

# Robust Beamforming Avoiding Satellite Interference in Integrated Terrestrial and Satellite Networks

Wenjing Cao<sup>\*†</sup>, Yafei Wang<sup>\*†</sup>, Wenjin Wang<sup>\*†</sup>, Symeon Chatzinotas<sup>‡</sup>, Björn Ottersten<sup>‡</sup>

<sup>\*</sup>National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China

<sup>†</sup>Purple Mountain Laboratories, Nanjing 211100, China

<sup>‡</sup>Interdisciplinary Centre for Security, Reliability and Trust (SnT), University of Luxembourg, Luxembourg

Email: {caowj, wangyf, wangwj}@seu.edu.cn, {symeon.chatzinotas, bjorn.ottersten}@uni.lu

**Abstract**—This paper investigates robust beamforming based on statistical channel state information (sCSI), against satellite-to-terrestrial user terminal (UT) interference arising from spectrum sharing in integrated terrestrial and satellite networks (ITSNs). First, we develop an integral-form interference model to characterize the interference from satellite to terrestrial base station (BS) UTs. Then, we propose a robust interference avoidance (IA) beamforming scheme under interference and power constraints. A closed-form solution is derived based on the minimum mean square error (MMSE) criterion, and a bisection method is employed to meet the interference threshold. Furthermore, we introduce a BS position-aided (PA) approximation scheme to eliminate the complex integral calculations. Simulation results validate the effectiveness of the proposed schemes.

**Index Terms**—Integrated terrestrial and satellite network, interference avoidance beamforming, minimum mean square error, position-aided.

## I. INTRODUCTION

SATELLITE communication is crucial for ubiquitous connectivity in the 6th generation (6G) [1]. However, spectrum scarcity, particularly below 6 GHz [2], challenges dedicated satellite frequency allocation. Satellite-terrestrial spectrum sharing is a promising solution [3], [4], which improves resource efficiency but introduces inter-system interference in their overlapping coverage [5]. Therefore, effective interference management is crucial in the integrated terrestrial and satellite network (ITSN).

Beamforming is an effective technique for interference management within a single communication system [6]. Conventional beamforming schemes, such as maximum ratio transmission (MRT), zero-forcing (ZF), and minimum mean square error (MMSE) beamforming [7]–[10], require perfect instantaneous CSI (iCSI). However, due to the long propagation distance and the high mobility, obtaining accurate iCSI is challenging for satellites, and beamforming design based on statistical CSI (sCSI) becomes a more practical approach. For example, [11] proposed a beamforming method based on sCSI, including angle and power information, to maximize the average signal-to-interference-plus-noise ratio. Although the beamforming schemes above can effectively reduce intra-system

user interference, they cannot be directly applied to mitigate inter-system interference in ITSNs. To eliminate interference from terrestrial base station (BSs) to satellite user terminals (UTs), [12] proposed a beamforming method at the terrestrial BS side under the minimum user rate constraints, and this terrestrial interference was further restricted to a power threshold in [13]. Based on shared CSI, a joint beamforming design for terrestrial and non-terrestrial systems is another research avenue for interference management [14]. Since satellites complement terrestrial communication systems [11], satellite UTs are usually located outside the coverage area of terrestrial BSs, making terrestrial interference to satellite UTs negligible [2]. However, due to the broader satellite beam coverage [15], interference to terrestrial users may still occur at the interface of the two systems. The beamforming in [5] focused on this satellite interference and constrained the minimum rate of each terrestrial UT. Most existing beamforming schemes for ITSNs rely on CSI sharing between satellites and terrestrial BSs [16]. However, as the dimension of sCSI-based discrete channels increases with the number of interfered terrestrial UTs, CSI sharing introduces significant communication overhead [17]. Consequently, beamforming to mitigate downlink interference from the satellite remains computationally challenging.

