Modelling of sparse lightweight composite structures using a geometrically discontinuous approach

Paul T. Kühner, Michal Habera, Andreas Zilian
Department of Engineering | University of Luxembourg

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UNIVERSITÉ DU LUXEMBOURG

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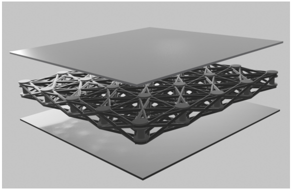
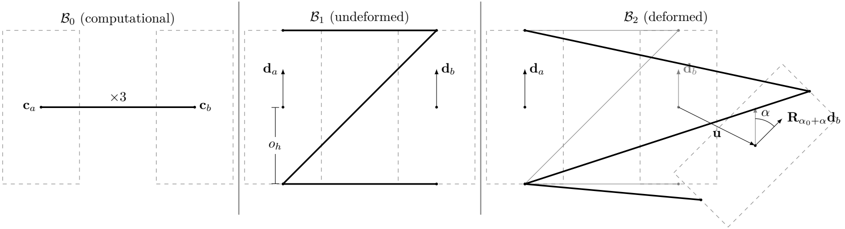
Motivation | Robotically wound CFRP structures



Winding cell (left, source: gradel.lu), GRAM sample (top right) and sandwich concept (bottom right).

- Robotic, endless filament winding
- Carbon fibre reinforced polymers (CFRP) wound around *bushings*
- Multiple passes between the same end-points form a *string*
- (Thermal) curing produces final string stiffness (wet winding)
- Similar approaches: GRAM®, IsoTruss® [1], Fibr® [2]

Model | Kinematics - Configurations



For *offset* $\boldsymbol{o} \in \mathcal{DG}_1(\mathcal{B}_0)^2$, *angle* $\boldsymbol{\alpha}^0, \boldsymbol{\alpha} \in \mathcal{P}_1(\mathcal{B}_0)$, *displacement* $\boldsymbol{u} \in V_u = \mathcal{P}_1(\mathcal{B}_0)^2$ the configurations are given by placements

$$\boldsymbol{x}_1 = \boldsymbol{x}_0 + I_{V_u} \boldsymbol{R}_{\boldsymbol{\alpha}^0} \boldsymbol{o}(\boldsymbol{x}_0) \tag{1}$$

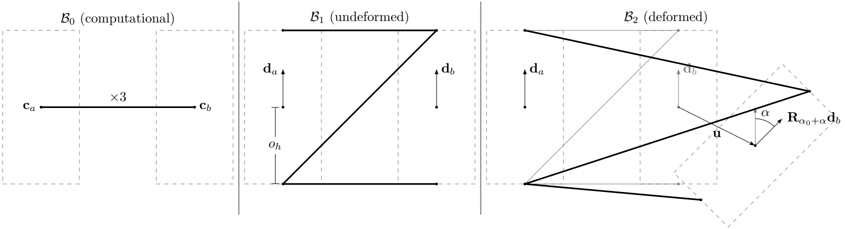
$$\boldsymbol{x}_2 = \boldsymbol{x}_0 + I_{V_u} \boldsymbol{R}_{\boldsymbol{\alpha}^0+\boldsymbol{\alpha}} \boldsymbol{o}(\boldsymbol{x}_0) + \boldsymbol{u}(\boldsymbol{x}_0). \tag{2}$$

- The (constant) offset is defined point-wise by

$$\boldsymbol{o}(\boldsymbol{c}_a) = o_h^a \boldsymbol{d}_a \quad \text{and} \quad \boldsymbol{o}(\boldsymbol{c}_b) = o_h^b \boldsymbol{d}_b. \tag{3}$$

- Equivalently extends to three dimensions ($\boldsymbol{o} \in \mathcal{DG}_1(\mathcal{B}_0)^3$, $\boldsymbol{\alpha} \in \mathcal{P}_1(\mathcal{B}_0)^2$)

