

# Coarse graining photo-isomerization reactions: Thermodynamic consistency and implications for molecular ratchets

Cite as: *J. Chem. Phys.* **163**, 134122 (2025); doi: [10.1063/5.0290281](https://doi.org/10.1063/5.0290281)

Submitted: 11 July 2025 • Accepted: 16 September 2025 •

Published Online: 7 October 2025



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## ABSTRACT

We formulate thermodynamically consistent coarse-graining procedures for molecular systems undergoing thermally and photo-induced transitions: starting from elementary vibronic transitions, we derive effective photo-isomerization reactions interconverting ground-state species. Crucially, the local detailed balance condition, which constrains reaction kinetics to thermodynamics, remains satisfied throughout the coarse-graining procedures. It applies to the effective photo-isomerization reactions just as it does to the elementary vibronic transitions. We then demonstrate that autonomous photo-driven molecular ratchets operate via the same fundamental mechanism as chemically driven ones. Because the local detailed balance remains satisfied, autonomous photo-driven molecular ratchets, similar to chemically driven ones, operate exclusively through an information ratchet mechanism. This reveals that their design and optimization should prioritize molecular properties governing the information ratchet mechanism, rather than those influencing energetic bias.

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## I. INTRODUCTION

Molecular systems are dissipative systems whose dynamics is ultimately governed by elementary transitions, interconverting molecular states and species, which must obey thermodynamic constraints. The fundamental constraint that all elementary transitions must satisfy is the local detailed balance condition:<sup>1–3</sup> the fluxes of each pair of forward and backward transitions do not take arbitrary values but are constrained by the corresponding thermodynamic force, i.e., the free energy variation along the transition. This condition guarantees that molecular systems relax to equilibrium unless they continuously harness free energy from their surroundings by exchanging energy and/or chemical species.<sup>4</sup> It also ensures the contribution of all elementary transitions to the total dissipation can be properly accounted for.

Molecular systems that are maintained out of equilibrium through the exchanges of chemical species with their surroundings

are said to be chemically driven. In this context, the local detailed balance enabled the development of general nonequilibrium thermodynamic theories.<sup>5–11</sup> These theories can quantify the energetic cost of, for instance, maintaining coherent oscillations<sup>12–15</sup> and patterns,<sup>16–19</sup> as well as powering chemical growth.<sup>20,21</sup> They can also identify the accessible chemical space,<sup>22,23</sup> the information encoded in polymers,<sup>24,25</sup> and speed limits.<sup>26,27</sup> Furthermore, they have been used to characterize the energetics of specific systems, such as the efficiency of molecular motors<sup>28–30</sup> and the energy stored by driven endergonic processes.<sup>31–35</sup> However, their direct application becomes quickly impractical when molecular systems reach high levels of complexity arising from a large number of species interconverted by a vast network of transitions. Indeed, they require knowing the concentrations of all species and the rate constants of every elementary transition, which is very seldom achieved. To address this issue, different strategies have been developed that either infer, from incomplete experimental data, the structure of the

underlying network of transitions<sup>36,37</sup> and the associated thermodynamic quantities<sup>38–40</sup> or provide thermodynamically consistent coarse-graining procedures.<sup>41–44</sup> In particular, these coarse-graining procedures yield simplified dynamic descriptions of complex molecular systems while retaining the exact dissipation by identifying effective transitions between slow-evolving species. Notably, thermodynamic consistency is retained even if the fluxes of the effective transitions do not, in general, satisfy the local detailed balance condition, hence hiding this fundamental constraint. These coarse graining procedures constitute a powerful tool to characterize the energetics of systems regardless of their size. Indeed, they have been used to study the efficiency of processes at the cellular scale,<sup>45,46</sup> but some recent attempts also explored applications at the ecosystem scale.<sup>47</sup>

Molecular systems that are instead maintained out of equilibrium through the exchanges of energy with light are said to be photo-driven. In this case, the photo-induced elementary transitions are intramolecular vibronic transitions, i.e., absorption, spontaneous, and stimulated emission of photons interconverting vibrational and electronic states of a molecule.<sup>48</sup> The local detailed balance condition still holds and corresponds to Einstein's relations,<sup>49</sup> which laid the foundation for a thermodynamic understanding of light–matter interactions. Nevertheless, thermodynamic theories for photo-driven molecular systems are not as well established as those for chemically driven ones. The current formulation considers only two-state molecules that interact with monochromatic light.<sup>50</sup> Molecular species are instead multi-state systems whose elementary photo-induced dynamics is driven by polychromatic light (with a continuous spectrum). Furthermore, many fast-evolving excited-state species, which are challenging to characterize experimentally, are involved in photo-induced dynamics, thus underscoring the need for thermodynamically consistent coarse-graining theories.

In this paper, we develop such thermodynamic consistent theories for photo-driven molecular systems. We begin by establishing a general framework for elementary transitions interconverting molecular species with an arbitrary number of vibrational and electronic states interacting with polychromatic light (Sec. II). We then develop two sequential coarse-graining procedures. The first one coarse grains the vibrational states (Sec. III), yielding a description in terms of electronic states only. The second one coarse grains electronic excited states (Sec. IV), yielding a description in terms of ground-state species only. This enables us to define effective photo-isomerization reactions (Sec. V) for a paradigmatic photochemical process, namely, the diabatic process.<sup>51</sup> Crucially, both these coarse-graining procedures are thermodynamically consistent, preserving the exact dissipation, and furthermore still satisfy the local detailed balance condition. On the one hand, thermodynamic consistency (Subsections III E and IV E) is achieved by defining the (effective) photo-isomerization reactions in terms of cycles between the ground-state species, similarly to what has been done for the coarse-graining of chemically driven systems.<sup>41,42</sup> On the other hand, the local detailed balance (Subsections III D and IV D) is retained by using the cycles identified by a modified version of the matrix-tree theorem<sup>52,53</sup> already used in Ref. 22. This is a major difference from the coarse-graining of chemically driven systems as it implies that the fluxes of the (effective) photo-isomerization reactions, not just those of the underlying elementary transitions, are also constrained by the corresponding free energy variation. The result is that both

the dynamics and the energetics of photo-driven systems can be characterized in terms of experimentally accessible quantities: the former through absorption spectra and quantum yields, while the latter through the chemical potentials of the photons and of the ground-state species.

The importance of having such thermodynamic theories cannot be overstated, given the ubiquity of photo-driven systems in inorganic<sup>54</sup> and organic matter<sup>55</sup> as well as the increasing attention they are receiving in current research. In biology, prominent examples of these systems include photosynthesis,<sup>55</sup> transducing photons into chemical energy, and animal vision,<sup>56</sup> where photons with certain frequencies initiate a signaling cascade. In systems chemistry, prominent examples include several classes of molecular motors,<sup>57,58</sup> pumps,<sup>59,60</sup> and active materials,<sup>61,62</sup> transducing photons into nanoscale directional motion or even macroscale work, as well as enhancement of catalytic activity under nonequilibrium conditions,<sup>63</sup> and active transport through a liquid membrane.<sup>64,65</sup>

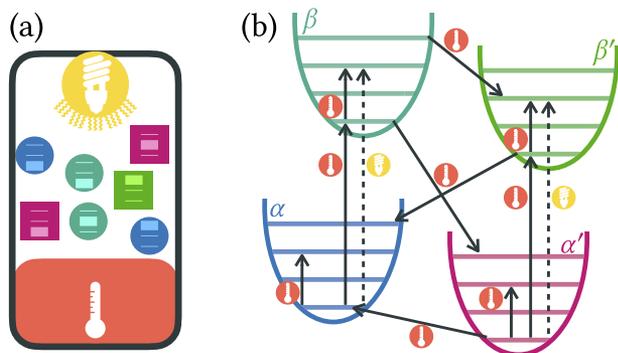
All these nonequilibrium chemical systems epitomize a wide class of systems known as molecular ratchets.<sup>66–68</sup> While no universally accepted definition exists (e.g., the ones in Refs. 66 and 67 overlap, but they are not equivalent), we adopt here the following definition: molecular ratchets are systems that couple free-energy-harnessing transitions to endergonic ones in a way that drives the latter against their spontaneous direction by exploiting an energy source (e.g., chemical potential gradients or light) with which only the former are in direct contact.<sup>66</sup> It is currently believed that photo-driven molecular ratchets operate via a different mechanism with respect to chemically driven ones.<sup>66–71</sup> In this paper, we show that this is not the case (Sec. VI) by applying our theories and in particular the local detailed balance condition of the (effective) photo-isomerization reactions, to photo-driven molecular ratchets. Namely, chemically and photo-driven ratchets operate via a pure information ratchet mechanism when powered by a constant energy source (named autonomous ratchets), thus unifying their description and unraveling a paradigmatic change in the design and optimization principles of photo-driven ratchets.

We conclude by briefly contextualizing our work within the historical development of the local detailed balance condition in Sec. VII and by summarizing and further discussing our results in Sec. VIII.

## II. ELEMENTARY TRANSITIONS

We consider a homogeneous ideal dilute solution with volume  $V$  immersed in incoherent light (photons), as illustrated in Fig. 1. The solution is maintained at constant temperature  $T$  by the solvent playing the role of a thermostat, while photons are generated by a light source. Different molecular species exist in different vibrational and electronic states, while photons can have different frequencies. Vibronic transitions within the same species and from one species to another can be thermally induced; namely, they can be promoted by the interactions with the thermostat. Vibronic transitions within the same species can be photo-induced too; namely, they can be promoted by the absorption/emission of photons.

*Notation.* In the following, Greek letters  $\alpha$  or  $\beta$  specify both the molecular species and its electronic state. In the rare case where we need to explicitly distinguish the electronic states of two

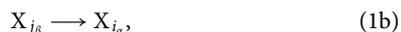


**FIG. 1.** (a) Pictorial illustration of an ideal dilute solution maintained at constant temperature by the solvent (represented by the thermometer) and immersed in incoherent light generated by a light source (represented by the bulb). Two different molecular species (represented by the disks and squares) can exist in different electronic (represented by the different colors) and vibrational (represented by the different position of the thickest line) states. (b) Some transitions (arrows) between different vibrational and electronic states of two species. The first (resp. second) species exists in two electronic states  $\alpha$  and  $\beta$  (resp.  $\alpha'$  and  $\beta'$ ). Transitions interconverting states of the same species (vertical arrows) can be thermally induced (arrows close to the thermometers) and/or photo-induced (dashed arrows close to the bulb). Transitions interconverting states of different species (oblique arrows) can only be thermally induced (arrows close to the thermometers). Each transition has always a backward counterpart, even if it is not represented.

different species,  $\alpha'$  or  $\beta'$  is used for the second one. On the other hand,  $i_\alpha$  or  $j_\alpha$  (resp.  $i_\beta$  or  $j_\beta$ ) specify the vibrational state corresponding to the species and electronic state  $\alpha$  (resp.  $\beta$ ). The chemical symbol and concentration of a species in a specific vibrational state are given by  $X_{i_\alpha}$  and  $[X_{i_\alpha}]$ , respectively. The chemical symbol and concentration of a species in a specific electronic state, regardless of the vibrational state, are given by  $X_\alpha$  and  $[X_\alpha] = \sum_{i_\alpha} [X_{i_\alpha}]$ , respectively. Photons with frequencies  $\nu$  are represented by the symbol  $p_\nu$ . The concentration of photons with frequencies in the interval  $[\nu, \nu + d\nu]$  is given by  $n(\nu)d\nu$ , while we will refer to  $n(\nu)$  as the density of photons with frequency  $\nu$ .

### A. Kinetics

Each thermally induced transition from  $i_\alpha$  to  $j_\beta$  has always its backward counterpart from  $j_\beta$  to  $i_\alpha$ . Together, these two transitions can be represented by the chemical equations,



whose fluxes read

$$R^q(i_\alpha \rightarrow j_\beta) = k_{i_\alpha \rightarrow j_\beta}^q [X_{i_\alpha}], \quad (2a)$$

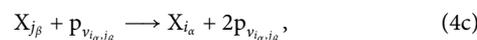
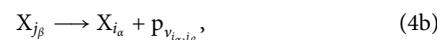
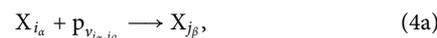
$$R^q(j_\beta \rightarrow i_\alpha) = k_{j_\beta \rightarrow i_\alpha}^q [X_{j_\beta}], \quad (2b)$$

respectively, with  $k_{i_\alpha \rightarrow j_\beta}^q$  and  $k_{j_\beta \rightarrow i_\alpha}^q$  being the kinetic constants. The corresponding net current is given by

$$J^q(i_\alpha \rightarrow j_\beta) = R^q(i_\alpha \rightarrow j_\beta) - R^q(j_\beta \rightarrow i_\alpha). \quad (3)$$

*Remark.* Thermally induced transitions can interconvert any pair of vibrational states. On the one hand, they are usually referred to as quenching transitions when they interconvert vibrational states of the same species, e.g.,  $i_\alpha$  and  $j_\beta$ . On the other hand, they define (unimolecular) isomerization transitions when they interconvert vibrational states of different species, e.g.,  $i_\alpha$  and  $j_{\alpha'}$ .