In this paper, we explore robust beamforming against satellite-to-terrestrial UT interference in ITSNs. We first establish an integral-form interference model for satellite downlink based on sCSI. Based on this, we design the robust interference avoidance (IA) beamforming scheme with an interference threshold constraint. Specifically, we formulate the MMSE problem, derive an optimal closed-form beamforming solution, and employ a dichotomy approach to satisfy the interference threshold for terrestrial UTs. Furthermore, we propose an interference approximation scheme that uses terrestrial BS position information to approximate the integral-form interference term, reducing the complex integral calculations.

## II. SYSTEM MODEL

In Fig. 1, we consider a spectrum-sharing downlink scenario in the ITSN. Herein, a LEO satellite and  $N_G$  terrestrial BSs serve  $K_S$  and  $K_G$  UTs, respectively, with coverage areas overlapping at the edges [5]. Both the terrestrial and satellite communication systems share the sub-6 GHz spectrum for

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downlink transmission in response to the requirements of the 3rd Generation Partnership Project (3GPP) [4], [18].

### A. Channel and Signal Model

We consider a satellite equipped with a large-scale uniform planar array (UPA) composed of  $M_S = M_x \times M_y$  antennas, where  $M_x$  and  $M_y$  denote the number of antenna elements along the  $x$ -axis and  $y$ -axis, respectively. The space-domain channel response vector for the satellite UT  $k$  at time instant  $t$  and frequency  $f$  can be modeled as [11]

$$\mathbf{h}_{ss,k}(t, f) = e^{j2\pi(tv_k^{\text{sat}} - f\tau_k^{\text{min}})} g_k^{\text{ss}}(t, f) \mathbf{v}_k^{\text{ss}} \in \mathbb{C}^{M_S \times 1}, \quad (1)$$

where  $g_k^{\text{ss}}$  represents the satellite downlink channel gain. Due to the line-of-sight propagation characteristics of satellite communication, the channel gain  $g_k^{\text{ss}}(t, f)$  follows Rician distribution with the Rician factor  $\kappa_k$  and power  $\mathbb{E}\{|g_k^{\text{ss}}(t, f)|^2\} = (\gamma_k^{\text{ss}})^2$ . Specifically, the real and imaginary parts of  $g_k^{\text{ss}}$  are independently and identically real-valued Gaussian distributed with mean  $\gamma_k^{\text{ss}} \sqrt{\frac{\kappa_k}{2(\kappa_k+1)}}$  and variance  $\frac{(\gamma_k^{\text{ss}})^2}{2(\kappa_k+1)}$ , respectively [19]. Additionally,  $v_k^{\text{sat}}$  refers to the Doppler shift caused by the motion of the LEO satellite, and  $\tau_k^{\text{min}}$  refers to the minimum propagation delay of the  $k$ -th satellite UT. The UPA response vector can be represented by  $\mathbf{v}_k^{\text{ss}} = \mathbf{v}_k^{\text{ss}}(\vartheta_k^x) \otimes \mathbf{v}_k^{\text{ss}}(\vartheta_k^y)$ . In Fig. 1, the channel space angles  $\vartheta_k^x = \sin \theta_k \cos \varphi_k$  and  $\vartheta_k^y = \cos \theta_k$  represent the direction cosines of the UT along the  $x$ -axis and  $y$ -axis, respectively. Herein,  $\theta_k$  denotes the elevation angle, and  $\varphi_k$  denotes the azimuth angle from the satellite to the  $k$ -th satellite UT.  $\mathbf{v}_k^{\text{ss}}(\vartheta_k^x)$  and  $\mathbf{v}_k^{\text{ss}}(\vartheta_k^y)$  are the steering vectors in the  $x$ -axis and  $y$ -axis as follows

$$\mathbf{v}_k^{\text{ss}}(\vartheta_k^x) = \frac{1}{\sqrt{M_x}} \left[ 1, e^{-j\frac{2\pi}{\lambda} d_x \vartheta_k^x}, \dots, e^{-j\frac{2\pi}{\lambda} d_x (M_x-1) \vartheta_k^x} \right]^T, \quad (2)$$

$$\mathbf{v}_k^{\text{ss}}(\vartheta_k^y) = \frac{1}{\sqrt{M_y}} \left[ 1, e^{-j\frac{2\pi}{\lambda} d_y \vartheta_k^y}, \dots, e^{-j\frac{2\pi}{\lambda} d_y (M_y-1) \vartheta_k^y} \right]^T, \quad (3)$$

where  $d_x = d_y = \frac{\lambda}{2}$  represent the distances between adjacent antenna elements along the  $x$ -axis and  $y$ -axis.

