Intelligent Target Maneuverability in Presence of Tracking with Multiple Radars

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ABSTRACT

A scenario with multiple radars connected to a fusion centre and tracking a target endowed with cognitive abilities is considered. The aim of the target is to degrade the performance of the radar network using its cognitive abilities. In the embodiment considered in this paper, the target injects interference that perturbs the measurements at the different radars. The injected interference is designed to maximize the trace of the error covariance matrix in each instance of the extended Kalman filter iterations used at the fusion centre. The optimal interference in such a setting is formulated as a convex problem and its structure reveals a low-rank correlated structure unlike the intuitive additive white noise. Relation to water-filling is drawn and the impact of such an interference is subsequently analysed using numerical simulations.

Index Terms— Cognitive radar, intelligent target, extended Kalman filter, colored interference, inverse cognition.

1. INTRODUCTION

A key function of any radar is to track the adversary/ target using the various degrees of freedom in its arsenal. Efficient use of the degrees of freedom is achieved by cognition where the radar can sense, learn and adapt [1]. This adaptive process involves modifying parameters like waveform, aperture, dwell time, and revisit rate based on current estimations. Several works have investigated various aspects in this context. Use of POMDPs for radar resource management was proposed in [2]. Waveform optimization in a single target tracking cognitive radar network (CRN) was proposed in [3]. In the context of multiple target tracking, resource optimization paradigms have been considered for collocated MIMO radar [4], [5] and for CRN [6]. Target tracking using age of information maximization and transmit sub-aperturing have been developed in [7] and [8] respectively. Moving beyond cognition, metacognitive radar [9] has been considered in [10] and hybrid cognition in [11]. Collection of recent works on Cognitive Radar can also be found in [12].

On the other hand, the target/ adversary aims to avoid radar detection by using a plethora of strategies ranging from stealth design to jamming. With the advent of cognitive radar, the adversary could also be endowed with capabilities to learn the cognitive strategies of the radar and devise novel mechanisms to degrade the performance of the radar. Inverse filtering towards estimating the posterior distribution of the adversary's tracker given its actions using a Bayesian approach is considered in [13]. This was extended to the verification of constrained utility maximization hypothesis in the *Inverse cognition* paradigm in [14], [15]. This *Inverse Reinforcement Learning* setting was based on *revealed preferences*; waveform optimization and beam allocation examples were discussed therein. A metacognitive set-up where in the radar aims to mask its cognitive abilities from an intelligent adversary has been recently considered in [16].

In the context of IRL, the cognitive target in [15] considers generating interference to confuse adversary radar. Therein, a single radar capable of optimizing its waveform based on the channel is considered and the channel information is perturbed towards degrading the signal to clutter plus noise ratio. A non-convex problem is formulated and a numerical example provided. Different from the aforementioned works, we consider a scenario where multiple adversary radars track a cognitive target. While the target continues to induce interference in measurements of the different radars, it differs from [15], since the statistical distribution of the interference across the different radars is an additional degree of freedom. Using the Extended Kalman Filter (EKF) setting to fuse the parameters, the paper provides a framework for optimal interference based on maximizing the trace of the posterior covariance matrix. This leads to a convex problem, whose solution structure is provided and closed-form solution detailed for special cases. The resulting interference is correlated among the radars, contrary to the convention of adding uncorrelated noise. The impact of this interference on range, velocity and acceleration is shown through numerical examples. The numerical results also demonstrate that the effect of target induced interference can not be adequately compensated even with multiple sensors.

2. SYSTEM MODEL AND PROBLEM FORMULATION

We consider the set-up where N radars, multiplexed in the frequency domain, are tracking a single target. Each radar estimates the standard range, angle and Doppler parameters and transmits them to a fusion centre (FC) where the estimates

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are fused to track the object. We further assume that the target is endowed with cognitive capabilities that aim to confuse the radar; in this work, the neither the radars nor the FC is aware of the cognitive capabilities of the target.