Each photo-induced transition from  $i_\alpha$  to  $j_\beta$  of the same species promoted by the absorption of a photon of frequency  $\nu_{i_\alpha, j_\beta}$  has always its backward counterpart in the spontaneous emission and stimulated emission from  $j_\beta$  to  $i_\alpha$ .<sup>72</sup> Together, these three transitions can be represented by the following chemical equations:



respectively. Here, we assumed, without loss of generality, that the energy of state  $j_\beta$  is larger than the energy of state  $i_\alpha$ , i.e.,  $u_{j_\beta}^\circ > u_{i_\alpha}^\circ$ . Furthermore, the frequency  $\nu_{i_\alpha, j_\beta}$  corresponds exactly to the energy difference between the two states, i.e.,  $h\nu_{i_\alpha, j_\beta} = |u_{j_\beta}^\circ - u_{i_\alpha}^\circ|$  (with  $h$  being the Planck constant). The fluxes of absorption and emission read

$$R^a(i_\alpha \rightarrow j_\beta) = k_{i_\alpha \rightarrow j_\beta}^a n(\nu_{i_\alpha, j_\beta}) [X_{i_\alpha}], \quad (5a)$$

$$R^e(j_\beta \rightarrow i_\alpha) = (k_{j_\beta \rightarrow i_\alpha}^e + k_{j_\beta \rightarrow i_\alpha}^{se} n(\nu_{i_\alpha, j_\beta})) [X_{j_\beta}], \quad (5b)$$

where  $k_{i_\alpha \rightarrow j_\beta}^a$ ,  $k_{j_\beta \rightarrow i_\alpha}^e$ , and  $k_{j_\beta \rightarrow i_\alpha}^{se}$  are the Einstein's coefficients, namely, the kinetic constants for absorption, spontaneous emission, and stimulated emission, respectively.<sup>49</sup> The corresponding net current is given by

$$J^p(i_\alpha \rightarrow j_\beta) = R^a(i_\alpha \rightarrow j_\beta) - R^e(j_\beta \rightarrow i_\alpha). \quad (6)$$

If  $u_{j_\beta}^\circ < u_{i_\alpha}^\circ$ , then  $J^p(i_\alpha \rightarrow j_\beta) = R^e(i_\alpha \rightarrow j_\beta) - R^a(j_\beta \rightarrow i_\alpha)$ .

*Remark.* Photo-induced transitions cannot interconvert states of different species, e.g.,  $i_\alpha$  and  $j_{\alpha'}$ , as a result of the Franck–Condon principle<sup>48</sup> implying that  $R^a(i_\alpha \rightarrow j_{\alpha'}) = R^e(j_{\alpha'} \rightarrow i_\alpha) = 0$ .

### B. Rate equations

The concentration of a species in a specific state, i.e.,  $[X_{j_\beta}]$ , is affected by both thermally and photo-induced transitions and thus evolves in time according to the following rate equation:

$$d_t [X_{j_\beta}] = \sum_\alpha \sum_{i_\alpha (i_\alpha \neq j_\beta)} (J^q(i_\alpha \rightarrow j_\beta) + J^p(i_\alpha \rightarrow j_\beta)). \quad (7)$$

The density of photons is instead not affected by the thermally induced transitions. It is affected by the photo-induced transitions and by the light source, with the latter represented by the net density current  $I(\nu)$  quantifying the net density of photon of frequency

$\nu$  entering [ $I(\nu) > 0$ ] or exiting [ $I(\nu) < 0$ ] the solution. Hence, the density of photons  $n(\nu)$  follows the rate equation,<sup>73</sup>

$$d_t n(\nu) = - \sum_{\alpha, \beta} \sum_{j_\beta > i_\alpha} \delta(\nu - \nu_{i_\alpha, j_\beta}) J^P(i_\alpha \rightarrow j_\beta) + I(\nu), \quad (8)$$

where  $\delta(\bullet)$  is the Dirac delta function, and we ordered the states, without loss of generality, in such a way that  $j_\beta > i_\alpha$  if  $u_{j_\beta}^\circ > u_{i_\alpha}^\circ$ .

*Remark.* In general, the net density current  $I(\nu)$  is non-homogeneous in the solution. In typical photochemical experiments, the solution is contained in a cuvette and irradiated from one side with a fix net density current.<sup>74</sup> According to the Beer–Lambert law, the magnitude of net density current then decreases along the direction of propagation because photons are absorbed. Nevertheless, the generalization of our framework to a non-homogeneous net density current  $I(\nu)$  is straightforward: the solution can be split in small volumes where  $I(\nu)$  is approximately homogeneous, as already shown in Refs. 29 and 60 for the case of monochromatic light.

### C. Local detailed balance

The fundamental constraint that all elementary transitions must satisfy is the local detailed balance condition,<sup>2–4</sup> establishing a correspondence between dynamics and thermodynamics.

For each thermally induced transition, the local detailed balance condition reads<sup>4,9</sup>

$$k_B T \ln \frac{R^q(i_\alpha \rightarrow j_\beta)}{R^q(j_\beta \rightarrow i_\alpha)} = -(\mu_{j_\beta} - \mu_{i_\alpha}), \quad (9)$$

where  $\mu_{i_\alpha}$  is the chemical potential<sup>75</sup> of the species in state  $i_\alpha$ ,

$$\mu_{i_\alpha} = \mu_{i_\alpha}^\circ + k_B T \ln [X_{i_\alpha}], \quad (10)$$

with  $\mu_{i_\alpha}^\circ = u_{i_\alpha}^\circ - T s_{i_\alpha}^\circ$  being the corresponding standard chemical potential and  $s_{i_\alpha}^\circ$  being the corresponding standard entropy. The local detailed balance condition (9) is the direct implication of the fact that thermally induced transitions must admit an equilibrium state when no chemical species are exchanged with the surroundings.<sup>4,76,77</sup>

For each photo-induced transition, the local detailed balance can be written as<sup>50</sup>

$$k_B T \ln \frac{R^a(i_\alpha \rightarrow j_\beta)}{R^e(j_\beta \rightarrow i_\alpha)} = -(\mu_{j_\beta} - \mu_{i_\alpha} - \mu_P(\nu_{i_\alpha, j_\beta})), \quad (11)$$

where we assumed, without loss of generality, that  $u_{j_\beta}^\circ > u_{i_\alpha}^\circ$  and the chemical potential of the photons with frequency  $\nu$  reads

$$\mu_P(\nu) = h\nu - k_B T \ln \frac{\varrho(\nu) + n(\nu)}{n(\nu)}, \quad (12)$$

with  $\varrho(\nu)$  being the density of photon states associated with the radiation with frequency  $\nu$  [reading  $\varrho(\nu) = 8\pi\nu^2/c^3$  in vacuum, with  $c$  the speed of light].<sup>78</sup> The local detailed balance condition (11) is a direct implication of Einstein's relations,<sup>49,79</sup>

$$\frac{k_{i_\alpha \rightarrow j_\beta}^a}{k_{j_\beta \rightarrow i_\alpha}^{se}} = \exp \frac{s_{j_\beta}^\circ - s_{i_\alpha}^\circ}{k_B}, \quad (13a)$$

$$\frac{k_{j_\beta \rightarrow i_\alpha}^e}{k_{i_\alpha \rightarrow j_\beta}^{se}} = \varrho(\nu_{i_\alpha, j_\beta}), \quad (13b)$$

as it can be proven by direct substitution of Eqs. (13), (12), and (10) into Eq. (11).

*Remark.* Any arbitrary  $n(\nu)$  can be expressed in terms of a black body distribution<sup>80</sup> with a frequency-dependent temperature  $T^{\text{bb}}(\nu)$  according to

$$n(\nu) = \frac{\varrho(\nu)}{e^{\frac{h\nu}{k_B T^{\text{bb}}(\nu)}} - 1}. \quad (14)$$

The chemical potential (12) can thus be written as

$$\mu_P(\nu) = h\nu \left( 1 - \frac{T}{T^{\text{bb}}(\nu)} \right), \quad (15)$$

and therefore, photons with frequency  $\nu$  have a non-vanishing chemical potential if the temperature of the corresponding black body  $T^{\text{bb}}(\nu)$  is different from the temperature of the solution  $T$  used as reference.<sup>81–83</sup> This means that, from a thermodynamic standpoint, each set of photons with frequency  $\nu$  plays the role of a thermostat with temperature  $T^{\text{bb}}(\nu)$ . The mismatch between the temperature  $T^{\text{bb}}(\nu)$  and the temperature  $T$  of the main thermostat, i.e., the solvent, defines the free energy carried by photons of frequency  $\nu$  that can be used to maintain transitions out of equilibrium.<sup>50</sup> If  $\mu_P(\nu) = 0$ , then  $T^{\text{bb}}(\nu) = T$ , which thermodynamically means that the set of photons with frequency  $\nu$  is in equilibrium with the solution and does not carry any free energy that can be used to maintain transitions out of equilibrium.

### D. Second law

The total free energy  $F$  of the solution is given by the sum of two contributions,

$$F = F_{\text{ch}} + F_{\text{ph}}, \quad (16)$$

where

$$F_{\text{ch}} = \sum_{\alpha} \sum_{i_\alpha} (\mu_{i_\alpha} - k_B T) [X_{i_\alpha}] \quad (17)$$

is the free energy contribution due to the molecular species,<sup>9,84</sup> while

$$F_{\text{ph}} = \int d\nu \left[ \underbrace{\mu_P(\nu) n(\nu) - k_B T \varrho(\nu) \ln \frac{\varrho(\nu) + n(\nu)}{\varrho(\nu)}}_{\equiv f_{\text{ph}}(\nu)} \right] \quad (18)$$

is the free energy contribution due to the photons,<sup>50,80</sup> with  $f_{\text{ph}}(\nu)$  being its free energy density per frequency. By taking the time derivative of the total free energy (16) according to the rate equations in Eqs. (7) and (8), we obtain the second law of thermodynamics in the form of

$$d_t F = -T \dot{\Sigma} + \dot{W}_P = -T \dot{\Sigma}_q - T \dot{\Sigma}_p + \dot{W}_p. \quad (19)$$

Here,  $\dot{\Sigma}$  is the total entropy production rate, which is given by the sum of two contributions. One is the entropy production rate  $\dot{\Sigma}_q$  of the thermally induced transitions (1) reading

$$T\dot{\Sigma}_q = -\sum_{\alpha,\beta} \sum_{j_\beta > i_\alpha} (\mu_{j_\beta} - \mu_{i_\alpha}) J^q(i_\alpha \rightarrow j_\beta) \geq 0, \quad (20)$$

which is non-negative because of the local detailed balance condition in Eq. (9). The other is the entropy production rate  $\dot{\Sigma}_p$  of the photo-induced transitions (4) reading

$$T\dot{\Sigma}_p = -\sum_{\alpha,\beta} \sum_{j_\beta > i_\alpha} (\mu_{j_\beta} - \mu_{i_\alpha} - \mu_p(v_{i_\alpha, j_\beta})) J^p(i_\alpha \rightarrow j_\beta) \geq 0, \quad (21)$$

which is non-negative because of the local detailed balance condition in Eq. (11). The last term featuring in Eq. (19), i.e.,  $\dot{W}_p$ , is the work rate quantifying the rate of free energy absorbed from the radiation,

$$\dot{W}_p = \int dv \mu_p(v) I(v), \quad (22)$$

maintaining transitions out of equilibrium.

*Remark.* If the chemical potential of the photons vanishes for every frequency  $\nu$ , namely, the temperature of the corresponding black body  $T^{\text{bb}}(\nu)$  is equal to the temperature of the solution  $T$  for every frequency  $\nu$ , the work rate also vanishes, namely,  $\dot{W}_p = 0$ . This physically means that the radiation is in equilibrium with the solution, and all transitions will eventually relax to equilibrium. This can be formally seen from the second law (19) that boils down to  $d_t F = -T\dot{\Sigma}_q - T\dot{\Sigma}_p \leq 0$  if  $\dot{W}_p = 0$  together with the fact that  $F$  is lower bounded by its equilibrium value  $F_{\text{eq}}$ .<sup>50</sup>

### III. COARSE GRAINING OF THE VIBRATIONAL STATES: EFFECTIVE TRANSITIONS BETWEEN ELECTRONIC STATES

We coarse grain here the dynamical description of Subsection II A to derive a dynamical description in terms of species in specific electronic states, namely,  $\{X_\alpha\}$ , regardless of their vibrational states. Crucially, we construct the effective transitions between electronic states in such a way that they are thermodynamically consistent and, furthermore, still satisfy a local detailed balance condition.

#### A. Equilibration within electronic states

We assume that the photo-induced transitions interconvert only vibrational states of different electronic states of the same species [such as the example shown in Fig. 1(b)]. This assumption is well justified for experiments where organic molecules are irradiated by UV-visible light. Indeed, UV-visible light sources generate photons with frequencies  $\nu_{i_\alpha, j_\beta}$  corresponding to the energy difference between vibrational states of different electronic states of organic molecules, i.e.,  $h\nu_{i_\alpha, j_\beta} = |\mu_{j_\beta}^\circ - \mu_{i_\alpha}^\circ|$ . Hence, we consider light sources that do not promote any photo-induced transitions between vibrational states of the same electronic state,<sup>85</sup>

$$J^p(i_\alpha \rightarrow j_\alpha) = 0 \quad (23)$$

for all  $i_\alpha$  and  $j_\alpha$ . We further assume that the transitions between vibrational states of the same electronic state occur on a much shorter timescale than the transitions between vibrational states of different electronic states. This assumption, often called *vibrational relaxation*, is well justified for experiments where organic molecules are irradiated by UV-visible light: vibrational transitions within the same electronic states occur on a time scale of the order of  $10^{-12}$  s, while transitions between different electronic states occur on a time scale of the order of  $10^{-8}$  s (in the absence of phosphorescent states).<sup>48</sup> Hence, for any  $\alpha$ , on the time scale at which the concentration  $[X_\alpha]$  remains almost constant, the concentrations  $\{[X_{i_\alpha}]\}$  dramatically change and the thermally induced transitions (1) between any  $i_\alpha$  and  $j_\alpha$  equilibrate. This has two main implications.

From a dynamical point of view,

$$R^q(i_\alpha \rightarrow j_\alpha) = R^q(j_\alpha \rightarrow i_\alpha), \quad (24)$$

at all times for every pair of vibration states  $i_\alpha$  and  $j_\alpha$  or, equivalently,  $J^q(i_\alpha \rightarrow j_\alpha) = 0$ . Hence, there exist

$$p(i_\alpha|\alpha) = \frac{e^{-\frac{\mu_{i_\alpha}^\circ}{k_B T}}}{Q_\alpha}, \quad (25)$$

with  $Q_\alpha = \sum_{i_\alpha} \exp(-\mu_{i_\alpha}^\circ/k_B T)$ , named hereafter the population of the vibrational state  $i_\alpha$ , such that

$$[X_{i_\alpha}] = p(i_\alpha|\alpha)[X_\alpha]. \quad (26)$$

This is a direct consequence of the fact that the transitions between vibration states of the same electronic state define a detailed balanced linear network. Indeed, by using Eq. (26) in the expression for the chemical potentials (10) together with the local detailed balance condition (9), one can easily verify that Eq. (24) is satisfied.

