition. After T, the carbon stock ranking is straightforward: under net zero it declines as net emissions drop to zero, while in unconstrained cases it converges to higher long-run level, with CCS below no-CCS.

Panel (*b*) of Figure 3 shows a related welfare pattern. Before T, both net-zero cases lie slightly above their unconstrained counterparts, suggesting that higher transition period emissions raise production benefits more than they increase climate damages during this period. After entry into the net-zero regime, welfare drops relative to the unconstrained paths, reflecting the cost of tighter emissions constraints. The decline is smaller under CCS, consistent with a lower inherited carbon stock, and both constrained paths recover gradually over time.

Overall, Figure 3 highlights a policy trade-off: net-zero improves environmental outcomes but reduces welfare relative to unconstrained cases. Within net-zero pathways, CCS partially mitigates this loss by lowering transition emissions and reducing the carbon stock carried into the post-transition phase.

4 Welfare criteria for assessing the desirability of CCS

This section establishes welfare-based criteria for assessing when CCS is socially desirable under an exogenously imposed net-zero date T. We examine how these criteria vary across policy-relevant time horizons (e.g., 2045, 2050, and 2060), and how they are affected by technological progress through learning-by-doing and by constraints on CCS storage capacity.

Building on the dynamic analysis in Sections 2 and 3, we formalize the conditions under which CCS remains welfare-desirable under a binding net-zero constraint. This section also explores how factors such as timing, technology progress, and storage limitations interact with these welfare criteria. In doing so, we complement the analysis of the optimal timing of CCS deployment by a social decision maker in Chen et al. (2024), as well as the reachability and free-riding outcomes explored in Chen et al. (2025), by incorporating a binding net-zero constraint. Additionally, our analysis builds on Hoel (2025), who studies the role of storage limits and the potential need for negative-emissions phases in the long-run transition to net-zero.

4.1 Welfare criterion under a net-zero constraint

Proposition 2 *Let $W_j^{\text{pre}}(0,T)$ denote the discounted welfare before T under regime $j \in \{zc, zr\}$, corresponding to the fossil-fuel-with-CCS and renewable-only pathways, respectively. Then the former is welfare-superior to the latter if and only if*

$$W_{zc}^{\text{pre}}(0,T) - W_{zr}^{\text{pre}}(0,T) \;>\; D_{zr}^{post} - D_{zc}^{post} = \frac{c\left[z_{zr}(T)^2 - z_{zc}(T)^2\right]}{2}\,\frac{e^{-rT}}{(r+2\delta)}. \tag{36}$$

Proof. Under the assumption that the cost of transitioning from fossil fuels to renewable energy is equivalent to the cost of CCS deployment from section 3, total welfare in each regime can be decomposed into transition-period welfare and post-transition environmental damages:

$$W_j = W_j^{\text{pre}}(0,T) + D_j^{\text{post}}, \qquad j \in \{c, r\}.$$

It follows that

$$W_{zc} - W_{zr} = [W_{zc}^{\text{pre}}(0,T) + D_{zc}^{post}] - [W_{zr}^{\text{pre}}(0,T) + D_{zr}^{post}].$$

Therefore, CCS is welfare-superior to the renewables if and only if $W_{zc} - W_{zr} > 0$, which is equivalent to $W_{zc}^{\text{pre}}(0,T) - W_{zr}^{\text{pre}}(0,T) > D_{zr}^{post} - D_{zc}^{post}$ in (36). This completes the proof.

Proposition 2 therefore provides a general condition for assessing when CCS improves welfare under a binding net-zero constraint. The welfare superiority of CCS depends on the difference between the relative social welfare generated before the net-zero deadline and the environmental damages incurred after the deadline. It is important to note that the right hand side of (36) can be either positive or negative.

Panel (d) in Figure 2 presents a case in which the fossil-fuel-with-CCS scenario is welfare-superior to the renewable-only pathway. Although the former under-performs the latter in the short run, this disadvantage is small in magnitude and is quickly reversed. CCS ultimately outperforms renewables because it results in a lower pollution stock both before and after reaching the deadline. Naturally, alternative parameter configurations could yield different welfare rankings.

Having established this welfare criterion, we next examine how the timing of the net-zero date T affects the condition in Proposition 2 and the associated welfare outcomes.

4.2 Sensitivity to the net-zero date T

The timing of the net-zero deadline has important welfare implications. In practice, neutrality deadlines vary across jurisdictions, with some countries in a better position to commit to earlier targets, while others have announced later ones. By varying dates T, we can examine how the timing constraint influences transition-period welfare and the date-T pollution stock $z(T)$.

Proposition 3 *Holding T exogenous but policy-relevant (e.g., 2045/2050/2060), the sign of* $\partial W_j / \partial T$ *depends on the trade-off between pre-T costs and benefits and the post-transition damage term* $D_j^{\text{post}}(T)$*. In our baseline:*

(a) A higher c (steeper environmental damage) implies that an earlier net-zero date T is preferred in both regimes.

(b) Higher deployments cost b_1, b_2 *can shift the welfare-maximizing T towards later dates in the CCS regime when pre-T abatement costs dominate.*

(c) A higher natural absorption capacity δ *raises welfare with T by reducing carry-over damages.*

A proof is provided in Appendix A.3. In economies facing high marginal climate damages c or limited natural absorption capacity δ, an earlier net-zero date yields higher welfare. By contrast, high upfront deployment costs b_1, b_2 tend to delay the optimal transition date. However, deployment costs are unlikely to remain constant over time and are expected to decline as the technology advances and its application becomes more widespread.[7]

[7] Chen et al. (2024) and Chen et al. (2025) examine optimal CCS adoption decisions in a setting with asymmetric deployment and damage costs across two countries, but do not consider cost dynamics.

4.3 Cost development due to learning effects

As the industry develops, technological learning gradually reduces the costs of capturing, transporting, and storing CO_2. Cost reductions in CCS and related abatement technologies following early deployment have been documented in the literature (IEA 2021; Rubin 2012; Rubin et al. 2015; Kearns et al. 2021; Amigues et al. 2016; Broek et al. 2009). To incorporate cost reductions driven by technological learning over time, we extend the analysis by modeling CCS unit costs to decline dynamically.

Specifically, we assume that the cost parameter b_1 follows an exponential learning path, $b_1(t) = b_{10}e^{-\kappa t}$, where $\kappa > 0$ is the learning rate, and examine how variations in κ affect the optimal net-zero date. The results are summarized in the following proposition.

