From Snow White to mathematical stories

Most of our students know the story of Snow White (from the Disney movie). Now comes the challenge: can you write that story in one sentence? You will be confronted with deciding what is important for you and discard the rest, and you will stretch the grammar to squeeze in everything that is possible. My attempt:

Snow White is a girl poisoned with a magic apple by a witch jealous of her beauty, coming disguised as an old ugly woman while the girl was undercover, hosted by seven dwarves: Snow White was believed to be dead but she was only in a permanent sleep, and in fact the kiss of a prince broke the spell and the girl married the prince, while the witch died escaping the dwarves.

And now, even more challenging, with only one short sentence allowed:

Snow White is the story of a girl, hosted by seven dwarves, whom a jealous witch tries to murder.

The point of the exercise is training basic skills that you need while preparing a mathematical presentation, given the fact that time is always very limited.
You have to select the content and make various cuts, even if you would prefer to say much more. Sometimes the selection is based on a personal preference. For example, others could have written:

Snow White is the story of a girl hosted by seven dwarves, poisoned by a witch and saved by a prince.

So “Cut & Squeeze in” is the art of preparing a presentation: you must cut a lot but not what is important; you must squeeze in all you can, but not to the point of losing your audience.
We should keep in mind that an in-presence presentation serves the main purpose of making “a mind map” for the audience, who should retain the key features and thus be guided while completing the learning as self-study, going more (or again) into the details. And, in any case, even a video presentation requires a selection and needs scaffolding.

A good presentation has some key takeaway, to be decided in advance by the presenter. For example, what is the key lesson of Snow White? Here some options:

- *Jealous people can be very dangerous.*
- *Never eat food offered by strangers.*
- *When you make a spell, do not underestimate improbable outcomes that may break it.*

While making a mathematical explanation, you need to engage the audience by making use of humor, pathos, or anything that is catchy. What is “mathematical drama”? Anything impressive, surprising, out of the ordinary, beautiful, or also ridiculously tedious. A theorem with dozens of applications. A very short or easy proof. A relatively easy proof that was undiscovered for decades. An almost magic formula like the one relating the volumes of a half-sphere, a cone, and a cylinder with the same basis and height (half-sphere+cone=cylinder).

For our metaphor, in Snow White the girl is extremely beautiful, the witch extremely wicked, the girl narrowly escapes death and so on. Possible catchy facts:

Snow White only lives because someone disobeys their orders (the hunter).
The beautiful witch, to pursue her fixation of being the most beautiful, makes herself ugly (for her disguise).

To make a good presentation, one also must ponder what may lead to confusion:

The girl was right in accepting the food of the dwarves but was not right in accepting the one of the old lady, although these were all strangers to her.

Moreover, one should anticipate some of the curiosity questions and prepare convincing answers (nowadays you can find reasonable answers to almost anything, but you still must ask the questions yourself):

Why are there 7 dwarves and not 8?
How tall is a dwarf?

The reason you have to "build a story" in a mathematical presentation is because everyone can listen to stories while it is hard to appreciate or remember the "chain-of-telegrams" version:

Witch is jealous of girl. Girl runs away. Dwarves host girl. Witch poisons apple. Witch offers apples. Girl eats apple …

Telling a story also means choosing what to highlight, like alternating low tides with high tides. Teachers always explain and entertain, use visuals as much as possible and metaphors and memory tricks, they make things catchy and "sell them". They do what it takes so that the message comes across.

A piece of advice for explanations is "be honest", namely "make use of disclaimers, identify black boxes". Like: *For brevity, I only explain the key parts of Snow White.* Or, with a mathematical example:

Cavalieri's principle for the volumes holds if the solid figure is sufficiently regular and, no, we are not going to prove it. The common solid figures are fine, don't worry, but do not apply the principle, say, to a fractal-like set. Also don't worry, there is a rigorous statement and a full proof, that is based on computing volumes as integrals.

Finally, don't be afraid to only sketch straight-forward steps or an analogous case to make space for real understanding, like why an assumption was needed for a statement or where an assumption was used in a proof.

Good luck with your theatre show… ehm, mathematical lesson.