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# Economics of Cultural Change: Openness, Interaction, and Intergenerational Transmission

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Skerdilajda Zanaj, DEM, Université du Luxembourg, UL
Emma Thill, DEM, Université du Luxembourg, UL
Anastasia Litina, University of Macedonia, GR

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For editorial correspondence, please contact: dem@uni.lu
University of Luxembourg
Faculty of Law, Economics and Finance
4, rue Alphonse Weicker
L-2721 Luxembourg

# Economics of Cultural Change: Openness, Interaction, and Intergenerational Transmission*

Emma Thill† Anastasia Litina‡ Skerdilajda Zanaj§

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**Abstract**

Culture shapes economic and social life, yet some traits erode quickly, while others persist across generations. Migrant experiences towards Europe or the United States illustrate this puzzle. In addition, some traits, such as fertility norms, tend to converge relatively quickly, whereas others, such as religiosity, exhibit substantially greater persistence. We develop a dynamic model of cultural transmission that endogenizes both cross-cultural group interaction and parental influence on cultural openness defined as a parentally transmitted willingness to adopt a new cultural trait when beneficial. Parents first shape cultural transmission by choosing their children's openness to alternative traits. As young adults, individuals then decide how much to interact with other groups and, conditional on interaction, whether to switch traits. This endogenizes peer exposure and makes cultural change a deliberate choice. Within a generation, a higher group-level propensity to switch reduces group size, while tighter norms can expand or shrink a group depending on the relative utility of its trait. Across generations, parental investments in openness generate three long-run equilibria: convergence to a single trait, coexistence with interaction, or segregation without interaction. By jointly modeling parental transmission and peer-driven switching, we show that cultural persistence or change reflects purposeful micro-level decisions.

## 1 Introduction

Culture shapes how people live, think, and interact with each other; it consists of the values, beliefs, and behaviors transmitted between or passed down across generations (Bisin and Verdier, 2001). Accordingly, it shapes fundamental aspects of economic and social life, ranging from trust and cooperation to gender roles, religious norms, and political preferences. Some cultural traits erode quickly, while others persist for generations. Migrant communities across the world provide striking illustrations. In Europe, Spanish and Portuguese families who moved to France in the $20^{\text{th}}$ century converged rapidly toward mainstream French norms, while North African communities more often maintained distinct cultural practices across generations (Simon, 2003a). During the Age of Mass Migration ($1850-1913$), immigrants to the United States from Norway, Sweden, and Denmark assimilated culturally more quickly than Italians, Russians or Finns (Abramitzky et al., 2020). More recently, in the United States, Mexican-American migrants have assimilated in language and education within one or two generations, yet fertility and family norms remain more persistent (Fernández and Fogli, 2009; Beine et al., 2025).

In this paper, we examine how family decisions and interactions among individuals from different cultural groups generate these divergent trajectories.

Empirical work has emphasized that culture is transmitted across generations and is shaped by economic incentives and peer pressure, as well as political and historical shocks (Giuliano and Nunn, 2021). Theoretical work has discussed

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†Department of Economics and Management, University of Luxembourg. Email: emma.thill@uni.lu

‡Department of Economics, University of Macedonia. Email: anastasia.litina@uom.edu.gr

§Department of Economics and Management, University of Luxembourg. Email: skerdilajda.zanaj@uni.lu

modes of horizontal and vertical transmission (e.g., Bisin and Verdier, 2001; Doepke and Zilibotti, 2017). What remains underdeveloped in the literature is to extend the models of parental influence to a framework that explains why some societies are more open to change and the adoption of new norms. To this end we treat openness itself as a transmitted trait. Moreover, we further extend the literature to offer micro-foundations for how children choose their peer interactions and, when relevant, adjust their cultural traits. Incorporating these individual choices, allows us to shed light on peer interactions and to study how parental guidance and peer dynamics together shape long-run cultural trajectories and aggregate cultural change.

Analytically, we assume two distinct cultural groups and develop a model of cross-group interactions and intergenerational transmission in which parents first choose how much to invest in shaping their children's cultural openness, a parentally transmitted willingness to adopt new cultural traits when beneficial. Individuals, once grown, decide whether to interact with other cultural groups and may adopt new traits accordingly. Each group has an associated level of *tightness*, defined as the strength of norm enforcement or stigma associated with leaving the group. The model follows three life-cycle stages in each generation. In *childhood*, children inherit their parent's trait and a level of openness shaped by parental effort. In *young adulthood*, individuals choose how much to interact with peers of another cultural group and may switch traits if the benefits outweigh the costs. Young adulthood is the period in which individuals create their own self and corresponds to what the psychology literature defines as the impressionable years (Krosnick and Alwin, 1989). This hypothesis posits that people are highly susceptible to attitude change during their late adolescence and early adulthood, after which their susceptibility decreases sharply and remains low throughout life [1]. In the last phase, in *parenthood*, adults pass on their trait to their child effortlessly and decide how much effort to put into shaping their level of openness to instill in their children. Individuals and in particular parents are heterogeneous. Those valuing adaptability raise children with a high openness to changing cultural traits, fostering rapid change and possible erosion of traditional cultures.[2] Parents prioritizing cultural continuity raise more conservative children, sustaining inherited traits but limiting the adoption of potentially more beneficial traits. Thus, culture is not merely inherited but deliberately maintained or reshaped in response to incentives.

Our main results pertain to both cultural change within one generation and the long-run dynamics. Within a generation, peer interactions can change the size of cultural groups. Higher cultural openness will always shrink group size. However, we find that tighter norms do not automatically enlarge a cultural group. Tightness reduces exits from the group but also makes the group less attractive to outsiders, lowering inflows. A cultural group only expands by tightening its norms when its trait has higher average payoff from holding the trait than the alternative one; otherwise, the drop in inflows outweighs the retention of incumbents and the group shrinks. Thus, within a generation, the size of cultural groups is a complex play of the payoff from holding the trait, the tightness of the group and openness to cultural change.

The model also produces different long-run cultural equilibria, depending on cross-group interactions and parental efforts to shape the propensity to adopt new cultural traits. Three stable outcomes can arise. *Convergence* emerges when switching traits and interactions are frequent. Individuals switch to traits that better fit their preferences and environment, with only one trait persising in the long run. This outcome mirrors permissive or "child-centered " parenting styles (Doepke and Zilibotti, 2017), where autonomy and adaptability are valued. Interestingly, the prevailing trait does not need to be the trait of the initial large cultural group: the minority group can erode the majority. Hence, the majority and minority culture can change within one generation. *Persistence* (i.e., stable coexistence of the two cultural groups) occurs when groups interact but switching is limited because either incentives are weak or parental control is moderate, yielding durable coexistence of traits. This outcome mirrors real-world patterns in multicultural societies, where distinct cultural groups coexist and interact extensively while maintaining their own identities. Finally, *segregation* arises when successive generations reduce openness to changing traits. As a consequence, interactions collapse, switching ceases, and traits become locked in. In this outcome two isolated cultural groups persist in the long run in absence of interactions. Segregation preserves continuity but fosters insularity, polarization, and stasis. This outcome can be observed in ethnic enclaves or polarized communities where separation endures not by law but by deliberate behavior, and may reflect authoritarian or protective parenting.

Our contribution is fourfold. First, we endogenize cultural openness, the willingness to adopt new traits, as a central driver of cultural change, rather than treating adaptability as exogenous. Second, we provide micro-foundations for both parental decisions and children's subsequent behavior: by modeling how parents instill openness in their children, we

[1] There is extensive literature that empirically confirms the impressionable years hypothesis, see, e.g., (Giuliano and Spilimbergo, 2009; Gavresi and Litina, 2023).

[2] Giuliano and Nunn (2021) examine one concrete channel of cultural adaptability, climatic variability across generations, and show theoretically and empirically that greater environmental instability is associated with weaker cultural persistence. Our framework formalizes and generalizes this evidence by providing micro-foundations for adaptability as an endogenous trait shaped by parental and individual choices, rather than linking it to a specific source. As such their evidence mirrors the adaptive process at the heart of our model.

allow young adults to actively engage in peer interactions and explicitly capture how these interactions shape switching behavior and group dynamics. Third, we characterize the full range of outcomes that can emerge from these choices, showing how the same society may converge to a single trait, sustain coexistence, or become segregated. Fourth, by incorporating cultural tightness, we explain why some traits or societies persist while others evolve more readily, offering a unified interpretation of heterogeneous cultural patterns.

Seminal models of cultural transmission show how parents invest in socialization while children may adopt alternative traits through social exposure (Bisin and Verdier, 2001; Akerlof and Kranton, 2000; Giavazzi et al., 2019; Doepke and Zilibotti, 2017; Giuliano and Nunn, 2021), but they typically treat the propensity to switch traits and cross-group contact as exogenous. The baseline model in Bisin and Verdier (2001) treats social exposure as exogenous, wherein children do not choose contact intensity or switching; in Doepke and Zilibotti (2017) parenting styles are endogenous, but cross-group contact and trait switching are not individual choices. Several other models of endogenous oblique transmission include parental influence on social circles, but again abstract from any individual level decisions (Bisin and Verdier (2001); Sáez-Martí and Sjögren (2008)). We close this gap by endogenizing both interaction and openness, and by introducing group tightness (group-specific switching costs) and heterogeneous utility supports across traits. This provides a direct mapping from micro decisions to equilibrium group sizes within a generation, and to long-run paths of convergence, persistence, or segregation.

The dynamics of cultural transmission matter for policy: they shape outcomes in integration, education, conflict recovery, and social cohesion. Our model highlights that cultural fragmentation or persistence is not simply the result of historical inertia or structural barriers, but can emerge endogenously from rational behavior. This has important implications. Policies aimed at promoting inclusion or cultural change must account not only for material incentives but also for how families shape cultural change across generations. Efforts to foster cross-group interaction through school integration, public discourse, or urban design may succeed only if they align with underlying parenting and social engagement. By clarifying the mechanisms that drive convergence or segregation, our framework helps explain why some societies experience high levels of cultural inertia, whilst others undergo rapid cultural transformation, and why in some cases society remains divided.

The roadmap of the paper is as follows. The next section situates our research within the existing literature. Section 3 presents relevant anecdotal evidence based on international representative surveys. Section 4 introduces the model, and Section 5 discusses how the framework of Bisin and Verdier (2001) fits within our setting. Section 6 concludes.

## 2 Literature Review

Our paper contributes to the literature on cultural economics, and in particular to models of cultural transmission and persistence. The starting point is the seminal model of Bisin and Verdier (2001), who formalized cultural transmission as the outcome of parental effort. In their framework, children inherit traits passively: if vertical transmission fails, they adopt the trait of a randomly matched peer. Parental involvement matters because greater effort increases the probability of vertical transmission. This model was the first to show how decentralized parental decisions could sustain persistent cultural heterogeneity in equilibrium. Our model departs from this setup in a simple but fundamental way. While Bisin and Verdier focus on the probability of children acquiring the parental trait, we assume children always inherit it initially (i.e. the trait is transmitted effortlessly. What parents focus on instead is shaping how open their children will be to alternative traits later in life. This difference, transmitting openness to changing cultural traits rather than just the trait itself, creates a mechanism by which parental influence extends into adulthood and across multiple generations.

Our framework also builds on Doepke and Zilibotti (2017), who show that parental involvement can shape not only children's preferences but also the set of choices they face. Parenting styles in their model reflect different trade-offs between preserving tradition and fostering adaptability. We capture a similar idea by introducing heterogeneity in parental openness: parents who value adaptability more strongly face higher costs of closing their children off, and may encourage interaction even at the risk of losing their own trait. In this sense, we preserve the core idea of Bisin and Verdier, that parents view the world through their own lens, while extending it to incorporate variation in parenting styles emphasized by Doepke and Zilibotti.

Relative to these two seminal frameworks, our contribution is threefold. First, we make social interactions explicit: individuals actively choose how intensively to interact with out-group members, rather than being randomly matched. This allows us to model different levels of cross-group contact, from integration to segregation. Second, when interactions occur, individuals weigh the costs and benefits of switching traits, which introduces the possibility that they adopt the trait best suited to their preferences and environment. Third, parents focus not on directly transmitting their own trait but on influencing future openness, thereby shaping both the likelihood of interactions and the probability of switching once interactions take place. Together, these elements allow us to explain why some traits persist, others erode, and why some cultural groups assimilate while others maintain distinct identities.

Our approach also relates to broader models of cultural evolution. Bisin and Verdier (2008) survey how cultural traits are transmitted vertically, obliquely, or horizontally, and emphasize how the surrounding cultural environment shapes long-run equilibria. Extensions of this framework consider education (Bénabou and Tirole, 2006), political preferences (Guiso et al., 2006), or assortative marriage (Fernández et al., 2005). Other contributions emphasize evolutionary approaches where culture changes slowly (Tabellini, 2008), or decision-based approaches where agents consciously choose among traits (Akerlof and Kranton, 2000). Within the identity literature, children may adjust their degree of identification with a trait, but parents still mainly attempt to transmit their own trait. Our model differs by shifting the parental choice variable itself: parents' decision of socialization is about the level of openness to cultural change to instill in their child, thereby influencing the very process of cultural change.

Finally, our work relates to the work of Giuliano and Nunn (2021). The authors examine a particular source of cultural adaptability emphasized in evolutionary-anthropology models: the extent to which environmental conditions remain stable across generations. When climatic conditions fluctuate more intensely, previously evolved traits become less reliable, increasing the value of adaptation and resulting in weaker cultural persistence. They test this mechanism empirically by combining historical climatic data from 500 to 1900 with contemporary measures of cultural attitudes, showing that populations whose ancestors experienced greater cross-generational climate variability place less emphasis on tradition today. Our approach complements this evidence by broadening the concept of adaptability. Rather than linking adaptability to a specific external factor such as climate, we model it as an endogenous trait—cultural openness, shaped by parental decisions, whereas cultural change is the result of individual peer interaction choices and switching probabilities. This allows us to develop explicit micro-foundations for how adaptability emerges and how it influences cultural change both within and across generations. In this sense, their empirical findings provide a concrete illustration of the broader mechanism we formalize: variability increases incentives for cultural change, and our model shows how these incentives translate into purposeful behavioral choices that shape long-run cultural dynamics.

# 3 Anecdotal Evidence

While our model is theoretical, it rests on mechanisms that are observable in the real world. Survey evidence from sources such as the Integrated Value Survey (IVS) consistently shows how parents influence their children's openness, values, and social attitudes, underscoring the central role of intergenerational socialization and interaction constraints in shaping cultural outcomes. In what follows, we first present illustrations on parenting styles across different world regions, and then turn to a case study that demonstrates how our framework can shed light on real-life cultural dynamics.

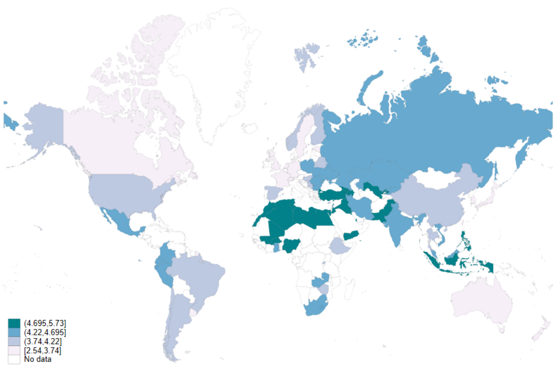
## 3.1 Different parenting styles across the world

Parenting styles vary across the world, with some putting more weight on tradition and obedience, whereas others place more emphasis on independence and tolerance towards others. We proxy these characteristics through a set of questions from the Integrated Value Survey (IVS). The different questions were not necessarily asked within the same year, and there is variation in the years in which the same question was asked across countries. To have the most accurate and representative data possible given the data limitations, we aggregate at the country level using the most recent year in which the question was asked for each country.