At the k^{th} frame instant, let the l^{th} radar (located at $[x_{l,ref}, y_{l,ref}]$) report the noisy measurement $\mathbf{z}_{l,k} \in \mathbb{R}^{3\times 1} = [\hat{r}_{l,k}, \hat{v}_{l,k}, \hat{\phi}_{l,k}]^T$], where $\hat{r}_{l,k}, \hat{\theta}_{l,k}, \hat{\phi}_{l,k} \in \mathbb{R}$ are the estimates of the target radial range, velocity, and azimuth angle, respectively. After observing $\{\mathbf{z}_{l,k}\}_{l=1}^{N}$, the fusion centre extracts the position ($[x_k, y_k]^T$), velocity ($[v_{x_k}, v_{y_k}]^T$), and acceleration ($[a_{x_k}, a_{y_k}]^T$) of the target in the Cartesian coordinates. This entails coordinate conversion and leads to a classical state-space estimation model.

On the other hand, the target, endowed with cognitive function, aims to confuse the radars by undertaking such operations that lead to the estimates $\overline{\mathbf{z}}_{l,k} \in \mathbb{R}^{m\times 1} = [\overline{r}_{l,k}, \overline{v}_{l,k}, \overline{\phi}_{l,k}]^T$. One possibility towards this is to use spoofing signals to interfere with the measurements. Two aspects need to be noted here: (i) As in [15], the target has limited power to effect the interference and (ii) The interference should be consistent across the radars so as to induce a false target and not averaged out by the fusion centre.

2.1. Target Motion and Measurement Models

We assume a constant acceleration (CA) model for the target with the n_s dimensional[1] state space in the global co-ordinate system [2] at the k^{th} instance being defined as, $\mathbf{s}_k \in \mathbb{R}^{n_s\times 1} = [x_k, v_{x_k}, a_{x_k}, y_k, v_{y_k}, a_{y_k}]^T$. The target states $\mathbf{s}_k$ evolve with k and the CA motion model relating the state vectors at k^{th} and $(k-1)^{st}$ instances can be represented as follows,

$$\mathbf{s}_k = [\mathbf{I}_2 \otimes \mathbf{F}]\mathbf{s}_{k-1} + \mathbf{u}_k, \tag{1}$$

where $\mathbf{F} = \begin{bmatrix} 1 & T & 0.5T^2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$, T is the CPI or filter update time, $\mathbf{u}_k$ is the model noise assumed to be $\mathcal{N}(\mathbf{0}, \mathbf{Q}_u)$ with, $\mathbf{Q}_u = [\mathbf{I}_3 \otimes \mathbf{T}]$. A representative characterization for $\mathbf{T}$ can be found in [17]. Letting $\dot{x}, \dot{y}$ refer to the time derivatives, the measurement vector $\mathbf{h}_l(\mathbf{s}_k) = [r_{l,k}, v_{l,k}, \phi_{l,k}]^T$ takes the classical form,

$$\mathbf{h}_l(\mathbf{s}_k) = [r_{l,k}, v_{l,k}, \phi_{l,k}]^T, \tag{2}$$

$$r_{l,k} = \sqrt{(x_k - x_{l,ref})^2 + (y_k - y_{l,ref})^2}, \tag{3}$$

$$v_{l,k} = \frac{(x_k - x_{l,ref})\,\dot{x}_k + (y_k - y_{l,ref})\,\dot{y}_k}{r_{l,k}}, \tag{4}$$

$$\phi_{l,k} = \arctan\frac{y_k - y_{l,ref}}{x_k - x_{l,ref}}. \tag{5}$$

In a classical set-up, the measurements, $\mathbf{z}_{l,k}$, are affected by noise, with

$$\mathbf{z}_{l,k} = \mathbf{h}_l(\mathbf{s}_k) + \mathbf{w}_{l,k}, \tag{6}$$

[1] as can be seen immediately $n_s = 6$, but we continue to use n_s.