Proposition 4 *Suppose that CCS unit costs decline exponentially according to* $b_1(t) = b_{10}e^{-\kappa t}$, $\kappa > 0$. *Then:*

(a) When the learning rate κ is sufficiently large, earlier CCS deployment can be welfare-improving, as declining future abatement costs tilt the welfare condition in (36) *in favor of CCS.*

(b) For policy-relevant net-zero dates $T \in \{20, 25, 35\}$, a higher learning rate (e.g., $\kappa = 0.03$ vs. $\kappa = 0.01$) increases welfare at all horizons by lowering the effective CCS cost $b_1(T)$ and transition-period abatement costs.

Proposition 4 highlights that an increase in the learning rate reduces future abatement costs and expands the parameter region in which CCS becomes welfare-desirable at mid-century horizons. As a result, learning-by-doing can mitigate the welfare trade-off between high upfront cost and abatement benefit. Hence, policies should aim at accelerating the learning process, for example, through R&D subsidies and public co-financing of CCS infrastructure.

4.4 Storage capacity and binding reservoir limits

The long-run feasibility of CCS also depends on the availability of geographical storage capacities. In this subsection, we introduce an explicit cumulative storage constraint, in line with Hoel (2025), and examine how it alters optimal CCS deployment, welfare outcomes and the welfare criterion derived above. Specifically, we impose a capacity cap $\bar{S}$ that is exogenously determined by geological conditions.

Proposition 5 *Suppose cumulative net carbon storage is subject to the constraint* $\int_0^\infty (1-\alpha)x(t)\,dt \le \bar{S}$. *If the constraint binds, the effective unit cost of CCS becomes*

$$b_1^{eff} = b_1 + (1-\alpha)\mu$$

where $\mu > 0$ is the Kuhn-Tucker multiplier. Then,

(a) The optimal CCS abatement levels are

$$x_{zc1}^{eff} = x_{zc1}^* - \frac{(1-\alpha)\mu}{b_2} < x_{zc1}^*, \quad x_{zc2}^{eff} = x_{zc2}^* - \frac{(1-\alpha)\mu}{a_2(1-\alpha)^2 + b_2} < x_{zc2}^*.$$

(b) *The steady-state pollution stock and welfare become functions of the storage cap $\bar{S}$. Although CCS may still reduce $z(T)$, the post-T trajectory cannot rely indefinitely on negative emissions when storage capacity is limited.*

(c) *There exists a minimum storage capacity $\bar{S}$ required for CCS to satisfy the welfare criterion in* (36). *When $\bar{S}$ falls below this threshold, the storage constraint eliminates CCS's welfare advantage, and the renewable-only pathway may dominate.*

The proof is given in Appendix A.4. Proposition 5 mirrors the storage-limit results in Hoel (2025), who shows that cumulative carbon dioxide removal (CDR) constraints affect extraction, long-run carbon stocks and the feasibility of negative-emissions phases under hard capacity limits. In our framework, storage scarcity weakens the ability of CCS to lower the inherited pollution stock at net zero and to generate post-transition welfare gains. When storage capacity is capped, welfare outcomes depend more strongly on CCS deployment efficiency (a lower α) and on cost reductions driven by learning-by-doing (a higher κ) (Hoel 2025).

Together, propositions 3 - 5 identify key policy dimensions, including target timing, cost reduction trajectories, and storage capacity, that jointly determine whether CCS can complement or even outperform renewable-only strategies under binding net-zero mandates.

5 Numerical analysis of the net-zero timing and CCS costs

Section 4 identifies two key factors determining the desirability of CCS under a binding net-zero target: the timing of the target and the cost of CCS deployment. The net-zero date determines the trade-off between transition-period welfare and post-transition damages by affecting the length of the adjustment period and the inherited pollution stock at net zero. CCS costs, in turn, affect its attractiveness as an abatement instrument, influencing emissions, transition dynamics, and welfare.

5.1 The parameters

Table 1 reports the baseline parameter values used throughout the paper. The rate of time preference r is set to 0.035, in line with Hoel and Karp (2002), Stern (2008), and Nordhaus (2007). The natural decay rate of atmospheric CO_2 is fixed at $\delta = 0.01$, capturing the slow absorption of carbon by natural sinks in reduced-form carbon-cycle representations (Reilly and Richards 1993; Mason et al. 2017). Recalling the benefit function $U(y) = a_1 y - \frac{a_2 y^2}{2}$, where the productivity parameters capture the efficiency with which emissions are converted into final output, we set $a_1 = 360$ and $a_2 = 2.2$ to ensure that emissions remain strictly positive along the optimal path and that marginal benefits from additional emissions diminish.[8] The parameter α is set to 0.2, which represents the additional emissions per unit of CCS activity due to energy requirements for capture, compression, and storage, reflecting imperfect efficiency. Moreover, the initial pollution stock is set to $z_0 = 900$, reflecting an already elevated atmospheric carbon concentration at

[8] As emphasized in Mason et al. (2017), the productivity parameter can take a wide range of values. Higher values correspond to an economy that generates greater consumption benefits per unit of carbon emissions, capturing higher overall production efficiency or cleaner production technologies.

the beginning of the planning horizon. We then fix $c = 0.002$ in the environmental damage function $D(z) = \frac{cz^2}{2}$, in line with Karp and Zhang (2012). CCS abatement costs are given by $B(x) = b_1 x + \frac{b_2 x^2}{2}$ with $b_2 = 2$; we set $b_1 = 5$ in the baseline and consider $b_1 = 0.1$ and $b_1 = 2$ as alternative scenarios. The net-zero date is fixed at $T = 25$, corresponding to a mid-century neutrality target within the model's normalized time scale. To analyze the impact of different net-zero dates on welfare, we compare this with scenarios involving $T = 20$ and $T = 35$, holding the other parameter values constant at the benchmark level.

Table 1: Baseline parameter values

a_1	a_2	b_1	b_2	α	z_0	r	δ	c	T
360	2.2	5	2	0.2	900	0.035	0.01	0.002	25

5.2 The impact of the net-zero date T

This subsection examines how the timing of the net-zero constraint affects transition dynamics and welfare when CCS is available. The net-zero date T is assumed to be exogenous, and we evaluate a range of alternative horizons, reflecting substantial cross-country differences in announced neutrality target dates, which span from mid-century to later dates such as 2060 or beyond.

