OVERDEMODULATION-AIDED ONE-BIT DOA ESTIMATION

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ABSTRACT

Sensing and communication systems commonly employ quadrature demodulation for signal down-conversion to baseband, using two orthogonal sinusoidal signals—an approach well-suited for systems equipped with high-resolution Analog-to-Digital Converters (ADCs). However, the performance of this approach is compromised when constrained to one-bit resolution ADCs. Motivated by this fact, in this paper, we explore the problem of one-bit DoA estimation, where the received signal is deliberately overdemodulated using multiple offset sinusoidal signals in the coherent analog down conversion before being processed by a one-bit ADC. Through numerical analysis, we reveal that the overdemodulation technique substantially enhances the accuracy of one-bit DoA estimation.

Index Terms— One-bit quantization, sparse linear arrays, direction of arrival (DoA) estimation, Overdemodulation.

1. INTRODUCTION

The problem of Direction of Arrival (DoA) estimation is of central importance in the field of array processing, and finds applications in radar, sonar, and wireless communications [1, 2]. While Uniform Linear Arrays (ULAs) are extensively studied for DoA estimation [3, 4], they require extensive hardware and receiver resources [4]. Sparse Linear Arrays (SLAs) have emerged as a solution, offering high resolution performance with more limited hardware resources [5, 6].

Various methods for DoA estimation via SLAs exist in the literature, broadly classified into Sparsity-Based Methods (SBMs) and Augmented Covariance-Based Methods (ACBMs) [7–13]. SBMs estimate DoAs by imposing sparsity constraints on source profiles and exploiting the compressive sensing recovery techniques [7–9]. On the other hand, in ACBMs, DoAs are estimated by applying conventional subspace methods such as MUSIC, ESPRIT on an Augmented Sample Covariance Matrix (ASCM) developed from the original sample covariance matrix by exploiting the difference co-array structure [10–13].

The aforementioned techniques for DoA estimation assume high-resolution Analog-to-Digital Converters (ADCs) for digital representation of analog array measurements, neglecting quantization errors. However, the use of high resolution ADCs is impractical in many modern applications due to limitations on power consumption and production cost. As a viable solution in such cases, deployment of low-resolution ADCs with a high sampling rate has been proposed in the literature [14–18]. Particularly, use of one-bit ADCs has received the most attention since they allow for sampling at an extremely high rate with a low cost and low power consumption.

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The problem of DoA estimation from one-bit data has recently attracted significant interest in the literature [19–26]. While it has been demonstrated that one-bit quantization leads to a moderate performance loss compared to high-resolution quantization, existing approaches typically rely on classic quadrature demodulation employing in-phase and quadrature-phase channels. However, classic quadrature demodulation, although optimal and incurring no information loss for receivers equipped with infinite-bit ADCs, is suboptimal from a statistical inference perspective when dealing with one-bit data [27]. On the other hand, the utilization of overmodulation, entailing the deployment of more than two modulation channels, is deemed closer to optimal for systems with one-bit ADCs [28]. Specifically, overdemodulation has been shown to enhance the performance of communication systems utilizing one-bit ADCs in terms of channel estimation and achievable rates [28]. This paper explores the use of overdemodulated data for DoA estimation from one-bit signals, with the objective of partially alleviating the information loss inherent in one-bit ADC systems. We propose an approach for DoA estimation from one-bit overdemodulated signals, demonstrating substantial enhancement in DoA estimation performance.

Notation: Vectors and matrices are referred to by lower- and upper-case bold-face, respectively. The superscripts T and H denote the transpose and Hermitian operations, respectively. $[\mathbf{A}]_{i,j}$ and $[\mathbf{a}]_i$ indicate the $(i,j)^{\text{th}}$ and i^{th} entry of $\mathbf{A}$ and $\mathbf{a}$, respectively. $\hat{\mathbf{A}}$ and $\hat{\mathbf{a}}$ denote the estimate of $\mathbf{A}$ and $\mathbf{a}$, respectively. $\operatorname{diag}(\mathbf{a})$ and $\operatorname{diag}(\mathbf{A})$ are a diagonal matrices whose diagonal entries are equal to the elements of $\mathbf{a}$ and diagonal elements of $\mathbf{A}$, respectively. $\mathbf{I}_M$ denotes the $M \times M$ identity matrix. $\operatorname{sgn}(x)$ denotes the sign function with $\operatorname{sgn}(x) = 1$ for $x \geq 0$ and $\operatorname{sgn}(x) = -1$ otherwise. The real and image part of a are denoted by $\Re\{a\}$ and $\Im\{a\}$, respectively. $\mathbb{E}\{.\}$ stands for the statistical expectation. $\otimes$ is Kronecker products. $\operatorname{vec}(\mathbf{A}) = [\mathbf{a}_1^T, \mathbf{a}_2^T, \cdots, \mathbf{a}_n^T]^T$ denotes the vectorization operation. $\mathbf{A}^\dagger$ is the pseudoinverse of the full column rank matrix $\mathbf{A}$.