[2] A common reference system considered for all actors in the system.

where $\mathbf{w}_{l,k} \in \mathbb{R}^{3\times 1}$ is the estimation error due to measurement noise and modelled as $\mathbf{w}_{l,k} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{w,k,l})$. Further, the measurement noise in different sensors are independent. Letting $\mathbf{w}_k = [\mathbf{w}_{1,k}^T, \mathbf{w}_{2,k}^T, \ldots, \mathbf{w}_{L,k}^T]^T$, the above assumption leads to $\mathbb{E}[\mathbf{w}_k\mathbf{w}_k^T] = \mathbf{R}_{w,k} = diag\,[\mathbf{R}_{w,k,1}, \mathbf{R}_{w,k,2}, \ldots, \mathbf{R}_{w,k,L}]$ being a block diagonal matrix.

2.2. Impact of the Cognitive Target

Leveraging on its cognitive abilities, the target interacts with the radars in way to induce errors in the measurement. In particular, it ensures that the resulting measurement, denoted as $\overline{\mathbf{z}}_{l,k} = [\overline{r}_{l,k}, \overline{v}_{l,k}, \overline{\phi}_{l,k}]^T$, takes the form,

$$\overline{\mathbf{z}}_{l,k} = \mathbf{h}_l(\mathbf{s}_k) + \mathbf{w}_{l,k} + \boldsymbol{\eta}_{l,k}, \tag{7}$$

where $\boldsymbol{\eta}_{l,k}$ is the random perturbation induced by the cognitive target. While $\boldsymbol{\eta}_{l,k}$ could be modelled as a Gaussian vector, there exist key differences between $\mathbf{w}_{l,k}$ and $\boldsymbol{\eta}_{l,k}$. Denoting $\boldsymbol{\eta}_k = [\boldsymbol{\eta}_{1,k}^T, \boldsymbol{\eta}_{2,k}^T, \ldots, \boldsymbol{\eta}_{L,k}^T]^T$, we let $\boldsymbol{\eta}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{\eta,k})$. Unlike as $\mathbf{R}_{w,k}$, the covariance matrix $\mathbf{R}_{\eta,k}$ is not assumed to have the block-diagonal structure and $\mathbf{R}_{\eta,k}$ could be low-rank as well. This leads to a correlated interference.

2.3. Fusion Model

Given the non-linear nature of the measurements, the Extended Kalman filter (EKF) is used at the Fusion Centre (FC) to estimate the states $\mathbf{s}_k$ from observable y_k. In this context, we approximate (7) using the Jacobian as,

$$\overline{\mathbf{z}}_{l,k} \approx \mathbf{H}_{l,k}\mathbf{s}_k + \mathbf{w}_{l,k} + \boldsymbol{\eta}_{l,k}, \tag{8}$$

where $\mathbf{H}_{l,k} = \frac{\partial \mathbf{h}_l(\mathbf{s})}{\partial \mathbf{s}}$ is the Jacobian of the measurement evaluated at the state predicted during k^{th} interval. Collating the measurements for $l \in [1, L]$ radars at the FC, we have,

$$\overline{\mathbf{z}}_k = \mathbf{H}_k\mathbf{s}_k + \mathbf{w}_k + \boldsymbol{\eta}_k, \tag{9}$$

where $\overline{\mathbf{z}}_k = [\overline{\mathbf{z}}_{1,k}^T, \overline{\mathbf{z}}_{2,k}^T, \ldots, \overline{\mathbf{z}}_{L,k}^T]^T$ is $3L \times 1$ vector, $\mathbf{H}_k = [\mathbf{H}_{1,k}^T, \mathbf{H}_{2,k}^T, \ldots, \mathbf{H}_{L,k}^T]^T$ is a $3L \times n_s$ ($n_s = 6$) dimensional matrix and the noise components are similarly stacked. In undertaking the stacking in (9), we assume that there exists synchronization between the different radars. We further assume that the FC assists the radars in this synchronization.