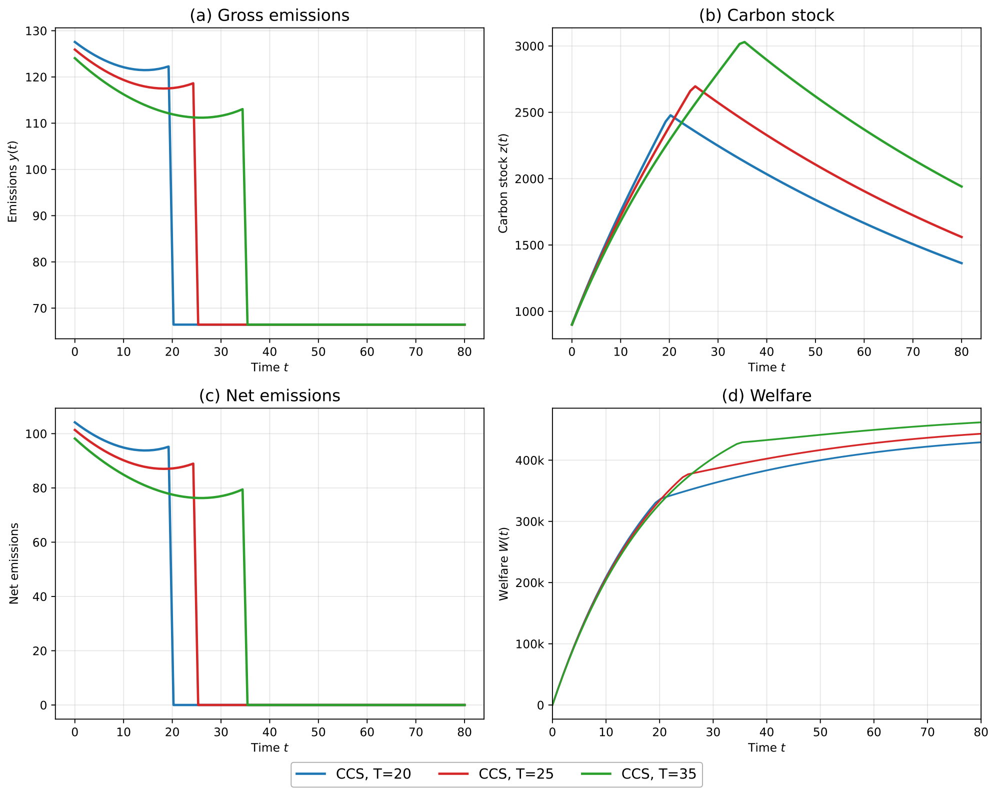

Figure 4: The impact of T on the transition dynamics.

Figure 4 illustrates the effects of alternative net-zero date on gross emissions, carbon stock, net emissions and welfare when CCS is available. Three different scenarios are presented, corresponding to $T = 20$, $T = 25$ and $T = 35$. Panels (a) and (c) show that a tighter deadline forces adjustment to take place sooner and more abruptly. The $T = 20$ case thus exhibits higher gross and net emissions at the beginning of the transition, followed by a sharper decline when the constraint is imposed.

The cost of delay is shown in panel (b). Because later-target cases maintain positive net emissions for longer, they accumulate a larger carbon stock by the time the net-zero constraint becomes binding. Once this point is reached, net emissions fall to zero in all cases, as shown in panel (c), and the carbon stock declines only through natural decay. As a result, delaying the transition leads to a persistently higher post-transition pollution path.

Panel (d) summarizes the welfare implications of this trade-off. Although a later net-zero date leads to a higher pollution stock, it is associated with higher cumulative welfare over much of the simulated horizon. This seemingly counterintuitive result reflects the intertemporal trade-off. Delaying the transition allows the society to spread the adjustment over a longer period and preserve production benefits for longer, while the resulting environmental damages are pushed further into the future and therefore discounted more heavily. Once net emissions fall to zero, the carbon stock declines through natural decay, which further reduces the present effect of post-transition damages.

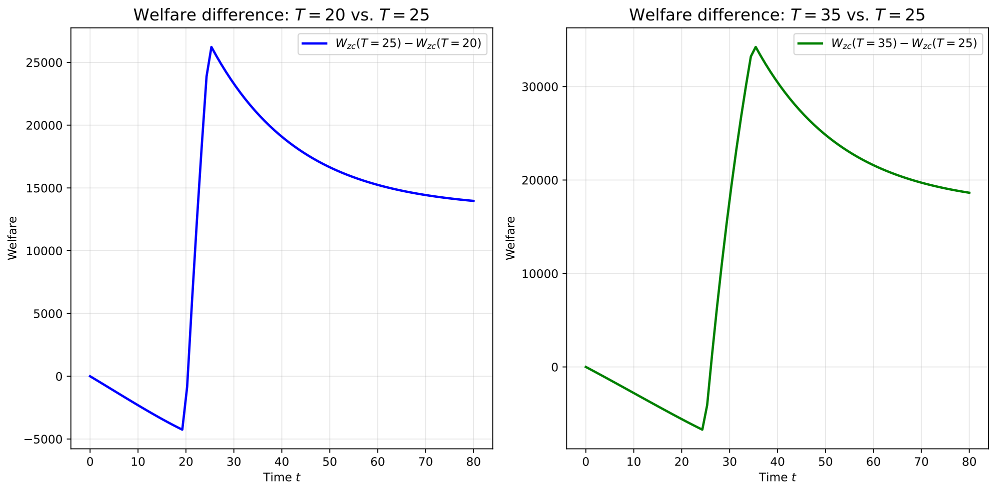

Figure 5: Welfare difference across net-zero dates T.

To make this trade-off more explicit, Figure 5 plots pairwise welfare differences across alternative net-zero dates. The figure shows that earlier net-zero dates yield higher welfare at the beginning of the transition period, because a tighter deadline induces a more front-loaded emissions path, as shown in panel (a) of Figure 4, and the associated short-run production benefits initially dominate. This ranking, however, reverses once the earlier-transition economy enters the net-zero regime. At that point, emissions must be reduced sooner, so the economy forgoes production benefits that remain available under a later net-zero date. As the transition unfolds, the later-dated case accumulates higher welfare through a more gradual adjustment path and a longer period of transition-period production. Although the earlier transition results in a lower carbon stock and hence lower future damages, under the baseline parameterization this environmental gain is not sufficient to offset the welfare loss of earlier adjustment.

A later date relaxes the emissions constraint for a longer transition period, allowing higher emissions and thus higher production prior to T, because after-transition emissions are forced to adjust so as to satisfy the net-zero condition, as shown in panel (a).