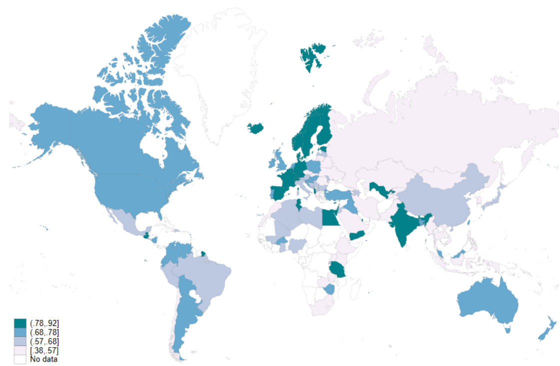
Figures 1a-1c depict world maps of the emphasis placed on each characteristic. Figure 1a depicts the answers to the question: "It is important to the person: Tradition". The importance placed on tradition varies greatly across countries. Whilst individuals in Europe, the US or Australia, value it relatively low, countries in Northern Africa and the Middle East see it as an important characteristic.

In order to motivate that the emphasis placed on tradition shapes the effort put into fostering cultural openness in the child, we look at 2 questions, asking about the importance of two characteristics in children for the respondent: (1) independence and (2) tolerance and respect towards others. Observing the map of the value of tradition to a map of (2) (figure 1b), we can see that they follow an inverse pattern to Figure 1: the countries where tradition is valued highly, tolerance in children is not. The patterns are similar for the value placed on independence in children, although we omit the corresponding figure for brevity. The results are not simply the inverse of those for tolerance, as tolerance may reflect not only openness toward other cultural groups but also a more general disposition of respectfulness, including toward members of one's own group. Nonetheless, the broader pattern indicates that this parameter is shaped by parental choices and is correlated with individuals' views on tradition. This supports our intuition that the perceived importance of cultural continuity influences the degree of tolerance parents instill in their children and the value they place on fostering independence.

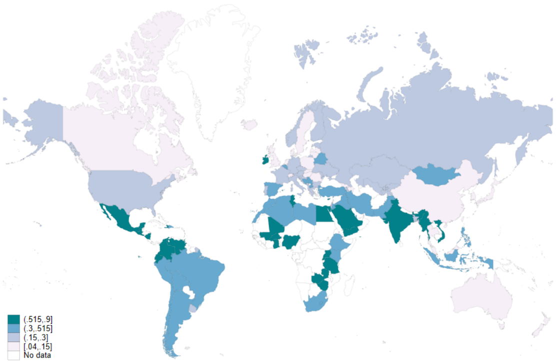
Finally, we look at whether these attitudes reflect different parenting styles. For this, we use a question that asks about the importance of obedience in children, the same question used by Doepke and Zilibotti (2017) to proxy for an authoritarian parenting style. Indeed, the importance of tolerance towards others for the parent is reflective of societies with less authoritarian parenting styles, as seen in Figure 1c.




(a) It is important to this person: Tradition.




(b) Important child qualities: Tolerance towards others.




(c) Important child qualities: Obedience.

Figure 1: Plots of worldwide attitudes towards (a) tradition, (b) tolerance in children, (c) obedience in children.

### 3.2 Immigration as a case study

The effect of parental involvement on children's cultural socialization and how long-run cultural outcomes such as convergence, persistence or segregation emerge from the decisions of individuals and families interacting with institutional and social constraints, are perhaps most clearly visible in the context of immigration. France provides a particularly striking example because of its strong state-led model of assimilation rooted in republican ideals and *laïcité*. While this approach has been presented as a path toward unity, integration outcomes have varied markedly across groups. While some communities, such as Spanish or Portuguese migrants in the 20th century, converged toward mainstream French norms over a generation or two, others, most notably North African communities, have maintained distinct cultural identities across generations (Simon, 2003b; Michalowski, 2017).

The reasoning behind this can be two-fold. On the one hand, adaptation for the former groups typically involved changes in traits that, while important, were not necessarily seen as central to personal or collective identity. In contrast, many North African Muslim communities require altering deeply identity-linked practices, such as religious dress or dietary customs, which can make change both personally costly and socially fraught. On the other hand, in line with the patterns in Figure 1, many North African societies place a high value on tradition and comparatively less on fostering independence and tolerance toward other groups in their children. When these value priorities are carried into the migration context, they can reinforce the tendency to preserve heritage traits. Over time, these intergenerational choices can reinforce social boundaries and segregation, and reduce meaningful interaction between groups.

Institutional policies that explicitly regulate cultural expression, such as the 2004 ban on conspicuous religious symbols in public schools (Abdelgadir and Fouka, 2020; Maurin and Navarrete, 2023; Uljarevic and Zanaj, 2025), or structural factors such as residential segregation in the banlieues and unequal access to education and jobs, can reinforce social boundaries and segregation as the effective cost of cross-group interaction increases. Whilst some parents may respond to this by integrating their children into French environments from an early age, knowing that they may benefit from exploring and adopting French traits, others may respond to perceived rejection by the host society or stigma from their own group by reducing their children's openness to French traits, deliberately socializing them to maintain heritage norms and avoid risky or costly interactions. This dynamic explains how cultural enclaves and behavioral segregation can persist even when formal opportunities for integration exist. This dominant pattern reflects what our model describes as a segregated equilibrium: a dynamic in which interactions decay over time, trait switching becomes increasingly infrequent, and parental efforts to shield children from the majority culture create self-reinforcing group boundaries.

The French case thus illustrates how formal equality and physical proximity can be insufficient for cultural integration when interaction costs remain high, out-group members have a low willingness to engage in interactions and parental socialization efforts reinforce segregation. Policies that ignore these endogenous dynamics risk deepening rather than bridging cultural divides.

# 4 The model

Consider a society composed of two groups of individuals. Each group is characterized by a distinct cultural trait or value, labeled $n$ or $m$, respectively. These traits are mutually exclusive and define specific norms and behaviors followed by individuals within each group. For example, trait $n$ might represent a religious identity, while trait $m$ represents a non-religious identity; alternatively, $n$ could represent left-wing political values, and $m$ right-wing political values. Individuals in this model are observed through three distinct phases of their lifecycle within each generation: childhood, young adulthood, and parenthood. A person may change their cultural trait during their lifecycle.

In the *childhood phase*, children are born and experience vertical cultural transmission. They inheret their parent's trait with certainty and parents actively engage in sozialising their offspring, shaping the probability that children will keep this trait within their lifecycle.

In the *young adulthood phase*, individuals become independent and engage in various forms of social interactions, including face-to-face conversations, group gatherings, online communications, or indirect interactions like media consumption. Through these interactions, individuals may reassess their cultural affiliations and decide whether to remain in their current cultural group or adopt the cultural trait of another group. We assume interactions among individuals within the same cultural group reinforce existing traits, meaning individuals do not switch groups as a result of interactions within their own community (i.e., no mutations). However, when interactions occur between members of different cultural groups, individuals evaluate the potential benefits of adopting the other group's traits. Since individuals cannot precisely predict their ultimate satisfaction upon changing groups, they base their decision on expected values of advantages in doing so. Thus, individuals are more inclined to switch groups if they perceive greater benefits in joining another group. Nonetheless, switching cultures incurs various social, psychological, or material costs, reflecting the stigma or penalties associated with leaving one's original cultural community.

In the *parenthood phase*, individuals reproduce asexually, each having a single child. By this stage, individuals are assumed to be satisfied with their current cultural affiliation and do not change any longer (Krosnick and Alwin, 1989). As parents, individuals transmit their cultural trait to their offspring with certainty and they invest effort into vertically transmitting a certain cultural openness - the propensity to adopt new cultural traits - to their child. This, in turn, shapes how likely their child is to change their trait and hence the likelihood for cultural continuity across generations. The vertically transmitted openess is determined by a tradeoff between the psychological and social cost sustained by the parent when the child holds a different trait to them vs. the child being worse off because it is reluctant to change, unable to explore and adopt alternative cultural traits due to being closed off through parental socialization. The cost sustained by the parent in each scenario is determined by their own openness. The parenthood phase marks the completion of a generation's lifecycle.

To clearly track the temporal progression of cultural evolution, we adopt a discrete generational model, with each generation labeled by the index $t$, experiencing exactly one full lifecycle:

1. Childhood at generation $t$: Individuals receive vertical transmission from parents of generation $t-1$.

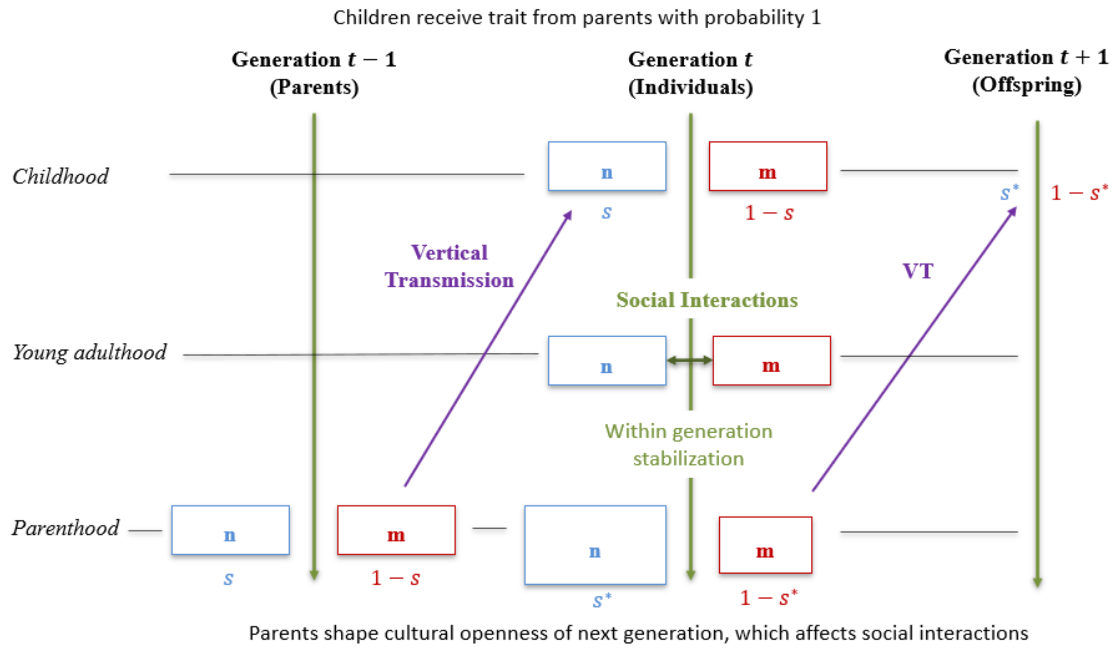



Figure 2: Overview of model timeline: generation $t-1 \rightarrow$ generation $t \rightarrow$ generation $t+1$

2. Young Adulthood at generation $t$: Individuals from generation $t$ engage in peer interactions (at school, university, work, on social media).

3. Parenthood at generation $t$ : After interactions stabilize at $t$, the same individuals become parents and vertically transmit traits/openness to children of the next generation $t+1$.

The mechanism is detailed in figure 2. For clarity and simplicity, we omit the explicit generational index $t$ when discussing phases within a single generation but reintroduce it in the long-term analysis when transitioning across generations.

## 4.1 Childhood at generation $t$: trait and preference formation

At a given generation $t$, every child is born with a trait $k \in \{n, m\}$ from their parent. We assume that children initially adopt their parents' cultural traits before engaging in any social interactions with individuals from other groups. The implications of an alternative scenario, where children are born without predefined traits, are analyzed later in section 5, following a model similar to Bisin and Verdier (2001).

Let group $k = n, m$ be denoted by $G_k$. The number of individuals born into groups $G_n$ and $G_m$ are denoted by $S_n$ and $S_m$, respectively, with corresponding population shares $s_n$ and $s_m$, and are determined by the group size of the parents of generation $t-1$. By definition

$$S_n + S_m = S \text{ and } s_n + s_m = 1,$$

where $S$ is the total population of born individuals. We simplify denoting $s_n = s$ and so $s_m = 1 - s$.

Individuals are born with a certain level of intrinsic utility that they derive from holding a particular trait. This individual utility captures a certain level of identity, individual benefits (both social and economic) and peer approval that an individual experiences when being part of this group. This is a personal experience and may vary across individuals, even when they share a common trait. Individuals (and their single parents) within each group are thus heterogeneous with respect to an individual utility that they acquire from holding a particular trait. The intrinsic utility derived from the cultural trait for individuals $i = 1, ..., S_k$ in group $G_k$ is distributed uniformly over the interval $[0, \Omega_k]$:

$$\mu_i \sim \text{Uniform}(0, \Omega_k), \;\; \Omega_k \in [0, 1].$$

For a group with a large utility support, $\Omega_k$, a larger share of individuals will experience these high levels of social and psychological rewards. This could be due to, for instance, a certain trait leading to better labor market access, education opportunities or social integration. Individuals derive utility both from their inherent utility $\mu_i$ and from the prevalence of their trait within society, $s$ or $1-s$. The utility of an individual for each group is then given by:

$$U_i = \mu_i + (1-\mu_i)s^2 \text{ for } i \in G_n \text{ and } U_i = \mu_i + (1-\mu_i)(1-s)^2 \text{ for } i \in G_m \tag{1}$$

The second component, the utility gain from group size, captures the fact that individuals value conformity (Bernheim, 1994; Akerlof and Kranton, 2000; Desmet and Wacziarg, 2021). Identifying with a group of people is a key part of self-identity and yields an important psychological benefit of belonging. The larger is the group of individuals sharing the same cultural trait, the larger the utility benefit. This component is moderated by the complement of the utility derived from the cultural trait. In other words, individuals assign a certain weight to their own cultural trait, which is simply the utility they derive from it, $\mu_i$, and to the size of the group. $1-\mu_i$. The idea is that if an individual gains a lot of utility from their trait, they care less about how many other people share this trait. However, if the trait provides less intrinsic utility, the individual places more importance on how widespread the trait is in the population. Both components factor into the total utility of the individual.

As children become young adults, they have the opportunity of interacting with individuals of the other group and of switching groups if there is a perceived benefit. A parent anticipates this potential switching, putting in effort to influence their child's propesity to adopt the other trait, which we denote by $p_{i,switch}$, and will be described in more detail in the next subsection (section 4.2). This parameter shapes both the child's social interaction choices and their likelihood of switching groups when they become young adults. The detailed process and implications of this vertical transmission are explored comprehensively in the Parenthood phase in Section 4.3.

## 4.2 Young Adulthood at generation $t$: peer interactions and trait switching

The phase of young adulthood spans a clearly defined segment of an individual's lifecycle, allowing sufficient opportunity to reassess and potentially alter their cultural traits. In some cases, traits that benefited parents may be less advantageous for their children. This may stem from exogenous differences in preferences; for example, children may attach less of their self-identity to religion or political affiliation than their parents ($\mu_{child} < \mu_{parent}$). Alternatively, such changes may arise from environmental shifts, such as migration to a new country, where different traits carry higher utility support by facilitating social integration and access to greater benefits. During this phase, individuals engage in various forms of peer interactions, including face-to-face conversations, group gatherings, online communications, or indirect interactions such as media consumption. These interactions may prompt individuals to reconsider their cultural trait, potentially switching to another group if they perceive sufficient benefits. Our approach to culture is adaptation and decisional.

When individuals from different groups interact, each individual must decide whether they could be better off by switching groups. To make this decision, individuals consider the *expected utility* of joining the other group, since we assume they cannot predict perfectly how well off they will be with the new trait. However, the anticipated expected utility of joining another group is biased by the trait the individual acquired from their parent; switching groups comes with a certain cost, denoted by $C_k$, where $k = n, m$ for the respective groups. The higher $C_k$, the higher the bias towards one's acquired trait. This cost reflects the level of tightness associated with a specific group, i.e. how strictly norms are reinforced within a cultural group and thus the psychological cost, social stigma or penalties associated with switching traits. This cost is a group-specific characteristic, which is experienced uniformly by all members of the group. It depends on the average utility within the group, $\frac{\Omega_k}{2}$, as well as other characteristics of the group captured by a parameter $c_n$ and $c_m$, respectively, with $c_n, c_m \geq 0$. This parameter is defined as the marginal cost of changing traits.[3] A higher value indicates stricter cultural norms, stronger in-group loyalty, or even institutionalized penalties for defectors (e.g., legal penalties, social ostracism, or moral sanctions). Conversely, lower values reflect more fluid cultural norms and fewer institutionalized barriers to trait switching.