To pursue the Kalman iterations in (9), the statistics of $\boldsymbol{\eta}_k$ need to be known. In our setting, the FC is not aware of the interference induced by the cognitive target and dimensions its tracking based on the statistics of $\mathbf{w}_k$. This leads to a sub-optimal approach. Since the aim is to design $\mathbf{R}_{\eta,k}$ to deteriorate the tracking performance, and noting that the aforementioned approach of omitting $\boldsymbol{\eta}_k$ is analytically intractable (due to model mismatch), the current work considers the hypothetical optimal setting for obtaining the performing metrics. Since the optimal performance is degraded, the actual performance is bound to suffer more. The resulting iterations would then be,

$$\mathbf{s}_{k|k-1} = [\mathbf{I}_2 \otimes \mathbf{F}]\mathbf{s}_{k-1|k-1}, \quad (10)$$

$$\mathbf{P}_{k|k-1} = [\mathbf{I}_2 \otimes \mathbf{F}]\mathbf{P}_{k-1|k-1}[\mathbf{I}_2 \otimes \mathbf{F}^T] + \mathbf{Q}_u, \quad (11)$$

$$\mathbf{S}_k = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}_{w,k} + \mathbf{R}_{\eta,k}, \quad (12)$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} - \mathbf{H}_k^T\mathbf{S}_k^{-1}, \quad (13)$$

$$\mathbf{s}_{k|k} = \mathbf{s}_{k|k-1} + \mathbf{K}_k[\overline{\mathbf{z}}_k - \mathbf{H}_k\mathbf{s}_{k|k-1}], \quad (14)$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k\mathbf{H}_k\mathbf{P}_{k|k-1}. \quad (15)$$

2.4. Problem Formulation

In pursuant of the aim to impart errors in the tracking, we consider $\mathrm{Trace}\left(\mathbf{P}_{k|k}\right)$ in (15) as the measure of performance. It should, however, be noted that $\mathrm{Trace}\left(\mathbf{P}_{k|k}\right)$ corresponds to the optimal EKF implementation and that the FC implements a sub-optimal solution where $\mathbf{R}_{\eta,k} = \mathbf{0}$ in (15). In this context, we could consider the following optimization,

$$\begin{aligned} \max_{\mathbf{R}_{\eta,k}} \quad & \mathrm{Trace}\left(\mathbf{P}_{k|k}\right), \\ \text{s.t.} \quad & \mathrm{Trace}(\mathbf{R}_{\eta,k}) \leq \gamma, \\ & \mathbf{R}_{\eta,k} \succ \mathbf{0} \end{aligned} \quad (16)$$

where γ is the available power for introducing interference.

3. OPTIMAL TARGET INDUCED INTERFERENCE

While (16) involves $\mathbf{R}_{\eta,k}$ through (12) and (15), a simpler mechanism to solve this equation lies in considering the following inverse recursion,

$$\mathbf{P}_{k|k}^{-1} = \mathbf{P}_{k|k-1}^{-1} + \mathbf{H}_k^T\left[\mathbf{R}_{w,k} + \mathbf{R}_{\eta,k}\right]^{-1}\mathbf{H}_k. \quad (17)$$

Initializing $\mathbf{P}_{0|0}$ as a positive definite matrix, it follows from (11) and (17) that $\mathbf{P}_{k|k}^{-1}$ exists. Noting that $\mathbf{P}_{k|k}$ is $n_s \times n_s$ dimensional matrix (here $n_s = 6$), and using the Arithmetic-Harmonic mean inequality, it follows that $n_s\left[\mathrm{Trace}\left(\mathbf{P}_{k|k}^{-1}\right)\right]^{-1} \leq \frac{\mathrm{Trace}\left(\mathbf{P}_{k|k}\right)}{n_s}$. Hence a lower bound on $\max\mathrm{Trace}\left(\mathbf{P}_{k|k}\right)$ can be obtained by $\max\mathrm{Trace}\left(\mathbf{P}_{k|k}^{-1}\right)^{-1}$. Further, it follows that $\max_{\mathbf{R}_{\eta,k}}\left(\left[\mathrm{Trace}\left(\mathbf{P}_{k|k}^{-1}\right)\right]^{-1}\right)$ is equivalent to $\min_{\mathbf{R}_{\eta,k}}\left[\mathrm{Trace}\left(\mathbf{P}_{k|k}^{-1}\right)\right]$. Using (17) and the aforementioned arguments, we use $\min_{\mathbf{R}_{\eta,k}}\left[\mathrm{Trace}\left(\mathbf{P}_{k|k}^{-1}\right)\right]$ as the objective function and reformulate (16) as,