These results do not imply that a later net-zero date is preferable. Instead, they suggest that, delaying the transition could be welfare-improving. This may help rationize why developing countries often announce later net-zero dates than developed countries: Sweden and Germany target 2045, the United States 2050, China 2060, and India 2070. For those developing economies, the economic gains from a more gradual transition may carry greater weight than the future welfare gains associated with earlier pollution reduction. However, the decision on when to set the net-zero date is beyond the scope of the current focus.

5.3 The impact of the CCS deployment cost b_1

Figure 6 illustrates how changes in the unit cost of CCS, b_1, affect optimal emissions, abatement, carbon accumulation, and welfare under a binding net-zero constraint. The figure reports differences relative to the baseline case $b_1 = 5$, comparing a low-cost scenario ($b_1 = 0.1$) and an intermediate-cost scenario ($b_1 = 2$). The vertical dashed line marks the net-zero date $T = 25$.

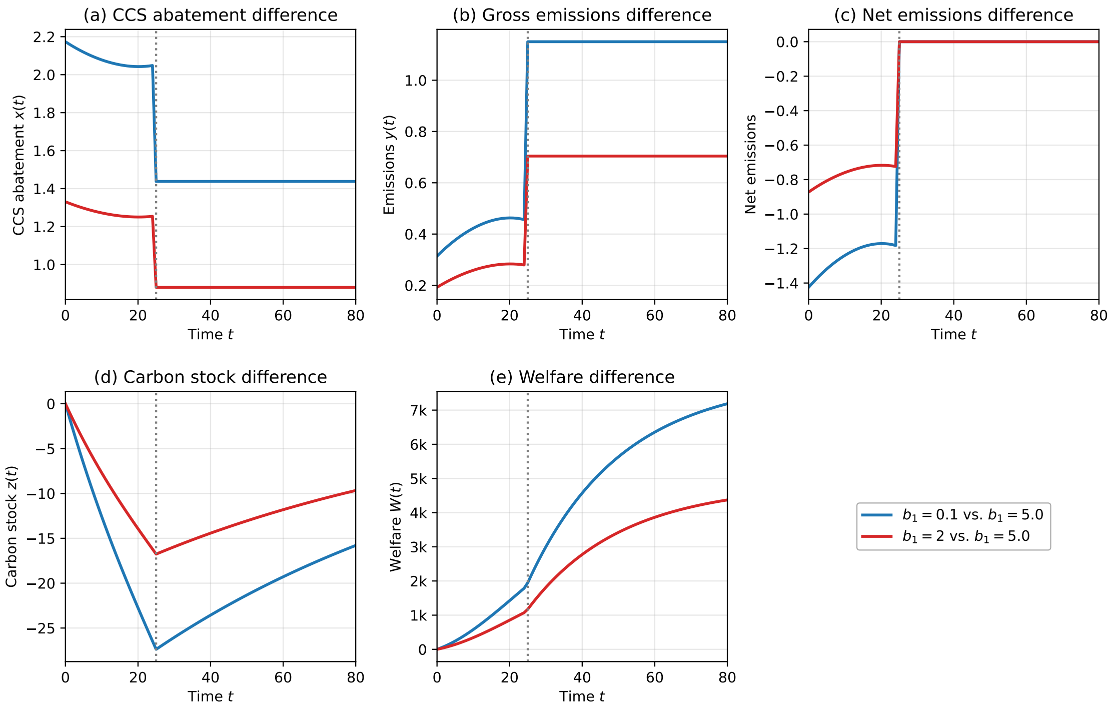

Figure 6: The impact of b_1.

Notes: The blue line compares low-cost scenario ($b_1 = 0.1$) with the baseline ($b_1 = 5.0$), and the red line compares the intermediate-cost scenario ($b_1 = 2.0$) with the baseline ($b_1 = 5.0$). Panels (a)–(e) show differences in CCS abatement $x(t)$, gross emissions $y(t)$, net emissions $y(t) - (1-\alpha)x(t)$, carbon stock $z(t)$, and welfare $W(t)$, respectively. The vertical dashed line marks the net-zero date $T = 25$. Parameter values are reported in Table 1.

Panel (a) shows that a reduction in b_1 leads to a higher level of CCS deployment prior to net-zero. When abatement becomes cheaper, the decision maker relies more heavily on capture activities in the transition phase. After T, CCS deployment remains permanently higher in the low-cost scenarios, reflecting the fact that, under a net-zero constraint, CCS becomes the important instrument for offsetting emissions. Panel (d) illustrates the corresponding differences in the carbon stock. Lower CCS costs translate into a lower inherited pollution stock $z(T)$. Although carbon stocks gradually converge after net-zero due to natural decay, the ranking across cost scenarios is preserved: economies with cheaper CCS enter the net-zero regime with a permanently lower carbon stock, which reduces post-transition climate damages. Panel (e) shows the cumulative welfare differences. Lower CCS costs raise welfare relative to the baseline. Welfare gains accumulate over time. Before net-zero, welfare gains are driven by lower abatement costs and higher production benefit. After net-zero, welfare continues to diverge because the lower inherited carbon stock at date T implies persistently lower discounted climate damages. Panels (b) and (c) describe the emissions channel. Gross emissions are higher when CCS is cheaper, both before and after net-zero, since capture enables a higher level of fossil-based activity while keeping net emissions lower (pre-T) and at zero (post-T), i.e. without increasing atmospheric carbon stock and without violating the net-zero constraint. Before T, CCS reduces net emissions despite higher fossil-based activity, while after T net emissions are mechanically zero in all net-zero scenarios; as a result, post-T welfare differences are driven exclusively by the inherited pollution stock $z(T)$.

Indeed, Figure 6 underscores the importance of cost-reducing policies, such as learning-by-doing, early deployment, and targeted subsidies, in enhancing the welfare performance of CCS-based net-zero pathways.

6 Conclusion

This paper studies the role of CCS under a binding net-zero emissions target that is imposed exogenously at a finite date. Unlike most of the dynamic climate-economy models, which treat carbon neutrality as a long-run outcome, we take the net-zero target date as a strict policy constraint.