From the perspective of an individual $i$ from group $G_k$, the expected utility of joining group $G_{-k}$ (the other group) is given by:

[3]These assumptions of the group switching cost are appealing for several reasons. First, the dependence on average utility within the group captures the psychological and social costs associated with leaving a well-integrated group. Individuals embedded in groups with high average utility experience substantial peer approval, shared identity, and collective benefits. Consequently, leaving such groups entails forfeiting substantial social and psychological rewards, which translates into higher switching costs. $c_n$ and $c_m$ captures group-specific rigidity or openness of cultural norms.

$$U_{-k}^E = E_i[\mu_j] + (1 - E_i[\mu_j])s_{-k}^2 - C_k, \tag{2}$$

with $C_k$ defined such that $U_{-k}^E \geq 0$, namely $C_k \leq E_i[\mu_j] + (1 - E_i[\mu_j])s_{-k}^2$. This condition ensures that some amount of switching is always possible. The expected utility represents an individual's anticipated satisfaction from adopting the other group's trait, adjusted by the costs incurred due to switching [4].

Assuming $C_k = c_k(\Omega_k/2)$, we have

$$U_{-k}^E = \frac{\Omega_{-k}}{2} + \left(1 - \frac{\Omega_{-k}}{2}\right) s_{-k}^2 - c_k \frac{\Omega_k}{2}.$$

The positivity condition above requires that $c_k \leq \frac{1}{\Omega_k}[\Omega_{-k} + (2 - \Omega_{-k})s_{-k}^2]$. Hence, we set $c_k \in [0, \frac{1}{\Omega_k}(\Omega_{-k} + (2 - \Omega_{-k})s_{-k}^2]$. Note that the expected utility evaluation of one's own group does not include the cost, i.e. $U_k^E = \frac{\Omega_k}{2} + \left(1 - \frac{\Omega_k}{2}\right) s_k^2$. We thus have that

$$U_{-k}^E = \begin{cases} \frac{\Omega_m}{2} + \left(1 - \frac{\Omega_m}{2}\right)(1-s)^2 - c_n \frac{\Omega_n}{2}, & \text{for all members in } G_n, \\ \frac{\Omega_n}{2} + \left(1 - \frac{\Omega_n}{2}\right) s^2 - c_m \frac{\Omega_m}{2}, & \text{for all members in } G_m. \end{cases}$$

with positivity conditions $c_n \leq \frac{1}{\Omega_n}[\Omega_m + (2 - \Omega_m)s^2]$ and $c_m \leq \frac{1}{\Omega_m}[\Omega_n + (2 - \Omega_n)(1-s)^2]$.

Trait switching depends significantly on two key factors: (1) the *effort* individuals are willing to invest into socializing and (2) their *intrinsic propensity to adopt new traits*. *Effort* captures the deliberate actions an individual takes to interact with members of another group, which could include moving to a new location, trying new hobbies, learning about different cultures, and so on, while the *propensity to adopt new traits* represents an intrinsic feature of the individual. The propensity to switch of an individual is influenced by their parent's decisions and efforts when the individual is a child. This will be discussed in more detail in section 4.3.

**Social Interactions.** The willingness to interact becomes a key factor in predicting the likelihood of switching traits - the fewer interactions with individuals who have different traits, the less likely switching will occur. Importantly, not every individual within a group has the same willingness to interact with people who are different to them. Depending on certain intrinsic, exogenous and endogenous characteristics, individuals choose how much to engage with others.

Let $e_i$ denote the effort that individual $i$ puts into interacting with individuals from a different cultural group. We assume that putting in effort to interact with people from another group is costly[5]. This cost is quadratic and increasing in the effort to reflect the realistic idea that exerting effort becomes increasingly costly at higher levels[6]:

$$k(e_i) = \frac{1}{2}\alpha e_i^2.$$

Here, $\alpha$ denotes a parameter that may change how costly it is to interact with others in the context of external shocks, e.g. due to residential segregation, a pandemic or social media. In absence of such disturbances, we assume $\alpha = 1$. Individuals face a tradeoff: engage and possibly improve their utility (via switching), at a certain personal cost, or avoid the cost by not interacting with out-group members at all, closing down the option of a potential utility gain.

Furthermore, we assume that in order for individuals to have the option of changing traits, they must *exchange* with an individual from the other group. If $p_i \in [0,1]$ is the *individual probability of interaction*, then we denote the *average* probability of meeting someone from the other group by $p_n$ or $p_m$ with

$$p_n = \frac{\sum_{i \in G_n} p_i}{S_n} \text{ and } p_m = \frac{\sum_{j \in G_m} p_j}{S_m}.$$

[4] Young adults maximize current expected utility based on beliefs about the other group and switching costs. They are atomistic and do not internalize how their choice affects future group sizes or their own later role as parents: teenagers ignore intergenerational consequences !

[5] Note that this cost is different from the cost of switching groups. The cost of interacting is an individual-level cost, whereas the cost of switching is a group-imposed cost, essentially a punishment for leaving. The cost of switching reflects the group's reinforcement of its norms, which the individual must take into account, alongside personal factors.

[6] see Holmström (1982).

The probability of individual $i$ from group $G_n$ meeting and exchanging with someone from group $G_m$ is thus given by $p_i p_m$. Then, the *probability of individual $i$ keeping* their own trait, $P_{ii}$, is given by

$$P_{ii} = (1 - p_i p_{-k}) + p_i p_{-k}(1 - p_{i,switch}) = 1 - p_i p_{-k} p_{i,switch},$$

and the *probability of individual $i$ switching their trait*, $P_{ij}$, is

$$P_{ij} = p_i p_{-k} p_{i,switch},$$

where $p_{i,switch} \in [0,1]$ denotes the propensity for switching traits of individual $i$. The term $(1 - p_i p_{-k})$ captures the probability of no exchange occurring (and thus no switching), while the term $p_i p_{-k}(1 - p_{i,switch})$ captures the probability of exchange without switching.

The decision problem faced by individual $i$ in selecting their level of effort $e_i$ is thus given by:

$$\max_{e_i} \left[P_{ii}U_i + P_{ij}U_{-k}^E\right] - k(e_i).$$

Each individual maximises the expected utility taking into account the actual probability of keeping their trait $P_{ii}$ and thus their utility level $U_i$, as well as the actual probability of switching traits $P_{ij}$ thus acquiring an expected utility $U_{-k}^E$, net of the cost of putting in effort.

Maximising in $e_i$ is equivalent to maximising in $p_i$[7]:

$$\max_{p_i}(1 - p_i p_{-k} p_{i,switch})U_i + p_i p_{-k} p_{i,switch} U_{-k}^E - \frac{1}{2}p_i^2.$$

Young adults maximize current expected utility given beliefs about the other group and switching costs. They are atomistic and do not internalize effects on future group sizes or their later parent role. The resulting optimal level for the probability of interaction of an individual $i$, $p_i^*$, is

$$p_i^* = p_{-k} p_{i,switch}(U_{-k}^E - U_i),$$

with $\partial p_i^*/\partial p_{-k} > 0$, $\partial p_i^*/\partial p_{i,swutch} > 0$, $\partial p_i^*/\partial U_{-k}^E > 0$ and $\partial p_i^*/\partial U_i < 0$. This implies that the higher the average effort of the other group to interact, captured by $p_{-k}$, and the higher the propensity that the individual $i$ switches groups, $p_{i,switch}$, the higher the individual's effort to engage with individuals from the other group. Additionally, if the relative expected utility from the other group, $U_{-k}^E$, is higher, the individual will put in more effort; and if the individual has a relatively high utility $U_i$, they will be less motivated to put in effort to meet someone from the other group.

For $p_i^*$ to be positive it must be that $U_{-k}^E \geq U_i$, i.e. the individual expects to be better off than if they kept the same trait. Moreover, for $p_{-k} p_{i,switch}(U_{-k}^E - U_i) \leq 0$, we have $p_i^* = 0$. This condition holds for $p_{i,switch} = 0 \cup p_{-k} = 0 \cup (U_{-k}^E - U_i) \leq 0$. Conversely, for $U_{-k}^E - U_i > 0 \cap p_{-k} > 0 \cap p_{i,switch} > 0$, we have $p_i^* > 0$. In other words, in order for the individual to make any effort to interact with others, the following conditions must be met: (i) the individual must have a non-zero intrinsic propensity to switch groups ($p_{i,switch} > 0$); (ii) the individual must expect to gain a higher utility from being part of the other group or acquiring the other group's trait ($U_{-k}^E - U_i > 0$); and (iii) there must be a positive probability of exchanging with other group members when the individual chooses to interact ($p_{-k} > 0$). If these conditions are not satisfied, the cost of interacting with individuals from the other group will outweigh the potential benefit, and the individual will choose not to interact.

Hence, parents may directly influence cultural openness: by shaping the propensity to switch to other traits in their children, they have a direct impact on the social interaction efforts their children make. They thus influence the actual probability of switching in two ways: once, by shaping the amount of social interactions taking place and second, the propensity of changing traits that re-enters in $P_{ij}$. Furthermore, we can see that there are spillover effects from the parental influence within the other group; if parents of the other group socialize their children to have a very low propensity of switching, $p_{-k}$ is reduced and with it, the effort put into socializing by individual $i$, $p_i^*$.

[7]Here, we follow the argument of Bisin and Verdier (2001) for their parental socialization problem. The map from effort $e_i$ to interaction probability $p_i$ is strictly increasing and strictly quai-concave; moreover $p_i = 0$ when $e_i = 0$. This assumption allows us to reformulate the decision problem as choosing $p_i$, rather than $e_i$.

**Switching Traits.** In order to determine the rate of cultural change within a generation, we need to know the amount of trait switching happening at the group level. We can now determine the *actual probability of changing traits of individual* $i$ from group $G_k$ to group $G_{-k}$, denoted by $P_{ij}$, where $i \epsilon G_k$, $j\epsilon G_{-k}$, and $k \in \{n, m\}$, given their chosen level of effort to interact and thus interaction probability $p_i^*$:

$$P_{ij} = \begin{cases} 0 & if \quad p_i^* = 0 \\ p_i^* p_{-k} p_{i,switch} & if \quad p_i^* > 0 \end{cases},$$

namely

$$P_{ij} = \begin{cases} 0 & if \quad p_i^* = 0 \\ (p_{-k} p_{i,switch})^2 (U_{-k}^E - U_i) & if \quad p_i^* > 0 \end{cases}.$$

The *group probabilities of switching* $P_{nm}$ and $P_{mn}$ respectively for group $G_n$ and for $G_m$ are then given by:

$$P_{nm} = p_n p_m p_{n,switch} = (p_m p_{n,switch})^2 (U_{-k=m}^E - U_{k=n}^E) \text{ if } p_n > 0, \tag{3}$$

and

$$P_{mn} = p_m p_n p_{m,switch} = (p_n p_{m,switch})^2 (U_{-k=n}^E - U_{k=m}^E) \text{ if } p_m > 0, \tag{4}$$

with

$$p_{n,switch} = \frac{\sum_{i \in G_n} p_{i,switch}}{S_n} \text{ and } p_{m,switch} = \frac{\sum_{j \in G_m} p_{j,switch}}{S_m}.$$

Then, given the individual optimal choices, the law of motion for the share of $G_n$ after $\tau + 1$ interaction rounds is

$$s^{\tau+1} = s^\tau - P_{nm} + P_{mn} \tag{5}$$

Using (3), (4) and (5), in discrete time, the change in the size of group $G_n$ between the set of interactions $\tau$ and $\tau + 1$ is

$$\Delta s^\tau = (p_n p_{m,switch})^2 (U_{-k=n}^E - U_{k=m}^E) - (p_m p_{n,switch})^2 (U_{-k=m}^E - U_{k=n}^E). \tag{6}$$

*Mutatis mutandis*, for group $G_m$. Eventually, social interactions will not bring any changes in group size when the composition of individuals within each group remains constant over rounds of interactions. More specifically, we have:

$$\Delta s^\tau = 0$$

Said differently, since young adults take the current group share as given when optimizing, after everyone makes their choice, the only way the next-round composition can be consistent with those choices is if net switching flows cancel, i.e. $\Delta s^\tau = s^{\tau+1} - s^\tau = -P_{nm} + P_{mn} = 0$. If $\Delta s^\tau \neq 0$, the realized $s^{\tau+1}$ would differ from the $s$ used to form $U_i$, $U_{-k}^E$ and thus $p_i^*$, creating a fixed-point inconsistency between the optimal individual behavior given $s$ and the aggregate composition generated by that behavior. Setting (6) to zero, we obtain

$$(p_n p_{m,switch})^2 \left(U_{-k=n}^E - U_{k=m}^E\right) = (p_m p_{n,switch})^2 \left(U_{-k=m}^E - U_{k=n}^E\right).$$

The full expression for the equilibrium condition is given by:

$$\begin{aligned} & (p_n p_{m,switch})^2 \left(\frac{\Omega_n}{2} + \left(1 - \frac{\Omega_n}{2}\right)(s^*)^2 - c_m \frac{\Omega_m}{2} - \frac{\Omega_m}{2} - \left(1 - \frac{\Omega_m}{2}\right)(1 - s^*)^2\right) \\ = & \\ & (p_m p_{n,switch})^2 \left(\frac{\Omega_m}{2} + \left(1 - \frac{\Omega_m}{2}\right)(1 - s^*)^2 - c_n \frac{\Omega_n}{2} - \frac{\Omega_n}{2} - \left(1 - \frac{\Omega_n}{2}\right)(s^*)^2\right) \end{aligned} \tag{7}$$

When $\Omega_m = \Omega_n$, the equilibrium group size within generation $t$ is given by

$$s^* = \frac{1}{2} + \frac{1}{2} \frac{\Omega}{\alpha (2 - \Omega)} \left((p_n p_{ms})^2 c_m - (p_m p_{ns})^2 c_n\right) \gtreqless 1/2, \tag{8}$$

where $\alpha \equiv (p_n p_{m,switch})^2 + (p_m p_{n,switch})^2$. For readibility of the paper, we provide the analysis of the scenarios $\Phi \lesseqgtr \Omega$ in the Appendix B. Therefore,

**Lemma 1** *A within-generation interaction equilibrium is a fixed point* $(s^*, p_n^*, p_m^*\ )$ *where each* $p_i^*$ *solves the individual problem given the corresponding aggregates* $s^*, p_n^*, p_m^*$*, and flow balance holds:* $P_{nm} = P_{mn}$ *(equivalently,* $\Delta s^\tau = 0$*).*

**Proof.** See Appendix A. ■

Two remarks are in order. First, independently of the magnitude of $s$, the solution $s^*$ can be larger or smaller than a half and can even be equal to zero. *This means that within one generation, the majority group can turn into the minority group and viceversa.* Second, note that $\frac{\partial}{\partial p_{n,switch}} s^* < 0$. This means that a higher average propensity of individuals from group $G_n$ to switch traits, $p_{n,switch}$, will always decrease the group size of $G_n$. Mutatis mutandis, for $p_{m,switch}$ and $G_m$. This is true for all scenarios $\Omega_n \lesseqgtr \Omega_m$. This feature highlights the effect of the propensity to switch on cultural change: a higher average propensity of individuals to switch traits always reduces the equilibrium size of their current group. Decreased attachment thus weakens group stability. It illustrates how parental efforts may directly shape the group size and how important cultural intolerance is for (majority) trait preservation. However, crucial are also the group-imposed cost of switching $c_k$ as well as the utility support associated with a trait $\Omega_k$. In particular, we have that $\frac{\partial}{\partial c_n} s^* < 0$, whereas the sign of $\frac{\partial}{\partial p_{n,switch}} \Omega$ is ambiguous. While parents may have great influence on the distribution of traits in the next generation, society-level persistent parameters may dampen or increase their effect.

