Distributed Precoding Design for Satellite Swarms under Imperfect Phase Synchronization

Biniam Tamiru, Konstantinos Ntontin, Vu Nguyen Ha, and Symeon Chatzinotas

Interdisciplinary Centre for Security, Reliability and Trust (SnT), University of Luxembourg
Email: {biniam.tamiru, kostantinos.ntontin, vu-nguyen.ha, symeon.chatzinotas}@uni.lu

***Abstract*—In this work, we consider the problem of distributed precoding design in a downlink transmission scenario from a swarm of low-Earth-orbit (LEO) satellites to multiple ground users, under imperfect phase synchronization. The formulated problem aims to maximize the expected sum rate under a probabilistic constraint on the signal-to-interference plus noise ratio (SINR). An analytical approach for solving the problem is provided, which avoids conventional approaches that rely on sample averaging and could result in a notable execution time. Simulation results reveal that our approach outperforms conventional precoding techniques and the case of collocated antennas on a single satellite up to a certain phase error variance.**

***Index Terms*—Satellite swarms, phase synchronization errors, precoding design**

I. Introduction

A. Background

Low-Earth-orbit (LEO) satellites are adopted in hybrid terrestrial and non-terrestrial communications to provide network coverage in areas that lack an adequate terrestrial infrastracture, such as remote/rural areas [1]. Similar to terrestrial base stations, LEO satellites can be deployed for multi-beam communications, where they transmit multiple beams simultaneously to support data transmission to different receivers [2]. In particular, a non-terrestrial massive multiple-input multiple-output (MIMO) communication system is described and analyzed in [3], where the authors consider a single LEO satellite equipped with multiple antennas that generates multiple beams directed at different users. Furtnermore, the necessity of efficient precoding techniques to reduce the effect of inter-beam interference in such multi-beam satellite communications is discussed in [4], whereas channel state information (CSI) estimation is addressed in [5].

Although the deployment of multiple-antenna LEO satellites for network coverage shows promising results, it has some limitations. In particular, to accommodate a large number of antennas on a single LEO satellite requires satellites of notable dimensions, especially for sub-6 GHz communication. This would go against the current trend of reducing the size of LEO satellites that in turn reduces their manufacturing and launching costs. Hence, satellite miniaturization and distributed satellite systems (DSS) have gained popularity over the last years [6], [7].

In pursuit of such DSSs, satellite swarms have been proposed [8], [9]. Swarms are a collection of satellites that fly in close proximity to each other to form a virtual array (VA). The size of satellites in a swarm can be small, which makes them easily manageable as well as maneuverable and significantly reduces the required operation cost. Additionally, the distance separation between the satellites can be adjusted so to allow for higher directive gain towards the intended receiver, and hence, enhancing the received signal power [10]. A downlink tranmission scenario from a VA to a ground station (GS) is presented in [11].

Despite the advantages of satellite swarms in reducing the manufacturing and launching costs of LEO satellites, there are practical challenges in deploying them. In particular, due to size limitations of small LEO satellites highly capable power amplifiers cannot be accommodated. Instead, the satellites in the swarm can be leveraged for jointly transmitting to ground users, thus avoiding the need for a high transmit power output per satellite. Towards this, proper coordination among the satellites in the swarm is needed in the form of synchronization in time, frequency, and phase which can be achieved by inter-satellite communication [12]. Despite the electronic advances in achieving synchronization among distributed nodes, a level of imperfect synchronization cannot be avoided. Among imperfect synchronization in time, frequency, and phase, particularly impactful is the lack of perfect synchronization in phase. This can normally arise due to imperfect inter-satellite ranging [8].

Due to the inevitable phase errors related to the inter-satellite ranging process, the precoding design for the swarm must be robust to minimize the performance loss as a result of this error. The knowledge about the phase error distribution can be leveraged to design robust precoders. Regarding relevant literature works, a stochastic weighted minimum-mean square error (SMMSE) approach is analyzed in [13] to reformulate a non-convex sum-rate maximization problem into an equivalent computationally less expensive objective function. Although this method is effective, it relies on sample realizations of the random variable and aggregation of instantaneous solutions until convergence. This could result in a considerable execution time, depending on the size of the parameters, such as the number of satellites, number of users, and the type of additional constraints to be included. The authors of [14], [15] devised a method to minimize the total power consumption of satellites under a probabilistic QoS constraint. This method utilizes the Taylor series to approximate a function of complex exponentials to polynomial expressions. The stochastic constraint is decomposed into a set of convex constraints as shown in [16]. The polynomial term is then re-defined into a set of convex expressions. Semidefinite programming (SDP) and Gaussian randomization are then used to solve the reformulated optimization problem.