Our main results can be summarized as follows. Modeling net-zero as a binding constraint allows us to derive an explicit welfare criterion that clarifies when CCS outperforms a renewable-only transition. The welfare comparison depends on the trade-off between the gains generated during the transition phase and the environmental damages arising from the pollution stock inherited at the net-zero date. Within this framework, we compare multiple policy scenarios and show that CCS can be welfare-enhancing even if it causes higher gross emissions prior to net-zero, provided that capture sufficiently reduces net emissions and lowers the inherited pollution stock at net zero. We then extend the analysis to examine the policy dimensions determining whether CCS can outperform a renewable-only pathway under a net-zero mandate, including the timing of the target, technology and cost parameters, and available storage capacity. In the numerical analysis, we examine how net-zero timing and CCS cost dynamics influence both the inherited pollution stock and intertemporal social welfare. First, when policy makers place a relatively high weight on production and instantaneous utility, a later date is preferred. By contrast, earlier transitions become more attractive when the marginal damages of CO_2 accumulation are large or when future welfare is weighted more heavily. Second, lower CCS deployment costs reduce inherited pollution stock at net zero, thereby raising welfare.

Our results complement recent work by Hoel (2025), who provides theoretical foundation for understanding the optimal use of CDR technologies under flexible climate objectives. Our framework departs from this approach by treating the net-zero emissions target as an exogenous and binding policy constraint, reflecting current policy landscape in which many countries have committed to fixed carbon-neutral timelines. Withing this setting, we derive explicit conditions under which CCS yields welfare improvements, translating the model's dynamics into interpretable thresholds governing the optimal timing and scale of abatement, cost-reduction efforts, and storage planning. The results suggest that CCS is not a substitute for decarbonization, but can serve a welfare-enhancing role in energy transitions where an immediate phase-out of fossil fuels remains technologically or economically infeasible.

A Appendix

A.1 Proof of the derivation of the optimal control equations in Section 2.1

Maximize the objective function:

$$W_b = \max_{y \geq 0} \int_0^{+\infty} \left[\left(a_1 y - \frac{a_2 y^2}{2} \right) - \frac{cz^2}{2} \right] e^{-rt} dt \tag{37}$$

subject to the state equation:

$$\dot{z} = y - \delta z$$

with $z(0) = z_0$ given. The Hamiltonian $\mathcal{H}$ for this problem is

$$\mathcal{H} = (a_1 y - \frac{a_2 y^2}{2} - \frac{cz^2}{2}) + \lambda(y - \delta z)$$

where λ is the costate variable. To find the optimal control y, we maximize $\mathcal{H}$ with respect to y:

$$\frac{\partial \mathcal{H}}{\partial y} = a - ay + \lambda = 0 \implies y = \frac{a_1 + \lambda}{a_2} \tag{38}$$

The costate equation is derived from the necessary conditions for optimality:

$$\dot{\lambda} = r\lambda - \frac{\partial \mathcal{H}}{\partial z} = r\lambda + cz + \lambda\delta = \lambda(r + \delta) + cz. \tag{39}$$

The shadow price λ represents the marginal value of the state variable z in current-value terms, because the problem is discounted at rate r, the shadow price must account for the opportunity cost of time. The shadow price grows at the discount rate r plus the natural decay rate δ. The state equation is thus given by:

$$\dot{z} = y - \delta z = \frac{a_1 + \lambda}{a_2} - \delta z. \tag{40}$$

In the steady state, $\dot{z} = \dot{\lambda}$=0, from which we can derive

$$\dot{z} = 0 \Rightarrow \lambda = a_2 \delta z - a_1.$$

$$\dot{\lambda} = 0 \Rightarrow \lambda(r + \delta) + cz = 0.$$

Substitute λ from the first equation into the second:

$$(r+\delta)(a_2\delta z - a_1) + cz = 0 \Rightarrow z_b^* = \frac{a_1(r+\delta)}{a_2\delta(r+\delta)+c} \tag{41}$$

where z_b^* denotes the long-run steady-state value of z. The long-run steady state of the shadow value λ, λ_b^*, can thus be expressed as:

$$\lambda_b^* = a_2\delta z_b^* - a_1 = \frac{-a_1 c}{a_2\delta(r+\delta)+c} < 0 \tag{42}$$

Substitute (42) into (38):

$$y_b^* = \frac{a_1+\lambda_b^*}{a_2} = \frac{\delta a_1(r+\delta)}{a_2\delta(r+\delta)+c} = \delta\, z_b^* > 0.$$

We then proceed to to calculate the convergence speed of the system toward the steady state by analyzing the dynamics near the steady state (z_b^*, λ_b^*). Using the state and costate equations (40) and (39), let $\tilde{z} = z - z_b^*$ and $\tilde{\lambda} = \lambda - \lambda_b^*$, then the linearized system is:

$$\begin{cases} \dot{\tilde{z}} = \dfrac{\tilde{\lambda}}{a_2} - \delta\tilde{z}, \\ \dot{\tilde{\lambda}} = c\tilde{z} + (r+\delta)\tilde{\lambda}. \end{cases}$$

In matrix form we have:

$$\begin{pmatrix} \dot{\tilde{z}} \\ \dot{\tilde{\lambda}} \end{pmatrix} = \begin{pmatrix} -\delta & \dfrac{1}{a_2} \\ c & r+\delta \end{pmatrix} \begin{pmatrix} \tilde{z} \\ \tilde{\lambda} \end{pmatrix}.$$

This gives:

$$(-\delta-\mu)(r+\delta-\mu) - \frac{c}{a_2} = 0,$$

$$\mu^2 - r\mu - \left(\delta(r+\delta) + \frac{c}{a_2}\right) = 0.$$

The solutions are:

$$\mu = \frac{r \pm \sqrt{r^2 + 4\left(\delta(r+\delta)+\frac{c}{a_2}\right)}}{2}. \tag{43}$$

Since $\delta(r+\delta)+\frac{c}{a_2} > 0$, $\sqrt{r^2+4\left(\delta(r+\delta)+\frac{c}{a_2}\right)} > r$, there exist two real eigenvalues: (1) $\mu_1 > 0$ unstable, (2) $\mu_2 < 0$ stable. We take $\mu_b = \mu_2$, which yields (5).
The value function W_b represents the maximized discounted welfare. In steady state, it can be expressed as:

$$W_b = \int_0^{+\infty} \left[\left(a_1 y_b^* - \frac{a_2(y_b^*)^2}{2}\right) - \frac{c(z_b^*)^2}{2}\right] e^{-rt}\,\mathrm{d}t \tag{44}$$

Since the integrand is constant in steady state, this simplifies to:

$$W_b = \frac{1}{r}\left[\left(a_1 y_b^* - \frac{a_2\left(y_b^*\right)^2}{2}\right) - \frac{c\left(z_b^*\right)^2}{2}\right], \tag{45}$$