2. SYSTEM MODEL

Consider a SLA consisting of M elements located at positions $\left(m_1\frac{\lambda}{2},\ m_2\frac{\lambda}{2}, \cdots, m_M\frac{\lambda}{2}\right)$ with $m_i \in \mathbb{M}$, as depicted in Fig. 1.

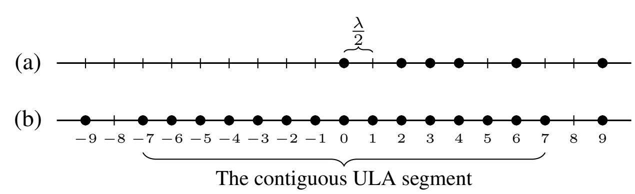


Fig. 1: Geometry of an SLA with $M = 6$ elements: (a) physical array; (b) difference co-array.

Here, $\mathbb{M}$ represents a set of integers with cardinality M, and λ denotes the wavelength of the incoming signals. K narrowband signals with distinct DoAs $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_K]^T$ are assumed to impinge on the SLA from far field. Let $\mathbf{r}(t) \in \mathbb{R}^{M\times 1}$ denote the vector of received passband signals at the time instant t, generated after amplification and filtering in the Radio Frequency (RF). Further, let $\mathbf{r}(t) = \Re\{\mathbf{y}(t)e^{\mathrm{j}2\pi f t}\}$ where f denotes the carrier frequency of the incoming signals and $\mathbf{y}(t)$ corresponds to the quadrature baseband representation of received signal at the time instant t, modeled as

$$\mathbf{y}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t) + \mathbf{n}(t) \in \mathbb{C}^{M\times 1}, \quad t = 1, \cdots, N, \tag{1}$$

where $\mathbf{s}(t) \in \mathbb{C}^{K\times 1}$ denotes the vector of source signals, $\mathbf{n}(t) \in \mathbb{C}^{M\times 1}$ is additive noise, and $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots \mathbf{a}(\theta_K)] \in \mathbb{C}^{M\times K}$ represents the SLA steering matrix with

$$\mathbf{a}(\theta_i) = \begin{bmatrix} e^{\mathrm{j}\pi \sin\theta_i m_1} & e^{\mathrm{j}\pi \sin\theta_i m_2} & \cdots & e^{\mathrm{j}\pi \sin\theta_i m_M} \end{bmatrix}^T, \tag{2}$$

being the SLA manifold vector for the i^{th} signal. The following assumptions are made on source signals and noise:

- **A1** $\mathbf{n}(t)$ follows a zero-mean circular complex Gaussian distribution with the covariance matrix $\mathbb{E}\{\mathbf{n}(t)\mathbf{n}^H(t)\} = \sigma^2\mathbf{I}_M$
- **A2** The source signal vector is modeled as a zero-mean circular complex Gaussian random vector with covariance matrix $\mathbb{E}\{\mathbf{s}(t)\mathbf{s}^H(t)\} = \mathrm{diag}(\mathbf{p})$ where $\mathbf{p} = [p_1, p_2, \cdots, p_K]^T \in \mathbb{R}^{K\times 1}_{>0}$ (i. e., $p_l > 0$, $\forall l$), representing uncorrelated source signals.
- **A3** Source and noise vectors are mutually independent.
- **A4** There is no temporal correlation between the snapshots, i.e., $\mathbb{E}\{\mathbf{n}(t_1)\mathbf{n}^H(t_2)\} = \mathbb{E}\{\mathbf{s}(t_1)\mathbf{s}^H(t_2)\} = \mathbf{0}$ when $t_1 \neq t_2$ and $\mathbf{0}$ is an all zero matrix of appropriate dimensions.
- **A5** An exact knowledge of the number of sources is available.