Model | Kinematics - Strain measure



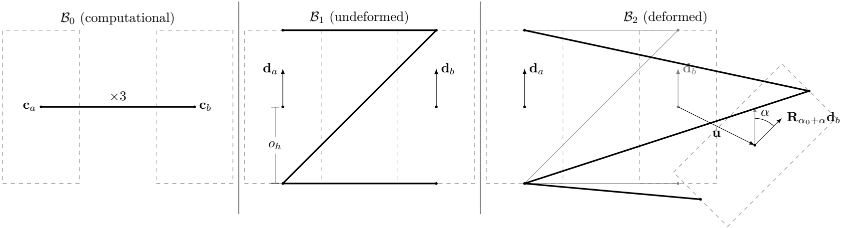
- Green-Lagrange strain (in undeformed configuration) reads

$$\boldsymbol{E} = \frac{1}{2}\left(\boldsymbol{C}_{12} - \boldsymbol{C}_{11}\right) = \frac{1}{2}\left(\left(\boldsymbol{J}_{12}\right)^T \boldsymbol{J}_{12} - \boldsymbol{I}\right), \quad \text{for} \quad \boldsymbol{J}_{ab} = \frac{\partial \boldsymbol{x}_b}{\partial \boldsymbol{x}_a}. \tag{4}$$

- Extract via Tangential Differential Calculus (TDC) [3–5] the tangential part of the kinematics

$$\boldsymbol{E}^m = \boldsymbol{P}\,\boldsymbol{E}\,\boldsymbol{P} \quad \text{with} \quad \boldsymbol{P} = \mathbf{t}_1 \otimes \mathbf{t}_1, \; \mathbf{t}_a = \frac{\boldsymbol{J}_{-1a}}{\|\boldsymbol{J}_{-1a}\|} \tag{5}$$

where $\boldsymbol{x}_{-1}$ is the placement of the one-dimensional reference element.
- Undeformed and computational (mesh) configuration do not coincide



St. Venant Kirchhoff strain energy density W, the total potential energy (on $\mathcal{B}_1$, and pulled-back to $\mathcal{B}_0$) is

$$\Pi(\boldsymbol{u}, \boldsymbol{\alpha}) = \int_{\mathcal{B}_1} A\, W(\boldsymbol{E}^m)\, \mathrm{d}\boldsymbol{x}_1 - P_{\text{ext}}(\boldsymbol{u}) \tag{6}$$

$$= \int_{\mathcal{B}_0} A\, W(\boldsymbol{E}^m) \underbrace{||\boldsymbol{J}_{01}\mathbf{t}_0||}_{\text{stretch ratio}} \mathrm{d}\boldsymbol{x}_0 - P_{\text{ext}}(\boldsymbol{u}). \tag{7}$$

Verification | Resolved truss model

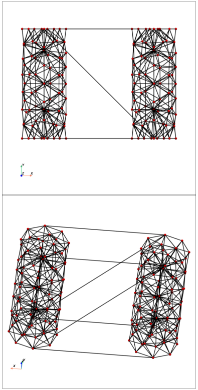
- Computational domain
  1. Mesh 3D geometry (bushing cylinders and strings)
  2. Extract wireframe/edge mesh
- Rigid bushings: large member stiffness ($\times 10^4$)
- Computational/undeformed $\boldsymbol{x}_0$, deformed $\boldsymbol{x}_1$

$$\boldsymbol{x}_1 = \boldsymbol{x}_0 + \boldsymbol{u} \tag{8}$$

$$\boldsymbol{E} = \frac{1}{2}(\boldsymbol{C}_{01} - \boldsymbol{C}_{00}) = \frac{1}{2}\left((\boldsymbol{J}_{01})^T \boldsymbol{J}_{01} - \boldsymbol{I}\right) \tag{9}$$

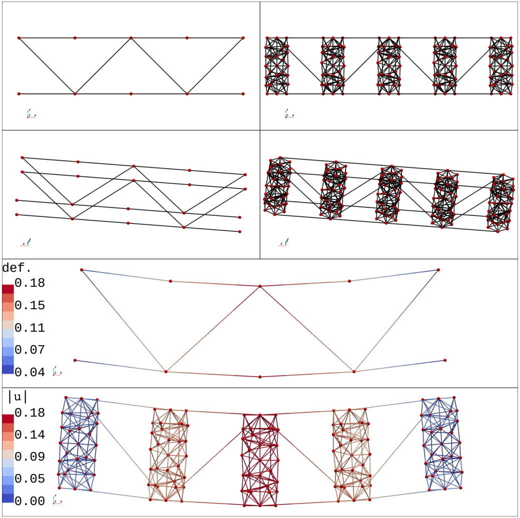
$$\boldsymbol{E}^m = \boldsymbol{P}\,\boldsymbol{E}\,\boldsymbol{P} \quad \text{with} \quad \boldsymbol{P} = \mathbf{t}_0 \otimes \mathbf{t}_0 \tag{10}$$

$$\Pi(\boldsymbol{u}) = \int_{\mathcal{B}_0} A\,W(\boldsymbol{E}^m)\,\mathrm{d}\boldsymbol{x}_0 - P_{\text{ext}}(\boldsymbol{u}). \tag{11}$$



Resolved mesh $\mathcal{B}_0$.