Substitute $y_b^* = \delta z_b^*$:

$$W_b = \frac{1}{r}\left[a_1\delta z_b^* - \frac{a_2\delta^2\left(z_b^*\right)^2}{2} - \frac{c\left(z_b^*\right)^2}{2}\right] = \frac{1}{r}\left[a_1\delta z_b^* - \frac{\left(z_b^*\right)^2}{2}(a_2\delta^2 + c)\right] \tag{46}$$

Substituting z_b^* from (41) into (46) yields the following:

$$W_b = \frac{a_1^2(\delta + r)\left(a_2\delta^2(\delta + r) + c(\delta - r)\right)}{2r(a_2\delta(\delta + r) + c)^2}.$$

■

A.2 Derivation of the optimal control in Section 2.2 and proof of Proposition 1

The current-value Hamiltonian to (1) is:

$$\mathcal{H} = \left(a_1 y - \frac{a_2 y^2}{2}\right) - \frac{cz^2}{2} - \left(b_1 x + \frac{b_2 x^2}{2}\right) + \lambda\left(y - \delta z + (1-\alpha)x\right)$$

First-order conditions w.r.t. y and x are:

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial y} &= a_1 - a_2 y + \lambda = 0 \implies y(t) = \frac{a_1 + \lambda(t)}{a_2} \\ \frac{\partial \mathcal{H}}{\partial x} &= -b_1 - b_2 x + \lambda(1-\alpha) = 0 \implies x(t) = \frac{\lambda(t)(1-\alpha) - b_1}{b_2} \end{aligned} \tag{47}$$

With the non-negativity constraint:

$$x(t) = \begin{cases} 0 & \text{if } \lambda(t) \geq \frac{b_1}{1-\alpha}, \\ \frac{\lambda(t)(1-\alpha) - b_1}{b_2} > 0 & \text{if } \lambda(t) < \frac{b_1}{1-\alpha} \end{cases}$$

The costate equation is:

$$\dot{\lambda}(t) = r\lambda - \frac{\partial \mathcal{H}}{\partial z} = r\lambda + cz + \lambda\delta$$

with the transversality condition:

$$\lim_{t \to \infty} e^{-rt}\lambda(t)z(t) = 0$$

Plugging (47) into the CO_2 accumulation dynamics $\dot{z} = y - \delta z + (1-\alpha)x$, we obtain the following: if $\lambda(t) < \frac{b_1}{1-\alpha}$, CCS is active, and

$$\dot{z} = -\delta z + \left(\frac{1}{a_2} + \frac{(1-\alpha)^2}{b_2}\right)\lambda + \frac{a_1}{a_2} - \frac{b_1(1-\alpha)}{b_2}$$

If $\lambda(t) \geq \frac{b_1}{1-\alpha}$, CCS is inactive, and

$$\dot{z} = \frac{a_1 + \lambda}{a_2} - \delta z.$$

We thus conclude,

$$\dot{z} = \begin{cases} -\delta z + \left(\frac{1}{a_2} + \frac{(1-\alpha)^2}{b_2}\right)\lambda + \frac{a_1}{a_2} - \frac{b_1(1-\alpha)}{b_2} & \text{if } \lambda(t) < \frac{b_1}{1-\alpha}, \\ \frac{a_1+\lambda}{a_2} - \delta z & \text{if } \lambda(t) \geq \frac{b_1}{1-\alpha}. \end{cases}$$

Thus, equation (10) is proved.

Solving for the convergence speed μ_c: Substitute the optimal controls $y(t)$ and $x(t)$ from the first-order conditions into the state equation $\dot{z} = y - \delta z + (1-\alpha)x$:

$$y(t) = \frac{a_1 + \lambda}{a_2}, \quad x(t) = \frac{\lambda(1-\alpha) - b_1}{b_2}.$$

Thus:

$$\dot{z} = \frac{a_1 + \lambda}{a_2} - \delta z + (1-\alpha)\left(\frac{\lambda(1-\alpha) - b_1}{b_2}\right).$$

The costate equation is:

$$\dot{\lambda} = (r+\delta)\lambda + cz.$$

Near the steady state (z_c^*, λ_c^*), the system is linearized as:

$$\begin{pmatrix} \dot{z} \\ \dot{\lambda} \end{pmatrix} = \mathbf{J} \begin{pmatrix} z - z_c^* \\ \lambda - \lambda_c^* \end{pmatrix},$$

where $\mathbf{J}$ is the Jacobian matrix:

$$\mathbf{J} = \begin{pmatrix} -\delta & \frac{1}{a_2} + \frac{(1-\alpha)^2}{b_2} \\ c & r+\delta \end{pmatrix}.$$

The eigenvalues μ satisfy $\det(\mathbf{J} - \mu \mathbf{I}) = 0$:

$$\mu^2 - r\mu - \left[\delta(r+\delta) + c\left(\frac{1}{a_2} + \frac{(1-\alpha)^2}{b_2}\right)\right] = 0.$$

The stable (negative) root is:

$$\mu_c = \frac{r - \sqrt{r^2 + 4\left[\delta(r+\delta) + c\left(\frac{1}{a_2} + \frac{(1-\alpha)^2}{b_2}\right)\right]}}{2} < 0.$$

In the steady state, $\dot{z} = 0$ and $\dot{\lambda} = 0$.

$$(r+\delta)\lambda_c^* + cz_c^* = 0 \implies \lambda_c^* = -\frac{cz_c^*}{r+\delta}.$$

Substitute y and x into $\dot{z} = 0$:

$$\frac{a_1 + \lambda_c^*}{a_2} - \delta z_c^* + (1-\alpha)\left(\frac{\lambda_c^*(1-\alpha) - b_1}{b_2}\right) = 0.$$

Substitute $\lambda_c^* = -\frac{cz_c^*}{r+\delta}$:

$$\frac{a_1 - \frac{cz_c^*}{r+\delta}}{a_2} - \delta z_c^* + (1-\alpha)\left(\frac{-\frac{cz_c^*}{r+\delta}(1-\alpha) - b_1}{b_2}\right) = 0 \implies z_c^* = \frac{a_1 b_2(r+\delta) - a_2 b_1(1-\alpha)(r+\delta)}{cb_2 + \delta a_2 b_2(r+\delta) + ca_2(1-\alpha)^2}. \tag{48}$$