B. Motivation and Contribution

1) Motivation: The few literature works on precoding design for distributed LEO satellite swarms in a multiple- user setup either consider perfect phase synchronization among the satellites or precoding design under imperfect phase synchronization with a probabilistic SINR constraint [15]. To the best of our knowledge, there have not been literature works that consider the practical case of robust precoding design for LEO satellite swarms where both the objective function and at least one of the constraints are statistical due to the presence of phase errors. In addition, as aforementioned, previous approaches consider sample averaging of Monte Carlo simulations for the phase errors instead of an analytical approach, which can lead to a considerable execution time.

2) Contribution: Based on the above, our contribution in this work is as follows:

- We formulate a novel distributed precoding design problem for LEO satellite swarms that targets the maximization of the expected user sum rate under a probabilistic SINR constraint in the presence of random phase errors from a Gaussian distribution.
- We provide analytical expression for the objective function using the phase error covariance. Due to the non-convexity of the problem, it has to be reformulated into a form that comprises an inner and outer optimization problems. We combine these expressions to form a single convex form in contrast to earlier works that iteratively alternate and solve these problems using samples from the error distribution.
- We provide numerical results for different scenarios and compare the performance of our approach with a conventional precoding method and also the robust precoder in the collocated antennas on a single satellite scenario. This allows us to identify the error threshold above which distributed satellite systems, with the specified system parameters, become no longer favorable.

The paper is organized as follows. Section II presents the system model and introduces the problem under consideration. Section III describes the proposed approach to solve the problem. It involves replacing the original non-convex problem with an equivalent and convex substitute problem. Section IV shows the simulation results for different cases considered. Section V summarizes the findings and concludes the work.

II. System model and Problem formulation

A. System Model

We consider K single-antenna users and a swarm of S single-antenna LEO satellites arranged, without loss of generality, in a spiral geometry [17]. The reason for considering the spiral geometry in this work is its ability to reduce the magnitude of the grating lobes compared to a plannar array formation. In addition, we consider single-antenna satellites since satellite swarms are expected to comprise small satellites, such as Cubesats, with limited size. Hence, they can only accommodate single antennas [18]. The wave vector, along the azimuth and elevation angles of φ and θ, respectively, is defined as $k(\varphi, \theta) = \frac{2\pi}{\lambda}[\cos\theta\cos\varphi,\ \cos\theta\sin\varphi,\ \sin\theta]^T$. For spiral formation, the position of the satellites is expressed as $\mathbf{u}_s = [r_s\cos\psi_s, r_s\sin\psi_s, 0]^T$ [17].

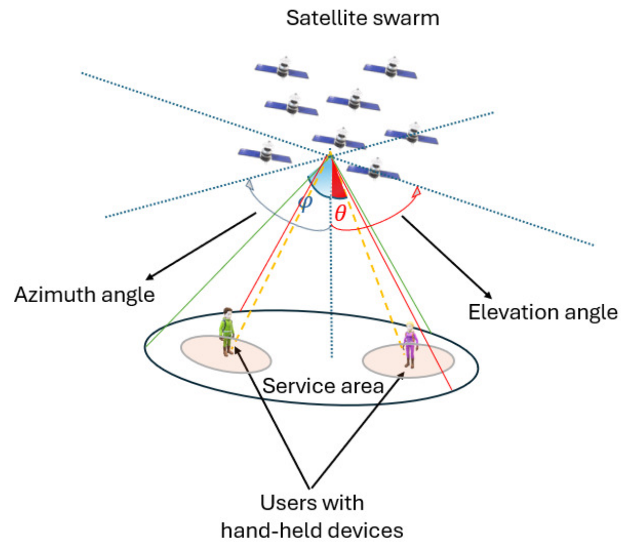


Fig. 1: Satellite swarm in spiral formation.

$$r_s = \frac{d}{\sqrt{\pi}} \times \sqrt{s}, \quad \psi_s = 2\pi\tau s, \tag{1}$$

where r_s, ψ_s are the distance and angle (respectively) of the center of satellite s with respect to the swarm center (or reference frame). τ is the golden ratio whose value is 1.618. As indicated in [9], the array factor is expressed as

$$\mathbf{a}(\varphi, \theta) = [\mathbf{a}_1(\varphi, \theta), \ldots, \mathbf{a}_S(\varphi, \theta)], \tag{2}$$

where $\mathbf{a}_s(\varphi, \theta) = g(\theta)\left[e^{jk^T(\varphi,\theta)\mathbf{u}_s}\right]$, and $g(\theta)$ is the element radiation pattern.