In the ITSN, the satellite and  $N_G$  terrestrial BSs share the spectrum to provide transmission to corresponding UTs. Since satellites typically complement terrestrial communication systems, when a terminal is within the overlapping coverage area of both the satellite and terrestrial networks, it can connect to the terrestrial network instead of the satellite. This suggests that satellite UTs are usually located outside the coverage area of terrestrial BSs, making terrestrial interference to satellite UTs negligible [2]. The signal  $y_{s,k}$  received by the  $k$ -th satellite UT can be expressed as

$$y_{s,k} = \mathbf{h}_{ss,k}^H \sum_{i=1}^{K_S} \mathbf{p}_{s,i} x_{s,i} + n_{s,k}, \quad (4)$$

herein,  $\mathbf{p}_{s,i} \in \mathbb{C}^{M_S \times 1}$  is the beamforming vector,  $x_{s,i}$  is the symbol sent by the satellite to the  $i$ -th satellite UT and  $n_{s,k}$  denotes the additive noise following  $\mathcal{CN}(0, \sigma_k^2)$ . For brevity,

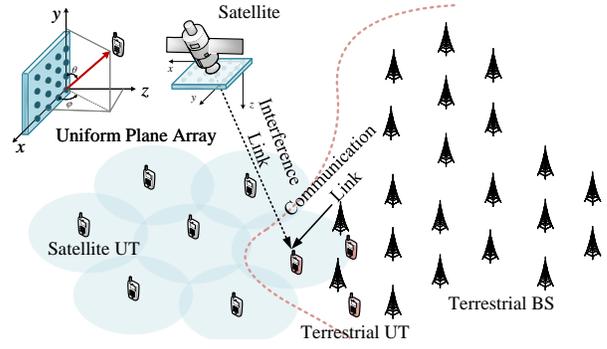


Fig. 1. The system architecture of the ITSN.

we combine the received signals of all satellite UTs and reformulate the signal model as

$$\mathbf{y}_s = \mathbf{H}_{ss}^H \mathbf{P}_s \mathbf{x}_s + \mathbf{n}_s \in \mathbb{C}^{K_S \times 1}, \quad (5)$$

where channel matrix  $\mathbf{H}_{ss} = [\mathbf{h}_{ss,1}, \dots, \mathbf{h}_{ss,K_S}] \in \mathbb{C}^{M_S \times K_S}$ , beamforming matrix  $\mathbf{P}_s = [\mathbf{p}_{s,1}, \dots, \mathbf{p}_{s,K_S}] \in \mathbb{C}^{M_S \times K_S}$ ,  $\mathbf{x}_s = [x_{s,1}, \dots, x_{s,K_S}]^T \in \mathbb{C}^{K_S \times 1}$  is the matrix of symbols sent by the satellite satisfying  $\mathbb{E}\{\mathbf{x}_s \mathbf{x}_s^H\} = \mathbf{I}$ . The average interference power to each terrestrial UT can be expressed as

$$I_{\text{avg}} = \mathbb{E}_{\mathbf{x}_s} \left\{ \frac{1}{K_G} \text{Tr} \{ \mathbf{H}_{sg}^H \mathbf{P}_s \mathbf{x}_s \mathbf{x}_s^H \mathbf{P}_s^H \mathbf{H}_{sg} \} \right\} \\ = \frac{1}{K_G} \text{Tr} \{ \mathbf{P}_s^H \mathbf{H}_{sg} \mathbf{H}_{sg}^H \mathbf{P}_s \}, \quad (6)$$

where  $\mathbf{H}_{sg} \in \mathbb{C}^{M_S \times K_G}$  denotes the satellite-to-terrestrial UT channel matrix based on iCSI. This interference should be minimized and constrained below a threshold, denoted as  $I_{\text{thr}}$  [13], [18].