From a thermodynamic point of view,

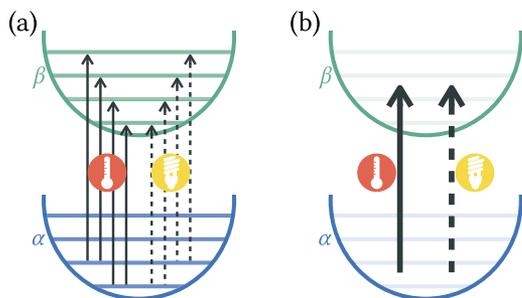
$$\mu_{i_\alpha} = \mu_{j_\alpha}, \quad (27)$$

at all times for every pair of vibration states  $i_\alpha$  and  $j_\alpha$ . This allows us to define the chemical potential of  $\alpha$  as

$$\mu_\alpha \equiv \mu_{i_\alpha} = \underbrace{\mu_{i_\alpha}^\circ + k_B T \ln p(i_\alpha|\alpha)}_{\equiv \mu_\alpha^\circ} + k_B T \ln [X_\alpha] \quad (28)$$

by using Eq. (26) in Eqs. (10) and (27). Note that the standard chemical potential  $\mu_\alpha^\circ$  is independent of  $i_\alpha$  because of Eq. (25) implying that  $\mu_\alpha^\circ = -k_B T \ln Q_\alpha$ .

*Remark.* The interconversions between vibrational states of the same electronic state would not equilibrate if they could occur via photon-induced transitions too. Indeed, the interplay between thermally induced transitions (promoted by the thermostat at temperature  $T$ ) and photo-induced transitions (promoted by photons playing the role of other thermostats at temperatures  $\{T^{\text{bb}}(\nu)\}$ ) would lead to a nonequilibrium steady state.<sup>50</sup> If this were the case, a thermodynamically consistent coarse-grained description in terms of transitions between electronic states only could not be derived.<sup>86</sup>



**FIG. 2.** Thermally induced (arrows close to the thermometer) and photo-induced (dashed arrows close to the bulb) transitions between two electronic states of the same species  $\alpha$  and  $\beta$ . (a) Thermally and photo-induced transitions between (some of) the vibrational states (represented by the horizontal lines) of the electronic states  $\alpha$  and  $\beta$ . (b) All thermally induced (resp. photo-induced) transitions are combined together to form an effective thermally induced (resp. photo-induced) transition between the electronic states of the same species  $\alpha$  and  $\beta$ . Each transition always has a backward counterpart, even if it is not represented.

## B. Kinetics

An effective thermally induced transition from the electronic state  $\alpha$  to the electronic state  $\beta$  results from the combination of all thermally induced transitions from the vibrational states  $\{i_\alpha\}$  to the vibrational states  $\{j_\beta\}$  (see Fig. 2). Thus, we can define the flux  $R^q(\alpha \rightarrow \beta)$  as

$$R^q(\alpha \rightarrow \beta) \equiv \sum_{i_\alpha, j_\beta} R^q(i_\alpha \rightarrow j_\beta), \quad (29)$$

which can be written as

$$R^q(\alpha \rightarrow \beta) = k_{\alpha \rightarrow \beta}^q [X_\alpha] \quad (30)$$

by using Eqs. (2) and (26) and defining the kinetic constant,

$$k_{\alpha \rightarrow \beta}^q \equiv \sum_{i_\alpha, j_\beta} k_{i_\alpha \rightarrow j_\beta}^q p(i_\alpha | \alpha). \quad (31)$$

The thermally induced transition from  $\alpha$  to  $\beta$  and its backward counterpart from  $\beta$  to  $\alpha$  can be represented by the following chemical equations:



and the corresponding net current reads

$$J^q(\alpha \rightarrow \beta) = R^q(\alpha \rightarrow \beta) - R^q(\beta \rightarrow \alpha). \quad (33)$$

*Remark.* These effective thermally induced transitions can interconvert the electronic states of the same species (such as in Fig. 2) as well as the electronic states of different species.

Similarly, an effective photo-induced transition from the electronic state  $\alpha$  to the electronic state  $\beta$  of the same species results from the combination of all photo-induced transitions from the vibrational states  $i_\alpha$  to the vibrational states  $j_\beta$  (see Fig. 2). Here, we assume that when the electronic state energies, defined as

$u_\alpha^0 = \sum_{i_\alpha} u_{i_\alpha}^0 p(i_\alpha | \alpha)$  and  $u_\beta^0 = \sum_{j_\beta} u_{j_\beta}^0 p(j_\beta | \beta)$ , satisfy  $u_\beta^0 > u_\alpha^0$ , then the only possible photo-induced transitions from  $\alpha$  to  $\beta$  (resp.  $\beta$  to  $\alpha$ ) are absorption (resp. emission) transitions. This physically means that there are no populated vibrational states of  $\alpha$  with energies higher than those of the populated vibrational states of  $\beta$ , which is a valid assumption at room temperature. We can hence define the density fluxes of absorption and emission between the electronic states  $\alpha$  and  $\beta$  (with  $u_\beta^0 > u_\alpha^0$ ) promoted by photons with frequency  $\nu$  as

$$r^a(\alpha \rightarrow \beta, \nu) \equiv \sum_{i_\alpha, j_\beta} \delta(\nu - \nu_{i_\alpha, j_\beta}) R^a(i_\alpha \rightarrow j_\beta), \quad (34a)$$

$$r^e(\beta \rightarrow \alpha, \nu) \equiv \sum_{i_\alpha, j_\beta} \delta(\nu - \nu_{i_\alpha, j_\beta}) R^e(j_\beta \rightarrow i_\alpha), \quad (34b)$$

respectively. They can be written as

$$r^a(\alpha \rightarrow \beta, \nu) = k_{\alpha \rightarrow \beta}^a(\nu) n(\nu) [X_\alpha], \quad (35a)$$

$$r^e(\beta \rightarrow \alpha, \nu) = (k_{\beta \rightarrow \alpha}^e(\nu) + k_{\beta \rightarrow \alpha}^{se}(\nu) n(\nu)) [X_\beta] \quad (35b)$$

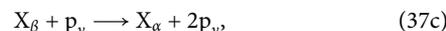
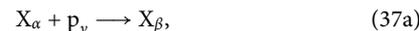
by using Eqs. (5) and (26) and defining the frequency-dependent kinetic constants,

$$k_{\alpha \rightarrow \beta}^a(\nu) = \sum_{i_\alpha, j_\beta} \delta(\nu - \nu_{i_\alpha, j_\beta}) k_{i_\alpha \rightarrow j_\beta}^a p(i_\alpha | \alpha), \quad (36a)$$

$$k_{\beta \rightarrow \alpha}^e(\nu) = \sum_{i_\alpha, j_\beta} \delta(\nu - \nu_{i_\alpha, j_\beta}) k_{j_\beta \rightarrow i_\alpha}^e p(j_\beta | \beta), \quad (36b)$$

$$k_{\beta \rightarrow \alpha}^{se}(\nu) = \sum_{i_\alpha, j_\beta} \delta(\nu - \nu_{i_\alpha, j_\beta}) k_{j_\beta \rightarrow i_\alpha}^{se} p(j_\beta | \beta). \quad (36c)$$

These transitions can be represented by the following chemical equations:



and the corresponding net density current reads

$$j^p(\alpha \rightarrow \beta, \nu) = r^a(\alpha \rightarrow \beta, \nu) - r^e(\beta \rightarrow \alpha, \nu). \quad (38)$$

Note that  $j^p(\alpha \rightarrow \beta, \nu) = r^e(\alpha \rightarrow \beta, \nu) - r^a(\beta \rightarrow \alpha, \nu)$  if  $u_\beta^0 < u_\alpha^0$ .

*Remark.* These effective photo-induced transitions can interconvert the electronic states of the same species only (such as in Fig. 2).

## C. Rate equations

The concentration of species in a specific electronic state, i.e.,  $[X_\beta]$ , is affected by both the effective thermally and photo-induced transitions and follows

$$d_t [X_\beta] = \sum_{\alpha (\alpha \neq \beta)} \left\{ J^q(\alpha \rightarrow \beta) + \underbrace{\int d\nu j^p(\alpha \rightarrow \beta, \nu)}_{=j^p(\alpha \rightarrow \beta)} \right\}, \quad (39)$$

as it can be verified by (i) using Eq. (7) in  $d_t[X_\beta] = \sum_{j_\beta \in \beta} d_t[X_{j_\beta}]$ , (ii) the definitions in Sec. III B, and (iii) recalling that  $J^q(i_\alpha \rightarrow j_\alpha) = 0$  and  $J^p(i_\alpha \rightarrow j_\alpha) = 0$  for all  $i_\alpha$  and  $j_\alpha$ .

On the other hand, the rate Eq. (8) for the density of photons with frequency  $\nu$  can be simply rewritten as

$$d_t n(\nu) = -\sum_{\beta > \alpha} j^p(\alpha \rightarrow \beta, \nu) + I(\nu), \quad (40)$$

where we ordered the electronic states, without loss of generality, in such a way that  $\beta > \alpha$  if  $u_\beta^0 > u_\alpha^0$ . Equation (40) can be simply obtained by using the definitions in Sec. III B and recalling that  $J^p(i_\alpha \rightarrow j_\alpha) = 0$  for all  $i_\alpha$  and  $j_\alpha$ .

#### D. Local detailed balance

We show now that the fluxes between electronic states defined in Eqs. (29) and (34) still satisfy a local detailed balance condition.

For thermally induced transitions, we can write

$$\frac{R^q(\alpha \rightarrow \beta)}{R^q(\beta \rightarrow \alpha)} = \frac{\sum_{i_\alpha, j_\beta} R^q(j_\beta \rightarrow i_\alpha) e^{-\frac{\mu_{j_\beta} - \mu_{i_\alpha}}{k_B T}}}{\sum_{i_\alpha, j_\beta} R^q(j_\beta \rightarrow i_\alpha)} \quad (41)$$

by using the definition in Eqs. (29) and (9). However, the equilibration of the thermally induced transitions within an electronic state implies that the chemical potentials  $\mu_{j_\beta}$  and  $\mu_{i_\alpha}$  are equal to the chemical potentials of the corresponding electronic state  $\mu_\beta$  and  $\mu_\alpha$  [see Eq. (28)]. Hence, we obtain the following local detailed balance condition:

$$k_B T \ln \frac{R^q(\alpha \rightarrow \beta)}{R^q(\beta \rightarrow \alpha)} = -(\mu_\beta - \mu_\alpha). \quad (42)$$

For photo-induced transitions, we can write

$$\frac{r^a(\alpha \rightarrow \beta, \nu)}{r^e(\beta \rightarrow \alpha, \nu)} = \frac{\sum_{i_\alpha, j_\beta} \delta(\nu - \nu_{i_\alpha, j_\beta}) R^e(j_\beta \rightarrow i_\alpha) e^{-\frac{\mu_{j_\beta} - \mu_{i_\alpha} - \mu_p(\nu)}{k_B T}}}{\sum_{i_\alpha, j_\beta} \delta(\nu - \nu_{i_\alpha, j_\beta}) R^e(j_\beta \rightarrow i_\alpha)}, \quad (43)$$

if  $u_\beta^0 > u_\alpha^0$ , by using the definition in Eqs. (34) and (11). As for the thermally induced transitions,  $\mu_{j_\beta} = \mu_\beta \forall j_\beta$  and  $\mu_{i_\alpha} = \mu_\alpha \forall i_\alpha$ , and therefore, we obtain the following local detailed balance condition:

$$k_B T \ln \frac{r^a(\alpha \rightarrow \beta, \nu)}{r^e(\beta \rightarrow \alpha, \nu)} = -(\mu_\beta - \mu_\alpha - \mu_p(\nu)). \quad (44)$$

*Remark.* As explained in Subsection II C, photons with frequency  $\nu$  correspond to a black body with temperature  $T^{bb}(\nu)$ , namely, a thermostat with temperature  $T^{bb}(\nu)$ . This allows us to interpret the  $\nu$ -dependence in Eq. (44), in terms of the local detailed balance condition resolving the specific thermostat promoting the transition from  $\alpha$  to  $\beta$ . If the light source were an actual black body at temperature  $T^{bb}$ , the chemical potential of the photons (15) would be frequency-independent since  $T^{bb}(\nu) = T^{bb}$  for all  $\nu$ . A local detailed balance condition would then hold also for the fluxes integrated over the frequency, namely,

$$k_B T \ln \frac{\int d\nu r^a(\alpha \rightarrow \beta, \nu)}{\int d\nu r^e(\beta \rightarrow \alpha, \nu)} = -(\mu_\beta - \mu_\alpha - \mu_p^{bb}), \quad (45)$$

with  $\mu_p^{bb} = h\nu(1 - T/T^{bb})$ .

#### E. Second law

The second law of thermodynamics (19) still holds after the coarse graining of the vibrational states. Indeed, all thermodynamic quantities introduced in Subsection II D can be expressed in terms of species in specific electronic states regardless of their vibrational states and of the density of photons. This is a direct consequence of the equilibration of the vibrational states of the same electronic state<sup>86</sup> as we show in the following.