Following the model given in [19], if we denote the array factor to user k as $\mathbf{a}^k(\varphi_k, \theta_k)$, the channel from the swarm to user k, $\mathbf{h}_k \in \mathbb{C}^{1\times S}$, can be modelled as

$$\mathbf{h}_k = \mathbf{a}^k(\varphi_k, \theta_k)L^{fs}(\theta_k) \tag{3}$$

where θ_k the elevation angle to user k. $L^{fs}(\theta_k) = \frac{\lambda}{4\pi r_k}$ is the free space attenuation from the satellite swarm to the user at a distance of r_k. The noise , n_k, is assumed to be a complex gaussian with zero mean and power of κBT, where κ is Boltzmann constant, T is the system noise temperature, and B is bandwidth. The received signal, $y_k \in \mathbb{C}$, at user k is modelled as

$$y_k = \mathbf{h}_k\mathbf{v}_k s_k + \sum_{l\neq k}\mathbf{h}_k\mathbf{v}_l s_l + n_k, \tag{4}$$

where $\mathbf{v}_k$ and s_k are the precoding vector and transmitted symbol for user k respectively. Moreover, we assume $\mathbb{E}[s_k s_k^H] = 1$. Then, the SINR for user k given as

$$\text{SINR}_k = \frac{|\mathbf{h}_k \mathbf{v}_k|^2}{P_n + \sum_{l \neq k} |\mathbf{h}_k \mathbf{v}_l|^2}, \tag{5}$$

where P_n is the noise variance. The true channel can be modelled as the hadamard product between the phase error vector, $\mathbf{e} = [e^{j\phi_1}, e^{j\phi_2}, \ldots e^{j\phi_S}]^H$(arising from an imperfect phase synchronization process) and the erroneous channel. $\mathbf{h}_k = \mathbf{h}'_k \odot \mathbf{e}_k^H = [e^{j\phi_1} h'_{k1}, e^{j\phi_2} h'_{k2}, \ldots e^{j\phi_S} h'_{kS}]$. We note that in this work we consider that the main factor of imperfect phase synchronization among the satellites are the limitations in the intersatellite ranging accuracy due to continuous perturbations of the satellites. Depending on the carrier frequency, even very small inaccuracies in the order of cm/mm can result in notable phase rotations.

B. Problem formulation

We aim to find optimal precoders maximizing the expected user sum rate in the presence of random Gaussian-distribution phase errors and under a probabilistic SINR constraint for each user as well as a per-satellite power constraint.

$$\max_{\mathbf{v}_1,\ldots,\mathbf{v}_K} \mathbb{E}_{\mathbf{e}} \left[\sum_{k=1}^{K} \frac{B}{K} \log \left(1 + \frac{|(\mathbf{h}'_k \odot \mathbf{e}_k{}^H)\mathbf{v}_k|^2}{P_n + \sum_{l \neq k} |(\mathbf{h}'_k \odot \mathbf{e}_k^H)\mathbf{v}_l|^2} \right) \right]$$

$$\begin{aligned} \text{subject to : } & \mathbf{Tr}(\mathbf{m}_s^H \mathbf{V}\mathbf{V}^H \mathbf{m}_s) \leq P_s \qquad \forall s, \\ & \mathbf{Pr}\{\text{SINR}_k > \mu_k\} \geq \epsilon_k, \end{aligned} \tag{P}$$

where $\mathbb{E}_{\mathbf{e}}(.)$ represents the expectation with respect to the phase error $\mathbf{e}$, $\mathbf{V} \in \mathbb{C}^{S \times S}$, such that the k^{th} column is $\mathbf{v}_k$, P_s is the per satellite power constraint, μ_k and ϵ_k are the lower bounds for the SINR and probability, respectively. $\mathbf{m}_s \in \mathbb{R}^{S \times 1}$ is a vector, where $\mathbf{m}_s(s) = 1$ and 0 elsewhere. The objective function in the above problem is non-convex and the deterministic version is shown to be NP-hard [20]. Since a stochastic form of this objective function, which maximizes the expectation over the error distribution, does not change the structure of the problem, it is also non-convex and NP-hard.

III. Proposed solution approach

Since problem (P) is NP-hard and cannot therefore be solved in polynomial time, an alternative approach is required to transform it into a form that is easier to solve. We adopt an equivalent objective function proposed by [13] in which additional optimization variables are introduced to decompose the problem into two parts. Considering that users are equipped with single antennas and ignoring the constant terms, we have the following.