Unlike a standard demodulator that obtains $\mathbf{y}(t)$ from $\mathbf{r}(t)$ using a single complex demodulator, an overdemodulation is considered herein. For the m^{th} antenna in the array, this methodology is illustrated in Fig. 2. Herein, instead of demodulation by in-phase and quadrature-phase carrier signals, the incoming signal is processed by a bank of demodulators. In particular, this bank multiplies the passband measurements at each array element separately with L sinusoidal signals, defined as $\cos(2ft + \frac{(l-1)\pi}{L})$ for $l = 1, \cdots, L$, where $L > 2$. It is important to note that this approach reduces to classic quadrature demodulation when $L = 2$. The resulting signal at each of L channels is low-pass filtered to produce the baseband overdemodulated signal $\mathbf{z}_m(t) \in \mathbb{R}^{L\times 1}$, given by

$$[\mathbf{z}_m(t)]_l = \Re\{[\mathbf{y}(t)]_m\}\cos(\frac{(l-1)\pi}{L}) + \Im\{[\mathbf{y}(t)]_m\}\sin(\frac{(l-1)\pi}{L}). \tag{3}$$

Here it is assumed that each component of the baseband overdemodulated signal $\mathbf{z}_m(t)$ is connected to a one-bit ADC which directly converts $\mathbf{z}_m(t)$ into binary data by comparing each of its components individually with zero. In such a case, the one-bit measurements at the l^{th} channel of the m^{th} array element are obtained as

$$[\mathbf{x}_m(t)]_l = Q([\mathbf{z}_m(t)]_l) = \mathrm{sgn}\left([\mathbf{z}_m(t)]_l\right). \tag{4}$$

The problem under consideration is the estimation of source DoAs using the one-bit quantized measurements, i.e., $\mathbf{X} = [\mathbf{x}(1), \mathbf{x}(2), \cdots, \mathbf{x}(N-1)] \in \{-1, 1\}^{ML\times N}$, where $\mathbf{x}(t) = [\mathbf{x}_1^T(t), \mathbf{x}_2^T(t), \cdots, \mathbf{x}_M^T(t)]^T \in \{-1, 1\}^{ML\times 1}$.

3. DOA ESTIMATION WITH ONE-BIT OVERDEMODULATED DATA

In this section, we introduce a method for estimating DoAs using one-bit overdemodulated measurements. We start by establishing a relation between the covariance matrix of the one-bit overdemodulated measurements, i.e., $\mathbf{X}$, and the quadrature baseband signals, represented by $\mathbf{Y} = [\mathbf{Y}(1), \mathbf{Y}(2), \cdots, \mathbf{Y}(N-1)] \in \mathbb{C}^{M\times N}$. This relationship allows us to derive a consistent estimate of the covariance matrix of $\mathbf{Y}$ from the one-bit overdemodulated measurements. Finally, we apply Co-Array-Based MUSIC (CAB-MUSIC) [11, 25, 29] to the resulting covariance matrix estimate to obtain DoA estimates.

It follows from the arcsin law that [25],

$$\mathbf{R}_{\mathbf{x}} = \mathbb{E}\{\mathbf{x}(t)\mathbf{x}^T(t)\} = \frac{2}{\pi}\arcsin(\overline{\mathbf{R}}_{\mathbf{z}}), \tag{5}$$

where $\overline{\mathbf{R}}_{\mathbf{z}}$ denotes the normalized covariance matrix of $\mathbf{z}(t) = [\mathbf{z}_1^T(t), \mathbf{z}_2^T(t), \cdots, \mathbf{z}_M^T(t)]^T \in \mathbb{R}^{ML\times 1}$, defined as

$$\overline{\mathbf{R}}_{\mathbf{z}} = \mathrm{diag}(\mathbf{R}_{\mathbf{z}})^{-1/2}\mathbf{R}_{\mathbf{z}}\mathrm{diag}(\mathbf{R}_{\mathbf{z}})^{-1/2}. \tag{6}$$

From (3), we have

$$\mathbf{z}(t) = \mathbf{G}\begin{bmatrix}\Re\{\mathbf{y}(t)\}\\ \Im\{\mathbf{y}(t)\}\end{bmatrix}, \tag{7}$$