First, the free energy contribution due to the molecular species (17) boils down,

$$F_{ch} = \sum_\alpha (\mu_\alpha - k_B T)[X_\alpha], \quad (46)$$

because of Eq. (28). Second, the entropy production rate of the thermally induced (20) and photo-induced (21) transitions become

$$T\dot{\Sigma}_q = -\sum_{\beta > \alpha} (\mu_\beta - \mu_\alpha) J^q(\alpha \rightarrow \beta) \geq 0, \quad (47a)$$

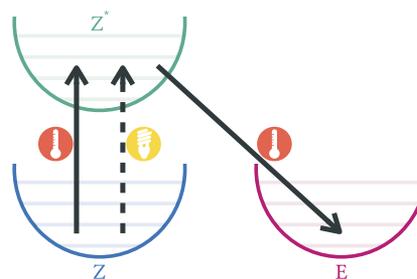
$$T\dot{\Sigma}_p = \int d\nu \left[ -\sum_{\beta > \alpha} (\mu_\beta - \mu_\alpha - \mu_p(\nu)) j^p(\alpha \rightarrow \beta, \nu) \right] \geq 0 \quad (47b)$$

$\equiv T\dot{\sigma}_p(\nu) \geq 0$

by using Eqs. (23), (24), (28), (29), and (34). Note that  $\dot{\Sigma}_q$  and  $\dot{\sigma}_p(\nu)$  in Eq. (47) are non-negative because of the local detailed balance conditions in Eqs. (42) and (44). Third, the free energy contribution due to the photons and the work rate are still given in Eqs. (18) and (22), respectively. Finally, one can easily verify (by direct substitution) that the thermodynamic quantities in Eqs. (18), (22), (46), and (47) satisfy the second law of thermodynamics in Eq. (19).

#### IV. COARSE GRAINING OF THE EXCITED STATES: ISOMERIZATION REACTIONS

We coarse grain here the dynamical description of Subsection III C to derive a dynamical description in terms of ground-state



**FIG. 3.** A species in the electronic ground state Z can be interconverted into its electronic excited state Z\* via a thermally induced (vertical arrow close to the thermometer) and a photo-induced (vertical dashed arrow close to the bulb) transition. The electronic excited state Z\* can also be interconverted via a thermally induced (oblique arrow close to the thermometer) transition into the the electronic ground state of a different species E. Chemically speaking, the species Z and E are different isomers. Recall that no photo-induced transition can interconvert the ground state of species E into the excited state of a different species Z\* as a result of the Franck–Condon principle.<sup>48</sup> Note that each transition has always a backward counterpart, even if it is not represented.

species only. Some of the resulting effective transitions interconvert a ground-state species into a different one, thus constituting an isomerization. We will refer to them as isomerization reactions. Other effective transitions interconvert the ground state of a species back into itself without realizing any transformation. We will refer to them as futile reactions. Crucially, we construct all these reactions in such a way that they are thermodynamically consistent and, furthermore, still satisfy a local detailed balance condition.

We focus here on the case illustrated in Fig. 3, where we replaced the general labels of the electronic states used until now (i.e.,  $\alpha$  and  $\beta$ ) with specific labels. The reactions resulting from the coarse graining are shown in Fig. 5. Two of them are isomerization reactions [Figs. 5(a) and 5(b)] while the other two are futile reactions [Figs. 5(c) and 5(d)].

*Remark.* From now on, we will refer to any transition involving ground-state species (independently of whether it is an effective transition or not) as a reaction.

### A. Relaxation of the electronic excited state

We assume that the evolution of the excited-state species  $Z^*$  is much faster than the evolution of the ground-state species  $Z$  and  $E$ , reflecting the fact that  $Z^*$  is generally short-lived. This is a valid assumption for experiments where organic molecules are irradiated by UV-visible light since the lifetime of excited-state species is of the order of  $10^{-8}$  s in the absence of long-lived phosphorescent states that we are not considering here.<sup>48</sup> Hence, on the time scale at which the concentrations  $[Z]$  and  $[E]$  remain almost constant, the concentration  $[Z^*]$  dramatically changes and relaxes to the corresponding steady state whose concentration reads

$$[Z^*]_{ss} = \frac{k_{Z \rightarrow Z^*}^q [Z] + \int d\nu k_{Z \rightarrow Z^*}^a(\nu) n(\nu) [Z] + k_{E \rightarrow Z^*}^q [E]}{\mathcal{D}_{Z^*}}, \quad (48)$$

with

$$\mathcal{D}_{Z^*} = k_{Z^* \rightarrow Z}^q + \int d\nu (k_{Z^* \rightarrow Z}^e(\nu) + k_{Z^* \rightarrow Z}^{se}(\nu) n(\nu)) + k_{Z^* \rightarrow E}^q. \quad (49)$$

Equation (48) is obtained by imposing  $d_t[Z^*] = 0$  in Eq. (39).

*Remark.* While in Subsection III A we assumed the equilibration of the vibrational states, here we are solely using a quasi-steady-state approximation. Nevertheless, we can construct reactions between ground-state species that still satisfy a local detailed balance condition as we prove in the following.

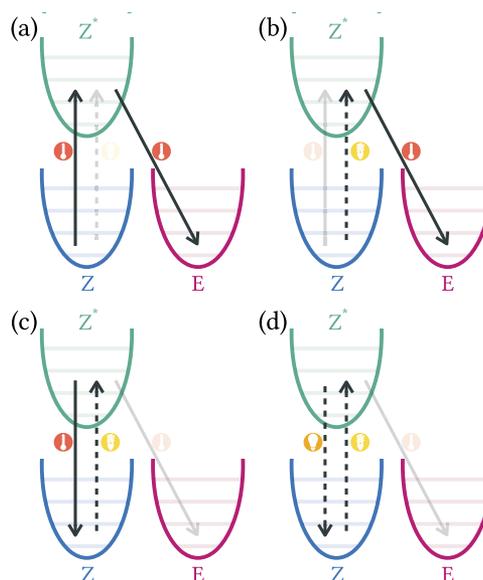
### B. Cycle kinetics

This time scale separation between excited-state species and ground-state species further implies that, as the concentrations of the ground-state species  $[Z]$  and  $[E]$  slowly evolve, the concentration of the excited-state species  $[Z^*]$  follows the steady-state concentration  $[Z^*]_{ss}$  in Eq. (48). The dynamics can, therefore, be

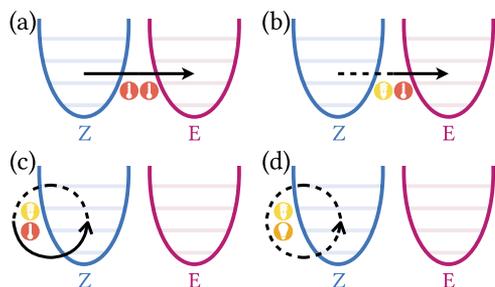
described in terms of so-called cycles.<sup>42</sup> In this context, a cycle is defined as a sequence of transitions that, upon completion, does not interconvert the excited-state species  $Z^*$  (which is and remains at steady state), but only interconverts the ground-state species  $Z$  and  $E$  (which are slowly evolving).<sup>87</sup> Its net effect is a (isomerization or futile) reaction between ground-state species determining their slow evolution (even though the underlying sequence of transitions involves the excited-state species). For the case in Fig. 3, the cycles and the corresponding (isomerization or futile) reactions are illustrated in Figs. 4 and 5, respectively.

We stress that (sets of linearly independent) cycles have been already used to construct thermodynamically consistent coarse-grained descriptions of chemically driven molecular systems<sup>41–43</sup> as well as photo-induced transitions.<sup>50</sup> However, the resulting reactions do not, in general, satisfy a local detailed balance condition.<sup>41,50</sup> Indeed, a local detailed balance condition can only be restored by using a conformal set of cycles,<sup>88</sup> which unfortunately might change during the dynamics since it is defined by the values of all net currents.

Here, we construct reactions between ground-state species, which still satisfy a local detailed balanced condition by considering all possible cycles (independently of whether they are linearly independent or not). This can be done because the thermally and photo-induced transitions we consider are linear in the concentrations  $[Z]$ ,  $[Z^*]$ , and  $[E]$ , and therefore, we can exploit a modified version of the matrix-tree theorem<sup>52,53</sup> already used in Ref. 22.



**FIG. 4.** Cycles (represented by sequences of arrows) are shown for the case in Fig. 3. Each cycle can be run in two directions: one direction is illustrated by the arrows while the other is the opposite direction. By following the sequences of transitions of each cycle, the excited-state species  $Z^*$  is never interconverted as it is always produced by one transition and consumed by another. Note that cycle (a) can be obtained by running cycle (c) in the opposite direction with respect to the one determined by the arrows, followed by cycle (b) [namely, cycle (a) is linearly dependent on cycle (b) and (c)]. Note also that cycle (d) results from a sequence of two photo-induced transitions involving photons with different frequency (represented by different bulbs of different colors).



**FIG. 5.** Reactions between ground-state species corresponding to the cycles shown in Fig. 4. Each reaction has always its backward counterpart even if it is not represented. Thermometers and bulbs specify whether the underlying transitions are thermally or photo-induced or a combination of both. (a) Isomerization reaction of Z into E via a sequence of only thermally induced transitions. (b) Isomerization reaction of Z into E via a sequence of both photo- and thermally induced transitions. (c) Futile reaction of Z into itself via a sequence of both photo- and thermally induced transitions. Its net effect is the interconversion of photons into heat. (d) Futile reaction of Z into itself via a sequence of only photo-induced transitions. Its net effect is the interconversion of photons with one frequency into photons with a different frequency. Recall that each reaction is defined by the sequence of transitions in Fig. 4 that involves the excited-state species  $Z^*$ .

We start by recognizing that the cycles in Fig. 4 constitute all possible cycles of the case in Fig. 3. Each cycle can be run in two directions: one is the direction illustrated by the arrows in Fig. 4 while the other is the opposite direction. One direction defines the forward reaction between ground-state species, while the other defines the backward one. According to the modified matrix-tree theorem<sup>52,53</sup> used in Ref. 22, the corresponding (density) fluxes are given by the product of the (density) fluxes of the underlying transitions along the cycle divided by  $\mathcal{D}_{Z^*}$  in Eq. (49) and the excited-state concentration  $[Z^*]$  (which will cancel out with an equivalent term featuring the numerator).

Hence, the fluxes of the cycle in Fig. 4(a) [or, equivalently, of the isomerization reaction in Fig. 5(a)] read

$$\Pi^q(Z \xrightarrow{Z^*} E) = \frac{k_{Z \rightarrow Z^*}^q [Z] k_{Z^* \rightarrow E}^q}{\mathcal{D}_{Z^*}}, \quad (50a)$$

$$\Pi^q(E \xrightarrow{Z^*} Z) = \frac{k_{E \rightarrow Z^*}^q [E] k_{Z^* \rightarrow Z}^q}{\mathcal{D}_{Z^*}}, \quad (50b)$$

while the corresponding net current is given by

$$\Psi^q(Z \xrightarrow{Z^*} E) = \Pi^q(Z \xrightarrow{Z^*} E) - \Pi^q(E \xrightarrow{Z^*} Z). \quad (51)$$

The density fluxes of absorption and emission of the cycle in Fig. 4(b) [or, equivalently, of the isomerization reaction in Fig. 5(b)] read

$$\pi^a(Z \xrightarrow{Z^*} E, \nu) = \frac{k_{Z \rightarrow Z^*}^a(\nu) n(\nu) [Z] k_{Z^* \rightarrow E}^q}{\mathcal{D}_{Z^*}}, \quad (52a)$$

$$\pi^e(E \xrightarrow{Z^*} Z, \nu) = \frac{(k_{Z^* \rightarrow Z}^e(\nu) + k_{Z^* \rightarrow Z}^{se}(\nu) n(\nu)) k_{E \rightarrow Z^*}^q [E]}{\mathcal{D}_{Z^*}}, \quad (52b)$$

while the corresponding net density current is given by

$$\psi^p(Z \xrightarrow{Z^*} E, \nu) = \pi^a(Z \xrightarrow{Z^*} E, \nu) - \pi^e(E \xrightarrow{Z^*} Z, \nu). \quad (53)$$

The density fluxes of absorption and emission of the cycle in Fig. 4(c) [or, equivalently, of the futile reaction in Fig. 5(c)] read

$$\pi^a(Z \xrightarrow{Z^*} Z, \nu) = \frac{k_{Z \rightarrow Z^*}^a(\nu) n(\nu) [Z] k_{Z^* \rightarrow Z}^q}{\mathcal{D}_{Z^*}}, \quad (54a)$$

$$\pi^e(Z \xrightarrow{Z^*} Z, \nu) = \frac{(k_{Z^* \rightarrow Z}^e(\nu) + k_{Z^* \rightarrow Z}^{se}(\nu) n(\nu)) k_{Z \rightarrow Z^*}^q [Z]}{\mathcal{D}_{Z^*}}, \quad (54b)$$

while the corresponding net density current is given by

$$\psi^p(Z \xrightarrow{Z^*} Z, \nu) = \pi^a(Z \xrightarrow{Z^*} Z, \nu) - \pi^e(Z \xrightarrow{Z^*} Z, \nu). \quad (55)$$

The density flux for the cycle in Fig. 4(d) when photons of frequency  $\nu$  are absorbed while photons of frequency  $\nu'$  are emitted [or, equivalently, the futile reaction in Fig. 5(d)] reads

$$\pi(Z \xrightarrow{Z^*} Z, \nu, \nu') = \frac{k_{Z \rightarrow Z^*}^a(\nu) n(\nu) [Z] (k_{Z^* \rightarrow Z}^e(\nu') + k_{Z^* \rightarrow Z}^{se}(\nu') n(\nu'))}{\mathcal{D}_{Z^*}}, \quad (56)$$

while the corresponding net density current is given by

$$\psi^p(Z \xrightarrow{Z^*} Z, \nu, \nu') = \pi(Z \xrightarrow{Z^*} Z, \nu, \nu') - \pi(Z \xrightarrow{Z^*} Z, \nu', \nu), \quad (57)$$

satisfying  $\psi^p(Z \xrightarrow{Z^*} Z, \nu, \nu') = -\psi^p(Z \xrightarrow{Z^*} Z, \nu', \nu)$ .