$$\begin{aligned} &\max_{\mathbf{v}_1,\ldots,\mathbf{v}_K} \mathbb{E}_{\mathbf{e}} \left[\sum_{k=1}^{K} \frac{B}{K} \log \left(1 + \frac{|\mathbf{h}_k \mathbf{v}_k|^2}{P_n + \sum_{l \neq k} |\mathbf{h}_k \mathbf{v}_l|^2} \right) \right] \\ &\equiv \min_{\mathbf{V}} \mathbb{E}_{\mathbf{e}} \left[\min_{\mathbf{W},\mathbf{Z}} \frac{B}{K} \sum_{k=1}^{K} - \log \mathbf{w}_k + \mathbf{w}_k \mathbf{E}_k(\mathbf{V}, \mathbf{H}) + \beta \|\mathbf{z}_k - \mathbf{v}_k\|^2 \right], \end{aligned} \tag{6}$$

where,

$$\begin{aligned} \mathbf{E}_k(\mathbf{V}, \mathbf{H}) =& (1 - (\mathbf{h}'_k \odot \mathbf{e}_k{}^H)\mathbf{v}_k)(1 - (\mathbf{h}'_k \odot \mathbf{e}_k{}^H)\mathbf{v}_k)^H \\ &+ \sum_{j \neq k} (\mathbf{h}'_k \odot \mathbf{e}_k{}^H)\mathbf{v}_j \mathbf{v}_j{}^H (\mathbf{h}'_k \odot \mathbf{e}_k{}^H)^H + P_n. \end{aligned} \tag{7}$$

Now, we formulate a single compact form expression to avoid alternating between the inner and outer optimization problems by generating samples.

Proposition 1. *The values of* $\mathbf{W}$ *and* $\mathbf{V}$ *that minimize inner optimization are:*

$$\mathbf{w}_k = \frac{1}{\ln 2} \mathbf{E}_k^{-1} \text{ and } \mathbf{z}_k = \mathbf{v}_k, \quad \forall k. \tag{8}$$

Proof: See Appendix A. ■

Substituting these values back into the inner function, and ignoring constant terms yields

$$\min_{\mathbf{v}_1,\ldots,\mathbf{v}_K} \mathbb{E}_{\mathbf{e}} \left[\sum_{k=1}^{K} \mathbf{E}_k(\mathbf{V}, \mathbf{h}_k) \right]. \tag{9}$$

Let $\mathbf{M}_k = \text{diag}(\mathbf{h}'_k)$, so that $\mathbf{h}'_k \odot \mathbf{e}_k{}^H = \mathbf{e}_k^H \mathbf{M}_k$, which yields

$$\begin{aligned} \mathbf{E}_k(\mathbf{V}, \mathbf{H}) =& (1 - \mathbf{e}_k^H \mathbf{M}_k \mathbf{v}_k)(1 - \mathbf{e}_k^H \mathbf{M}_k \mathbf{v}_k)^H \\ &+ \sum_{j \neq k} (\mathbf{e}_k^H \mathbf{M}_k)\mathbf{v}_j \mathbf{v}_j{}^H (\mathbf{e}_k^H \mathbf{M}_k)^H + P_n \\ =& 1 - \mathbf{e}_k^H \mathbf{M}_k \mathbf{v}_k - \mathbf{v}_k^H \mathbf{M}_k^H \mathbf{e}_k + \mathbf{e}_k^H \mathbf{M}_k \mathbf{v}_k \mathbf{v}_k^H \mathbf{M}_k^H \mathbf{e}_k \\ &+ \sum_{j \neq k} \mathbf{e}_k^H \mathbf{M}_k \mathbf{v}_j \mathbf{v}_j{}^H \mathbf{M}_k^H \mathbf{e}_k + P_n \\ =& 1 - 2\mathbf{Re}(\mathbf{e}_k^H \mathbf{M}_k \mathbf{v}_k) + \mathbf{e}_k^H \mathbf{M}_k \mathbf{v}_k \mathbf{v}_k^H \mathbf{M}_k^H \mathbf{e}_k \\ &+ \sum_{j \neq k} \mathbf{e}_k^H \mathbf{M}_k \mathbf{v}_j \mathbf{v}_j{}^H \mathbf{M}_k^H \mathbf{e}_k + P_n. \end{aligned} \tag{10}$$

Using the above equation, the expectation of $\mathbf{E}_k$ is as follows:

$$\begin{aligned} \mathbb{E}_{\mathbf{e}} \left[\sum_{k=1}^{K} \mathbf{E}_k(\mathbf{V}, \mathbf{h}_k) \right] =& \mathbb{E}_{\mathbf{e}} \left[\sum_{k=1}^{K} \Big(-2\text{Re}(\mathbf{e}_k^H \mathbf{M}_k \mathbf{v}_k) \right. \\ &\left. + \mathbf{e}_k^H \mathbf{M}_k \mathbf{v}_k \mathbf{v}_k^H \mathbf{M}_k^H \mathbf{e}_k + \sum_{j \neq k} \mathbf{e}_k^H \mathbf{M}_k \mathbf{v}_j \mathbf{v}_j{}^H \mathbf{M}_k^H \mathbf{e}_k \Big) \right] \\ =& \mathbb{E}_{\mathbf{e}} \left[\sum_{k=1}^{K} -2\text{Re}(\mathbf{e}_k^H \mathbf{M}_k \mathbf{v}_k) + \sum_{j=1}^{K} \mathbf{e}_k^H \mathbf{M}_k \mathbf{v}_j \mathbf{v}_j{}^H \mathbf{M}_k^H \mathbf{e}_k \right]. \end{aligned} \tag{11}$$