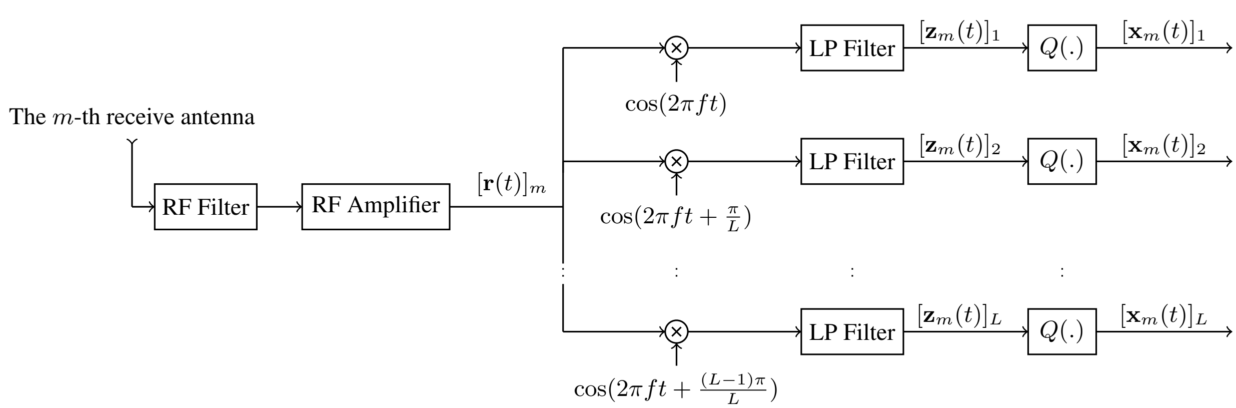


Fig. 2: The receiver architecture with overdemodulation and one-bit quantization at the m-th array element.

where

$$\mathbf{G} = \begin{bmatrix}\mathbf{I}_M \otimes \mathbf{v} & \mathbf{I}_M \otimes \mathbf{u}\end{bmatrix}, \in [-1,1]^{ML\times 2M}, \tag{8}$$

with $\mathbf{v} = [0, \cos(\frac{\pi}{L}), \cdots, \cos(\frac{(l-1)\pi}{L}), \cdots, \cos(\frac{(L-1)\pi}{L})]^T \in [-1,1]^{L\times 1}$ and $\mathbf{u} = [0, \sin(\frac{\pi}{L}), \cdots, \sin(\frac{(l-1)\pi}{L}), \cdots, \sin(\frac{(L-1)\pi}{L})]^T \in [-1,1]^{L\times 1}$. From (7), the covariance matrix of $\mathbf{z}(t)$ is obtained as

$$\mathbf{R_z} = \frac{1}{2}\mathbf{G}\begin{bmatrix}\Re\{\mathbf{R_y}\} & -\Im\{\mathbf{R_y}\} \\ \Im\{\mathbf{R_y}\} & \Re\{\mathbf{R_y}\}\end{bmatrix}\mathbf{G}^T, \tag{9}$$

where $\mathbf{R_y}$ denotes the covariance matrix of $\mathbf{y}(t)$. Under assumptions **A1** - **A4**, $\mathbf{R_y}$ is expressed as

$$\mathbf{R_y} = \mathbf{A}(\boldsymbol{\theta})\mathrm{diag}(\mathbf{p})\mathbf{A}^H(\boldsymbol{\theta}) + \sigma^2\mathbf{I}_M \in \mathbb{C}^{M\times M}. \tag{10}$$

Lemma 1. *The matrices* $\mathrm{diag}(\mathbf{R_z})$ *is a scaled identity matrix, given by* $\mathrm{diag}(\mathbf{R_z}) = (\sum_{k=1}^K p_k + \sigma^2)\mathbf{I}_{ML}$.