More specifically, proposition 1 gives insight into how group-imposed costs may influence cultural change:

**Proposition 1** *Let the utility support for traits n and m be denoted by* $\Omega_n$ *and* $\Omega_m$*, respectively, and let* $s_k^*$ *denote the equilibrium size of group* $G_k$ *for* $k \in \{n, m\}$*. At any interior equilibrium (with positive interaction probabilities), the sign of the effect of group-specific tightness* $c_k$ *on equilibrium group size* $s_k^*$ *depends only on the ordering of* $\Omega_n$ *and* $\Omega_m$*. If* $\Omega_k > \Omega_{-k}$*, then* $\frac{\partial}{\partial c_k} s_k^* > 0$*, i.e., the tighter trait k (the higher* $c_k$*), the larger* $G_k$*. However, if* $\Omega_k \leq \Omega_{-k}$*, the effects of tightness are reversed, namely* $\frac{\partial}{\partial c_k} s^* < 0$*, i.e. the tighter the trait k, the smaller the group* $G_k$*.*
***Proof.*** *See appendix B for the proof of proposition 1.* ■

Proposition 1 delivers one intuitive and one less intuitive result. As expected, tighter norms should increase group size, since fewer individuals leave due to higher switching costs. However, this outcome critically depends on relative utility supports. When one group enjoys a greater utility support, it means some individuals derive such high utility from belonging to this group that they have no comparable counterparts in the other group. Such individuals rarely abandon their trait. In this scenario, tightening norms indeed increases the size of the group with higher utility support, as expected. Conversely, a counterintuitive result emerges for the group with lower utility support (or when utility supports are equal). Here, tightening norms actually reduces equilibrium group size. Although tighter norms discourage members from leaving, they simultaneously discourage newcomers from joining. In equilibrium, the loss of incoming members outweighs the retention of current ones, shrinking the group. *Thus, tightening norms does not universally boost group size; rather, its effect depends critically on whether the group's trait confers a utility advantage relative to alternatives.*

The mechanism behind this result is as follows. The decision to switch traits hinges on the net utility gain from switching, which depends both on the stigma cost of leaving one's current group (linked directly to its tightness) and on the anticipated stigma costs in the alternative group. While tighter norms always deter exits, they also reduce the attractiveness of the group to outsiders. In equilibrium, tighter norms only translate into a larger group if the group's trait is associated with higher relative utility, providing sufficient incentive for outsiders to join despite the increased stigma cost. Otherwise, the increased retention effect is overshadowed by the decreased inflow of newcomers, causing the group size to decline. Hence, the group's tightness $c_k$ hits two channels in opposite directions: it discourages outflow from $k$ (which helps $G_k$) but, by lowering the group's average interaction effort $p_k$, it also reduces inflow from $-k$ (which hurts $G_{-k}$). Which channel dominates is governed by the relative intrinsic support of the traits, $\Omega_k$ vs $\Omega_{-k}$.We show in Appendix C that when $\Omega_n = \Omega_m = \Omega$, then if the utility of both groups increases to $\Omega'$, group size $s^*$ becomes larger in equilibrium if only if $c_m > c_n \left( \frac{p_m p_{n,switch}}{p_n p_{m,switch}} \right)^2$. In other words, for equal utility-support, a cultural group grows only when the other group is relatively tight.

Together, these results offer insights into how norms and interactions shape cultural groups within a single generation. Tighter norms, often adopted in response to external threats, can paradoxically lead to a smaller equilibrium group size, thereby weakening the group's overall societal presence. This unintended effect can only be avoided when the cultural trait itself becomes more socially rewarding, aligning individual incentives with the collective tightening of norms.

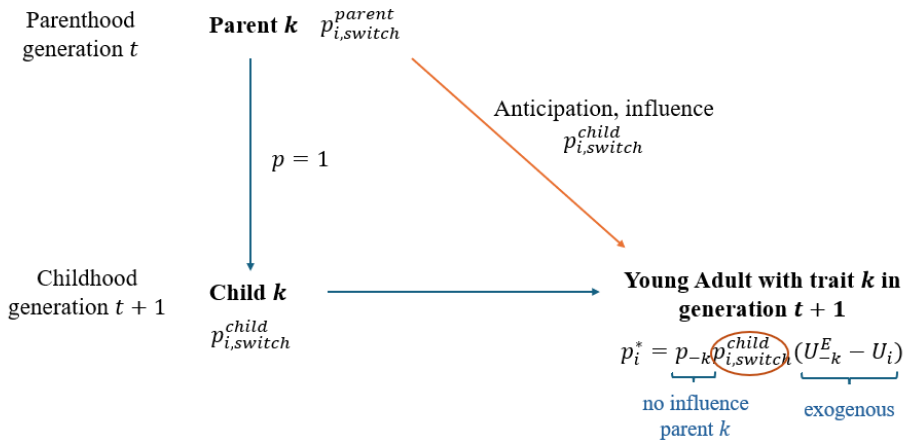



Figure 3: Mechanism of Parental Transmission.

### 4.3 Parenthood and vertical transmission

In this phase, society is at the within-generation interaction equilibrium, so group shares are constant at $s^*$ for $G_n^*$ and $1 - s^*$ for $G_m^*$, with balanced switching flows $P_{nm} = P_{mn}(\Delta s^\tau = 0)$. Individuals reproduce asexually, each having one child and then exiting, so the total population $S$ is constant and there are no fertility differences across groups. Although the aggregate shares are constant at reproduction, individual trait switching remains possible; but balanced flows imply no net change in $s^*$. We assume parents are content with their current trait at reproduction and intend to pass it on.

Children inherit their parent's trait with probability one before any horizontal interactions. Hence, the inherited trait at birth is certain; horizontal exposure comes after inheritance. Following Bisin and Verdier (2001), we posit that parents are altruistic towards their children and so they have an incentive to socialize their child to a specific trait if they *think* this may increase their welfare. Let $V_{ii}$ denote the utility parent $i$ receives if the child keeps the their trait and $V_{ij}$ if the child changes to the alternative trait. We define the difference in these utilities $\Delta V_i = V_{ii} - V_{ij}$ as the level of cultural intolerance the parent holds towards their child holding a different trait to themselves. Bisin and Verdier (2001) make an assumption of imperfect empathy, wherein $\Delta V_i > 0$. We abstract from such assumptions in order to allow for different parenting styles - altruistic and permissive or authoritarian - following the idea of Doepke and Zilibotti (2017). Parents, anticipating their children's future social interactions and potential trait switches, invest in socialization to influence the child's propensity to switch, denoted $p_{i,switch}^{child}$. This choice shapes two margins in the child's later problem: (i) the child's optimal interaction effort $p_i^*$, and (ii) the child's switching probability $P_{ij}$. Parental socialization directly affects whether children explore and possibly adopt a trait that yields them higher utility. Note that $\Delta V_i$ is conceptually distinct from $p_{i,switch}$, the parent's own openness. While the last reflects an individual's personal attitude toward their own trait, the first captures the extent to which a parent seeks to instill those values in their children. Although it may appear intuitive to assume a relation between the two, such a link is not necessary. We therefore treat them as distinct and do not impose any functional dependence.

For simplicity, we denote $p_{i,switch}^{parent} \equiv p_{i,switch}$. This is the propensity to switch that the parents were endowed with when they entered the phase of social interactions, treated in the above sections. For an individual $i \in G_k$,the probabilities of keeping or switching the inherited trait are

$$P_{ii} = 1 - p_i^* p_m p_{i,switch}^{child} \text{ and } P_{ij} = p_i^* p_m p_{i,switch}^{child},$$

where we assume that $p_i^*$ and $p_m$ are the values that have been internalized by the parents during their lifetime, i.e. $p_i^* p_m$ is the probability of exchange between the parent and someone from group $G_m$ when they were a young adult. With this, we assume that parents view the potential for social interactions of their children through the lens of their own experience. These probabilities most likely change for the child, but cannot be internalized correctly by the parent. We illustrate the mechanism of our model of vertical transmission in figure 3.

As we described above, parents aim to influence whether their child keeps or changes their trait during young adulthood, which requires an active effort. For instance, parents may be selective in where their children go to school, who they allow them to play with or how they talk about out-group individuals at home. As such, they may regulate the exposure of the kid at a young age and influence how open their child may be to adopting other traits. The more effort they put into changing their child's openness, or their propensity to switch traits, the more costly this is for the parent. The magnitude of the cost of effort depends on how far the child's openness is from the parent's. This captures both the psychological cost felt by the parent if their kid has a different attitude towards other cultures than them as well as the amount of effort parents have to put in for their child to have different attitudes to them. In the *laissez-faire* scenario, children will observe the parent and adopt their attitudes effortlessly. We assume parents face a quadratic socialization cost that penalizes deviations of the child's openness from the parent's own:

$$C(p_{i,switch}, p_{i,switch}^{child}) = \frac{1}{2}\left(p_{i,switch} - p_{i,switch}^{child}\right)^2$$

The decision problem for a parent $i \in G_k$ is thus:

$$\max_{p_{i,switch}^{child}} U_i + (1 - p_i^* p_{-k} p_{i,switch}^{child}) V_{ii} + p_i^* p_{-k} p_{i,switch}^{child} V_{ij} - \frac{1}{2}\left(p_{i,switch} - p_{i,switch}^{child}\right)^2$$

and the first order condition yields:

$$p_{i,switch}^{*child} = p_{i,switch} - p_i^* p_{-k} \Delta V_i, \tag{9}$$

with $\frac{\partial p_{i,switch}^{*child}}{\partial p_{i,switch}} > 0$, $\frac{\partial p_{i,switch}^{*child}}{\partial (p_i^* p_{-k})} < 0$ and $\frac{\partial p_{i,switch}^{*child}}{\partial \Delta V_i} < 0$. The child's openness moves toward the parent's: higher parental openness raises $p_{i,switch}^{*child}$; higher intolerance $\Delta V_i$ lowers it; stronger anticipated exposure $p_i^* p_m$ lowers it. If $\Delta V_i = 0$, the child inherits the parent's openness. Thus, parents who highly value trait continuity ($\Delta V_i > 0$) or expect intensive cross-group contact (high $p_i^* p_{-k}$) invest more to reduce the child's openness. This can limit the child's ability to adopt a more beneficial trait, since since lower openess $p_{i,switch}^{*child}$ implies lower interactions probabilities $p_i^*$ and actual switching probabilities $P_{ij}$ for the same level of expected utility gain $U_{-k}^E - U_i$. When such behavior is widespread, it can lower the next generation's welfare even in the absence of legal barriers. Shifts in intolerance after collective trauma (Alexander et al., 2004) may amplify this mechanism. Notice that in Appendix D, we extend the parental problem to account not only for concerns about their children's future traits but also for those of their grandchildren. Interestingly, when parents internalize the welfare of two future generations, cultural persistence becomes even stronger, as they further reduce $p_{i,switch}^{*child}$ compared to $p_{i,switch}$. It follows that

**Proposition 2** *When parents exert effort to reduce the likelihood that their children change their original cultural trait, whilst keeping their child's characteristics as close as possible to their own, the children's propensity to switch traits is always lower than that of the parent when $\Delta V_i > 0$ and higher when $\Delta V_i < 0$. If the parent is indifferent to their child's cultural trait ($\Delta V_i = 0$), the child will simply inherit their parent's propensity to switching.*

Higher, positive $\Delta V_i$ might indicate more desirable, traditional or culturally important traits (e.g. religious or political views, living arrangements, etc.), explaining why these cultural traits have been shown to be highly persistent. It could also simply be a reflection of a more traditional (authoritarian) parenting style. In contrast, negative $\Delta V_i$ may capture traits that are not as important to the parent, or are specifically important to the parent not to transmit; instead they want to push their child to copy peers, with the goal of their child fitting in with the people around them or live more happily in society. For instance, patience may be a trait that the parent wants the child to adopt even if the parent may be impatient. Similarly, the parent may push the child to adopt cooperation attitudes similar to their peers so they can form more successful relationships with others in the broader society. It may also reflect a more permissive or altruistic parenting style, in that the parent has a positive tolerance towards their child exploring and potentially adopting more beneficial traits.

On a group level we have that

$$p_{k,switch}^{*child} = p_{k,switch} - p_k^* p_{-k} \Delta V_k,$$

where $\Delta V_k = \frac{\sum_{i \in G_k} \Delta V_i}{S_k}$. This captures the aggregate parental influence on group openness from one generation to the next.

**The effect on generation $t+1$.** Now, we turn to the effects of the results in Proposition 2 for social interactions in the next generation $t+1$. The child (now young adult) solves:

$$p_i^* = p_{-k} p_{i,switch}^{child} (U_{-k}^E - U_i).$$

It is not clear whether each individual probability of interaction $p_i^*$ will automatically be reduced (or increased) since this depends on the sign of $U_{-k}^E - U_i$. However, on a group level, we have:

$$p_k^* = \frac{\sum_{i \in G_k} p_i}{S_k} = p_{-k} p_{k,switch}^{child} (U_{-k}^E - U_k^E).$$

Since we know the difference in expected utilities is the same as in the previous generation, whether the amount of people interacting grows or shrinks depends on $p_{k,switch}^{child}$ and the interaction potential with the other group $p_{-k}$. This interaction potential is, in turn, shaped by the aggregate probability of switching of the other group, $p_{-k,switch}^{child}$. If $p_{k,switch}^{child} > p_{k,switch}^{parent}$, $p_k^*$ will increase if $p_{-k}$ has not changed from one generation to the next. However, if $p_{-k,switch}^{child}$ has become lower, then the increase in $p_{k,switch}^{child}$ may not necessarily result in higher group-level interactions for group $G_k$ as it is counteracted by a lower level of $p_{-k}$. In order to observe the change in group size in generation $t+1$ as a result of parental influence, we look at the actual probability of switching, given by:

$$P_{nm} = (p_m p_{n,switch}^{child})^2 (U_{-k=m}^E - U_{k=n}^E), \tag{10}$$

and

$$P_{mn} = (p_n p_{m,switch}^{child})^2 (U_{-k=n}^E - U_{k=m}^E). \tag{11}$$

It follows that:

**Corollary 1** *If, on average, parents in cultural group $k$ prefer continuity ($\Delta V_k > 0$), then $p_{k,switch}^{child} < p_{k,switch}$, so switching from $k$ to $-k$ falls in the next generation; if $\Delta V_k < 0$, switching from $k$ to $-k$ rises.*

Indeed, if, on average, a group of parents derives positive utility from their child adopting their own trait ($\Delta V_k > 0$), the overall openness of the group will decrease. Formally, this occurs because the next generation's switching probability becomes: $p_{k,switch}^{*child} = p_{k,switch} - p_k^* p_{-k} \Delta V_k < p_{k,switch}^{parent}$. In other words, even individuals who themselves are relatively open to switching traits will, if they care sufficiently about their children keeping the same cultural trait $n$ or $m$, invest effort in socializing their offspring in ways that reduce the child's likelihood of switching. This means that the stronger the parent's preference for trait continuity, the lower the child's openness and the lower the child's probability of switching. Thus, a direct relationship arises between the intensity of parental preferences for trait preservation and the reduced openness of future generations towards alternative traits. We know that this indeed leads to a larger equilibrium group size in the next generation ($\frac{\partial s^*}{\partial p_{n,switch}} < 0$).

## 4.4 Long Term Trait Distribution

In our model that integrats both social interactions and vertical transmission, parental efforts to modify their children's propensity to change traits shape the long-term trajectory of group population sizes by affecting the propensity for future generations to switch traits during social interactions. Through this mechanism, parental influence extends beyond a single generation, altering the evolution of cultural traits and group dynamics over time.