As long as the time scale separation between excited-state species and ground-state species holds, i.e.,  $[Z^*] = [Z^*]_{ss}$ , the net cycle currents univocally define the current of all underlying transitions in Fig. 3. Mathematically speaking, the cycle currents are a basis set for the currents of all underlying transitions. Indeed, the current of the thermally induced transitions can be written as

$$J^q(Z \rightarrow Z^*) = - \int d\nu \psi^p(Z \xrightarrow{Z^*} Z, \nu) + \Psi^q(Z \xrightarrow{Z^*} E), \quad (58a)$$

$$J^q(Z^* \rightarrow E) = \int d\nu \psi^p(Z \xrightarrow{Z^*} E, \nu) + \Psi^q(Z \xrightarrow{Z^*} E), \quad (58b)$$

while the net density current of the photo-induced transition can be written as

$$j^p(Z \rightarrow Z^*, \nu) = \psi^p(Z \xrightarrow{Z^*} Z, \nu) + \psi^p(Z \xrightarrow{Z^*} E, \nu) + \int d\nu' \psi^p(Z \xrightarrow{Z^*} Z, \nu, \nu'). \quad (59)$$

*Remark.* In order to apply the modified matrix-tree theorem<sup>52,53</sup> used in Ref. 22, one has to recognize that photo-induced transitions promoted by different frequencies have to be treated as independent transitions.

### 1. Quantum yields

The cycle fluxes in Eqs. (50), (52), (54), and (56) can be expressed in terms of the following quantum yields:

$$\Phi^q(Z^* \rightarrow E) \equiv k_{Z^* \rightarrow E}^q / \mathcal{D}_{Z^*}, \quad (60a)$$

$$\Phi^q(Z^* \rightarrow Z) \equiv k_{Z^* \rightarrow Z}^q / \mathcal{D}_{Z^*}, \quad (60b)$$

$$\phi^p(Z^* \rightarrow Z, \nu) \equiv (k_{Z^* \rightarrow Z}^e(\nu) + k_{Z^* \rightarrow Z}^{se}(\nu) n(\nu)) / \mathcal{D}_{Z^*}, \quad (60c)$$

quantifying the probability that the excited-state species  $Z^*$  interconverts to E via the thermally induced transition or to Z via the thermally induced transition or to Z via the photo-induced transition, respectively. Indeed, by using the definitions (60) in Eqs. (50), (52), (54), and (56), the cycle fluxes become

$$\Pi^q(Z \xrightarrow{Z^*} E) = \Phi^q(Z^* \rightarrow E) k_{Z \rightarrow Z^*}^q [Z], \quad (61a)$$

$$\Pi^q(E \xrightarrow{Z^*} Z) = \Phi^q(Z^* \rightarrow Z) k_{E \rightarrow Z^*}^q [E], \quad (61b)$$

$$\pi^a(Z \xrightarrow{Z^*} E, \nu) = \Phi^q(Z^* \rightarrow E) k_{Z \rightarrow Z^*}^a(\nu) n(\nu) [Z], \quad (61c)$$

$$\pi^e(E \xrightarrow{Z^*} Z, \nu) = \phi^p(Z^* \rightarrow Z, \nu) k_{E \rightarrow Z^*}^q [E], \quad (61d)$$

$$\pi^a(Z \xrightarrow{Z^*} Z, \nu) = \Phi^q(Z^* \rightarrow Z) k_{Z \rightarrow Z^*}^a(\nu) n(\nu) [Z], \quad (61e)$$

$$\pi^e(Z \xrightarrow{Z^*} Z, \nu) = \phi^p(Z^* \rightarrow Z, \nu) k_{Z \rightarrow Z^*}^q [Z], \quad (61f)$$

$$\pi(Z \xrightarrow{Z^*} Z, \nu, \nu') = \phi^p(Z^* \rightarrow Z, \nu') k_{Z \rightarrow Z^*}^a(\nu) n(\nu) [Z], \quad (61g)$$

thus proving that they are functions of the absorption spectra and quantum yields together with the concentrations of the ground-state species only.

*Remark.* The quantum yields in Eq. (60) satisfy

$$\Phi^q(Z^* \rightarrow E) + \Phi^q(Z^* \rightarrow Z) + \int d\nu \phi^p(Z^* \rightarrow Z, \nu) = 1. \quad (62)$$

### C. Rate equations

As long as the time scale separation between excited-state species and ground-state species holds, the concentrations  $[Z]$  and  $[E]$  follow

$$d_t[Z] = -d_t[E] = -\Psi^q(Z \xrightarrow{Z^*} E) - \int d\nu \psi^p(Z \xrightarrow{Z^*} E, \nu), \quad (63)$$

as it can be verified by using Eqs. (58) and (59). Namely, the dynamics of  $[Z]$  and  $[E]$  can be expressed in terms of the net currents of the cycles (a) and (b) in Fig. 4 or equivalently of the isomerization reactions (a) and (b) in Fig. 5.

On the other hand, the density of photons with frequency  $n(\nu)$  follows

$$d_t n(\nu) = -\left[ \psi^p(Z \xrightarrow{Z^*} Z, \nu) + \psi^p(Z \xrightarrow{Z^*} E, \nu) + \int d\nu' \psi^p(Z \xrightarrow{Z^*} Z, \nu, \nu') \right] + I(\nu), \quad (64)$$

as it can be verified by using Eq. (59). Namely, the dynamics of  $n(\nu)$  can be expressed in terms of the net density currents of the cycles (b), (c), and (d) in Fig. 4 or equivalently of the isomerization reaction (b) and the futile reactions (c) and (d) in Fig. 5.

### D. Local detailed balance

By simply plugging Eqs. (42) and (44) into the cycle fluxes in Eqs. (50), (52), (54), and (56), we obtain the following local detailed balance conditions for the reactions in Fig. 5:

$$k_B T \ln \frac{\Pi^q(Z \xrightarrow{Z^*} E)}{\Pi^q(E \xrightarrow{Z^*} Z)} = -(\mu_E - \mu_Z), \quad (65a)$$

$$k_B T \ln \frac{\pi^a(Z \xrightarrow{Z^*} E, \nu)}{\pi^e(E \xrightarrow{Z^*} Z, \nu)} = -(\mu_E - \mu_Z - \mu_p(\nu)), \quad (65b)$$

$$k_B T \ln \frac{\pi^a(Z \xrightarrow{Z^*} Z, \nu)}{\pi^e(Z \xrightarrow{Z^*} Z, \nu)} = \mu_p(\nu), \quad (65c)$$

$$k_B T \ln \frac{\pi(Z \xrightarrow{Z^*} Z, \nu, \nu')}{\pi(Z \xrightarrow{Z^*} Z, \nu', \nu)} = -(\mu_p(\nu') - \mu_p(\nu)). \quad (65d)$$

*Remark.* Like in Subsection III D, if the light source were an actual black body at temperature  $T^{bb}$ , the chemical potential of the photons (15) would be frequency independent and local detailed balance condition would then also hold for the fluxes integrated over the frequency, namely,

$$k_B T \ln \frac{\int d\nu \pi^a(Z \xrightarrow{Z^*} E, \nu)}{\int d\nu \pi^e(E \xrightarrow{Z^*} Z, \nu)} = -(\mu_E - \mu_Z - \mu_p^{bb}), \quad (66a)$$

$$k_B T \ln \frac{\int d\nu \pi^a(Z \xrightarrow{Z^*} Z, \nu)}{\int d\nu \pi^e(Z \xrightarrow{Z^*} Z, \nu)} = \mu_p^{bb}, \quad (66b)$$

$$k_B T \ln \frac{\int d\nu \int d\nu' \pi(Z \xrightarrow{Z^*} Z, \nu, \nu')}{\int d\nu \int d\nu' \pi(Z \xrightarrow{Z^*} Z, \nu', \nu)} = 0, \quad (66c)$$

with  $\mu_p^{bb} = h\nu(1 - T/T^{bb})$ .

### E. Second law

We prove here that the dynamical description in terms of reactions between ground-state species is thermodynamically consistent. Namely, the entropy production rate of the case in Fig. 3, i.e.,

$$T\dot{\Sigma} = -(\mu_{Z^*} - \mu_Z)J^q(Z \rightarrow Z^*) - (\mu_{Z^*} - \mu_E)J^q(E \rightarrow Z^*) - \int d\nu (\mu_{Z^*} - \mu_Z - \mu_p(\nu))j^p(Z \rightarrow Z^*, \nu), \quad (67)$$

obtained by specializing Eqs. (47a) and (47b), can be expressed in terms of the dissipation along the reactions shown in Fig. 5 (or, equivalently, along the cycles in Fig. 4). Indeed, by plugging Eqs. (58) and (59) into Eq. (67), we obtain

$$T\dot{\Sigma} = -(\mu_E - \mu_Z)\Psi^q\left(Z \xrightarrow{Z^*} E\right) - \int dv (\mu_E - \mu_Z - \mu_P(v)) \\ \times \Psi^p\left(Z \xrightarrow{Z^*} E, v\right) - \int dv (-\mu_P(v))\Psi^p\left(Z \xrightarrow{Z^*} Z, v\right) \\ - \frac{1}{2} \int dv \int dv' (\mu_P(v') - \mu_P(v))\Psi^p\left(Z \xrightarrow{Z^*} Z, v, v'\right), \quad (68)$$

where we used  $\Psi^p\left(Z \xrightarrow{Z^*} Z, v, v'\right) = -\Psi^p\left(Z \xrightarrow{Z^*} Z, v', v\right)$ . Each term featuring Eq. (68) is non-negative because of the local detailed balance conditions (65) and specifies the dissipation of the corresponding reaction in Fig. 5. The first (resp. second) one accounts for the dissipation of the isomerization reaction of Z into E via an underlying sequence of transitions that are thermally induced only (resp. both thermally and photo-induced). The third one accounts for the dissipation of the futile reaction of Z back to itself via an underlying sequence of transitions that are both thermally and photo-induced. The net effect is the interconversion of photons into heat. The last one accounts for the dissipation of the futile reaction of Z back to itself via an underlying sequence of transitions that are photo-induced only. The net effect is the interconversion of photons of frequency  $v$  into photons of a different frequency  $v'$ .

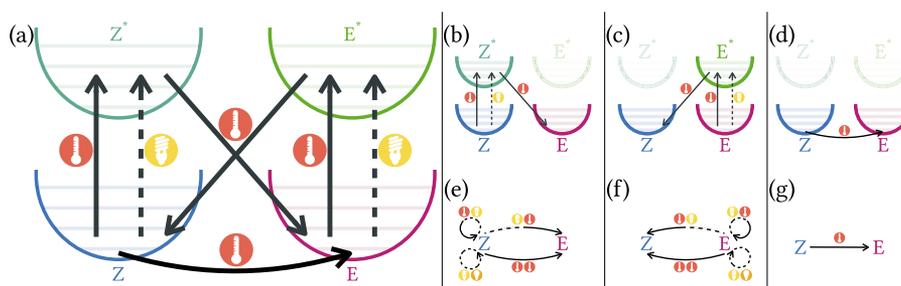
## V. COARSE GRAINING OF THE DIABATIC PROCESS

We now use the results of Sec. IV to coarse grain the photo-isomerization scheme in Fig. 6(a), known as diabatic process,<sup>48,51</sup> in terms of reactions between ground-state species only. The diabatic process describes the interconversion of one species

Z (in the electronic ground state) into a different isomer E (in the electronic ground state), which involves the electronic excited-state species  $Z^*$  and  $E^*$ . It is widely common in organic molecules that undergo photo-isomerizations involving, for example, alkene<sup>56</sup> or azobenzene<sup>89</sup> derivatives.

We start by recognizing that the transitions in Fig. 6(a) can be grouped into three sets illustrated in Figs. 6(b)–6(d), respectively. The first set [Fig. 6(b)] collects the same transitions discussed in Sec. IV, namely, the thermally and photo-induced transitions involving the excited-state species  $Z^*$  and leading to the interconversion of Z into E. The second set [Fig. 6(c)] collects the thermally and photo-induced transitions involving the excited-state species  $E^*$  and leading to the interconversion of E into Z. In practice, it collects the same transitions discussed in Sec. IV, after relabeling the species according to Z into E, E into Z, and  $E^*$  into  $Z^*$ . The third set [Fig. 6(d)] collects the direct interconversion of Z into E via a thermally induced transition, which does not involve any excited-state species.

The first two sets of transitions [in Figs. 6(b) and 6(c), respectively] can be coarse grained by repeating the procedure illustrated in Sec. IV. By assuming that the evolution of the excited-state species  $Z^*$  and  $E^*$  is much faster than the evolution of the ground-state species Z and E, the latter can be described in terms of cycles defining (isomerization and futile) reactions between the ground-state species Z and E only. The reactions for the first set [Fig. 6(e)], as well as their fluxes and net currents, are those derived in Sec. IV. Similarly, the reactions for the second set [Fig. 6(f)], as well as their fluxes and net currents, correspond to those derived in Sec. IV after relabeling the species according to Z into E, E into Z, and  $E^*$  into  $Z^*$ . Crucially, all these reactions still satisfy a local detailed balance condition as proved in Sec. IV. The third and last set of reactions is the direct interconversion of Z into E and satisfies a local detailed balance condition, as discussed in Sec. III.



**FIG. 6.** Diabatic process. (a) The species in the electronic ground state Z (resp. E) can be interconverted into its electronic excited state  $Z^*$  (resp.  $E^*$ ) via thermally induced (arrow close to the thermometer) and photo-induced (dashed arrow close to the bulb) transitions. The electronic excited state  $Z^*$  (resp.  $E^*$ ) can also be interconverted via a thermally induced (oblique arrow close to the thermometer) transition into the electronic ground state E (resp. Z) of the other species. The ground-state species Z and E can directly interconvert into the ground-state species E via a thermally induced transition (curved arrow close to the thermometer). Chemically speaking, the species Z and E are different isomers. (b) Set of transitions of the diabatic process involving the electronic excited-state species  $Z^*$ . When the dynamics of  $Z^*$  is faster compared to Z and E, these transitions can be coarse grained into the (isomerization and futile) reactions in panel (e) between Z and E involving the excited-state species  $Z^*$ . (c) Set of transitions of the diabatic process involving the electronic excited-state species  $E^*$ . When the dynamics of  $E^*$  is faster compared to Z and E, these transitions can be coarse grained into the (isomerization and futile) reactions in panel (f) between Z and E involving the excited-state species  $E^*$ . (d) Transition that does not involve electronic excited-state species  $Z^*$  and  $E^*$  and corresponds to the direct isomerization reaction of Z into E in panel (g). Note that each transition and reaction has always a backward counterpart, even if it is not represented.