Proof. For $1 \le m \le M$ and $1 \le l \le L$, the $((m-1)L+l)^{\mathrm{th}}$ diagonal element of $\overline{\mathbf{R}}_\mathbf{z}$ is obtained as

$$[\mathbf{R_z}]_{(m-1)L+l,(m-1)L+l} = \frac{1}{2}\mathbf{g}^T_{(m-1)L+l}\begin{bmatrix}\Re\{\mathbf{R_y}\} & -\Im\{\mathbf{R_y}\} \\ \Im\{\mathbf{R_y}\} & \Re\{\mathbf{R_y}\}\end{bmatrix} \times \mathbf{g}_{(m-1)L+l}, \tag{11}$$

where $\mathbf{g}_{(m-1)L+l]} \in [-1,1]^{2M\times 1}$ is the transpose of the $((m-1)L+l)^{\mathrm{th}}$ row of $\mathbf{G}$. From the definition of $\mathbf{G}$ in (8), it is readily observed that

$$\mathbf{g}_{(m-1)L+l} = \begin{bmatrix}\mathbf{e}_m\cos(\frac{(L-1)\pi}{L}) & \mathbf{e}_m\sin(\frac{(L-1)\pi}{L})\end{bmatrix}, \tag{12}$$

where $\mathbf{e}_m$ is the m^{th} column of $\mathbf{I}_M$. Substituting (12) into (11) and using (10) yields

$$[\mathbf{R_z}]_{(m-1)L+l,(m-1)L+l} = \frac{1}{2}\mathbf{e}_m^T\Re\{\mathbf{R_y}\}\mathbf{e}_m = \frac{1}{2}(\sum_{k=1}^K p_k + \sigma^2) \tag{13}$$

□

Making use of Lemma 1, (9) and (6), we obtain

$$\frac{1}{2}\mathbf{G}\begin{bmatrix}\Re\{\overline{\mathbf{R}}_\mathbf{y}\} & -\Im\{\overline{\mathbf{R}}_\mathbf{y}\} \\ \Im\{\overline{\mathbf{R}}_\mathbf{y}\} & \Re\{\overline{\mathbf{R}}_\mathbf{y}\}\end{bmatrix}\mathbf{G}^T = \sin(\frac{\pi}{2}\mathbf{R_x}), \tag{14}$$

where $\overline{\mathbf{R}}_\mathbf{y} = \frac{\mathbf{R}}{\sigma^2 + \sum_{k=1}^K p_k} = \mathbf{A}(\boldsymbol{\theta})\mathrm{diag}(\overline{\mathbf{p}})\mathbf{A}^H(\boldsymbol{\theta}) + (1 - \sum_{k=1}^K \overline{p}_k)\mathbf{I}_M$, with $\overline{\mathbf{p}} = [\overline{p}_1, \overline{p}_2, \cdots, \overline{p}_K]^T$ and $\overline{p}_k = \frac{p_k}{\sigma^2 + \sum_{k=1}^K p_k}$.

Lemma 2. *The matrices* $\mathbf{G}$ *is a semi-orthogonal matrix, i.e.,* $\mathbf{G}^T\mathbf{G} = \frac{L}{2}\mathbf{I}_{2M}$.

Proof. Considering (8), we have

$$\mathbf{G}^T\mathbf{G} = \begin{bmatrix}\|\mathbf{v}\|^2\mathbf{I}_M & \mathbf{v}^T\mathbf{u}\mathbf{I}_M \\ \mathbf{v}^T\mathbf{u}\mathbf{I}_M & \|\mathbf{u}\|^2\mathbf{I}_M.\end{bmatrix} \tag{15}$$

It follows from the definition of $\mathbf{v}$ and $\mathbf{u}$ that

$$\|\mathbf{v}\|^2 = \sum_{l=1}^L \cos^2(\frac{(l-1)\pi}{L}) = \frac{1}{2}\sum_{l=1}^L [1 + \cos(\frac{2(l-1)\pi}{L})] = \frac{L}{2}, \tag{16}$$

$$\|\mathbf{u}\|^2 = \sum_{l=1}^L \sin^2(\frac{(l-1)\pi}{L}) = \frac{1}{2}\sum_{l=1}^L [1 - \cos(\frac{2(l-1)\pi}{L})] = \frac{L}{2}, \tag{17}$$

$$\begin{aligned}\mathbf{v}^T\mathbf{u} &= \sum_{l=1}^L \cos(\frac{(l-1)\pi}{L})\sin(\frac{(l-1)\pi}{L}) \\ &= \frac{1}{2}\sum_{l=1}^L \sin(\frac{2(l-1)\pi}{L}) = 0.\end{aligned} \tag{18}$$