Plugging z_c^* into $\lambda_c^* = -\frac{cz_c^*}{r+\delta}$:

$$\lambda_c^* = -\frac{c\left[a_1 b_2(r+\delta) - a_2 b_1(1-\alpha)(r+\delta)\right]}{(r+\delta)\left[cb_2 + \delta a_2 b_2(r+\delta) + ca_2(1-\alpha)^2\right]}.$$

The explicit solution to the linear dynamic system, (9) and (11), is the following: for $t \geq T_c$,

$$\begin{cases} z_c(t) = (z(T_c) - z_c^*)e^{\mu_c(t-T_c)} + z_c^*, \\ \lambda_c(t) = \dfrac{c}{\mu_c - (r+\delta)}(z(T_c) - z_c^*)e^{\mu_c(t-T_c)} + \lambda_c^*, \end{cases} \tag{49}$$

by taking into account the transversality condition, where

$$\mu_c = \frac{1}{2}\left[r - \sqrt{(r+2\delta)^2 + 4c\left(\frac{1}{a_2} + \frac{(1-\alpha)^2}{b_2}\right)}\right] (< 0);$$

and for $0 \leq t \leq T_c$,

$$\begin{cases} z_c(t) = c_1 e^{\mu_{b1} t} + c_2 e^{\mu_{b2}} + z_b^*, \\ \lambda_c(t) = \frac{c}{\mu_{b1} - (r+\delta)}\, c_1 e^{\mu_{b1} t} + \frac{c}{\mu_{b2} - (r+\delta)}\, c_2 e^{\mu_{b2} t} + \lambda_b^*, \end{cases} \tag{50}$$

where $\mu_{b1} = \mu_b$ and

$$\mu_{b2} = \frac{1}{2}\left[r + \sqrt{(r+2\delta)^2 + \frac{4c}{a_2}}\right] (< 0).$$

■

A.3 Proof of Proposition 3

We present the detailed proof for the CCS case; the corresponding arguments for the renewable-only scenario are analogous.

To shorten the notation, let $W_{zc1}(T)$ and $W_{zc2}(T)$ denote social welfare accrued before and after the net-zero date T, respectively. Substituting the optimal choice $y_{zc1}(t)$, $x_{zc1}(t)$, and x_{zc2} into the value functions and taking the derivative with respect to T yields:

$$\frac{dW_{zc1}(T)}{dT} = \frac{e^{-rT}}{2}\left[\frac{a_1^2}{a_2} + \frac{b_1^2}{b_2} - \left(\frac{1}{a_2} + \frac{(1-\alpha)^2}{b_2}\right)\lambda_{zc1}^2(T) - cz_{zc}^2(T)\right],$$

and

$$\frac{dW_{zc2}(T)}{dT} = \frac{e^{-rT}}{2}\left[cz_{zc}^2(T) - \frac{(a_1(1-\alpha) - b_1)^2}{a_2(1-\alpha)^2 + b_2}\right].$$

Thus, combining the above two expressions together and simplifying them, we obtain

$$\frac{dW_{zc}(T)}{dT} = \frac{dW_{zc1}(T)}{dT} + \frac{dW_{zc2}(T)}{dT} = \frac{e^{-rT}}{2}\left[\frac{a_1^2}{a_2} + \frac{2a_1b_1(1-\alpha)}{a_2(1-\alpha)^2+b_2} + \frac{b_1^2a_2(1-\alpha)^2}{b_2(a_2(1-\alpha)^2+b_2)}\right].$$

The results in Proposition 3 follow directly from the last expression. That completes the proof.

A.4 Proof of Proposition 5

To solve the optimal control problem (17) subject to (18) equipped with the capacity constraint $\int_0^\infty (1-\alpha)x(t)\,dt \leq \bar{S}$, we define the Lagrangian as

$$\mathcal{L} = W_{zc} + \mu\left(\bar{S} - \int_0^T (1-\alpha)x(t)\,dt - \int_T^\infty (1-\alpha)x(t)\,dt\right).$$

The current-value Hamilitonian is given by

$$\mathcal{H}^{cv} = \begin{cases} a_1y - \frac{a_2y^2}{2} - \frac{cz^2}{2} - \left(b_1x + \frac{b_2x^2}{2}\right) - \mu(1-\alpha)x, & 0 \leq t \leq T, \\ (a_1(1-\alpha) - b_1)x - \frac{(a_2(1-\alpha)^2+b_2)x^2}{2} - \mu(1-\alpha)x, & t \geq T, \end{cases}$$

with μ being the Kuhn-Tucker multiplier.

It is straightforward to show that the first order condition for optimal emissions and the corresponding co-state variable are the same as in Section 3.2; while the optimal choice of CCS abatement is the one given in Proposition 5(a) with the following complementary slackness condition:

$$\mu \geq 0, \quad \int_0^T (1-\alpha)x_{zc1}^{eff}dt + \int_T^\infty (1-\alpha)x_{zc1}^{eff} \leq \bar{S},$$

and

$$\mu\left[\int_0^T (1-\alpha)x_{zc1}^{eff}dt + \int_T^\infty (1-\alpha)x_{zc1}^{eff} - \bar{S}\right] = 0.$$

We can thus define the effective unit cost of CCS as $b_1^{eff} = b_1 + (1-\alpha)\mu$. When the cumulative storage constraint $\bar{S}$ is sufficiently large so that it does not bind, $\mu = 0$. By contrast, when $\bar{S}$ is binding, $\mu = \mu(\bar{S}) > 0$ and $b_1^{eff} = b_1^{eff}(\bar{S}) > b_1$. The remaining results stated in Proposition 5 then follow directly.

That completes the proof.

References

Amigues, Jean-Pierre, Gilles Lafforgue, and Michel Moreaux (2016). Optimal timing of carbon capture policies under learning-by-doing. en. Journal of Environmental Economics and Management 78, pp. 20–37.

Ayong Le Kama, Alain, Mouez Fodha, and Gilles Lafforgue (2013). Optimal carbon capture and storage policies. Environmental Modeling & Assessment 18.4, pp. 417–426.

Bacilieri, Andrea, Richard Black, and Rupert Way (2023). Assessing the relative costs of high-CCS and low-CCS pathways to 1.5 degrees. Oxf. Smith Sch. Enterp. Environ 4, pp. 8–23.

Benchekroun, Hassan and Ngo Van Long (1998). Efficiency Inducing Taxation for Polluting Oligopolists. en. Journal of Public Economics 70.2, pp. 325–342.