Therefore, as long as the time scale separation between excited-state species and ground-state species holds, the concentrations  $[Z]$  and  $[E]$  follow

$$\begin{aligned} d_t[Z] = -d_t[E] = & -\Psi^q\left(Z \xrightarrow{Z^*} E\right) - \int d\nu \psi^p\left(Z \xrightarrow{Z^*} E, \nu\right) \\ & + \Psi^q\left(E \xrightarrow{E^*} Z\right) + \int d\nu \psi^p\left(E \xrightarrow{E^*} Z, \nu\right) \\ & - J^q(Z \rightarrow E), \end{aligned} \quad (69)$$

where the first, second, and third lines feature the currents [Eqs. (51), (53), and (33)] of the isomerization reactions involving  $Z^*$  [Fig. 6(e)],  $E^*$  [Fig. 6(f)], and no excited-state species [Fig. 6(g)], respectively. However, the fluxes of the isomerization reactions involving thermally induced transitions from a ground-state species to an excited-state species [see Eq. (61)] are usually negligible compared to the others. The rate Eq. (69) can thus be simplified to

$$\begin{aligned} d_t[Z] = -d_t[E] \approx & - \int d\nu \Phi^q(Z^* \rightarrow E) k_{Z \rightarrow Z^*}^a(\nu) n(\nu) [Z] \\ & + \int d\nu \Phi^q(E^* \rightarrow Z) k_{E \rightarrow E^*}^a(\nu) n(\nu) [E] \\ & - (k_{Z \rightarrow E}^q [Z] - k_{E \rightarrow Z}^q [E]) \end{aligned} \quad (70)$$

by expressing the reaction fluxes according to Eqs. (30) and (61). We notice that Eq. (70) corresponds to the standard rate equation used to describe the dynamics of the diabatic process in terms of the experimentally measurable kinetic constants, absorption spectra, and quantum yields.<sup>74</sup> By factorizing the concentrations  $[Z]$  and  $[E]$ , Eq. (70) is often interpreted as the rate equation resulting from the two effective reactions  $Z \rightarrow E$  and  $E \rightarrow Z$ , whose

kinetic constants read  $[\int d\nu \Phi^q(Z^* \rightarrow E) k_{Z \rightarrow Z^*}^a(\nu) n(\nu) + k_{Z \rightarrow E}^q]$  and  $[\int d\nu \Phi^q(E^* \rightarrow Z) k_{E \rightarrow E^*}^a(\nu) n(\nu) + k_{E \rightarrow Z}^q]$ , respectively.

## VI. IMPLICATIONS OF THE LOCAL DETAILED BALANCE FOR PHOTO-DRIVEN MOLECULAR RATCHETS

We now consider a prototypical autonomous photo-driven molecular ratchet, whose network of transitions is illustrated in Fig. 7(a). On the one hand, thermally induced reactions interconvert the (ground-state) species  $Z_1$  into  $Z_2$  and  $E_1$  into  $E_2$ . On the other hand, diabatic processes (discussed in Sec. V and illustrated in Fig. 6) interconvert the (ground-state) species  $Z_1$  into  $E_1$  (via transitions also involving the excited-state species  $Z_1^*$  and  $E_1^*$ ) and  $Z_2$  into  $E_2$  (via transitions also involving the excited-state species  $Z_2^*$  and  $E_2^*$ ). This scheme is routinely coarse grained in terms of reactions between ground-state species only and used to exemplify the conceptual and practical elements of photo-driven molecular ratchets.<sup>66–68</sup> Its validity has been verified against experimentally realized molecular motors<sup>58</sup> and pumps.<sup>59,60</sup> The photoisomerization and futile reactions resulting from the (thermodynamically consistent) coarse graining (developed in Sec. V) of the diabatic processes shown in Fig. 7(a) are illustrated in Fig. 7(b).

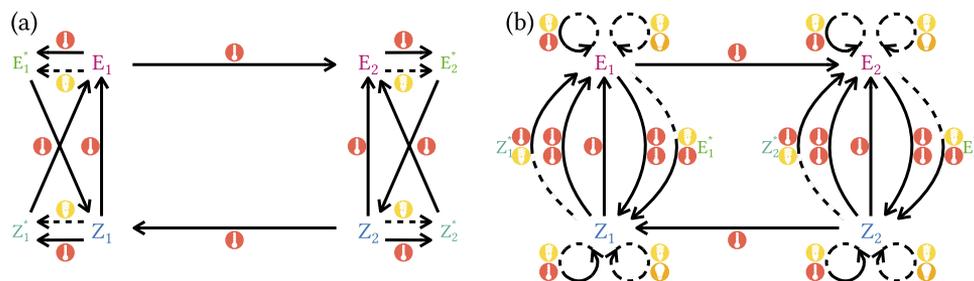
We examine the implications that the existence of a local detailed balance condition for (photo-isomerization and futile reactions of) the diabatic processes has on the kinetic asymmetry of the network,<sup>70</sup> quantified by the ratcheting constant  $K_r$ .<sup>66,67,71</sup> This constant serves as a directionality parameter, identifying whether, at steady state, the net currents across the network of reactions shown in Fig. 7(b) are preferentially directed clockwise (corresponding to  $K_r > 1$ ), counterclockwise (corresponding to  $K_r < 1$ ), or show no net bias (corresponding to  $K_r = 1$ ). For the network of reactions shown in Fig. 7(b),  $K_r$  can be written as

$$\begin{aligned} K_r = & \frac{\left( \int d\nu \pi^a\left(Z_1 \xrightarrow{Z_1^*} E_1, \nu\right) + \Pi^q\left(Z_1 \xrightarrow{Z_1^*} E_1\right) + R^q(Z_1 \rightarrow E_1) + \Pi^q\left(Z_1 \xrightarrow{E_1^*} E_1\right) + \int d\nu \pi^e\left(Z_1 \xrightarrow{E_1^*} E_1, \nu\right) \right) R^q(E_1 \rightarrow E_2)}{\left( \int d\nu \pi^e\left(E_1 \xrightarrow{Z_1^*} Z_1, \nu\right) + \Pi^q\left(E_1 \xrightarrow{Z_1^*} Z_1\right) + R^q(E_1 \rightarrow Z_1) + \Pi^q\left(E_1 \xrightarrow{E_1^*} Z_1\right) + \int d\nu \pi^a\left(E_1 \xrightarrow{E_1^*} Z_1, \nu\right) \right) R^q(E_2 \rightarrow E_1)} \\ & \times \frac{\left( \int d\nu \pi^e\left(E_2 \xrightarrow{Z_2^*} Z_2, \nu\right) + \Pi^q\left(E_2 \xrightarrow{Z_2^*} Z_2\right) + R^q(E_2 \rightarrow Z_2) + \Pi^q\left(E_2 \xrightarrow{E_2^*} Z_2\right) + \int d\nu \pi^a\left(E_2 \xrightarrow{E_2^*} Z_2, \nu\right) \right) R^q(Z_2 \rightarrow Z_1)}{\left( \int d\nu \pi^a\left(Z_2 \xrightarrow{Z_2^*} E_2, \nu\right) + \Pi^q\left(Z_2 \xrightarrow{Z_2^*} E_2\right) + R^q(Z_2 \rightarrow E_2) + \Pi^q\left(Z_2 \xrightarrow{E_2^*} E_2\right) + \int d\nu \pi^e\left(Z_2 \xrightarrow{E_2^*} E_2, \nu\right) \right) R^q(Z_1 \rightarrow Z_2)}, \end{aligned} \quad (71)$$

where the four terms in parentheses are the net fluxes resulting from the sum of the fluxes of all reactions leading to the interconversions  $Z_1 \rightarrow E_1$ ,  $E_1 \rightarrow Z_1$ ,  $E_2 \rightarrow Z_2$ , and  $Z_2 \rightarrow E_2$ , respectively. Namely, the numerator features the product of the net fluxes of the reactions cycling clockwise, i.e.,  $Z_1 \rightarrow E_1 \rightarrow E_2 \rightarrow Z_2 \rightarrow Z_1$ , while the denominator features the product of the net fluxes of the reactions cycling counterclockwise, i.e.,  $Z_1 \leftarrow E_1 \leftarrow E_2 \leftarrow Z_2 \leftarrow Z_1$ .

We first stress that the expression of the ratcheting constant only resembles—but is not—that of (the exponential of) a thermodynamic affinity. Indeed, only if each net flux (in parentheses) in Eq. (71) were the flux of a single reaction satisfying local detailed balance—which is not the case for the photo-driven molecular ratchet in Fig. 7(b)—then would  $k_B T \ln K_r$  correspond to the thermodynamic cycle affinity.

We then noticed that the expression of the ratcheting constant in Eq. (71) might seem different from the usual one.<sup>66,71</sup> On the one hand,  $K_r$  is usually written in terms of kinetic constants rather than reaction fluxes, but the two expressions are equivalent. Indeed, by using the fluxes in Eqs. (30) and (61), the dependence on the concentrations of the ground-state species, i.e.,  $[Z_1]$ ,  $[Z_2]$ ,  $[E_1]$ , and  $[E_2]$ , cancels out and the ratcheting constant becomes a function of kinetic constants only or, equivalently, the absorption spectra and quantum yields [see Eqs. (30) and (61)]. We chose not to write  $K_r$  in Eq. (71) [as well as in Eq. (73)] in terms of kinetic constants to maintain a more compact notation. On the other hand, the terms in parentheses accounting for to the interconversions  $Z_1 \rightarrow E_1$ ,  $E_1 \rightarrow Z_1$ ,  $E_2 \rightarrow Z_2$ , and  $Z_2 \rightarrow E_2$ , are usually written as the sum of few fluxes,<sup>66,71,90</sup> while each of these terms in Eq. (71) is given by the sum of five



**FIG. 7.** Mechanism of prototypical autonomous photo-driven molecular ratchets. (a) Network of transitions explicitly accounting for the excited-state species  $Z_1^*$ ,  $E_1^*$ ,  $Z_2^*$ , and  $E_2^*$ . The two horizontal long arrows (close to the thermometers) represent the thermally induced reactions interconverting  $Z_1$  into  $Z_2$ , on the one hand, and  $E_1$  into  $E_2$ , on the other hand. The other arrows represent the diabatic processes interconverting  $Z_1$  into  $E_1$  (via transitions also involving the excited-state species  $Z_1^*$  and  $E_1^*$ ), on the one hand, and  $Z_2$  into  $E_2$  (via transitions also involving the excited-state species  $Z_2^*$  and  $E_2^*$ ), on the other hand. (b) Network of reactions between ground-state species resulting from the coarse graining of the diabatic processes (discussed in Sec. V and illustrated in Fig. 6). The horizontal arrows (close to the thermometers) still represent the thermally induced reactions interconverting  $Z_1$  into  $Z_2$ , on the one hand, and  $E_1$  into  $E_2$ , on the other hand. The other arrows represent the (isomerization and futile) reactions [given in Figs. 6(e)–6(g)] resulting from the coarse graining of the diabatic processes interconverting  $Z_1$  into  $E_1$ , on the one hand, and  $Z_2$  into  $E_2$ , on the other hand. Note that all transitions and reactions have always their backward counterparts, even if they are not represented.

fluxes. The reason for this difference arises from the coarse graining of the diabatic process. As explained in Subsection IV B, we identified the (isomerization and futile) reactions that still satisfy the local detailed balance condition by constructing them using all possible cycles between ground-state species. This led to five reactions. Nevertheless, the usual expression of  $K_r$  can be recovered from Eq. (71) by (i) neglecting the (usually small) fluxes of the isomerization reactions involving thermally induced transitions from a ground-state species to an excited-state species [see Eq. (61)] and (ii) simplifying the concentration dependences using Eqs. (30) and (61). Indeed, this leads to

$$K_r \approx \frac{\left( \int dv \Phi^q(Z_1^* \rightarrow E_1) k_{Z_1^* \rightarrow Z_1}^a(v) n(v) + k_{Z_1^* \rightarrow E_1}^q \right) k_{E_1 \rightarrow E_2}^q}{\left( k_{E_1 \rightarrow Z_1}^q + \int dv \Phi^q(E_1^* \rightarrow Z_1) k_{E_1^* \rightarrow E_1}^a(v) n(v) \right) k_{E_2 \rightarrow E_1}^q} \times \frac{\left( k_{E_2 \rightarrow Z_2}^q + \int dv \Phi^q(E_2^* \rightarrow Z_2) k_{E_2^* \rightarrow E_2}^a(v) n(v) \right) k_{Z_2 \rightarrow Z_1}^q}{\left( \int dv \Phi^q(Z_2^* \rightarrow E_2) k_{Z_2^* \rightarrow Z_2}^a(v) n(v) + k_{Z_2^* \rightarrow E_2}^q \right) k_{Z_1 \rightarrow Z_2}^q}. \quad (72)$$

While Eqs. (71) and (72) (approximately) provide the same numerical value of the ratcheting constant, only the former allows us to identify crucial properties of the working mechanism of photo-driven molecular ratchets.