Substituting (16), (17) and (18) into (15) completes the proof. □

It follow from Lemma 2, (14) and some straightforward algebraic manipulation that

$$\begin{aligned}\overline{\mathbf{R}}_\mathbf{y} = &\frac{4}{L^2}\Big(\big[\mathbf{I}_M \otimes \mathbf{v}^T\big]\sin(\frac{\pi}{2}\mathbf{R_x})\big[\mathbf{I}_M \otimes \mathbf{v}\big] \\ &+ \big[\mathbf{I}_M \otimes \mathbf{u}^T\big]\sin(\frac{\pi}{2}\mathbf{R_x})\big[\mathbf{I}_M \otimes \mathbf{u}\big]\Big) \\ &+ \frac{4\mathrm{j}}{L^2}\Big(\big[\mathbf{I}_M \otimes \mathbf{u}^T\big]\sin(\frac{\pi}{2}\mathbf{R_x})\big[\mathbf{I}_M \otimes \mathbf{v}\big] \\ &- \big[\mathbf{I}_M \otimes \mathbf{v}^T\big]\sin(\frac{\pi}{2}\mathbf{R_x})\big[\mathbf{I}_M \otimes \mathbf{u}\big]\Big).\end{aligned} \tag{19}$$

It is deduced from the strong law of large numbers [30, ch. 8] that the sample covariance matrix of one-bit data provides a consistent estimate of $\mathbf{R_x}$ with probability 1, i.e., $\Pr\left(\lim_{N\to\infty}\widehat{\mathbf{R}}_\mathbf{x} = \mathbf{R_x}\right) = 1$, where $\widehat{\mathbf{R}}_\mathbf{x} = \frac{1}{N}\mathbf{X}\mathbf{X}^H$. Accordingly, a consistent estimate of $\overline{\mathbf{R}}_\mathbf{y}$ is obtained by replacing $\mathbf{R_x}$ with $\widehat{\mathbf{R}}_\mathbf{x}$ in the expression for $\overline{\mathbf{R}}_\mathbf{y}$, given in (19). Therefore, the normalized ASCM can be constructed from $\widehat{\overline{\mathbf{R}}}_\mathbf{y}$ as [23]

$$\widehat{\overline{\mathbf{R}}}_v = \begin{bmatrix}\mathbf{T}_v\mathbf{J}^\dagger\widehat{\overline{\mathbf{r}}}_\mathbf{y} & \mathbf{T}_{v-1}\mathbf{J}^\dagger\widehat{\overline{\mathbf{r}}}_\mathbf{y} & \cdots & \mathbf{T}_1\mathbf{J}^\dagger\widehat{\overline{\mathbf{r}}}_\mathbf{y}\end{bmatrix} \in \mathbb{C}^{v\times v}, \tag{20}$$

where $\widehat{\overline{\mathbf{r}}}_\mathbf{y} = \mathrm{vec}(\widehat{\overline{\mathbf{R}}}_\mathbf{y})$ and the matrices $\mathbf{T}_i \in \{0,1\}^{v\times(2D-1)}$ and $\mathbf{J} \in \{0,1\}^{M^2\times(2D-1)}$ are defined as follows [25]

$$\mathbf{T}_i = \begin{bmatrix}\mathbf{0}_{v\times(i+D-v-1)} & \mathbf{I}_v & \mathbf{0}_{v\times(D-i)}\end{bmatrix}, \tag{21}$$

$$\mathbf{J} = \begin{bmatrix}\mathrm{vec}(\mathbf{L}_{D-1}^T) & \cdots & \mathrm{vec}(\mathbf{L}_0) & \cdots & \mathrm{vec}(\mathbf{L}_{D-1})\end{bmatrix}, \tag{22}$$

with $[\mathbf{L}_n]_{p,q} = \begin{cases}1, & \text{if } m_p - m_q = \ell_n, \\ 0, & \text{otherwise,}\end{cases}$ with $1 \le p, q \le M$ and $0 \le n \le D-1$. Here D and v refer to the number of non-negative elements in the difference co-array of the SLA and its contiguous ULA segment, respectively (See Fig. 1). It follows from the consistency of $\widehat{\overline{\mathbf{r}}}$ that