$$\begin{aligned} \min_{\mathbf{R}_{\eta,k}} \quad & \mathrm{Trace}\left(\mathbf{H}_k^T\left[\mathbf{R}_{w,k} + \mathbf{R}_{\eta,k}\right]^{-1}\mathbf{H}_k\right), \\ \text{s.t.} \quad & \mathrm{Trace}(\mathbf{R}_{\eta,k}) \leq \gamma, \mathbf{R}_{\eta,k} \succ \mathbf{0}. \end{aligned} \quad (18)$$

Note that we have dropped the term $\mathbf{P}_{k|k-1}^{-1}$ from the optimization in (18) since that has been derived from previously optimized $\mathbf{P}_{k-1|k-1}$ and quantities not dependent on $\mathbf{R}_{\eta,k}$.

Theorem 1. *When $\mathbf{R}_{w,k}$ is positive definite, the problem in* (18) *is convex.*

Proof: The proof follows by considering the classical line restriction and noting that $\mathbf{R}_{w,k}$ is invertible.

Following Theorem 1, it is possible to use convex programming methods to solve the problem. In the sequel, additional investigations into the structure of the solution is provided.

Availability of information for effecting the optimization: The optimization is undertaken at the target at each processing instance. For this, the target needs to know the measurement matrix $\mathbf{H}_k$ at that k^{th} instance as well as the naive measurement noise covariance matrix, $\mathbf{R}_{w,k}$. The former implies the knowledge of the radars' position and the estimate of the target position at the FC. In this work, we assume the knowledge of the position of the radars and that the target has precise information about its own position; these would be used to generate $\mathbf{H}_k$. In the absence of information about $\mathbf{R}_{w,k}$, the target can use a scaled identity matrix, $\mathbf{R}_{w,k} = \sigma_w^2\mathbf{I}$ with σ_w^2 obtained from known device characteristics.

Majorization based solutions: While one may be tempted to use the majorization tools proposed in [18] to solve (18), it should be noted that the structure of the problems considered are different. A simple transformation between the two seems difficult to realize. In this context, we proceed further with the investigation of (18).

3.1. Solution Structure

To gain further insights into the solution, we use $\mathbf{H}_k^T\left[\mathbf{R}_{w,k} + \mathbf{R}_{\eta,k}\right]^{-1}\mathbf{H}_k = \mathbf{H}_k^T\mathbf{R}_{w,k}^{-\frac{1}{2}}\left[\mathbf{I} + \mathbf{R}_{w,k}^{-\frac{1}{2}}\mathbf{R}_{\eta,k}\mathbf{R}_{w,k}^{-\frac{1}{2}}\right]^{-1}\mathbf{R}_{w,k}^{-\frac{1}{2}}\mathbf{H}_k$. Let $\mathbf{U}_k$ denote the basis of the (utmost) n_s dimensional range space of $\mathbf{R}_{w,k}^{-\frac{1}{2}}\mathbf{H}_k$. Further let $\mathbf{\Pi}_H$ and $\mathbf{\Pi}_H^{\perp}$ denote the projections on $\mathbf{U}_k$ and its orthogonal complement. It then follows that, $\mathbf{R}_{w,k}^{-\frac{1}{2}}\mathbf{H}_k = \mathbf{U}_k\mathbf{\Gamma}_k$, where $\mathbf{\Gamma}_k$ is a $n_s \times n_s$ invertible matrix. Further, for any matrix $\mathbf{X}$, $\mathbf{X}\mathbf{X}^T = \mathbf{\Pi}_H\mathbf{X}\mathbf{X}^T\mathbf{\Pi}_H + \mathbf{\Pi}_H^{\perp}\mathbf{X}\mathbf{X}^T\mathbf{\Pi}_H^{\perp}$. Specializing $\mathbf{X}\mathbf{X}^T = \mathbf{R}_{w,k}^{-\frac{1}{2}}\mathbf{R}_{\eta,k}\mathbf{R}_{w,k}^{-\frac{1}{2}}$, it follows that the component of $\mathbf{R}_{w,k}^{-\frac{1}{2}}\mathbf{R}_{\eta,k}\mathbf{R}_{w,k}^{-\frac{1}{2}}$ in the subspace spanned by $\mathbf{U}_k$ contributes to the objective. Further noting that the $\mathbf{R}_{w,k}^{-\frac{1}{2}}\mathbf{R}_{\eta,k}\mathbf{R}_{w,k}^{-\frac{1}{2}}$ appears with a matrix inverse, it implies that the minimization occurs when $\mathbf{R}_{\eta,k}$ is chosen such that $\mathbf{R}_{w,k}^{-\frac{1}{2}}\mathbf{R}_{\eta,k}\mathbf{R}_{w,k}^{-\frac{1}{2}}$ lies in the subspace spanned by $\mathbf{U}_k$. This sketch of proof leads to the following theorem,