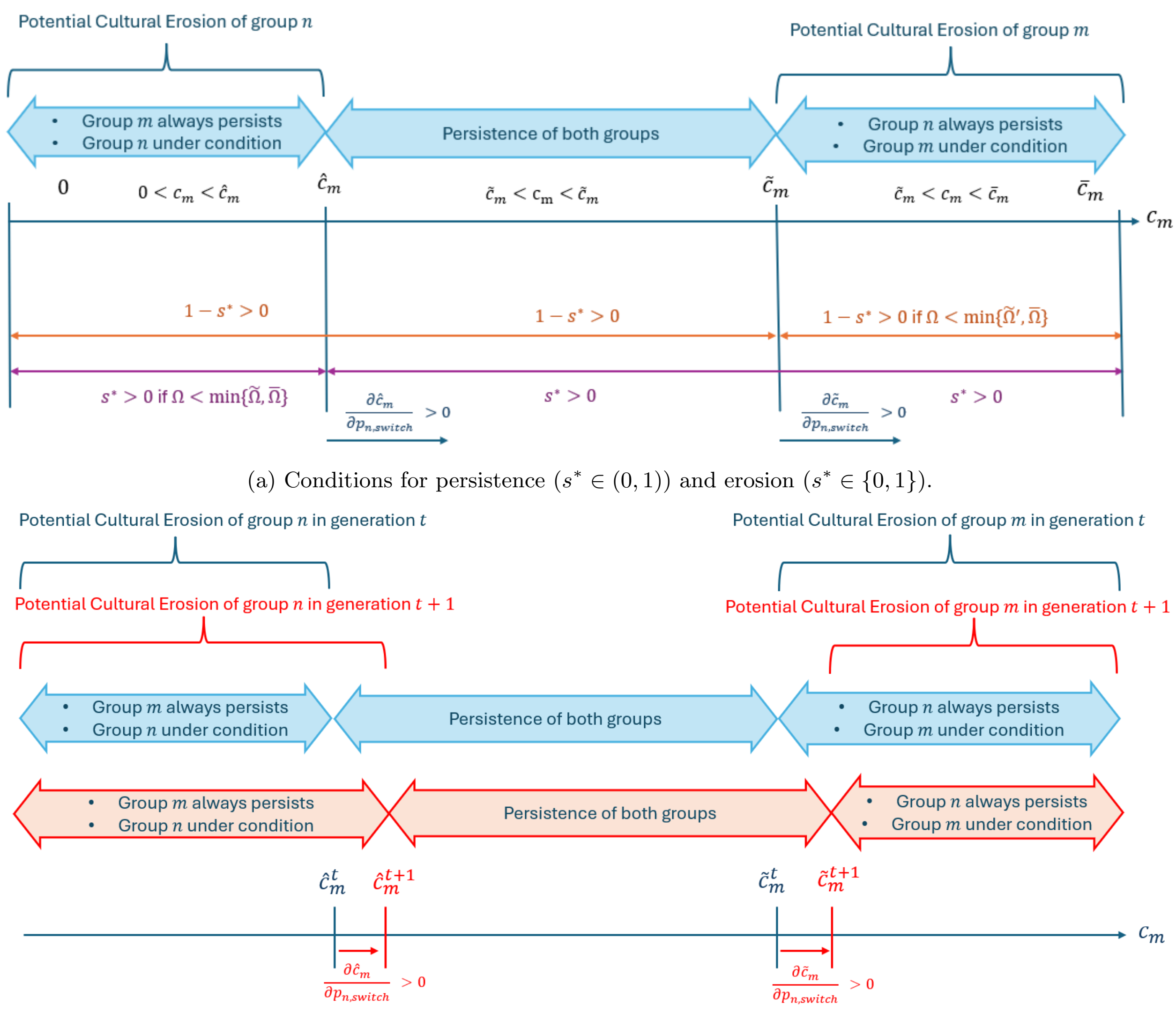
As a consequence of the cultural openness of the different groups, $p_{k,switch}$, potentially changing across generations, a group that managed to persist until generation $t$, may erode in generation $t+1$. In order to understand how the composition of groups may change in the long-run due to parental involvement and social interactions, we analyze equation (8) and the possible values that $s^*$ may take.

Let us define $\tilde{c}_m \equiv 1 + (c_n + 1)\frac{(p_m p_{n,switch})^2}{(p_n p_{m,switch})^2}$ and $\hat{c}_m \equiv -1 + (c_n - 1)\frac{(p_m p_{n,switch})^2}{(p_n p_{m,switch})^2}$, $\tilde{\Omega}' \equiv \frac{-\left(2(p_{mn} p_{n,switch})^2 + 2(p_{nm} p_{m,switch})^2\right)}{(1-c_n)(p_{mn} p_{n,switch})^2 + (1+c_m)(p_{nm} p_{m,switch})^2}$ and $\tilde{\Omega} \equiv -\frac{2(p_{mn} p_{n,switch})^2 + 2(p_{nm} p_{m,switch})^2}{(c_m - 1)(p_{nm} p_{m,switch})^2 - (c_n + 1)(p_{mn} p_{n,switch})^2}$. Then,

**Proposition 3** *Let the utility support for traits $n$ and $m$ be denoted by $\Omega_n$ and $\Omega_m$, respectively, and let $\Omega_n = \Omega_m$. For relativey intermediate levels of the marginal cost of changing traits $c_n$ and $c_m$, i.e. $\hat{c}_m < c_m, c_n < \tilde{c}_m$, then $s^* \in (0,1)$, i.e.*

*both traits persist. However, if $c_m < \hat{c}_m$, then if $\Omega \geq \tilde{\Omega}$, we have that $s^* \leq 0$, i.e. trait $n$ erodes. Similarly, trait $m$ erodes meaning $s^* \geq 1$, if $c_m > \tilde{c}_m$ and $\Omega \geq \tilde{\Omega}'$.*
***Proof.*** *See Appendix E for the proof of proposition 3.* ∎

Note that the mutual exclusivity of traits $n$ and $m$ and no outside option means that the level $s^* \in (0,1)$ guarantees the persistence of both traits, whereas $s^* \in \{0,1\}$ is indicative of either the erosion of $n$ and society only consisting of group $m$ ($s^* = 0$, $1 - s^* = 1$), or the erosion of $m$ and only trait $n$ persisting in society ($s^* = 1$, $1 - s^* = 0$).




(b) Parental influence on boundaries of persistence and/or erosion.

Figure 4: Long-run cultural erosion vs. persistence.

All conditions and outcomes as well as the effect of parental influence are shown in the illustrations in figures 4a and 4b. As expected, a higher propensity to switch of a group, $p_{k,switch}$, makes cultural erosion a more likely outcome for all other parameters held constant, since, for instance for group $m$, $\frac{\partial \tilde{c}_m}{\partial p_{m,switch}} < 0$, which makes the interval of possible cultural erosion for group $m$ larger ($c_m > \tilde{c}_m$ becomes more likely). This illustrates how parental efforts may directly shape the continuity of the their cultural trait across generations through the shaping of the propensity of switching of their child. Notice that a higher marginal tightness of group $m$, $c_m$, may lead to potential cultural erosion of the group, more specifically, once it crosses the threshold of $\tilde{c}_m$. In other words, increased group rigidity and enforcement of norms may counterintuitively lead to the erosion of the trait. This follows from the result in proposition 1: if the tightening of norms leads to a smaller group size $s^*$, we can see how this could lead to cultural erosion, more specifically if $s^*$ shrinks to 0. Increasing the tightness of norms is often the response of a group when facing outside threats. Since cultural survival is the key motivation for this, our result shows that this might in reality have the opposite to the desired effect if the tightness crosses a certain threshold. Rather than solidifying the group's position, such defensive tightening might initiate a negative feedback loop: reduced group size decreases the trait's perceived value, further discouraging new entrants, which might eventually lead to the trait eroding completely. However, a tighter or more open out-group (higher $c_n$ or $p_{n,switch}$) may counteract this effect since $\frac{\partial \tilde{c}_m}{\partial c_n} > 0$ and $\frac{\partial \tilde{c}_m}{\partial p_{n,switch}} > 0$. Moreover, we have that $\frac{\partial}{\partial p_{n,switch}}\tilde{\Omega} < 0$, i.e. a higher propensity to switch at the group level will shorten the interval $\Omega < \tilde{\Omega}$, increasing the likelihood that trait $n$ erodes. Similarly, a higher cost of switching $c_n$ augments the probability that trait $n$ will erode, as expected ($\frac{\partial}{\partial c_n}\tilde{\Omega} < 0$).

Hence, our model yields three possible long-run outcomes: (i) equilibrium with one trait only persisting (cultural erosion); (ii) an adaptive equilibrium with horizontal transmission (cultural persistence with interactions) and finally (iii) Segregated equilibrium with two traits (cultural persistence without interactions).

While the conditions for cultural erosion and persistence within a generation are described above in proposition 3, the equilibrium outcomes crucially depend on the socialization efforts of the parents, which influence both interaction potential and the amount of people switching between groups. We have that:

**Proposition 4** *In the long run, parental efforts will cause social interactions to cease altogether if $\Delta V_k > 0$ because $\lim_{t->T} p^t_{k,switch} = 0$ $(k = n, m)$. In this case, the steady state group size of group $G_n$ is given by*

$$s^*_{seg} = \frac{1}{2} + \frac{\Omega}{2}\frac{(p_n^{t-1}p_{m,switch}^{t-1})^2 c_m - (p_m^{t-1}p_{n,switch}^{t-1})^2 c_n}{\alpha\,(2-\Omega)}$$

*for $\Omega_n = \Omega_m = \Omega$. The state is reached once $\sum_{i=1}^{t-1} p_n^i p_m^i = \frac{p_{n,switch}^{t=1}}{\Delta V_n}$, i.e. when the sum of the interactions of each generation equals the ratio of initial oppenness to the intolerance of the parents. If $\Delta V_k > 0$ for one group but $\Delta V_{-k} < 0$ for the other, then group $-k$ may erode over time, before $p^t_{k,switch} = 0$ is reached. However, if $\Delta V_k < 0$ for both groups, then a dynamic outcome with social interactions exists with*

$$s^*_{dyn} = \frac{1}{2} + \frac{\Omega}{2}\frac{(p_{nm}^{t-1})^2 c_m - (p_{mn}^{t-1})^2 c_n}{\left((p_{nm}^{t-1})^2 + (p_{mn}^{t-1})^2\right)(2-\Omega)}$$

*because $\lim_{t->T} p^t_{k,switch} = 1$ $(k = n, m)$. Finally, if in both groups parental intolerance and tolerance cancel out on average, i.e. for $\Delta V_k = 0$, we have a stable equilibrium defined by $s^*_{t=1}$ of the first generation, since $p_{k,switch}$ remains unchanged across generations.*

**Proof.** See appendix F. ■

Proposition 4 highlights key mechanisms for scenarios of stable cultural erosion or persistence. On the one hand, culture may erode if the conditions in proposition 3 become fulfilled, i.e. once $p_{k,switch}$ and $p_{-k,switch}$ change in a way that pushes society to the left of $\hat{c}_m$ or right of $\tilde{c}_m$. This is especially likely if one group becomes more culturally open over time due to negative group-level parental intolerance ($\Delta V_k < 0$). This represents a scenario where one group fosters openness and cultural change, but with the consequence of their culture eroding. Cultural erosion may also arise if both groups become more open over time, but the conditions make it more favourable that only one of them survives. However, this scenario can also lead to a dynamic equilibrium with both groups persisting and intense social interactions. This is the case when the conditions in society are such that for any values of $p_{k,switch} \in [p^{t=1}_{k,switch}, 1]$ for both groups $G_n$ and $G_m$, the conditions for cultural persistence are always fulfilled. Eventually, both groups are as open as possible, the propensity to switch attaining a value of 1 for both groups, and interactions, as well as switching, fully reflect potential utility gains, with $p^*_k = U^E_{-k} - U^E_k$ and $P_{k,-k} = U^E_{-k} - U^E_k$ and the equilibrium group size given by $s^*_{dyn}$ in Proposition 4. This represents a culturally rich and open society, with many exchanges but the possibility of both groups retaining their identity.

Finally, the segregated equilibrium emerges when openness drops so low across generations that the expected benefits of interactions no longer outweigh the costs or simply when intergenerational openness declines to zero for at least one group ($p_{k,switch} = 0$).Mathematically, this is an unavoidable equilibrium path in the long-run, in absence of shocks, if parental intolerance of at least one group is positive ($\Delta V_k > 0$). If in the last generation before seggregation occurs, the conditions for persistence were ensured, then both traits can persist ($s^* \in (0, 1)$) in the segregated equilibrium. In this outcome, no one switches traits anymore because no one interacts across group lines, giving us a long-term steady state defined by the unique equilibrium $s^*_{seg}$ in Proposition 4, where $p_{n,switch}$ and $p_{m,switch}$ are defined as those from the last generation before they both turn zero. At this point, cultural trait distributions freeze: society becomes static. In the absence of shocks, this segregation persists. The initial openness of a group, given by $p^{t=1}_{k,switch}$, is exogenous and specific to the trait. The higher this is, the longer it takes to reach a state of total segregation and the higher the aggregate intolerance of the parents from group $k$, $\Delta V_k$, the sooner this state is reached. Once the sum of the interactions of each generation equals the ratio of initial openness to the intolerance of the parents, society will become segregated. This highlights how the timing of segregation depends on the initial oppeness of the group that held this trait when it came about as well as on all previous interaction preferences between the groups, showing the importance of the intensity of social interactions in predicting cultural stability or cultural stasis.

Propositions 3 and 4 shed light on why culture persists and why some cultural groups may become increasingly isolated while others erode completely. The more open a group is to change in the next generation, the more vulnerable to erosion it becomes. This creates tension between preserving inherited traditions and being open to cultural change in order to adopt traits that are more beneficial in a given environment.

# 5 Vertical transmission à la Bisin and Verdier

In this section, we compare our results with the more traditional models of cultural evolution and transmission, derived from the seminal work by Bisin and Verdier (2001). In their model, parents choose a level of effort to transmit the *trait* $k \in \{n, m\}$. Furthermore, children are born traitless and receive their trait passively either through vertical or oblique transmission. With a certain probability, they receive the parent's cultural trait (vertical transmission), which is directly linked to the parental efforts put into transmitting the trait. If parental efforts fail, children are instead paired with a random individual in society and adopt their trait (oblique transmission).