Since $\mathbf{e}_k^H \mathbf{M}_k \mathbf{v}_j \mathbf{v}_j{}^H \mathbf{M}_k^H \mathbf{e}_k$ is a scalar number, we can rewrite it as $\mathbf{v}_j{}^H \mathbf{M}_k^H \mathbf{e}_k \mathbf{e}_k^H \mathbf{M}_k \mathbf{v}_j = \mathbf{v}_j^H \Phi_k \mathbf{v}_j$, where $\Phi_k = \mathbf{M}_k^H \mathbf{e}_k \mathbf{e}_k^H \mathbf{M}_k$. Let $\mathbf{E} = \mathbb{E}_{\mathbf{e}}[\mathbf{e}_k \mathbf{e}_k^H]$, and $\mathbf{R}_k = \mathbb{E}_{\mathbf{e}}[\Phi_k] = \mathbf{M}_k^H \mathbf{E} \mathbf{M}_k$, then (11) can be written as

$$\begin{aligned} &\mathbb{E}_{\mathbf{e}} \left[\sum_{k=1}^{K} \mathbf{E}_k(\mathbf{V}, \mathbf{h_k}) \right] \\ &= -\sum_{k=1}^{K} 2\text{Re}\left(\mathbb{E}_{\mathbf{e}} \left[\mathbf{e}_k^{\text{H}} \right] \text{M}_k \mathbf{v}_k \right) + \sum_{k=1}^{K} \sum_{j=1}^{K} \mathbf{v}_j^H \mathbb{E}_{\mathbf{e}} \left[\Phi_k \right] \mathbf{v}_j \\ &= \sum_{k=1}^{K} \left[-2\mathbb{E}_{\mathbf{e}} \left(\mathbf{e}_k^H \right) \text{Re} \left(\mathbf{M}_k \mathbf{v}_k \right) + \mathbf{v}_k^H \Big(\sum_{j=1}^{K} \mathbf{R}_j \Big) \mathbf{v}_k \right]. \end{aligned} \tag{12}$$

Proposition 2. *(12) is convex for uncorrelated Gaussian phase errors.*

Proof: see Appendix B. ∎

Since the users share the same set of transmitter antennas, we can assume $\mathbf{e}_k = \mathbf{e}, \quad \forall k$.

Regarding the probabilistic SINR constraint, we adopt the method used in [15] to approximate the stochastic constraint with a set of multiple deterministic constraints by leveraging the knowledge about the nature of Gaussian phase error distributions. We have,

$$\mathbf{Pr}\{\text{SINR}_k > \mu_k\} \geq \epsilon_k \equiv \mathbf{Pr}_{(\Delta)}\{\mathbf{e}_k^H \Omega_k \mathbf{e_k} \geq \mu_k\} \geq \epsilon_k, \quad (13)$$

where

$$\Omega_k = \text{diag}\left(\hat{h}_k^H\right)\left(\frac{1}{\eta_k}\mathbf{P}_k - \sum_{j\neq k}\mathbf{P}_j\right)\text{diag}\left(\hat{h}_k\right), \quad (14)$$

$$\mathbf{\Delta} = [\phi_1, \phi_2, ...\phi_S], \quad (15)$$

in which $\mathbf{P}_k = \mathbf{v}_k\mathbf{v}_k^H$. Let's define

$$[f(\mathbf{X})]_{m,n} = \begin{cases}\mathbf{X}_{m,n} - \sum_l \mathbf{X}_{m,l}, & \text{for} m = n,\\ \mathbf{X}_{m,n}, & \text{for } m \neq n,\end{cases} \quad (16)$$

$$\text{and} \qquad [g(\mathbf{Y})]_{m,n} = 2\sum_l \mathbf{Y}_{m,l}. \quad (17)$$