Theorem 2. *The optimal $\mathbf{R}_{w,k}$ has a low-rank structure with* $\mathrm{rank}(\mathbf{R}_{\eta,k}) \leq \mathrm{rank}(\mathbf{H}_k)$.

The optimal interference lies in a subspace whose dimensions depend on the number of states and is independent of the number of sensors. In this context, Theorem 2 indicates that the interference power is concentrated in the measurement subspace and is not spread in all dimensions as would have been the case of a white noise addition. The exact nature of the sub-space can be specified for the special case given next.

3.2. Special Case: White Measurement Noise

When $\mathbf{R}_{w,k} = \sigma_w^2 \mathbf{I}$, we can get a closed-form expression for $\mathbf{R}_{\eta,k}$. Let $\mathbf{A} = \mathbf{H}_k \mathbf{H}_k^T, \mathbf{B} = \left[\sigma_w^2 \mathbf{I} + \mathbf{R}_{\eta,k}\right]^{-1}$ and $\{\lambda_i\}, \{\mu_i\}$ be the eigenvalues of $\mathbf{A}$ and $\mathbf{B}$ arranged in the ascending and descending orders respectively. Further, let $\{\gamma_i\}$ be the eigenvalues of $\mathbf{R}_{\eta,k}$ in the ascending order; it is straightforward to see that $\mu_i = \frac{1}{\sigma_w^2 + \gamma_i}$.

Using the trace property, the objective in (18) takes the form $\mathrm{Trace}\left(\mathbf{AB}\right)$. Then using well-known trace inequality [18], it follows that $\mathrm{Trace}\left(\mathbf{AB}\right) \geq \sum_i \lambda_i(\mathbf{A})\mu_i(\mathbf{B})$. The equality occurs when $\mathbf{A}$ and $\mathbf{B}$ are simultaneously diagonalizable [18]. Using these, it then follows that, the eigenvalue decomposition of $\mathbf{R}_{\eta,k}$ takes the form $\mathbf{R}_{\eta,k} = \mathbf{U}_k \mathbf{\Lambda} \mathbf{U}_k^T$, with $\mathbf{U}_k$ being the eigenvectors corresponding to the non-zero eigenvalues of $\mathbf{H}_k \mathbf{H}_k^T$. Further $\mathbf{\Lambda} = diag(\{\gamma_i\})$, with utmost n_s values being non-zero based on Theorem 2. The cost function then takes the form,

$$\begin{aligned} \min_{\{\gamma_i\}} \quad & \sum_{i=1}^{n_s} \frac{\lambda_i}{\sigma_{w,k}^2 + \gamma_i}, \\ \text{s.t.} \quad & \gamma_i \geq 0, \forall i, \sum_{i=1}^{n_s} \gamma_i = \gamma. \end{aligned} \tag{19}$$

Recall that λ_i is the i^{th} largest eigenvalue of $\mathbf{H}\mathbf{H}^T$. The solution then takes a water-filling form with, $\gamma_i = \left(\delta\sqrt{\lambda_i} - \sigma_{w,k}^2\right)^+$, where $\delta \geq 0$ is the Lagrange multiplier satisfying $\sum_i \gamma_i = \gamma$.