In our model, for $\Omega_n = \Omega_m = \Omega$, within a generation, peer interactions deliver a unique and globally stable composition

$$s^*_t = \frac{1}{2} + \frac{\Omega}{2}\frac{(p_{nm}p_{m,switch})^2 c_m - (p_{mn}p_{n,switch})^2 c_n}{\alpha\,(2-\Omega)},$$

where $s^*_t$ is indirectly shaped by parents through the propensity of their children to switch trait in young adulthood, $p_{k,switch}$. In Bisin and Verdier's model of cultural transmission, parents do not choose the child's openness, and so the within-generation equilibrium group size $s^*_t$ is unaffected by parental efforts $d_k$. In Appendix G, we showcase this result by deriving the entire setting using the framework of Bisin and Verdier. It follows that pairing this transmission framework à la Bisin and Verdier with our model of endogeneous social interactions does not lead to the same parental influence on cultural evolution. We have that,

**Remark 1** ***Parental effort*** $d_k$ ***shifts the pre-interaction composition of the next cohort, but the post-interaction composition returns to*** $s^*_t$***. Hence, a transmission à la Bisin and Verdier in our setting does not change long-run shares unless it changes the parameters that govern interaction (e.g., openness).***

Differently said, in a model incorporating explicitly social interactions, wherein children do not inherit their parent's trait with probability 1, but instead parents exert efforts to socialize their offspring to their cultural trait $n$ or $m$ as in Bisin and Verdier (2001) and none on preventing switchings in social interactions, parental efforts only influence the distribution of traits in the subsequent generation before any interaction takes place. It is instead the social interactions among young adults that ultimately determines the long-term trajectory of group population sizes by traits $n$ and $m$. The intuition behind this is the following. If parents focus on transmitting the cultural trait itself, $n$ or $m$, rather than influencing the child's openness to the trait, children no longer inherit their parent's trait with probability 1, leading to different group sizes between parents of generation $t-1$ and children of generation $t$ ($s^{t-1} \neq s^t$), in general. Thus, parental efforts may temporarily shift the cultural composition of the immediate next generation (for the first lifecycle phase, i.e. childhood). However, all other characteristics of the children are exogenous, including $p_{k,switch}$. Since the equilibrium group size after social interactions, $s^*_t$, does not depend on the initial $s_t$ of the children of this generation, there is no factor of influence from the parent that can affect $s^*_t$. Thus, the equilibrium $s^*_t$ in generation $t$ after social interactions is solely determined by exogenous parameters, leading to the same equilibrium value as if no parental efforts had taken place at all. In other words, as children become adults and interact socially, society naturally returns to its unique equilibrium distribution of traits ($s^*_t$) regardless of the temporary effects of parental transmission, as parental effects are not internalised by the child. Focusing only on transmitting the trait itself, rather than shaping the child's intrinsic characteristics, does not influence the decisions that children make later on in life, and thus cannot shape the long-run trajectory of the distribution of traits in society. In contrast, our model of vertical transmission explicitly incorporates decision-making of young adults within the parental choice. This comparison highlights a crucial difference between the two settings. When parents explicitly transmit openness or adaptability itself, as in our framework, they directly shape their children's future social choices and therefore permanently influence long-run cultural dynamics. However, when parents merely transmit a cultural trait without affecting openness, the effect remains temporary. In the absence of external shocks altering societal parameters, society will inevitably revert to its original equilibrium driven by social interactions alone.

Last, consider the scenario in which the transmitted cultural trait of the parent is concretely the open-mindedness to possibly change traits. In the scenario where the trait under analysis is open-mindedness, the fundamental distinction between our model and the framework of Bisin and Verdier (2001) hinges on precisely what is transmitted from parents to children. In our setting, parents not only pass on their specific level of open-mindedness but actively shape their child's intrinsic openness and adaptability to change. This means parental experiences and preferences have lasting effects across generations, embedding openness directly into their children's future social interactions. Conversely, in Bisin and Verdier's model, parents transmit only the cultural trait itself, without influencing the child's intrinsic openness towards being openminded. In very simple words: while Bisin and Verdier allows parental transmission of a level of openness, our model transmits an intrinsic openness to children that lets them decide how open they want to be. Furthermore, the children as modelled in Bisin and Verdier passively inherit either the parent's or an alternative level of open-mindedness, the degree of which cannot be influenced by the parent.

# 6 Conclusion

This paper has developed a unified framework to explain why some cultural traits persist while others erode. By combining within-generation social interactions with intergenerational parental choices, we move beyond models where cultural change results from mechanical exposure or historical inertia. The central mechanism lies in the endogeneity of two key margins, cross-group interaction and parental transmission of cultural openness, which together determine both short-term cultural change and long-run cultural equilibria.

Within a generation, social interaction endogenously produces cultural dynamics even in the absence of shocks. Tightening norms reduces the probability of exit but also deters entry, so that a group expands only when its trait provides higher relative utility support. This dual channel offers a microfoundation for why defensive norm tightening, often intended to preserve a culture, can, under certain conditions, shrink its social base. Across generations, the interaction between parental socialization and children's decision-making produces path-dependent cultural trajectories. Parents who value continuity transmit low openness, gradually reducing interaction and leading to segregation, whereas parents who value adaptability foster convergence through assimilation. Between these extremes lies a stable coexistence equilibrium, where groups interact yet retain distinct identities.

By explicitly embedding parental effort and individual choice within the same model, we bridge the theoretical gap between the Bisin and Verdier (2001) framework of vertical transmission and models of horizontal interaction. In contrast to settings where parents only transmit the trait itself, here parents shape their children's intrinsic openness, an enduring characteristic that influences all subsequent interaction and switching behavior. This feature explains how parental preferences can have lasting consequences for social structure, extending their influence beyond a single generation and determining whether cultural diversity endures, erodes, or becomes entrenched.

The model also carries broader implications. It suggests that cultural fragmentation or integration can arise endogenously from rational behavior, even in the absence of institutional or legal barriers. It highlights a policy tension: interventions that tighten norms or promote cultural preservation may inadvertently accelerate erosion or create depper divisions, while policies that encourage openness and contact can foster adaptability but risk undermining identity. Effective strategies therefore require aligning institutional incentives with parental socialization and opportunities for meaningful cross-group engagement.

Ultimately, culture in this framework is neither purely inherited nor exogenously imposed. It is the outcome of purposeful micro-level decisions, how individuals interact and how parents prepare their children to engage with difference. These choices accumulate across generations to produce macro-level patterns of assimilation, coexistence, or segregation. By endogenizing both social interaction and parental transmission, our model offers a tractable way to link the evolution of cultural traits to incentives, behavior, and policy, and provides a foundation for future empirical work on how openness and norm tightness shape the long-run diversity of societies.

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# Appendices

## A Proof of Lemma 1

**Proof.** Fix $s^\tau$. Individuals are atomistic and maximize current expected utility, so each $i \in G_k$ chooses

$$p_i^* \in \arg\max_{p_i} \left\{(1 - p_i p_{-k} p_{i,\text{switch}})U_i + p_i p_{-k} p_{i,\text{switch}}\, U_{-k}^E - \tfrac{1}{2}p_i^2\right\},$$

treating $s^\tau$ (and thus $U_i$, $U_{-k}^E$, $p_{-k}$) as given. Aggregating yields $p_k^*$ and hence group-level switching probabilities $P_{nm} = p_n^* p_m^* p_{n,switch}$ and $P_{mn} = p_m^* p_n^* p_{m,switch}$. The law of motion for the group share is

$$s^{\tau+1} = s^\tau - P_{nm} + P_{mn}.$$

An interaction equilibrium requires consistency between (i) optimal behavior computed at $s^\tau$ and (ii) the composition that results from that behavior. Thus $s^{\tau+1}$ must equal the $s^\tau$ used in optimization, which is equivalent to $\Delta s^\tau = 0$, i.e. $P_{nm} = P_{mn}$. If $\Delta s^\tau \neq 0$, then $s^{\tau+1} \neq s^\tau$, contradicting consistency. Hence the flow-balance condition holds at equilibrium. ■

## B Equilibrium condition and proof of Proposition 1

**Proof.** When $\Omega_n = \Omega_m$, the equilibrium solution is

$$s^* = \frac{1}{2} + \frac{\Omega}{2}\frac{(p_{nm}p_{m,switch})^2 c_m - (p_{mn}p_{n,switch})^2 c_n}{\alpha\,(2-\Omega)}$$

with $\frac{\partial}{\partial c_n}s^* < 0$; $\frac{\partial}{\partial c_m}s^* > 0$, where $\alpha \equiv (p_{nm}p_{m,switch})^2 + (p_{mn}p_{n,switch})^2$.

Rewrite for simplicity $\Omega_n = k\Omega_m$ where $k \neq 0$ measures the difference between the two utility supports, when $k > 1$, then $\Omega_n < \Omega_m$, and when when $k < 1$, then $\Omega_n > \Omega_m$. With $\Omega_n \neq \Omega_m$, there are two possible roots for $s^*$

$$s_1^* = \frac{2\left(\sqrt{p_{mn}p_{n,switch})^2\Omega^2\,(1-k)\,c_n - (p_{nm}p_{m,switch})^2 k\Omega^2\,(1-k)\,c_m + A} + \frac{(2-k\Omega)}{2}\right)}{\Omega\sqrt{\alpha}\,(1-k)}$$

and

$$s_2^* = \frac{2\left[\sqrt{\alpha}\sqrt{(p_{mn}p_{n,switch})^2\Omega^2\,(1-k)\,c_n + (p_{nm}p_{m,switch})^2 k\Omega^2\,(k-1)\,c_m + A} + \frac{1}{2}\alpha\,(k\Omega-2)\right]}{\Omega\alpha\,(k-1)}$$

depending on whether $k$ is higher or lower than 1, with $A \equiv \left(-2\Omega + k^2\Omega^2 - 2k\Omega + \Omega^2 - k\Omega^2 + 4\right)\alpha$. If $k < 1 \Leftrightarrow \Phi < \Omega$ : $s_1^*$ is the positive acceptable solution. Then, $\frac{\partial}{\partial c_n}s^* > 0$; $\frac{\partial}{\partial c_m}s^* < 0$. If $k > 1 \Leftrightarrow \Phi > \Omega$ : $s_2^*$ is the positive acceptable solution. Then, $\frac{\partial}{\partial c_n}s^* < 0$; $\frac{\partial}{\partial c_m}s^* > 0$. ■

## C Proof of the sign of the derivative $\frac{\partial s^*}{\partial \Omega}$

**Proof.** Let $\Omega_n = \Omega_m = \Omega$. Then if the utility of both groups increases to $\Omega'$, group size $s^*$ becomes larger in equilibrium if only if $c_m > c_n\left(\frac{p_m p_{n,switch}}{p_n p_{m,switch}}\right)^2$.

We take the derivative of $s^*$ with respect to $\Omega$:

$$\begin{aligned}
\frac{\partial}{\partial\Omega}\Big(\frac{1}{2} + \frac{1}{2}\frac{\Omega}{\alpha\,(2-\Omega)}\left((p_n p_{m,switch})^2 c_m - (p_m p_{n,switch})^2 c_n\right)\Big) &> 0 \\
\frac{1}{\alpha\,(\Omega-2)^2}\left(c_m p_n^2 p_{m,switch}^2 - c_n p_m^2 p_{n,switch}^2\right) &> 0 \\
c_m &> c_n\left(\frac{p_m p_{n,switch}}{p_n p_{m,switch}}\right)^2
\end{aligned}$$

■

## D Dynastic extension: grandchildren

Let $\beta \in (0,1)$ be a dynastic weight. Define the child-and grandchild-level trait premia

$$\Delta V_i \equiv V_{ii} - V_{ij}, \qquad \Delta W_i \equiv W_{ii} - W_{ij},$$

and the childrens' switching probability (given parental beliefs)

$$P_i \equiv p_i^* \, p_{-k} \, p_{i,switch}^{child}.$$

Let $\theta_k \in (0,1)$ summarize expected meeting intensity for the child in adulthood, and let $\bar{G}_k \in (0,1)$ be a baseline probability that the grandchild shares the parents trait when $p_{i,switch}^{child} = 0$. Parents' problem writes as

$$\max_{p_{i,switch}^{child}} \; U_i + (1 - P_i) V_{ii} + P_i V_{ij} + \beta \left(\bar{G}_k - \theta_k \, p_{i,switch}^{child}\right) W_{ii} + \beta \left(1 - \bar{G}_k + \theta_k \, p_{i,switch}^{child}\right) W_{ij} - \tfrac{1}{2} \left(p_{i,switch}^{child} - p_{i,switch}\right)^2 .$$

Setting $\beta = 0$ recovers the baseline model without dynastic concerns.
The first order conditions gives:

$$p_{i,switch}^{*child} = p_{i,switch} - p_i^* p_{-k} \, \Delta V_i - \beta \, \theta_k \, \Delta W_i.$$

Group-level aggregation for $k \in \{n, m\}$:

$$p_{k,switch}^{*child} = p_{k,switch} - p_k^* p_{-k} \, \Delta V_k - \beta \, \theta_k \, \Delta W_k,$$

where $\Delta V_k \equiv \frac{1}{S_k} \sum_{i \in G_k} \Delta V_i, \quad \Delta W_k \equiv \frac{1}{S_k} \sum_{i \in G_k} \Delta W_i.$

Caring about grandchildren amplifies cultural persistence: when $\Delta W_i > 0$, parents push children's openness even lower, strengthening trait retention across two generations; if $\Delta W_i < 0$, they tilt toward greater openness to help future descendants fit the best trait.

## E Proof of Proposition 3

**Proof.** For cultural persistence of both traits, we need two conditions to hold simultaneously: (1) $s^* > 0$ (simultaneously ensures that $1 - s^* < 1$) and (2) $1 - s^* > 0$ (simultaneously ensures that $s^* < 1$). We know that for $\Omega_n = \Omega_m = \Omega$, $s^* = \frac{1}{2} + \frac{1}{2} \frac{\Omega}{\alpha(2-\Omega)} \left((p_{nm} p_{m,switch})^2 c_m - (p_{mn} p_{n,switch})^2 c_n\right)$ with $\alpha \equiv (p_{nm} p_{m,switch})^2 + (p_{mn} p_{n,switch})^2$. Then, we have

(1)

$$s^* > 0$$

$$\iff$$

$$\frac{1}{2} + \frac{1}{2} \frac{\Omega}{\left((p_{nm} p_{m,switch})^2 + (p_{mn} p_{n,switch})^2\right)(2 - \Omega)} \left((p_{nm} p_{m,switch})^2 c_m - (p_{mn} p_{n,switch})^2 c_n\right) > 0$$

Let $p_{n,switch} = p_X$ and $p_{m,switch} = p_Y$. Then, we have

$$\frac{1}{2} \frac{2p_X^2 p_m^2 + 2p_Y^2 p_n^2 - \Omega p_X^2 p_m^2 - \Omega p_Y^2 p_n^2 - \Omega c_n p_X^2 p_m^2 + \Omega c_m p_Y^2 p_n^2}{\left(p_X^2 p_m^2 + p_Y^2 p_n^2\right)(2 - \Omega)} > 0$$

$$\iff$$

$$\left((c_m - 1) p_Y^2 p_n^2 - (c_n + 1) p_X^2 p_m^2\right) \Omega + \left(2p_X^2 p_m^2 + 2p_Y^2 p_n^2\right) > 0$$

(1) (i) The above inequality is always true for $c_m p_Y^2 p_n^2 - p_Y^2 p_n^2 - (c_n + 1) p_X^2 p_m^2 > 0 \iff c_m > \frac{p_Y^2 p_n^2 + (c_n+1) p_X^2 p_m^2}{p_Y^2 p_n^2} = 1 + (c_n + 1) \frac{p_X^2 p_m^2}{p_Y^2 p_n^2} \equiv \tilde{c}_m$. Thus, we have that

If $c_m > \tilde{c}_m$, then $s^* > 0$.

(1) (ii) Let $c_m < \tilde{c}_m$. Then, $s^* > 0$ if $\left((c_m - 1) p_Y^2 p_n^2 - (c_n + 1) p_X^2 p_m^2\right) \Omega + \left(2p_X^2 p_m^2 + 2p_Y^2 p_n^2\right) > 0 \iff \Omega < -\frac{\left(2p_X^2 p_m^2 + 2p_Y^2 p_n^2\right)}{(c_m-1) p_Y^2 p_n^2 - (c_n+1) p_X^2 p_m^2} \iff \Omega < -\frac{2p_X^2 p_m^2 + 2p_Y^2 p_n^2}{(c_m-1) p_Y^2 p_n^2 - (c_n+1) p_X^2 p_m^2} \equiv \tilde{\Omega}$.