$$\begin{aligned}\lim_{N\to\infty}\widehat{\overline{\mathbf{R}}}_v &= \begin{bmatrix}\mathbf{T}_v\mathbf{J}^\dagger\overline{\mathbf{r}} & \mathbf{T}_{v-1}\mathbf{J}^\dagger\overline{\mathbf{r}} & \cdots & \mathbf{T}_1\mathbf{J}^\dagger\overline{\mathbf{r}}\end{bmatrix} \in \mathbb{C}^{v\times v} \\ &= \mathbf{A}_v(\boldsymbol{\theta})\mathrm{diag}(\overline{\mathbf{p}})\mathbf{A}_v^H(\boldsymbol{\theta}) + (1 - \sum_{k=1}^K \overline{p}_k)\mathbf{I}_v,\end{aligned} \tag{23}$$

where $\mathbf{A}_v(\boldsymbol{\theta}) = [\mathbf{a}_v(\theta_1), \mathbf{a}_v(\theta_2), \cdots, \mathbf{a}_v(\theta_K)] \in \mathbb{C}^{v\times K}$ denotes the steering matrix of a contiguous ULA with v elements located at $(0, \frac{\lambda}{2}, \cdots, (v-1)\frac{\lambda}{2})$.Hence, the DoA estimates can be obtained by applying MUSIC to $\widehat{\overline{\mathbf{R}}}_v$. Algorithm 1 summarizes the proposed method for DoA estimation with one-bit Overdemodulated data.

4. SIMULATION RESULTS

In this section, we provide some numerical results to assess the performance of the proposed approach for DoA estimation from one-bit overdemodulated data. In all simulation results, each simulated point has been computed by 1000 Monte Carlo repetitions. In addition, it is assumed that the K independent sources are located at $\{-60^\circ + 120^\circ k/(K-1) : k = 1, \cdots, K-1\}$. All sources have an equal power, i.e., $p_k = p$ for all k, and the SNR is defined

Algorithm 1

Input: one-bit overdemodulated data, i.e., $\mathbf{X}$.

Output: The estimates of source DoAs.

1: Compute the sample covariance matrix of one-bit overdemodulated data as $\widehat{\mathbf{R}}_{\mathbf{x}} = \frac{1}{N}\mathbf{X}\mathbf{X}^H$.

2: Compute $\widehat{\widehat{\mathbf{R}}}_{YV}$ from (19).

3: Compute $\widehat{\widehat{\mathbf{r}}}_{\mathbf{y}} = \text{vec}(\widehat{\widehat{\mathbf{R}}}_{\mathbf{y}})$.

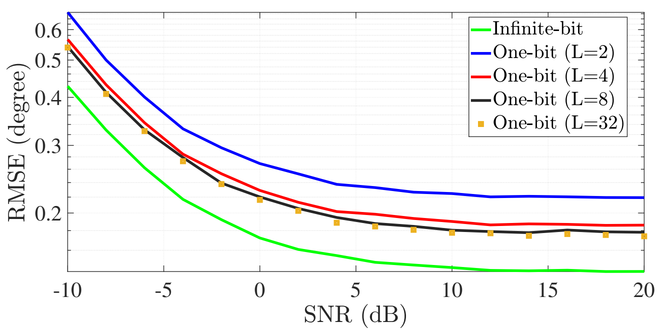
4: Compute $\widehat{\widehat{\mathbf{R}}}_{v}$ from (20).

5: Apply MUSIC to $\widehat{\widehat{\mathbf{R}}}_{v}$ to estimate DoAs.

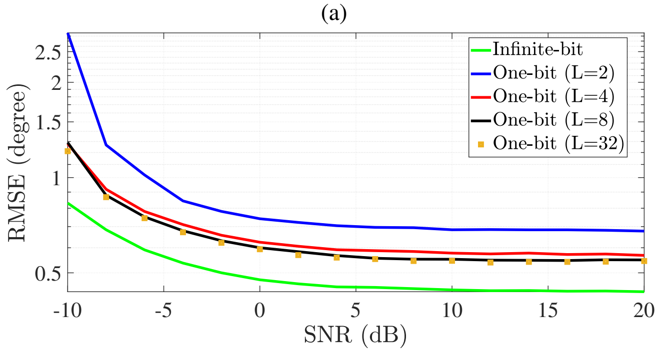
as $10 \log \frac{p}{\sigma^2}$. Throughout this section, we use a nested array with $M = 10$ physical elements and the following geometry