4. SIMULATIONS

In the following, we ascertain the loss in performance at the FC due to correlated interference in a simulated set-up. Two different experiments are considered. In both these experiments, the filter update time is set to $T = 10\ ms$ and the process noise covariance is obtained from [17] with $\sigma_a = 0.01$. The Monte Carlo simulations involve a time span corresponding to 500 iterations and averaged over 150 different noise realizations. A single target having the following initial state vector $\mathbf{s}_0 = [2, 0.1, 0.01, 2, 0.1, 0.01]]^T$ is used.

4.1. Impact of Interference power

In the first experiment, three radars are considered to be located at $(1, 0), (0, 1)$ and $(0, 0)$. In this study, the impact of γ on the estimation of the $x-$ position is considered in Fig. 1 for different values of $\eta = \frac{\mathrm{trace}(\mathbf{R}_{\eta,k})}{\mathrm{trace}(\mathbf{R}_{w,k})}$. Two cases on considered: the plots on the left refer to the proposed method while the right plot refers to the equivalent uncorrelated additive white noise as interference, i.e., $\mathbf{R}_{\eta,k}$ is a scaled identity. Fig. 1 clearly shows the performance degradation due to proposed correlated structure as opposed to the intuitive white noise.

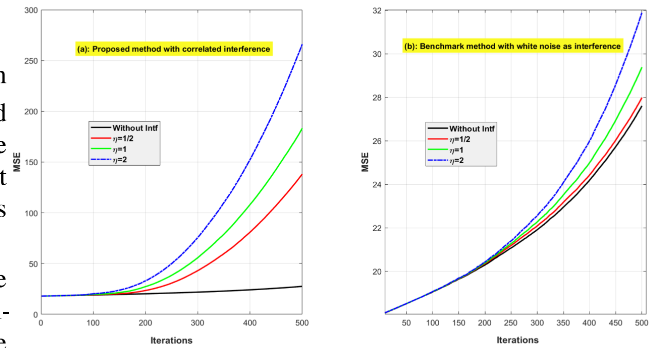

Fig. 1. Impact of interference power on MSE performance of x estimate: (a) Proposed method with correlated interference, (b) Uncorrelated interference of similar power

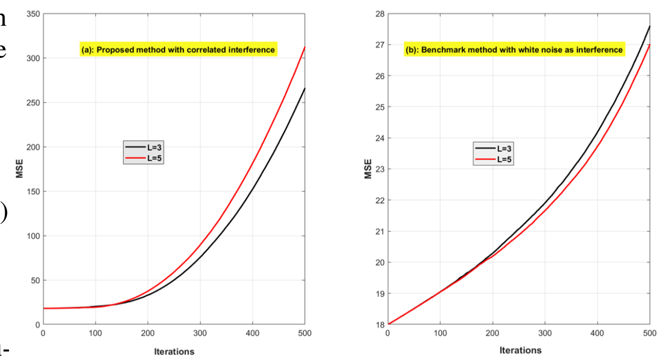

Fig. 2. Impact of number of sensors, L, on estimation of x position, $\eta = 2$. (a) Proposed method, (b) No interference

4.2. Impact of number of sensors

In this study, the impact of L, the number of sensors, on the estimation of the x position for $\eta = 2$. In addition to the earlier positions, we also consider sensors at $(1, 2), (2, 1)$. The motivation arises from the fact that $\mathbf{R}_{w,k}$ are independent across the sensors and can be combated by averaging over a larger number of sensors. However, the designed $\mathbf{R}_{\eta,k}$ are correlated across different k and cannot be compensated by an increase in the number of sensors; in fact, they tend to increase as seen in Fig. 2.

5. CONCLUSIONS

This work presents a cognitive setting with multiple radars tracking an intelligent target. The target, using its cognitive ability, aims to improve its maneuverability by imparting interference to the measurements at different radars to degrade their performance. This work investigates a setting where the minimization of the chosen radar performance is achieved through a convex problem formulation. The counter-intuitive nature of the correlated interference is brought out and its impact highlighted using numerical examples.

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