Bertinelli, Luisito, Carmen Camacho, and Benteng Zou (Sept. 1, 2014). Carbon Capture and Storage and Transboundary Pollution: A Differential Game Approach. European Journal of Operational Research 237.2, pp. 721–728.

Bos, Kyra and Joyeeta Gupta (2018). Climate change: the risks of stranded fossil fuel assets and resources to the developing world. Third World Quarterly 39.3, pp. 436–453.

Boucekkine, Raouf, Weihua Ruan, and Benteng Zou (2023). The Irreversible Pollution Game. en. Journal of Environmental Economics and Management 120, p. 102841.

Broek, Machteld van den, Rick Hoefnagels, Edward S. Rubin, Wim C. Turkenburg, and André P. C. Faaij (2009). Effects of Technological Learning on Future Cost and Performance of Power Plants with CO_2 Capture. Progress in Energy and Combustion Science 35.6, pp. 457–480.

Chen, Yiwen, Nora Paulus, Xi Wan, and Benteng Zou (2024). Optimal timing of carbon capture and storage policies—A social planner's view. Energy Economics 136, p. 107656.

— (2025). Timing carbon capture and storage (CCS) deployment across borders: A game-theoretic analysis. Economic Modelling, p. 107191.

Clark, Victoria R and Howard J Herzog (2014). Can "stranded" fossil fuel reserves drive CCS deployment? Energy Procedia 63, pp. 7261–7271.

Dockner, Engelbert (2000). *Differential games in economics and management science*. Cambridge University Press.

Dockner, Engelbert J. and Ngo Van Long (1993). International Pollution Control: Cooperative versus Noncooperative Strategies. Journal of Environmental Economics and Management 25.1, pp. 13–29.

Dolphin, Geoffroy, Michael Pahle, Dallas Burtraw, and Mirjam Kosch (2023). A net-zero target compels a backward induction approach to climate policy. Nature climate change 13.10, pp. 1033–1041.

Fikru, Mahelet G, Fateh Belaid, and Hongyan Ma (2024). Carbon capture and renewable energy policies: Could policy harmonization be a puzzle piece to solve the electricity crisis? Energy Economics 136, p. 107753.

Grimaud, André and Luc Rouge (2014). Carbon sequestration, economic policies and growth. Resource and Energy Economics 36.2, pp. 307–331.

Hepburn, Cameron, Matthew C Ives, Sam Loni, Penny Mealy, Pete Barbrook-Johnson, J Doyne Farmer, Nicholas Stern, and Joseph Stiglitz (2025). Economic models and frameworks to guide climate policy. Oxford Review of Economic Policy 41.2, pp. 616–652.

Hoel, Michael and Larry Karp (2002). Taxes versus quotas for a stock pollutant. Resource and Energy Economics 24.4, pp. 367–384.

Hoel, Michael Olaf (2025). The path to net zero emissions. Journal of Environmental Economics and Management 132, p. 103177.

Holz, Franziska, Tim Scherwath, Pedro Crespo del Granado, Christian Skar, Luis Olmos, Quentin Ploussard, Andrés Ramos, and Andrea Herbst (2021). A 2050 perspective on the role for carbon capture and storage in the European power system and industry sector. Energy Economics 104, p. 105631.

IEA (2021). *Is carbon capture too expensive?* International Energy Agency.

IPCC (2022). *Climate Change 2022: Impacts, Adaptation and Vulnerability*. Intergovernmental Panel on Climate Change.

Karp, Larry and Jiangfeng Zhang (2012). Taxes versus quantities for a stock pollutant with endogenous abatement costs and asymmetric information. Economic Theory 49.2, pp. 371–409.

Kearns, David, Hao Liu, and Chris Consoli (2021). *Technology Readiness and Costs of CCS*. Tech. rep. 3. Global CCS Institute.

Lafforgue, Gilles, Bertrand Magné, and Michel Moreaux (2008). Energy substitutions, climate change and carbon sinks. Ecological Economics 67.4, pp. 589–597.

Mason, Charles F, Stephen Polasky, and Nori Tarui (2017). Cooperation on climate-change mitigation. European Economic Review 99, pp. 43–55.

Moreaux, Michel, Jean-Pierre Amigues, Gerard Van Der Meijden, and Cees Withagen (2024). Carbon Capture: Storage vs. Utilization. Journal of Environmental Economics and Management 125, p. 102976. DOI: 10/gttz2b.

Moreaux, Michel and Cees Withagen (2015). Optimal abatement of carbon emission flows. Journal of Environmental Economics and management 74, pp. 55–70.

Nordhaus, William D (2007). A review of the Stern review on the economics of climate change. Journal of economic literature 45.3, pp. 686–702.

Ploeg, Frederick van der (2013). Cumulative Carbon Emissions and the Green Paradox. Annual Review of Resource Economics 5, pp. 281–300.

Reilly, John M and Kenneth R Richards (1993). Climate change damage and the trace gas index issue. Environmental and Resource Economics 3.1, pp. 41–61.

Rubin, Edward S (2012). Understanding the pitfalls of CCS cost estimates. International journal of greenhouse gas control 10, pp. 181–190.

Rubin, Edward S, John E Davison, and Howard J Herzog (2015). The cost of CO2 capture and storage. International Journal of Greenhouse gas control 40, pp. 378–400.

Sinn, Hans-Werner (2008). Public Policies against Global Warming: A Supply Side Approach. International Tax and Public Finance 15.4, pp. 360–394.

Stern, Nicholas (2008). The economics of climate change. American economic review 98.2, pp. 1–37.

Van der Ploeg, Frederick and Armon Rezai (2020). Stranded assets in the transition to a carbon-free economy. Annual Review of Resource Economics 12.1, pp. 281–298.

Van der Ploeg, Frederick and Cees Withagen (2015). Global warming and the green paradox: A review of adverse effects of climate policies. Review of Environmental Economics and Policy.

Van der Zwaan, Bob and Reyer Gerlagh (2009). Economics of geological CO2 storage and leakage. Climatic change 93.3, pp. 285–309.

Vergragt, Philip J, Nils Markusson, and Henrik Karlsson (2011). Carbon capture and storage, bio-energy with carbon capture and storage, and the escape from the fossil-fuel lock-in. Global Environmental Change 21.2, pp. 282–292.

Welsby, Dan, James Price, Steve Pye, and Paul Ekins (2021). Unextractable fossil fuels in a 1.5 C world. Nature 597.7875, pp. 230–234.