$$\mathbb{M}_{\text{nested}} : \{1, 2, 3, 4, 5, 8, 10, 15, 20\}\,. \tag{24}$$

Fig. 3 depicts the Root-Mean-Squares-Error (RMSE) in degree versus the SNR for the nested array configuration specified in (24), with the number of snapshots fixed at $N = 500$. Considering $M = 10$, we explore two distinct scenarios: (a) $K = 5 < M$, representing fewer sources than sensors, and (b) $K = 12 > M$, where the number of sources exceeds the sensors. The figure reveals that for both scenarios, the RMSE for unquantized (infinite-bit) data and one-bit data demodulated at factors of $L = 2, 4, 8, 32$ tends to converge to a constant non-zero value in the high SNR.



(a)



(b)

Fig. 3: RMSE in degrees versus SNR for a nested array with $M = 10$ elements and configuration given in (24), $N = 500$, and: (a) $K = 5 < M$; (b) $K = 12 > M$.

Further insights from Figure 3 indicate that quartic demodulation offers a substantial improvement in RMSE over quadratic demodulation for both $K = 5 < M$ and $K = 12 > M$ scenarios. Specifically, at an SNR of 5 dB, quartic demodulation yields RMSE improvements of approximately $0.1°$ for $K = 5 < M$ and $0.2°$ for $K = 12 > M$, compared to quadratic demodulation. However, it is observed that increasing the demodulation order beyond quartic results in only minimal improvements in RMSE. This demonstrates the significant potential of quartic demodulation in enhancing DoA estimation accuracy across different source-sensor configurations.

Fig. 4 depicts the probability of resolution versus the source separation for DoA estimation, comparing unquantized (infinite-bit) data with one-bit data demodulated at factors of $L = 2, 4, 8, 32$. This analysis utilizes a nested array configuration as specified in (24), with parameters set at $N = 100$ SNR of 0 dB. We consider a scenario involving two equally powered sources located at $\theta_1 = 20° - \frac{\Delta\theta}{2}$ and $\theta_2 = 20° + \frac{\Delta\theta}{2}$. A criterion for source resolvability is established such that the sources are deemed resolvable if the maximum deviation between the estimated and actual source angles, $\max_{i \in 1,2} |\hat{\theta}_i - \theta_i|$, is less than half the angular separation $\frac{\Delta\theta}{2}$ [31]. The results demonstrate a marked improvement in resolution probability with quartic demodulation over quadratic, with a notable increase of 14% at a separation of $\Delta\theta = 1.5°$. Further, it is evident that employing demodulation orders higher than quartic yields only marginal benefits in resolution probability.

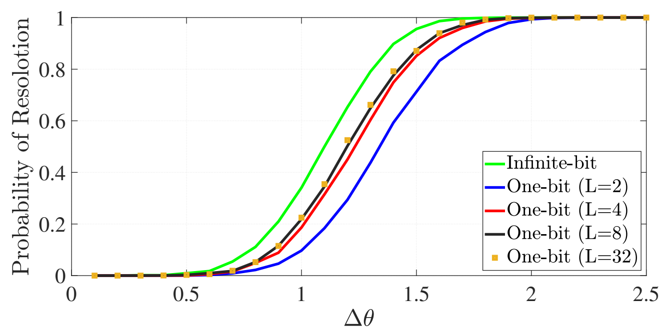


Fig. 4: Probability of resolution versus source separation in degree for a nested array with $M = 10$ elements and configuration given in (24), $N = 500$ and SNR = 0 dB.

5. CONCLUSION

In this paper, we explored the potential of Direction of Arrival (DoA) estimation using overdemodulated one-bit data. We introduced a novel approach for DoA estimation in this context and demonstrated that adopting quartic demodulation significantly enhances the accuracy and resolution of one-bit DoA estimation compared to traditional quadratic demodulation. Our findings also suggest that the benefits of demodulation orders beyond quartic are minimal, indicating that quartic demodulation strikes an optimal balance.

Looking ahead, a critical avenue for future research involves developing a comprehensive analytical framework to assess the performance of the proposed DoA estimation method. Such an investigation will enable a deeper understanding of its key characteristics and potential limitations, paving the way for further enhancements and applications in signal processing.

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