To do so, we now use the local detailed balance conditions in Eqs. (42) and (65) in the ratcheting constant in Eq. (71), thus obtaining

$$K_r = \frac{\left( \int dv \pi^a \left( Z_1 \xrightarrow{Z_1^*} E_1, v \right) + \Pi^q \left( Z_1 \xrightarrow{Z_1^*} E_1 \right) + R^q(Z_1 \rightarrow E_1) + \Pi^q \left( Z_1 \xrightarrow{E_1^*} E_1 \right) + \int dv \pi^e \left( Z_1 \xrightarrow{E_1^*} E_1, v \right) \right)}{\left( \int dv \pi^a \left( Z_1 \xrightarrow{Z_1^*} E_1, v \right) e^{-\frac{\mu_p(v)}{k_B T}} + \Pi^q \left( Z_1 \xrightarrow{Z_1^*} E_1 \right) + R^q(Z_1 \rightarrow E_1) + \Pi^q \left( Z_1 \xrightarrow{E_1^*} E_1 \right) + \int dv \pi^e \left( Z_1 \xrightarrow{E_1^*} E_1, v \right) e^{\frac{\mu_p(v)}{k_B T}} \right)} \times \frac{\left( \int dv \pi^a \left( Z_2 \xrightarrow{Z_2^*} E_2, v \right) e^{-\frac{\mu_p(v)}{k_B T}} + \Pi^q \left( Z_2 \xrightarrow{Z_2^*} E_2 \right) + R^q(Z_2 \rightarrow E_2) + \Pi^q \left( Z_2 \xrightarrow{E_2^*} E_2 \right) + \int dv \pi^e \left( Z_2 \xrightarrow{E_2^*} E_2, v \right) e^{\frac{\mu_p(v)}{k_B T}} \right)}{\left( \int dv \pi^a \left( Z_2 \xrightarrow{Z_2^*} E_2, v \right) + \Pi^q \left( Z_2 \xrightarrow{Z_2^*} E_2 \right) + R^q(Z_2 \rightarrow E_2) + \Pi^q \left( Z_2 \xrightarrow{E_2^*} E_2 \right) + \int dv \pi^e \left( Z_2 \xrightarrow{E_2^*} E_2, v \right) \right)}, \quad (73)$$

which is independent of the concentrations of the ground-state species, i.e.,  $[Z_1]$ ,  $[Z_2]$ ,  $[E_1]$ , and  $[E_2]$  even if it is written in terms of reaction fluxes [see Eqs. (30) and (61)].

Three important properties of the working mechanism of photo-driven ratchets follow from Eq. (73). The first one is well known, while the other two result from the fact that the isomerization reactions satisfy the local detailed balance condition. We now examine each of these properties and explain their physical meaning. First, if  $\mu_p(v) = 0$ , then  $K_r = 1$  for any value of the reaction fluxes and directionality cannot emerge. This is a direct implication of fact that the radiation is in equilibrium with the

solution when  $\mu_p(v) = 0$ . Hence, no (nonconservative<sup>9</sup>) thermodynamic forces maintain reactions out of equilibrium and the steady state is an equilibrium state satisfying detailed balance. Second, the fluxes of the thermally induced reactions  $Z_1 \rightleftharpoons Z_2$  and  $E_1 \rightleftharpoons E_2$ , i.e.,  $R^q(E_1 \rightarrow E_2)$ ,  $R^q(E_2 \rightarrow E_1)$ ,  $R^q(Z_2 \rightarrow Z_1)$ , and  $R^q(Z_1 \rightarrow Z_2)$ , do not enter the expression of  $K_r$  in Eq. (73). This implies that the equilibrium constants of these reactions, i.e.,

$$K_{E_1 \rightarrow E_2}^{\text{eq}} = \frac{k_{E_1 \rightarrow E_2}^q}{k_{E_2 \rightarrow E_1}^q}, \quad (74a)$$

$$K_{Z_1 \rightarrow Z_2}^{\text{eq}} = \frac{k_{Z_1 \rightarrow Z_2}^{\text{q}}}{k_{Z_2 \rightarrow Z_1}^{\text{q}}}, \quad (74b)$$

do not play any role in determining the directionality. Third, if all the kinetic constants (or, equivalently, the absorption spectra and quantum yields) of the reactions interconverting  $Z_1$  into  $E_1$  were equal to those of the reactions interconverting  $Z_2$  into  $E_2$ , namely,

$$\pi^{\text{a}}\left(Z_1 \xrightarrow{Z_1^*} E_1, \nu\right) / [Z_1] = \pi^{\text{a}}\left(Z_2 \xrightarrow{Z_2^*} E_2, \nu\right) / [Z_2], \quad (75a)$$

$$\Pi^{\text{q}}\left(Z_1 \xrightarrow{Z_1^*} E_1\right) / [Z_1] = \Pi^{\text{q}}\left(Z_2 \xrightarrow{Z_2^*} E_2\right) / [Z_2], \quad (75b)$$

$$R^{\text{q}}(Z_1 \rightarrow E_1) / [Z_1] = R^{\text{q}}(Z_2 \rightarrow E_2) / [Z_2], \quad (75c)$$

$$\Pi^{\text{q}}\left(Z_1 \xrightarrow{E_1^*} E_1\right) / [Z_1] = \Pi^{\text{q}}\left(Z_2 \xrightarrow{E_2^*} E_2\right) / [Z_2], \quad (75d)$$

$$\pi^{\text{e}}\left(Z_1 \xrightarrow{E_1^*} E_1, \nu\right) / [Z_1] = \pi^{\text{e}}\left(Z_2 \xrightarrow{E_2^*} E_2, \nu\right) / [Z_2], \quad (75e)$$

then  $K_r = 1$  and directionality could not emerge.

We conclude by rewriting Eq. (73) in a more standard form. To do so, we (i) neglect the (usually small) fluxes of the thermally induced isomerization reactions (involving thermally induced transitions from a ground-state species to an excited-state species) and (ii) express the remaining fluxes using Eqs. (30) and (61). We thus obtain

$$K_r \approx \frac{\left[ \frac{\int d\nu \Phi^{\text{q}}(Z_1^* \rightarrow E_1) k_{Z_1 \rightarrow Z_1^*}^{\text{a}}(\nu) n(\nu)}{k_{Z_1 \rightarrow E_1}^{\text{q}}} + \int d\nu \phi^{\text{p}}(E_1^* \rightarrow E_1, \nu) k_{Z_1 \rightarrow E_1^*}^{\text{q}} + 1 \right]}{\left[ \frac{\int d\nu \Phi^{\text{q}}(Z_1^* \rightarrow E_1) k_{Z_1 \rightarrow Z_1^*}^{\text{a}}(\nu) n(\nu) e^{-\frac{\mu_{\text{p}}(\nu)}{k_{\text{B}}T}} + \int d\nu \phi^{\text{p}}(E_1^* \rightarrow E_1, \nu) k_{Z_1 \rightarrow E_1^*}^{\text{q}} e^{-\frac{\mu_{\text{p}}(\nu)}{k_{\text{B}}T}} + 1 \right]}{\left[ \frac{\int d\nu \Phi^{\text{q}}(Z_2^* \rightarrow E_2) k_{Z_2 \rightarrow Z_2^*}^{\text{a}}(\nu) n(\nu)}{k_{Z_2 \rightarrow E_2}^{\text{q}}} + 1 \right]} \left[ \frac{\int d\nu \Phi^{\text{q}}(Z_2^* \rightarrow E_2) k_{Z_2 \rightarrow Z_2^*}^{\text{a}}(\nu) n(\nu) e^{-\frac{\mu_{\text{p}}(\nu)}{k_{\text{B}}T}} + \int d\nu \phi^{\text{p}}(E_2^* \rightarrow E_2, \nu) k_{Z_2 \rightarrow E_2^*}^{\text{q}} e^{-\frac{\mu_{\text{p}}(\nu)}{k_{\text{B}}T}} + 1 \right]}{\left[ \frac{\int d\nu \Phi^{\text{q}}(Z_2^* \rightarrow E_2) k_{Z_2 \rightarrow Z_2^*}^{\text{a}}(\nu) n(\nu)}{k_{Z_2 \rightarrow E_2}^{\text{q}}} + 1 \right]} \right]} \quad (76)$$

Equation (76) still shows that directionality is not affected by the equilibrium constants in Eq. (74) and does not emerge if  $\mu_{\text{p}}(\nu) = 0$ . It also explicitly shows that the following equivalence between (experimentally measurable) kinetic constants and quantum yields,

$$\Phi^{\text{q}}(Z_1^* \rightarrow E_1) k_{Z_1 \rightarrow Z_1^*}^{\text{a}}(\nu) = \Phi^{\text{q}}(Z_2^* \rightarrow E_2) k_{Z_2 \rightarrow Z_2^*}^{\text{a}}(\nu), \quad (77a)$$

is not a sufficient condition to get  $K_r = 1$ .

### A. Ratchet mechanism

We now discuss the meaning of the properties of photo-driven ratchets derived from Eq. (73) in terms of ratchet mechanisms.

To do so, we start by recalling that molecular ratchets primarily operate through two distinct (but not mutually exclusive) mechanisms: the energy ratchet and the information ratchet mechanisms.<sup>66–68,71</sup> In the former, directionality is dictated by differences in the free energy of reaction intermediates or, equivalently, equilibrium constants.<sup>91</sup> In the latter, directionality is dictated by differences in the fluxes of the free-energy-harnessing reactions,

e.g., the vertical reactions in Fig. 7(b), determined by the information encoded in the intermediates they interconvert.<sup>90</sup> The independence of the ratcheting constant (73) from the equilibrium constants (74) shows that the mechanism underlying directionality in autonomous photo-driven ratchets does not rely on the energy differences between reaction intermediates. Namely, there is no energy ratchet contribution. The fact that the kinetic constants of the diabatic processes interconverting  $Z_1$  into  $E_1$  and  $Z_2$  into  $E_2$  must differ [i.e., Eq. (75) must not hold] to ensure  $K_r \neq 1$  means that the mechanism underlying directionality in autonomous photo-driven ratchets relies on differences in the fluxes of the free-energy-harnessing reactions. Namely, the ratchet mechanism is a pure information ratchet mechanism.

Until now, autonomous chemically driven and photo-driven ratchets were considered fundamentally different because only the former were believed to operate via a pure information ratchet mechanism. Indeed, so far, only thermally induced reactions underpinning chemically driven ratchets were known to satisfy the local detailed balance condition, implying that differences in the

free energy of reaction intermediates are irrelevant to determine directionality<sup>70,92</sup> (also see Appendix). At the same time, (approximated) expressions such as the one in Eq. (72) led to the (erroneous) idea that the ratchet mechanism of autonomous photo-driven ratchets might include an energy ratchet contribution.<sup>66–69,71</sup> By establishing a local detailed balance condition also for the (isomerization and futile) reactions of the diabatic process, we proved that this is not the case: the ratchet mechanism of autonomous photo-driven ratchets is exactly the same as that of chemically driven ones, i.e., an information ratchet mechanism.

This result reveals a paradigmatic shift in the design and optimization principles of photo-driven ratchets that can be now directly built on top of the insights already well established for chemically driven ratchets. In particular, rather than focusing on increasing energy differences between reaction intermediates, design and optimization strategies should aim to maximize the differentiation between the reaction fluxes of the free-energy-harnessing reactions, as this is the primary factor affecting the ratcheting constant (73) and, therefore, directionality.

Finally, we notice that the conclusion that autonomous photo-driven molecular ratchets operate exclusively through an information ratchet mechanism is derived from the network of reactions shown in Fig. 7(b). This does not diminish its relevance since (i) many autonomous photo-driven molecular ratchets are in fact well described by this network<sup>58–60</sup> and (ii) the design and optimization principles inferred from this network have been shown to remain valid for more complex cases.<sup>35,91</sup> Furthermore, the definition of the ratcheting constant  $K_r$  in Eq. (71) extends naturally to multi-cycle photochemical networks as well,<sup>93</sup> and so does our conclusion. That said, we acknowledge that there also exist photo-driven molecular ratchets whose underlying mechanism does not involve photo-isomerizations.<sup>94</sup> In such cases, our conclusion does not apply, at least not in its present formulation.

## VII. A BRIEF HISTORY OF LOCAL DETAILED BALANCE

Before concluding, we offer a short historical perspective on the local detailed balance condition to contextualize our work.

The local detailed balance condition is a constraint on the ratio of the fluxes of each pair of forward and backward elementary transitions between suitably coarse-grained states describing a system. It guarantees the thermodynamic consistency of the description and, in the absence of (nonconservative) thermodynamic forces, the relaxation of the system to equilibrium. This constraint has emerged independently in different contexts and has been referred to by various names across disciplines. Indeed, it is often called “microscopic reversibility”<sup>95,96</sup> or “generalized detailed balance.”<sup>73</sup> Since systems undergoing thermally and photo-induced transitions/reactions provide two early contexts where the local detailed balance was introduced, we take this opportunity to briefly revisit the historical development of this constraint.

To the best of our knowledge, the earliest instance of a local detailed balance condition is due to Boltzmann, who introduced it in the context of the Boltzmann equation describing the dynamics of dilute gases.<sup>97</sup> There, Boltzmann assumed fundamental time-reversal symmetry between the rates describing the change in momentum caused by molecular collisions that ensures the validity of the H-theorem and thus relaxation toward equilibrium.<sup>98</sup>

Boltzmann’s original work was formulated within the microcanonical ensemble. Later generalizations to the canonical ensemble, relevant for thermally or photo-induced transitions/reactions, emerged independently across multiple fields.