Then, the constraint in (13) is broken down into [15]

$$\begin{aligned}
(C1): \quad & \sum_{m,n} \mathbf{\Omega}_{k,[m,n]} - P_n + \text{Tr}(\sigma_\phi^2 \mathbf{F}_k)\\
& \qquad \geq 2\sqrt{-\ln(1-\epsilon_k)}(x_k + y_k),\\
(C2): \quad & \frac{1}{2\sqrt{2}}||\sigma_\phi \mathbf{G}_k||_2 \leq x_k,\\
(C3): \quad & \alpha_k ||\mathbf{vec}(\sigma_\phi^2 \mathbf{F}_k)||_2 \leq y_k,\\
(C4): \quad & \Omega_k = \mathbf{X}_k + j\mathbf{Y}_k,\\
(C5): \quad & \mathbf{F}_k = f(\mathbf{X}_k), \mathbf{G}_k = g(\mathbf{Y}_k),\\
(C6): \quad & \mathbf{P}_k \succcurlyeq 0.
\end{aligned}$$

where α_k must satisfy the condition $\left(1 - \frac{1}{2\alpha_k^2}\right)\alpha_k = \sqrt{-\ln(1-\epsilon_k)}$ [16]. The SINR constraint parameter is $\mathbf{P}$, while the precoding vector in the objective function is $\mathbf{v}$. Since we cannot directly impose the relationship $\mathbf{P}_k = \mathbf{v}_k\mathbf{v}_k^H$, which would not be a convex constraint, we relax the condition and add a penalty expression into the objective function to ensure that the squared-norm of $\mathbf{v}_k$ is close to the trace of $\mathbf{P}_k$. Let β be our scaling coefficient for the penalty. We can redefine our convex objective function function as follows,

$$\begin{aligned}
\min_{\mathbf{V},\mathbf{P},\mathbf{x},\mathbf{y},t} \; & \mathbb{E}_\mathbf{e}\left[\sum_{k=1}^K \mathbf{E}_k(\mathbf{V},\mathbf{h}_k) + \beta t\right]\\
\text{subject to}: \; & -\text{Tr}(\mathbf{P}_k) + \mathbf{v}_k{}^{\text{H}}\mathbf{v}_k \leq t,\\
& \text{Tr}(\mathbf{m}_s^H \mathbf{V}\mathbf{V}^H \mathbf{m}_s) \leq P_s \qquad \forall s,\\
& t \geq 0,\\
& \text{constraints } (C1) - (C6).
\end{aligned} \quad (Q)$$

Remark 1. *If $\phi_s \rightharpoonup N(\mu_\phi, \sigma_\phi^2)$, the expected value of the complex exponential is computed as,*

$$\mathbb{E}_\mathbf{e}[e^{j\phi_s}] = \int_{-\infty}^{\infty} e^{j\phi_s}\frac{1}{\sigma_\phi\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\phi_s-\mu_\phi}{\sigma_\phi}\right)^2} d\phi_s = e^{j\mu_\phi - \frac{\sigma_\phi{}^2}{2}}, \quad (18)$$

where μ_ϕ, and $\sigma_\phi{}^2$ are mean and variances of ϕ_s respectively. When $\mu_\phi = 0$, we have $\left(\mathbb{E}_\mathbf{e}[e^{j\phi_s}]\right)^2 = e^{-\sigma_\phi{}^2}$.

From the remark given above, the covariance matrix for the complex exponential is as follows:

$$[\mathbf{E}]_{m,n} = \begin{cases}1 & \text{for } m = n,\\ e^{-\sigma_\phi{}^2} & \text{for } m \neq n.\end{cases} \quad (19)$$

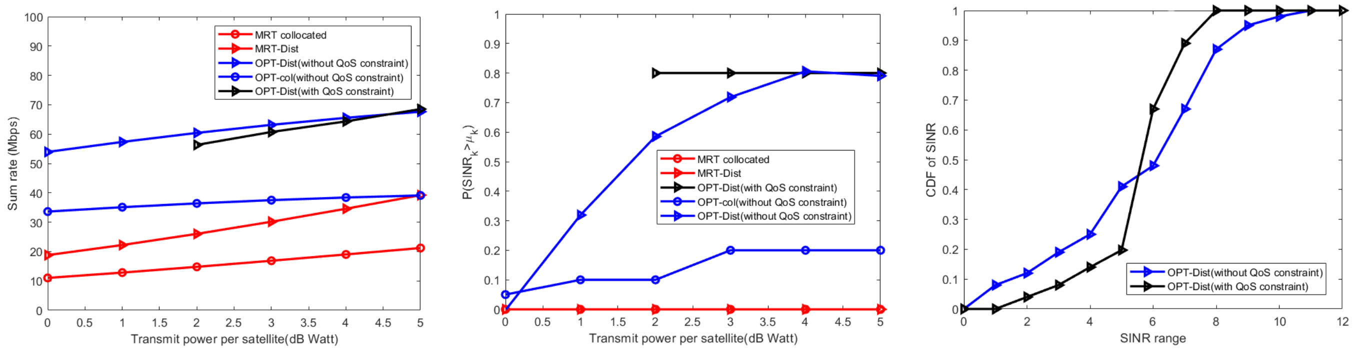
Problem (Q) can be solved in polynomial time using interior point method (IPM). The general approach to approximate the solution of problem (P) is summarized in Algorithm 1.