Since $\Omega \in [0,1]$, this is always true if $\tilde{\Omega} > 1$, namely $-\frac{2p_X^2p_m^2+2p_Y^2p_n^2}{(c_m-1)p_Y^2p_n^2-(c_n+1)p_X^2p_m^2} > 1 \iff -c_mp_Y^2p_n^2 - p_Y^2p_n^2 + (c_n-1)p_X^2p_m^2 < 0 \iff c_m > \frac{-p_Y^2p_n^2+(c_n-1)p_X^2p_m^2}{p_Y^2p_n^2} = -1+(c_n-1)\frac{p_X^2p_m^2}{p_Y^2p_n^2} \equiv \hat{c}_m < \tilde{c}_m$. (We have that $\tilde{c}_m - \hat{c}_m = 1+(c_n+1)\frac{p_X^2p_m^2}{p_Y^2p_n^2} + 1 - (c_n-1)\frac{p_X^2p_m^2}{p_Y^2p_n^2} = 2\frac{p_X^2p_m^2+p_Y^2p_n^2}{p_Y^2p_n^2} > 0$), Therefore, we have that

$$\text{If } \hat{c}_m < c_m < \tilde{c}_m\text{, then } s^* > 0 \text{ and } 1-s^* < 1.$$

$$\text{If } c_m < \hat{c}_m\text{, then } s^* > 0 \text{ and } 1-s^* < 1 \text{ if } \Omega < \tilde{\Omega}.$$

Furthermore, $\Omega < -\frac{2p_X^2p_m^2+2p_Y^2p_n^2}{(c_m-1)p_Y^2p_n^2-(c_n+1)p_X^2p_m^2} \equiv \tilde{\Omega}$ is never true if $\tilde{\Omega} < 0 \implies -\frac{2p_X^2p_m^2+2p_Y^2p_n^2}{(c_m-1)p_Y^2p_n^2-(c_n+1)p_X^2p_m^2} < 0 \iff \frac{2p_X^2p_m^2+2p_Y^2p_n^2}{(c_m-1)p_Y^2p_n^2-(c_n+1)p_X^2p_m^2} > 0 \iff 2p_X^2p_m^2 + 2p_Y^2p_n^2 < 0$ since

$(c_m-1)p_Y^2p_n^2 - (c_n+1)p_X^2p_m^2 < 0 \iff p_Y^2 < -\frac{p_X^2p_m^2}{p_n^2}$

We have that the left-hand side is positive and the right-hand side is negative, which implies that $\tilde{\Omega} \geq 0$ always.

(2)

$$1 - s^* > 0$$

$\iff$

$$1 - \left(\frac{1}{2} + \frac{1}{2}\frac{\Omega}{\left((p_{nm}p_{m,switch})^2 + (p_{mn}p_{n,switch})^2\right)(2-\Omega)}\left((p_{nm}p_{m,switch})^2c_m - (p_{mn}p_{n,switch})^2c_n\right)\right) > 0$$

$\iff$

$$-\frac{1}{2}\frac{-2p_X^2p_m^2 - 2p_Y^2p_n^2 + \Omega p_X^2p_m^2 + \Omega p_Y^2p_n^2 - \Omega c_np_X^2p_m^2 + \Omega c_mp_Y^2p_n^2}{\left(p_X^2p_m^2 + p_Y^2p_n^2\right)(2-\Omega)} > 0$$

$\iff$

$$((1-c_n)p_X^2p_m^2 + (1+c_m)p_Y^2p_n^2)\Omega - \left(2p_X^2p_m^2 + 2p_Y^2p_n^2\right) < 0$$

(2) (i) The above inequality is always true for $(1-c_n)p_X^2p_m^2 + (1+c_m)p_Y^2p_n^2 < 0 \iff c_m < \frac{-(1-c_n)p_X^2p_m^2-p_Y^2p_n^2}{p_Y^2p_n^2} = -1-(1-c_n)\frac{p_X^2p_m^2}{p_Y^2p_n^2} \equiv \tilde{c}'_m$. Note that $\tilde{c}'_m = \hat{c}_m$.

$$\text{If } c_m < \tilde{c}'_m \text{ then } 1-s^* > 0 \text{ and } s^* < 1.$$

(2) (ii) Let $c_m > \tilde{c}'_m$. Then $1-s^* > 0$ if $((1-c_n)p_X^2p_m^2 + (1+c_m)p_Y^2p_n^2)\Omega - \left(2p_X^2p_m^2 + 2p_Y^2p_n^2\right) < 0 \iff \Omega < \frac{2p_X^2p_m^2+2p_Y^2p_n^2}{(1-c_n)p_X^2p_m^2+(1+c_m)p_Y^2p_n^2} \equiv \tilde{\Omega}'$.

This is always true if $\tilde{\Omega}' > 1$, namely if $\frac{2p_X^2p_m^2+2p_Y^2p_n^2}{(1-c_n)p_X^2p_m^2+(1+c_m)p_Y^2p_n^2} > 1 \iff 2p_X^2p_m^2 + 2p_Y^2p_n^2 - (1-c_n)p_X^2p_m^2 - (1+c_m)p_Y^2p_n^2 > 0 \iff c_m < \frac{(1+c_n)p_X^2p_m^2+p_Y^2p_n^2}{p_Y^2p_n^2} = 1+(1+c_n)\frac{p_X^2p_m^2}{p_Y^2p_n^2} \equiv \hat{c}'_m > \tilde{c}'_m > 0$. Note that $\hat{c}'_m = \tilde{c}_m$. Therefore, we have that

$$\text{If } \tilde{c}'_m < c_m < \hat{c}'_m \text{ then } 1-s^* > 0 \text{ and } s^* < 1.$$

$$\text{If } c_m > \hat{c}'_m \text{ then } 1-s^* > 0 \text{ and } s^* < 1 \text{ if } \Omega < \tilde{\Omega}'.$$

Furthermore, $\Omega < \frac{2p_X^2p_m^2+2p_Y^2p_n^2}{(1-c_n)p_X^2p_m^2+(1+c_m)p_Y^2p_n^2} \equiv \tilde{\Omega}'$ is never true if $\tilde{\Omega}' < 0 \iff 2p_X^2p_m^2 + 2p_Y^2p_n^2 < 0 \iff p_Y^2 < -\frac{p_X^2p_m^2}{p_n^2}$. Since the left-hand side is always positive and the right-hand side always negative, this is never true, implying that we have $\tilde{\Omega}' \geq 0$ always.

Initial conditions for $c_m$ tell us that $0 \leq c_m \leq \frac{1}{\Omega}[\Omega + (2-\Omega)s^2] \equiv \bar{c}_m$. We know for sure that $\tilde{c}_m > 0$. However, it is ambiguous whether $\hat{c}_m \lesseqgtr 0$, $\hat{c}_m, \tilde{c}_m \lesseqgtr \frac{1}{\Omega}[\Omega + (2-\Omega)s^2]$. Let us analyze under which conditions the boundary conditions for $c_m$ are satisfied.

When is $\hat{c}_m < 0$?

$$\hat{c}_m < 0$$

$\iff$

$$-1-(1-c_n)\frac{p_X^2 p_m^2}{p_Y^2 p_n^2} < 0$$

$\iff$

$$c_n < 1 + \frac{p_Y^2 p_n^2}{p_X^2 p_m^2}.$$

Then $\hat{c}_m > 0$ when $c_n > 1 + \frac{p_Y^2 p_n^2}{p_X^2 p_m^2}$.

When is $\hat{c}_m > \frac{1}{\Omega}[\Omega + (2-\Omega)s^2]$?

$$\hat{c}_m > \frac{1}{\Omega}[\Omega + (2-\Omega)s^2]$$

$\iff$

$$-1-(1-c_n)\frac{p_X^2 p_m^2}{p_Y^2 p_n^2} - \frac{1}{\Omega}(\Omega + (2-\Omega)s^2) > 0$$

$\iff$

$$c_n > 1 - \frac{s^2 p_Y^2 p_n^2 - 2\Omega(s^2+1)p_Y^2 p_n^2}{\Omega p_X^2 p_m^2}$$

When is $\tilde{c}_m < \frac{1}{\Omega}[\Omega + (2-\Omega)s^2]$?

$$\tilde{c}_m < \frac{1}{\Omega}[\Omega + (2-\Omega)s^2]$$

$\iff$

$$1+(1+c_n)\frac{p_X^2 p_m^2}{p_Y^2 p_n^2} - \frac{1}{\Omega}(\Omega + (2-\Omega)s^2) < 0$$

$\iff$

$$c_n < -1 + \frac{s^2(2-\Omega)p_Y^2 p_n^2}{\Omega p_X^2 p_m^2}$$

Thus, the ICs are satisfied if

$$1 + \frac{p_Y^2 p_n^2}{p_X^2 p_m^2} < c_n < -1 + \frac{s^2(2-\Omega)p_Y^2 p_n^2}{\Omega p_X^2 p_m^2}.$$

This inequality holds if

$$1 + \frac{p_Y^2 p_n^2}{p_X^2 p_m^2} - \left(-1 + \frac{s^2(2-\Omega)p_Y^2 p_n^2}{\Omega p_X^2 p_m^2}\right) < 0$$

$\iff$

$$\Omega < \frac{2s^2 p_Y^2 p_n^2}{2p_X^2 p_m^2 + p_Y^2 p_n^2 + s^2 p_Y^2 p_n^2} \equiv \bar{\Omega}$$

To determine when *both cultures persist*, (1) $s^* > 0$ and (2) $1 - s^* > 0$ must be simultaneously fulfilled. We thus have persistence of both traits if

$$\begin{aligned}
\text{(i) } c_m &< \hat{c}_m \text{ if } \Omega < \tilde{\Omega} \\
\text{(ii) } \hat{c}_m &< c_m < \tilde{c}_m \\
\text{(iii) } c_m &> \tilde{c}_m \text{ if } \Omega < \tilde{\Omega}'
\end{aligned}$$

We can characterise the persistent states in the following way, using the ICs:

Case (1): $\tilde{c}_m < \bar{c}_m$, $\hat{c}_m > 0$. Then, we have persistence of both traits if:

$$\begin{aligned}
\text{(i) } 0 &< c_m < \hat{c}_m \text{ if } \Omega < \tilde{\Omega} \\
\text{(ii) } \hat{c}_m &< c_m < \tilde{c}_m \\
\text{(iii) } \tilde{c}_m &< c_m < \bar{c}_m \text{ if } \Omega < \tilde{\Omega}'
\end{aligned}$$

Case (2): $\tilde{c}_m < \bar{c}_m$, $\hat{c}_m < 0$. Then, we have persistence of both traits if:

$$\begin{aligned}
\text{(i) } 0 &< c_m < \tilde{c}_m \\
\text{(ii) } \tilde{c}_m &< c_m < \bar{c}_m \text{ if } \Omega < \tilde{\Omega}'
\end{aligned}$$

Case (3): $\tilde{c}_m > \bar{c}_m$, $\bar{c}_m > \hat{c}_m > 0$. Then, we have persistence of both traits if:

$$\begin{aligned}
\text{(i) } 0 &< c_m < \hat{c}_m \text{ if } \Omega < \tilde{\Omega} \\
\text{(ii) } \hat{c}_m &< c_m < \bar{c}_m
\end{aligned}$$

Case (4): $\tilde{c}_m > \bar{c}_m$, $\hat{c}_m < 0$. Then, we have persistence of both traits if:

$$0 < c_m < \bar{c}_m$$

Case (5): $\tilde{c}_m > \bar{c}_m$, $\hat{c}_m > \bar{c}_m$. Then, we have persistence of both traits if:

$$0 < c_m < \bar{c}_m \text{ if } \Omega < \tilde{\Omega}$$

In order for all 3 cases to be satisfied we are in case (1): $\tilde{c}_m < \bar{c}_m$, $\hat{c}_m > 0$. This holds if

$$1 + \frac{p_Y^2 p_n^2}{p_X^2 p_m^2} < c_n < -1 + s^2 \frac{(2-\Omega)}{\Omega} \frac{p_Y^2 p_n^2}{p_X^2 p_m^2}.$$

Thus, we have that traits $n$ and $m$ may persist, i.e. $s^* > 0$ and $1 - s^* > 0$, if:

$$\begin{aligned}
\text{(i) } 0 &< c_m < \hat{c}_m \text{ if } \Omega < \min\{\tilde{\Omega}, \bar{\Omega}\} \\
\text{(ii) } \hat{c}_m &< c_m < \tilde{c}_m \text{ if } \Omega < \bar{\Omega} \\
\text{(iii) } \tilde{c}_m &< c_m < \bar{c}_m \text{ if } \Omega < \min\{\tilde{\Omega}', \bar{\Omega}\}
\end{aligned}$$

Naturally, we may have possible trait erosion if these conditions are not fulfilled.

Trait $n$ erodes if $s^* = 0$, i.e. when $\left((c_m - 1)p_Y^2 p_n^2 - (c_n + 1)p_X^2 p_m^2\right)\Omega + \left(2p_X^2 p_m^2 + 2p_Y^2 p_n^2\right) = 0 \iff \Omega = \tilde{\Omega}$. However, we need $\Omega < \bar{\Omega}$ for this situation to exist. Therefor, we have for

$$c_m < \hat{c}_m \text{ and } \Omega = \tilde{\Omega} \text{ and } \Omega < \bar{\Omega}, \text{ then } s^* = 0.$$

Similarly, for trait $m$ we have that if

$$c_m > \tilde{c}_m \text{ and } \Omega = \tilde{\Omega}' \text{ and } \Omega < \bar{\Omega}, \text{ then } 1 - s^* = 0.$$

The proof follows in the same way as for trait $n$.

In our model, we assume that people move between groups (no outside option). Thus, the only condition under which a trait can erode is if all members move to the other group. For instance, trait $n$ erodes and $m$ persists if $s^* = 0$ and $1 - s^* = 1$. As we saw above, the former is the case for

$$c_m < \tilde{c}'_m = \hat{c}_m \text{ and } \Omega = \tilde{\Omega}, \text{ then } s^* = 0.$$

To check our model is consistent, we make sure that this overlaps with the condition of $1 - s^* = 1$.

When is $1 - s^* = 1$?

$$1 - s^* = 1 \iff -\frac{1}{2} \frac{-2p_X^2 p_m^2 - 2p_Y^2 p_n^2 + \Omega p_X^2 p_m^2 + \Omega p_Y^2 p_n^2 - \Omega c_n p_X^2 p_m^2 + \Omega c_m p_Y^2 p_n^2}{\left(p_X^2 p_m^2 + p_Y^2 p_n^2\right)(2 - \Omega)} = 1$$

$\iff$

$$\left(c_m p_Y^2 p_n^2 - p_Y^2 p_n^2 - c_n p_X^2 p_m^2 - p_X^2 p_m^2\right)\Omega + \left(2p_X^2 p_m^2 + 2p_Y^2 p_n^2\right) = 0$$

If $c_m p_Y^2 p_n^2 - p_Y^2 p_n^2 - c_n p_X^2 p_m^2 - p_X^2 p_m^2 > 0$, this is never true, i.e. $c_m > \frac{p_Y^2 p_n^2 + (c_n+1) p_X^2 p_m^2}{p_Y^2 p_n^2} = 1 + (c_n+1)\frac{p_X^2 p_m^2}{p_Y^2 p_n^2} \equiv \tilde{c}_m$ (i.e. condition for no erosion). This is in line with our previous finding that $s^* > 0$ when $c_m > \tilde{c}_m$. Since under these conditions, group $n$ persists, the size of group $m$ will not reach 1. For this condition to hold we thus need $c_m < \tilde{c}_m$ and $\Omega = -\frac{2p_X^2 p_m^2 + 2p_Y^2 p_n^2}{(c_m-1)p_Y^2 p_n^2 - (c_n+1)p_X^2 p_m^2} \equiv \tilde{\Omega}$. This result shows that $s^* = 0$ is a sufficient condition for $1 - s^* = 1$. This in turn shows that, within our framework, only one trait can erode and it must be that the other group encompasses the whole population. The proof for $s^* = 1$ follows from this.