Verification | Model comparison

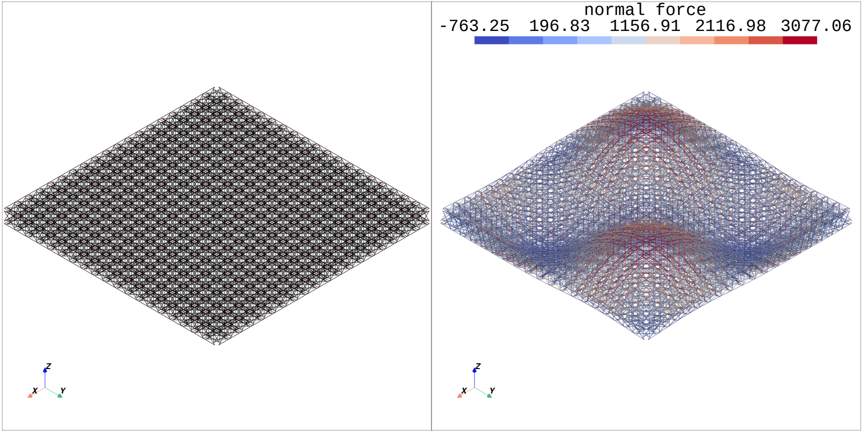


Offset- (left) and resolved (right) models, deformation (bottom).

- Fixed bushing centers left/right
- Uniform top loading of non-boundary bushings
- No z-components in displacement
- Equivalent for finer resolved meshes

For int. strain energy $U = \Pi - P_{\text{ext}}$ one gets

$$\frac{|U^{\text{off.}} - U^{\text{res.}}|}{|U^{\text{res.}}|} \approx 2.37\times10^{-5}.$$



Sandwich configuration with 20×20 bushings, 11856 strings, undeformed and deformed with normal force.

- Finite strain kinematics characterise (global) response
- Low DOF count enables simulation of large architectured structures

Summary and Outlook

Advantages

- Fully variational
- Couplings are a priori enforced via the prescribed kinematics
- Low DOF count

Disadvantages

- Elaborate kinematics
- TDC: expensive quadrature degree

Future work

- Application to optimisation tasks: shape-/topology optimisation
- Extension to beam string models
- Incorporation of non-rigid bushing response
- Open source functionality and demo through `dolfiny` [6]

References

[1] Benjamin K.S. Woods, Ioan Hill and Michael I. Friswell. 'Ultra-efficient wound composite truss structures'. In: *Composites Part A: Applied Science and Manufacturing* 90 (2016), pp. 111–124. ISSN: 1359-835X. DOI: 10.1016/j.compositesa.2016.06.022.

[2] Steffen Reichert et al. 'Fibrous structures: An integrative approach to design computation, simulation and fabrication for lightweight, glass and carbon fibre composite structures in architecture based on biomimetic design principles'. In: *Computer-Aided Design* 52 (2014), pp. 27–39. ISSN: 0010-4485. DOI: 10.1016/j.cad.2014.02.005.

[3] Adam Sky et al. 'Intrinsic mixed-dimensional beam-shell-solid couplings in linear Cosserat continua via tangential differential calculus'. In: *Computer Methods in Applied Mechanics and Engineering* 432 (2024), p. 117384.

[4] Michael Neunteufel. 'Mixed finite Element Methods For Nonlinear Continuum Mechanics and Shells'. en. PhD thesis. TU Wien, 2021.

[5] Peter Hansbo and Mats G. Larson. 'Finite element modeling of a linear membrane shell problem using tangential differential calculus'. In: *Computer Methods in Applied Mechanics and Engineering* 270 (2014), pp. 1–14.

[6] Andreas Zilian and Michal Habera. 'dolfiny: Convenience wrappers for DOLFINx'. English. In: Cambridge, United Kingdom, 23 March 2021. HDL: https://orbilu.uni.lu/10993/47422.