### B. sCSI for Robust Beamforming and Interference Model

Benefiting from the large-scale UPA equipped in the satellite, we leverage the beamforming technique to mitigate this interference in ITSNs. While conventional beamforming design typically relies on CSI availability, obtaining accurate iCSI is generally infeasible due to the long propagation delay, large Doppler shift, and limited pilot overhead [19], [20]. Frequent updating of beamforming vectors based on the iCSI of  $\mathbf{H}_{ss}$  and  $\mathbf{H}_{sg}$  also presents challenges for practical implementation on satellite payloads. Consequently, we utilize sCSI for the satellite to design the IA beamforming in the ITSN, including

$$\mathbb{E}\{g_k^{\text{ss}}(t, f)\} = \bar{\gamma}_k^{\text{ss}} = \gamma_k^{\text{ss}} \sqrt{\frac{\kappa_k}{2(\kappa_k+1)}} (1+j), \quad (7)$$

$$\mathbb{E}\{\mathbf{H}_{ss} \mathbf{H}_{ss}^H\} = \sum_{k=1}^{K_S} (\gamma_k^{\text{ss}})^2 \mathbf{v}_k^{\text{ss}} \mathbf{v}_k^{\text{ss}H}, \quad (8)$$

$$\mathbb{E}\{\mathbf{H}_{sg} \mathbf{H}_{sg}^H\} = \sum_{k=1}^{K_G} (\gamma_k^{\text{sg}})^2 \mathbf{v}_k^{\text{sg}} \mathbf{v}_k^{\text{sg}H}, \quad (9)$$

which are assumed slowly varying in ITSNs [11]. The average interference power can be further expressed in statistical form

$$\begin{aligned} & \mathbb{E}_{\mathbf{H}_{\text{sg}}, \mathbf{x}_s} \left\{ \frac{1}{K_G} \text{Tr} \{ \mathbf{H}_{\text{sg}}^H \mathbf{P} \mathbf{x}_s \mathbf{x}_s^H \mathbf{P}^H \mathbf{H}_{\text{sg}} \} \right\} \\ &= \frac{1}{K_G} \text{Tr} \{ \mathbf{P}^H \mathbb{E} \{ \mathbf{H}_{\text{sg}} \mathbf{H}_{\text{sg}}^H \} \mathbf{P} \} \\ &= \frac{1}{K_G} \text{Tr} \{ \mathbf{P}^H \mathbf{\Upsilon}_{\text{sg}} \mathbf{P} \}, \end{aligned} \quad (10)$$

where  $\mathbf{\Upsilon}_{\text{sg}} = \mathbb{E} \{ \mathbf{H}_{\text{sg}} \mathbf{H}_{\text{sg}}^H \}$ . Due to the numerous interfered terrestrial UTs, obtaining all their sCSI may still incur significant pilot overhead, so we convert the sum of the discrete terrestrial UT channels into an integral of the user distribution over the terrestrial BS coverage. The integral is given by

$$\begin{aligned} \mathbf{\Upsilon}_{\text{sg}} &= \sum_{n=1}^{N_G} \sum_{k=1}^{\bar{K}_G} \mathbb{E} \left\{ |g_{n,k}^{\text{sg}}(t, f)|^2 \mathbf{v}_{n,k}^{\text{sg}} \mathbf{v}_{n,k}^{\text{sg}H} \right\} \\ &= \sum_{n=1}^{N_G} \int_0^{2\pi} \int_0^{R_{\text{bs}}} \frac{c^2 M_x M_y G_T G_R \mathbf{V}_n^{\text{sg}}(r_n, \phi_n) f(r_n, \phi_n) r_n dr_n d\phi_n}{(4\pi f d_n(r_n, \phi_n))^2}, \end{aligned} \quad (11)$$