■

**Comparative Statics.** We have that $\tilde{c}_m \equiv 1 + (c_n+1)\frac{p_X^2 p_m^2}{p_Y^2 p_n^2}$. The fact that we have conditions for $s^* > 0$ when $c_m > \tilde{c}_m$ (i.e. $\Omega < \tilde{\Omega}'$) and none for $c_m < \tilde{c}_m$ for group $m$ reveals that higher costs (tighter norms) may lead to cultural erosion if the conditions are not fulfilled. However, if the cost of group $n$ is larger, $c_m$ may take higher values without eroding since the threshold increases ($\frac{\partial \tilde{c}_m}{\partial c_n} > 0$). Furthermore, a group's own switching probability $p_{m,switch}$ leaves less wiggle-room for increased tightness ($\frac{\partial \tilde{c}_m}{\partial p_{m,switch}} < 0$).

On the other side of the interval, we have $\hat{c}_m \equiv -1 + (c_n - 1)\frac{p_X^2 p_m^2}{p_Y^2 p_n^2}$. Rewriting, we have $\hat{c}_n = 1 + \frac{p_Y^2 p_n^2}{p_X^2 p_m^2}(c_m + 1)$. Conditions $c_m < \hat{c}_m$ imply the same conditions as $c_n > \hat{c}_n$. Again, lower tightness $c_n$ implies a higher stability for group $n$, as was the case with groups $m$ above, which can be balanced by a tighter group $m$ ($\frac{\partial \tilde{c}_n}{\partial c_m} > 0$). Similarly, a higher propensity to switch lowers the thresshold ($\frac{\partial \tilde{c}_n}{\partial p_{n,switch}} < 0$).

# F Proof of Proposition 4

**Proof.** If both parental groups have positive intolerance ($\Delta V_k > 0$ for $k \in \{n, m\}$), one or both groups will be completely cultural intolerant after a period $T$, meaning $p_{n,switch}^{child}$ goes to zero of at least one group. In this moment, social interactions grind to a halt. This is because $p_k^*$ will go to zero, i.e. no interactions will take place and thus no one will switch ($P_{k,-k} = 0$). Formally, taking as an example group $n$: $p_{n,switch}^{*child} = 0 \iff p_{n,switch} - p_{nm}^* p_{mn}\Delta V_n = 0 \iff p_{n,switch} = p_{nm}^* p_{mn}\Delta V_n$. In other words, if the $p_{n,switch}$ of the previous generation for group $G_n$ is exactly equal to the product between the probability of exchange and intolerance of the parents, then we will have a seggregated society in the next generation, and hence $\frac{ds^*}{dt} = 0$. Then $s^*$ is given by $s^* = \frac{1}{2} + \frac{\Omega}{2}\frac{(p_{nm}p_{m,switch})^2 c_m - (p_{mn}p_{n,switch})^2 c_n}{\alpha(2-\Omega)}$ for $\Omega_n = \Omega_m = \Omega$. More specifically, replacing *child* by $t+1$ and the parent's characteristics by $t$, we have

$$p_{n,switch}^{t+1} = p_{n,switch}^t - p_{nm}^t p_{mn}^t \Delta V_n = p_{n,switch}^{t-1} - p_{nm}^{t-1} p_{mn}^{t-1}\Delta V_n - p_{nm}^t p_{mn}^t \Delta V_n \iff p_{n,switch}^{t+1} = p_{n,switch}^1 - \Delta V_n \sum_{i=1}^{t-1} p_{nm}^i p_{mn}^i.$$

If $T$ is the generation where the switching probability goes to zero, then we have

$$\lim_{t\to T} p_{n,switch}^t = 0 \iff p_{n,switch}^1 - \Delta V_n \sum_{i=1}^{T-1} p_{nm}^i p_{mn}^i = 0 \iff \sum_{i=1}^{T-1} p_{nm}^i p_{mn}^i = \frac{p_{n,switch}^1}{\Delta V_n}.$$

If instead one group grows more open across generations, while the other grows more closed-off, the more open group may erode in the long-term if group size goes to zero before the openness of the other group goes to zero (which would halt social interactions and thus group size changes). More specifically, this happens for group $G_n$ if $c_m < \hat{c}_m$ and when

$$\Omega \geq \tilde{\Omega} = \frac{2p_{n,switch}^2 p_m^2 + 2p_{m,switch}^2 p_n^2}{(c_m - 1)p_{m,switch}^2 p_n^2 - (c_n + 1)p_{n,switch}^2 p_m^2}$$

In this case the group that remains encompasses the entire population, i.e. $s_k^* = 1$. For group $G_m$ this scenario occurs if $c_m > \tilde{c}_m$ and when

$$\Omega \geq \tilde{\Omega}' = \frac{2p_{n,switch}^2 p_m^2 + 2p_{m,switch}^2 p_n^2}{(1 - c_n)p_{n,switch}^2 p_m^2 + (1 + c_m)p_{m,switch}^2 p_n^2}$$

Lastly, if both groups grow more open across generations, i.e. for $\Delta V_k > 0$ for $k \in \{n, m\}$, then, eventually $p_{k,switch} = 1$ for both groups and $s^* = \frac{1}{2} + \frac{\Omega}{2} \frac{(p_{nm}^{t-1})^2 c_m - (p_{mn}^{t-1})^2 c_n}{\left((p_{nm}^{t-1})^2 + (p_{mn}^{t-1})^2\right)(2-\Omega_k)}$. ■

# G Vertical Transmission à la Bisin and Verdier

In this section, we provide the detailed calculation introducing the model à la Bisin and Verdier in our setting. The vertical transmission probability $d_n$ is defined as a function of the expected utility of the parental trait relative to the other trait, moderated by a function of the effort put into socialising the child. For a parent in group $G_n$, the probability of direct vertical transmission of trait $n$, $d_n$, is given by $d_n = D(e_n, \left(U_{k=m}^E\right)^*, \left(U_{k=n}^E\right)^*)$. Similarly, for a parent in group $G_m$, the probability of direct vertical transmission of trait $m$, is given by $d_m = D(e_m, \left(U_{k=n}^E\right)^*, \left(U_{k=m}^E\right)^*)$.

If vertical transmission does not occur, which occurs with probability $1 - d_n$, the child acquires the trait horizontally from a member of the parent's generation, which can be someone from group $G_n$ or $G_m$. This horizontal transmission probability is weighted by the expected utility of each trait in society. In this setting, the child is initially traitless and does not make a decision about which trait to adopt. The probability of a child adopting trait $n$ horizontally when they have parent $n$, $p_{nn}$, is:

$$p_{nn} = (1 - d_n) \cdot \left(U_{k=n}^E\right)^*.$$

The probability of a child adopting trait $m$ horizontally when they have parent $n$, $p_{nm}$, is:

$$p_{nm} = (1 - d_n) \cdot \left(U_{k=m}^E\right)^*.$$

It is worth noting that at this point there is no cost of switching involved, because the child has no trait yet so it is not biased and sees no cost.

Similarly, for trait $m$, we have:

$$p_{mm} = (1 - d_m) \cdot \left(U_{k=m}^E\right)^* \text{ and } p_{mn} = (1 - d_m) \cdot \left(U_{k=n}^E\right)^*$$

The final probability $T_{nn}$ that a child acquires a particular trait (e.g., $n$) when it has a parent of the same trait is a combination of the direct vertical probability and the horizontal probability:

$$T_{nn} = d_n + p_{nn}.$$

Probability $T_{nm}$ that a child acquires trait $m$ when it has a parent of trait $n$:

$$T_{nm} = p_{nm}$$

Probability $T_{mm}$ that a child acquires trait $m$ when it has a parent of the same trait:

$$T_{mm} = d_m + p_{mm}.$$

Probability $T_{mn}$ that a child acquires trait $n$ when it has a parent of trait $m$:

$$T_{mn} = p_{mn}$$

Parents get a certain utility from their child having the same trait as them. However, they see the benefit of their child having a certain trait through the lens of their own preferences (imperfect empathy). We assume that the draw $\mu$ does not influence the parental decisions, e.g. each parent of a certain group sees this in the same way. In addition, since parents have a bias towards their own trait, there is a certain incentive to put effort into socialising their child to their own trait. However, putting in effort is associated with a personal cost, $C(e_n)$.

The parental decision problem boils down to the following. Parents choose how much effort to put into socialising their children, $e_n$, given their personal utility, the utility they get from their kid's acquring their own trait or the other, as well as the cost. Bisin and Verdier (2001) show that under certain assumptions, chosing $e_n$ boils down to choosing $d_n$. We assume a quadric form of the cost:

$$C(d_n) = \frac{1}{2} d_n^2.$$

Thus, the parental decision problem of parent $n$ is given by:

$$\max_{d_n} U_i + T_{nn}V_{nn} + T_{nm}V_{nm} - C(d_n)$$

that becomes:

$$\max_{d_n} U_i + \left[d_n + (1-d_n)\cdot\left(U^E_{k=n}\right)^*\right]V_{nn} + (1-d_n)\cdot\left(U^E_{k=m}\right)^* V_{nm} - \frac{1}{2}d_n^2$$

The maximization problem yields:

$$d_n^* = (1 - \left(U^E_{k=n}\right)^*)V_{nn} - \left(U^E_{k=m}\right)^* V_{nm}$$

Using the expressions for expected utilities, we obtain:

$$\begin{aligned} d_n^* &= \left[1 - \left(\frac{\Omega_n}{2} + \left(1 - \frac{\Omega_n}{2}\right)(s^*)^2\right)\right]V_{nn} - \left(\frac{\Omega_m}{2} + \left(1 - \frac{\Omega_m}{2}\right)(1-s^*)^2\right)V_{nm} \\ &= \left(\frac{1}{2}(s^*)^2\Omega_n - \frac{1}{2}\Omega_n - (s^*)^2 + 1\right)V_{nn} - \left(2s^* - s^*\Omega_m + \frac{1}{2}(s^*)^2\Omega_m - (s^*)^2 - 1\right)V_{nm} \end{aligned}$$

We have that $\frac{\partial d_n}{\partial \Omega_n} < 0$, which means the higher the support on the intrinsic utility of the group, the less effort the parent puts into socialising their child. This can be understood as follows: the higher $\Omega_n$, the more likely the child is to acquire the trait from society, as opposed to the other, since $p_{nn} = (1-d_n)\cdot\left(U^E_{k=n}\right)^*$ and so the parent can save some socialisation costs as they have a piece of mind that it is quite likely the kid will acquire their trait anyway. A similar argument goes for group size, with $\frac{\partial d_n}{\partial s} < 0$, which reflects the notion of cultural substituability as was derived in Bisin and Verdier. Finally, $\frac{\partial d_n}{\partial \Omega_m} > 0$ tells us that if the other group has an objectively higher support on their utility, the parent will put in more effort to socialise their child to their own trait, since they fear that if the child does not acquire the trait vertically, there is a higher chance it might not acquire it at all.

**Group size evolution.** We have a new population of children with a (potentially) different distribution of traits. The new size of group $G_n$ , denoted by $s_{t+1}$, is given by

$$\begin{aligned} s_{\tau+1} &= T_{nn}s^* + T_{mn}(1-s^*) = (d_n + p_{nn})s^* + p_{mn}(1-s^*) \\ &= (d_n + (1-d_n)\cdot\left(U^E_{k=n}\right)^*)s^* + (1-d_m)\cdot\left(U^E_{k=m}\right)^*(1-s^*) \end{aligned}$$

Substituting all our expressions, we get:

$$\begin{aligned} s_{\tau+1} &= \left[(1 - \left(U^E_{k=n}\right)^*)V_{nn} - \left(U^E_{k=m}\right)^* V_{nm} + (1 - (1 - \left(U^E_{k=n}\right)^*)V_{nn} + \left(U^E_{k=m}\right)^* V_{nm})\cdot\left(U^E_{k=n}\right)^*\right]s^* \\ &\quad + (1 - (1 - \left(U^E_{k=m}\right)^*)V_{mm} + \left(U^E_{k=n}\right)^* V_{mn})(1-s^*) \end{aligned}$$

and taking a linear approximation of the above expression for $s_{t+1}$ we find

$$s_{t+1} = \frac{1}{4}(2(\Omega_m - 2)V_{nm} + (\Omega_m^2 - 4(\Omega_m - 1))V_{nn} - 4(2-\Phi)V_{mm} + 4\Omega_m V_{mn} - 2\Omega_m)s^*$$

This means that $s_{t+1} > s^*$ iff $\left(\frac{1}{4}(2(\Omega_m - 2)V_{nm} + (\Omega_m^2 - 4(\Omega_m - 1))V_{nn} - 4(2-\Omega_m)V_{mm} - 4\Omega_m V_{mn} - 2\Omega_m)\right) > 1$, then parents effort to rear their offspring with the trait $n$ increase the number of individuals holding that trait $n$. However, if $\left(\frac{1}{2}\Omega_m\left((\Omega_m - 2)V_{mm} + \frac{1}{2}\Omega_m V_{mn}\right) + V_{nn}\left(\frac{1}{2}\Omega_m - 1\right)^2 - V_{mn}\left(\frac{1}{2}\Omega_m - 1\right)\right) < 1$, then parents effort decrease the size of the belonging group. It can be proved numerically that both conditions define non empty sets.

The relationship between parental efforts and population size hinges on the relative magnitudes of $V_{nn}$ , $V_{nm}$, $V_{mn}$ and $V_{mm}$. While increasing $V_{nn}$ and $V_{nm}$ with $\Omega_m$ can lead to an increase in the number of individuals holding a trait, too large a value for $V_{mm}$ and $V_{mn}$ could reverse this effect and reduce the overall number of individuals with the trait in the next generation. Note that $\frac{\partial s_{t+1}}{\partial V_{nn}} > 0$ and $\frac{\partial s_{t+1}}{\partial V_{nm}} < 0$, whilst $\frac{\partial s_{t+1}}{\partial V_{mm}} < 0$ and $\frac{\partial s_{t+1}}{\partial V_{mn}} > 0$. In other words, the higher is the utility gained for a parent $n$ from their kid having the same trait, $V_{nn}$, and the lower for their kid having the other trait, $V_{nm}$, the higher will be $s_{t+1}$. Similarly, the higher its the utility of a parent $m$ from their kid acquiring the same trait, $V_{mm}$, and the lower for their kid having the other trait, $V_{mn}$, the lower will be $s_{t+1}$.

Let us define $\Delta V_n = \frac{1}{4}\left((\Omega_m^2 - 4(\Omega_m - 1))V_{nn} - 2(\Omega_m - 2)V_{nm}\right)$ as the intolerance of parent $n$ toward their child having a different trait and $\Delta V_m = -(2 - \Omega_m)V_{mm} + \Omega_m V_{mn}$ as the intolerance of parent $m$ toward their child having a different trait. Then $\frac{\partial s_{t+1}}{\partial \Delta V_n} > 0$ and $\frac{\partial s_{t+1}}{\partial \Delta V_m} < 0$, i.e. the higher the intolerance of group $n$ parents, the larger the group size $s$ in the next generation and vice-versa for group $m$. Then group $G_n$ grows in the next generation as long as

$$\Delta V_n - \Delta V_m > \frac{\Omega_m}{2}$$

i.e. as long as the difference in intolerance of group $n$ parents and group $m$ parents is at least the average utility of the groups.

**Result:** *Whether or not the parent's group grows in the next generation depends on both the size of the different utilities gained by the parents when their child acquires a certain trait, $V$, for both types of parents, as well as on the support on the utility of the groups, $\Omega_n = \Omega_m$, and thus the coefficients on the different utilities. Group $G_n$ will grow as long as the difference in intolerances for both parents is larger than the average utility ($\Delta V_n - \Delta V_m > \frac{\Omega_m}{2}$).*

Indeed, this result shows a potential change in group sizes for the next generation of children. However, as we can see, *only* group size is affected and all other parameters are not. Thus, the equilibrium group size as given by equation (8) will remain unchanged by parental influence.

It is worth mentioning that the impact of this result will not change if we extend this model to individual level dynamics. This is because by putting effort into transferring only the trait, parents will only influence the group size in the short term: if none of the parameters of the system change due to parental transmission, then the equilibrium state in the next generation will look the same.