For systems undergoing (thermally induced) chemical reactions, Marcelin formulated an early version of the local detailed balance condition in Eq. (9). He expressed this in terms of what he introduced as the standard Gibbs free energy of activation, namely, the free energy of the common activated state that the pathways of both the forward and backward (thermally induced) chemical reactions must overcome.<sup>99</sup> For this reason, Marcelin’s work is also regarded as the first theoretical treatment that justifies the Arrhenius equation from first principles, by laying the groundwork for the transition state theory.<sup>100</sup> Following similar approaches, other researchers, most notably Tolman<sup>76</sup> and Lewis,<sup>77</sup> introduced the principle of microscopic reversibility for (thermally induced) chemical reactions, leading to the same constraint. It is worth noting that in their original formulations,<sup>76,77</sup> the principle of microscopic reversibility was introduced as a dynamic equilibrium condition, namely, that the net current of each elementary transition/reaction vanishes at equilibrium, rather than in the form of Eq. (9), although the latter follows directly from the former. Subsequently, Van Ryselberghe proposed the first general framework for incorporating thermodynamic constraints in networks of (thermally induced) elementary transitions/reactions arbitrarily far from equilibrium.<sup>101</sup>

For systems undergoing photo-induced transitions/reactions, as emphasized in the main text, Einstein’s thermodynamic argument for postulating the existence of the spontaneous emission can be considered as another early derivation of the local detailed balance condition. Indeed, although perturbation theory of an atom in a classical radiation field predicts only absorption and stimulated emission, Einstein argued that thermodynamic consistency requires an additional emission process.<sup>49,102</sup> By requiring that an atom immersed in a radiation field reaches thermal equilibrium with the blackbody spectrum, Einstein derived that the ratio of the fluxes of absorption and total (spontaneous and stimulated) emission are governed by a Boltzmann factor, as expressed in Eq. (11). Notably, in quantum electrodynamics, both spontaneous and stimulated emissions arise from the same elementary interaction between matter and the quantized electromagnetic field. From this modern perspective, spontaneous and stimulated emissions are not fundamentally distinct processes; namely, they represent different limits of the same physical mechanism.<sup>103</sup> This realization remains underappreciated in most of the chemistry community, where photo-induced transitions are often treated as if they were unconstrained by microscopic reversibility.<sup>104</sup>

Finally, the terminology “local detailed balance” itself was introduced in a 1983 paper by Katz, Lebowitz, and Spohn.<sup>105</sup> However, the conceptual foundations trace back to earlier efforts within the statistical mechanics community to construct consistent models of nonequilibrium systems interacting with multiple reservoirs.<sup>1,2,106</sup> The key insight is that when multiple reservoirs promote distinct transitions, local thermodynamic constraints naturally arise at the level of individual transitions. These constraints depend on the specific quantities, such as energy, entropy, or matter, which are exchanged with the corresponding reservoir. They provide a constructive principle for building physically sound models of nonequilibrium behavior and ensure detailed balance, i.e., the existence of

an equilibrium state, is recovered in the absence of nonconservative forces.

Nowadays, local detailed balance lies at the core of nonequilibrium formulations of thermodynamics,<sup>2,3,107–112</sup> including in chemical processes.<sup>7,9,10</sup>

## VIII. DISCUSSION AND FURTHER PERSPECTIVES

In this paper, we established a general thermodynamic theory of molecular systems undergoing thermally and photo-induced transitions. By building on the last historical developments highlighted in Sec. VII, we showed that local detailed balance can be preserved even when combining thermally and photo-induced transitions, thus unifying two historically important forms of local detailed balance in the context of chemistry and radiation.

We further developed two sequential thermodynamically consistent coarse-graining procedures. The first eliminates vibrational states by assuming their rapid equilibration within electronic states. The second eliminates electronic excited states by assuming their rapid relaxation to a, in general, nonequilibrium steady state. The final result is a thermodynamic theory for photo-isomerization reactions interconverting ground-state species, which are proven to satisfy the local detailed balance conditions just as the elementary transitions do. This theory allowed us to shed light on the working mechanism of photo-driven ratchets. Namely, we showed that they operate exclusively through an information ratchet mechanism when powered by a constant light source in the same way that chemically driven ratchets do when powered by a constant chemical potential gradient.

Although we explicitly considered here only unimolecular transitions, thermally induced reactions between ground state species of arbitrary molecularity can be included in the theory too (similarly to what has been done in Ref. 29). At the current stage, the theory cannot consider multimolecular transitions between excited- and ground-state species such as those involved in bimolecular quenchings, photo-additions, and photo-dissociations.

We furthermore focused on photo-driven systems, where light is the sole source of free energy. Nevertheless, our theory can be straightforwardly combined with thermodynamic theories for chemically driven systems,<sup>5–11</sup> thus allowing the characterization of systems with both photo-driven and chemically driven reactions. In such systems, photons can be transduced into chemical energy and vice versa. Analyzing these transduction processes in terms of thermodynamic gears<sup>88,113</sup> may pave the way toward systematic studies of efficiency in photosynthetic systems.

Finally, we stress that the working mechanism of photo-driven ratchets could only be correctly identified by applying the local detailed balance condition to the exact expression of the ratcheting constant in Eq. (71). Applying the same condition to the approximate expression in Eq. (72) fails to reveal the same working mechanism. This discrepancy arises from neglecting certain reaction fluxes in Eq. (72) that, although usually negligible in magnitude, are not independent of the remaining ones. As a result, their omission obscures the constraints imposed by thermodynamics on the ratcheting constant in the same way as overlooking local detailed balance conditions can lead to incorrect conclusions about chemically driven systems.<sup>95,96</sup>

## ACKNOWLEDGMENTS

F.A. was supported by Project No. P-DiSC#BIRD2023-UNIPD funded by the Department of Chemical Sciences of the University of Padova (Italy). M.E. was supported by the Fond National de la Recherche-FNR, Luxembourg, by the Project Chem-Complex (Grant No. C21/MS/16356329) and by Project No. INTER/FNRS/20/15074473 funded by F.R.S.-FNRS (Belgium) and FNR.

## AUTHOR DECLARATIONS

### Conflict of Interest

The authors have no conflicts to disclose.

### Author Contributions

**Francesco Avanzini:** Conceptualization (equal); Data curation (equal); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Software (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **Massimiliano Esposito:** Conceptualization (equal); Data curation (equal); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Software (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **Emanuele Penocchio:** Conceptualization (equal); Data curation (equal); Formal analysis (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Software (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal).

## DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

## APPENDIX: RATCHETING CONSTANT OF CHEMICALLY DRIVEN RATCHETS

We consider a prototypical chemically driven molecular ratchet,<sup>66–68,90</sup> whose network of reactions is illustrated in Fig. 8, and recap how the existence of a local detailed balance condition affects its ratcheting constant  $K_r$ .

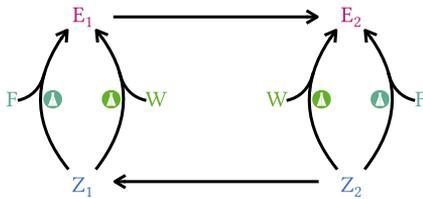
We start by recalling that, according to mass-action kinetics, the reaction fluxes read

$$R^F(Z_1 \rightarrow E_1) = k_{E_1 \rightarrow E_1}^F [F][Z_1], \quad (\text{A1a})$$

$$R^F(E_1 \rightarrow Z_1) = k_{E_1 \rightarrow Z_1}^F [E_1], \quad (\text{A1b})$$

$$R^W(Z_1 \rightarrow E_1) = k_{Z_1 \rightarrow E_1}^W [W][Z_1], \quad (\text{A1c})$$

$$R^W(E_1 \rightarrow Z_1) = k_{E_1 \rightarrow Z_1}^W [E_1], \quad (\text{A1d})$$



**FIG. 8.** Reaction mechanism of a prototypical chemically driven molecular ratchet. All reactions are thermally induced and involve only ground-state species. The horizontal arrows represent the reactions interconverting  $Z_1$  into  $Z_2$ , on the one hand, and  $E_1$  into  $E_2$ , on the other hand. The other arrows (close to the flasks) represent reactions coupled to chemostats (i.e., chemical reservoirs of either F or W) interconverting  $Z_1$  into  $E_1$ , on the one hand, and  $Z_2$  into  $E_2$ , on the other hand. Each reaction always has a backward counterpart, even if it is not represented.

$$R(E_1 \rightarrow E_2) = k_{E_1 \rightarrow E_2} [E_1], \quad (\text{A1e})$$

$$R(E_2 \rightarrow E_1) = k_{E_2 \rightarrow E_1} [E_2], \quad (\text{A1f})$$

$$R^F(Z_2 \rightarrow E_2) = k_{Z_2 \rightarrow E_2}^F [F][Z_2], \quad (\text{A1g})$$

$$R^F(E_2 \rightarrow Z_2) = k_{E_2 \rightarrow Z_2}^F [E_2], \quad (\text{A1h})$$

$$R^W(Z_2 \rightarrow E_2) = k_{Z_2 \rightarrow E_2}^W [W][Z_2], \quad (\text{A1i})$$

$$R^W(E_2 \rightarrow Z_2) = k_{E_2 \rightarrow Z_2}^W [E_2], \quad (\text{A1j})$$

$$R(Z_1 \rightarrow Z_2) = k_{Z_1 \rightarrow Z_2} [Z_1], \quad (\text{A1k})$$

$$R(Z_2 \rightarrow Z_1) = k_{Z_2 \rightarrow Z_1} [Z_2], \quad (\text{A1l})$$

and must satisfy the following local detailed conditions:

$$k_B T \ln \frac{R^F(Z_1 \rightarrow E_1)}{R^F(E_1 \rightarrow Z_1)} = -(\mu_{E_1} - \mu_{Z_1} - \mu_F), \quad (\text{A2a})$$

$$k_B T \ln \frac{R^W(Z_1 \rightarrow E_1)}{R^W(E_1 \rightarrow Z_1)} = -(\mu_{E_1} - \mu_{Z_1} - \mu_W), \quad (\text{A2b})$$

$$k_B T \ln \frac{R(E_1 \rightarrow E_2)}{R(E_2 \rightarrow E_1)} = -(\mu_{E_2} - \mu_{E_1}), \quad (\text{A2c})$$

$$k_B T \ln \frac{R^F(Z_2 \rightarrow E_2)}{R^F(E_2 \rightarrow Z_2)} = -(\mu_{E_2} - \mu_{Z_2} - \mu_F), \quad (\text{A2d})$$

$$k_B T \ln \frac{R^W(Z_2 \rightarrow E_2)}{R^W(E_2 \rightarrow Z_2)} = -(\mu_{E_2} - \mu_{Z_2} - \mu_W), \quad (\text{A2e})$$

$$k_B T \ln \frac{R(Z_1 \rightarrow Z_2)}{R(Z_2 \rightarrow Z_1)} = -(\mu_{Z_2} - \mu_{Z_1}). \quad (\text{A2f})$$

Here, the species F and W are chemostats, namely, they represent the source of free energy that the chemically driven ratchet uses to stay out of equilibrium. Their concentrations, as well as

their chemical potentials, are, therefore, treated as constant (control) parameters. Furthermore, all reactions are thermally induced, but we dropped the superscript q to specify the chemostats they are coupled to (F or W or neither).

For the chemically driven molecular ratchet in Fig. 8, the ratcheting constant reads

$$K_r = \frac{(R^F(Z_1 \rightarrow E_1) + R^W(Z_1 \rightarrow E_1))R(E_1 \rightarrow E_2)}{(R^F(E_1 \rightarrow Z_1) + R^W(E_1 \rightarrow Z_1))R(E_2 \rightarrow E_1)} \times \frac{(R^F(E_2 \rightarrow Z_2) + R^W(E_2 \rightarrow Z_2))R(Z_2 \rightarrow Z_1)}{(R^F(Z_2 \rightarrow E_2) + R^W(Z_2 \rightarrow E_2))R(Z_1 \rightarrow Z_2)} \quad (\text{A3})$$

and becomes

$$K_r = \frac{\left(\frac{R^F(Z_1 \rightarrow E_1)}{R^W(Z_1 \rightarrow E_1)} + 1\right) \left(\frac{R^F(Z_2 \rightarrow E_2)}{R^W(Z_2 \rightarrow E_2)} e^{\frac{\mu_W - \mu_F}{k_B T}} + 1\right)}{\left(\frac{R^F(Z_1 \rightarrow E_1)}{R^W(Z_1 \rightarrow E_1)} e^{\frac{\mu_W - \mu_F}{k_B T}} + 1\right) \left(\frac{R^F(Z_2 \rightarrow E_2)}{R^W(Z_2 \rightarrow E_2)} + 1\right)} \quad (\text{A4})$$

by applying the local detailed balance conditions in Eq. (A2). Notice that  $K_r$  in Eqs. (A3) and (A4) is independent of the concentrations  $[Z_1]$ ,  $[Z_2]$ ,  $[E_1]$ , and  $[E_2]$  even if it is written in terms of reaction fluxes rather than kinetic constants [see Eq. (A1)]. We chose to write  $K_r$  in terms of reaction fluxes for consistency with what we have done in the main text for photo-driven ratchets in Eqs. (71) and (73).

Finally, we recap the properties of the working mechanism of chemically driven ratchets that follow from Eq. (A4). Crucially, these are the same properties we identified for photo-driven ratchets. First, if  $\mu_W - \mu_F = 0$ , then  $K_r = 1$ . Namely, if the chemostats are in equilibrium, there are no (nonconservative) thermodynamic forces maintaining reactions out of equilibrium and directionality cannot emerge. Second,  $K_r$  is independent of the reaction fluxes  $R(E_1 \rightarrow E_2)$ ,  $R(E_2 \rightarrow E_1)$ ,  $R(Z_2 \rightarrow Z_1)$ , and  $R(Z_1 \rightarrow Z_2)$  and, therefore, of the equilibrium constants,

$$K_{E_1 \rightarrow E_2}^{\text{eq}} = \frac{k_{E_1 \rightarrow E_2}}{k_{E_2 \rightarrow E_1}}, \quad (\text{A5a})$$

$$K_{Z_1 \rightarrow Z_2}^{\text{eq}} = \frac{k_{Z_1 \rightarrow Z_2}}{k_{Z_2 \rightarrow Z_1}}. \quad (\text{A5b})$$

Namely, directionality in chemically driven ratchets does not rely on the energy differences between reaction intermediates. Third, if

$$R^F(Z_1 \rightarrow E_1)/[Z_1] = R^F(Z_2 \rightarrow E_2)/[Z_2], \quad (\text{A6a})$$

$$R^W(Z_1 \rightarrow E_1)/[Z_1] = R^W(Z_2 \rightarrow E_2)/[Z_2], \quad (\text{A6b})$$

then  $K_r = 1$ . Namely, directionality in chemically driven ratchets relies on differences in the fluxes of the free-energy-harnessing reactions.

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