Algorithm 1 Summary of proposed approach

1: Given σ_ϕ, H, ϵ_k, μ_k, P_s $\forall k$.
2: Compute the error covariance matrix → (19).
3: Compute $\mathbf{R}_k = \mathbb{E}_e[\Phi_k] = \mathbf{M}_k^H \mathbf{E}\mathbf{M}_k$, $\forall k$.
4: Define reformulated objective function → (12).
5: Solve (Q).

IV. Simulation results

For the simulations, a LEO satellite swarm is considered in which each satellite has an independent phase error sampled from a Gaussian distribution with zero mean and standard deviation σ_ϕ. TABLE I presents the simulation parameters. We note that for the considered value of $d = 1000\lambda$=100 m, where λ is the wavelength, can be supported by current flying formation technologies [9]. To support this even further, recently the European Space Agency has launched the Proba-3 mission that will demonstrate formation station keeping at distances ranging between 25 m and 250 m [21]. optimization problem (Q) is solved with CVX modelling, and optimal precoding weights are obtained. Fig. 2 (a) compares the performance of the proposed scheme with the maximum-ratio-transmission (MRT) precoder in the distributed and single satellite (collocated) antenna cases. It can be seen that the optimized scheme outperforms MRT for both collocated and distributed cases. It should be noted that the case of collocated antennas on a single satellite does not exhibit a phase synchronization error. However it achieves a notably lower sum-rate performance due to a higher channel correlation that arises from collocated antenna in a line-of-sight scenario. In addition, from Fig. 2 (a) we observe that in the range (0,2) dBW, the QoS constraint cannot be met, hence there is no output. On the other hand, when the range in the (2,5) dBW range, the constraint is met. Moreover, we observe that the performance is slightly lower than the case without QoS for that specific range and the gap reduces as the transmit power increases until they almost converge for 5 dBW. Fig 2. (b) depicts the empirical probabilities of the SINR from



(a) (b) (c)

Fig. 2: (a) - Total sum-rate comparison for transmit power per satellite range of (0,5), $\sigma_\phi = 20^0, \mu_k = 5dB, \epsilon_k = 0.8 \ \forall k$. (b) - Corresponding empirical probabilities. (c) CDF for the cases with and without QoS constraint at 2dB watt

TABLE I: Simulation parameters

Parameters	Value
Number of satellites, S	50
Number of users, K	10
Antennas per satellite	1
Satellite altitude	600km
Carrier frequency	3 GHz
Bandwidth, B	30MHz
Satellite spacing unit (d)	1000λ
Satellite antenna gain	10dB
Service area coverage	$\theta = (-25^0, 25^0)$ $\varphi = (-25^0, 25^0)$
System noise temperature	250K

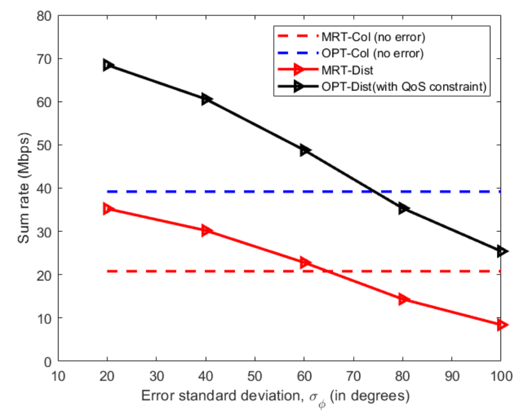


Fig. 3: Performance under different error standard deviations at 5 dB Watt, $\mu_k = 2dB, \epsilon_k = 0.5$.

the samples, which is approximated by running Monte-Carlo simulations using the optimal precoders obtained from the solution of (Q). Fig 2. (c) shows the cumulitive distribution function (CDF) comaprison for the proposed approach with and without QoS constraint at 2 dBW total transmit power. The approach without QoS sacrifices the SINR of some users in order to maximize the sum rate. Although the case with QoS has a slightly lower total sum rate, it satisfies the SINR constraint.

Fig. 3 shows the error standard deviation threshold above which the sum-rate of the proposed scheme falls below the performance achieved by a collocated set of antennas with no error for both proposed and MRT schemes, and for $\mu_k = 2dB, \epsilon_k = 0.5$. For the selected set of parameters, this threshold is around 78^0 at 5 dBW. The value depends on the number of users and their distributions in the service area. Since one benefit of a distributed system is high beamforming gain, it outperforms collocated antennas until the gain is overwhelmed by the loss in performance from phase synchronization errors.

V. Conclusion

In this work, we have considered the problem of maximizing the total sum-rate in multi-user LEO satellite communications where there is a random Gaussian phase synchronization error. It is shown that the proposed solution outperforms MRT for both collocated and distributed cases. Moreover, Monte-carlo simulations demonstrated that the empirical probabilities agree with the criteria defined for QoS. Despite satellite swarms suffering from imperfect phase synchronization, they still offer better performance than collocated antennas upto a certain phase error variance. This is a useful information for system designers that can be used as a maximum threshold to be considered in order to design a distributed system which is more advantageous than collocated antennas.

Future work will account for different phase- error distributions, e.g. uniform, as well as error correlations.

Appendix

A. *Proof of Proposition 1*

We prove (8) as follows. Let

$$f(\mathbf{W}, \mathbf{Z}) = \sum_{k=1}^{K} -\log \mathbf{w}_k + \mathbf{w}_k \mathbf{E}_k + \beta \|\mathbf{z}_k - \mathbf{v}_k\|^2. \quad (20)$$

We use the first derivative to find the values of $\mathbf{W}$ and $\mathbf{Z}$ that minimize the objective function.

Taking the derivative with respect to $\mathbf{z}_k$ and equating to zero, gives $\frac{df(\mathbf{W},\mathbf{Z})}{d\mathbf{z_k}} = 0 \rightarrow \mathbf{z}_k - \mathbf{v}_k = 0 \rightarrow \mathbf{z}_k = \mathbf{v}_k$. Similarly, $\frac{df(\mathbf{W},\mathbf{Z})}{d\mathbf{w_k}} = 0 \rightarrow -\frac{1}{\ln 2}\frac{1}{\mathbf{w}_k} + \mathbf{E}_k = 0 \rightarrow \mathbf{w}_k = \frac{1}{\ln 2}\mathbf{E}_k^{-1}$.

B. Proof of Proposition 2

We show that (12) is convex by dividing it into two parts. The first part, which is $\sum_{k=1}^{K} -2\mathbb{E}_{\mathbf{e}}\left(\mathbf{e}_k^H\right) \mathrm{Re}\left(\mathbf{M}_k \mathbf{v}_k\right)$ is a linear function of $\mathbf{v}_k$, and therefore, is convex.

We prove that second part, $\sum_{k=1}^{K} \mathbf{v}_k^H \left(\sum_{j=1}^{K} \mathbf{R}_k\right) \mathbf{v}_k$ is convex as follows:

Note that $\mathbf{R}_k = \mathbf{M}_k^H \mathbf{E} \mathbf{M}_k$, and since $\mathbf{M}_k$ is a diagonal matrix, we can rewrite the above equation as $\mathbf{R}_k = \mathbf{M}_k^H \mathbf{M}_k \odot \mathbf{E}$. $\mathbf{M}_k^H \mathbf{M}_k$ is positive semidefinite (PSD) given that it is a diagonal matrix with positive diagonal entries.

Assuming a Gaussian phase error, $\mathbf{E}$, given in (19), it can be seen that all diagonal elements are 1, and non-diagonal elements are $e^{-\sigma_\phi{}^2}$. Hence, $\mathbf{E1} = (1 + e^{-\sigma_\phi{}^2}(S-1))\mathbf{1}$, where $\mathbf{1}$ is a column vector of ones, and if we consider the vector $\mathbf{x} \in \mathbb{R}^{S\times 1}$, where $\sum_i \mathbf{x}_i = 0$, (meaning $\mathbf{x}$ has dimension $S-1$), then $\mathbf{Ex} = (1 - e^{-\sigma_\phi{}^2})\mathbf{x}$, Therefore, $(1 + e^{-\sigma_\phi{}^2}(S-1))$ and $(1 - e^{-\sigma_\phi{}^2})$ are the only eigen values of $\mathbf{E}$ as we have a total of $S - 1 + 1 = S$ eigen vector dimensions (multiplicity).

Both eigen values are non-negative since $e^{-\sigma_\phi{}^2} \leq 1$ for $\forall\ \sigma_\phi \geq 0$. Invoking the Schur product theorem (which states that the element-wise product of two PSD matrices is also PSD), we conclude that $\mathbf{R}_k = \mathbf{M}_k^H \mathbf{M}_k \odot \mathbf{E} = \mathbf{M}_k^H \mathbf{E} \mathbf{M}_k$ is PSD. Therefore, the sum of the quadratic forms $\sum_{k=1}^{K} \mathbf{v}_k^H \left(\sum_{j=1}^{K} \mathbf{R}_j\right) \mathbf{v}_k$ is convex. Since we showed that the first and second part of (12) are convex, it implies that it is convex, and this concludes the proof.

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