where  $N_G$  and  $\bar{K}_G$  are the numbers of terrestrial BSs and UTs per BS, respectively. We have  $\mathbb{E} \left\{ |g_{n,k}^{\text{sg}}(t, f)|^2 \right\} = (\gamma_{n,k}^{\text{sg}})^2 = \frac{M_x M_y G_T G_R c^2}{(4\pi f d_n)^2}$  and  $\mathbf{V}_n^{\text{sg}}(r_n, \phi_n) = \mathbf{v}_n^{\text{sg}}(r_n, \phi_n) \mathbf{v}_n^{\text{sg}H}(r_n, \phi_n)$ .  $G_T$  and  $G_R$  are the per-antenna transmit gain and receive gain, respectively.  $f(r_n, \phi_n)$  is the probability density function of user distribution. The propagation distance between the satellite and the UT at the  $n$ -th terrestrial BS, given the UT's polar coordinates  $\phi_n$  and  $r_n$ , can be computed as  $d_n(r_n, \phi_n) = \sqrt{h_{\text{sat}}^2 + R_n^2 + r_n^2 + 2R_n r_n \cos(\psi_n - \phi_n)}$ . Herein,  $h_{\text{sat}}$  denotes the satellite height,  $\psi_n$  the polar angle of the  $n$ -th terrestrial BS, and  $R_n$  the distance from the  $n$ -th terrestrial BS to the sub-satellite point. Then we compute the  $(i, j)$ -th element of  $\mathbf{V}_n^{\text{sg}}(r_n, \phi_n)$  as

$$[\mathbf{V}_n^{\text{sg}}(r_n, \phi_n)]_{i,j} = \frac{e^{j\pi[(m_a - m_p)\vartheta_n^x + (n_b - n_q)\vartheta_n^y]}}{M_x M_y}, \quad (12)$$

the element indices are determined by the expressions  $i = n_q M_x + m_p + 1$  and  $j = n_b M_x + m_a + 1$ , where the indices follow the ordering:  $n_q = 0, \dots, M_y - 1$ ,  $m_p = 0, \dots, M_x - 1$ ,  $n_b = 0, \dots, M_y - 1$  and  $m_a = 0, \dots, M_x - 1$ . The channel space angles are computed as  $\vartheta_n^x(r_n, \phi_n) = \frac{R_n \cos \psi_n + r_n \cos \phi_n}{R_{\text{sat}}}$  and  $\vartheta_n^y(r_n, \phi_n) = \frac{R_n \sin \psi_n + r_n \sin \phi_n}{R_{\text{sat}}}$  where  $R_{\text{sat}}$  is the satellite coverage radius. We simplify  $\varpi_n(r_n, \phi_n) = (m_a - m_p)\vartheta_n^x + (n_b - n_q)\vartheta_n^y$ . Thus, we obtain the integral form of the statistical interference channel term  $\mathbf{\Upsilon}_{\text{sg}}^{\text{int}}$ , and its  $(i, j)$ -th element can be rewritten as

$$[\mathbf{\Upsilon}_{\text{sg}}^{\text{int}}]_{i,j} = \sum_{n=1}^{N_G} \int_0^{2\pi} \int_0^{R_{\text{bs}}} \frac{G_T G_R c^2 e^{j\pi \varpi_n(r_n, \phi_n)}}{(4\pi f d_n(r_n, \phi_n))^2} \times f(r_n, \phi_n) r_n dr_n d\phi_n. \quad (13)$$

This integral-form statistical interference model reduces reliance on satellite-to-terrestrial UT iCSI, mitigating the need for pilot overhead, latency, and computational complexity.

### III. CLOSED-FORM IA BEAMFORMING BASED ON MMSE

#### A. Problem Formulation

In this section, we consider the IA beamforming design in the ITSN based on the MMSE criterion, i.e., we minimize the MSE of the received and the transmitted signal, constrained with the average interference threshold and power budget. The IA beamforming optimization problem for the MMSE criterion is formulated as follows

$$\begin{aligned} \min_{\mathbf{P}, \beta} & \mathbb{E}_{\mathbf{H}_{\text{ss}}, \mathbf{x}_s, \mathbf{n}_s} \left\{ \left\| \frac{\mathbf{\Psi}_s (\mathbf{H}_{\text{ss}}^H \mathbf{P} \mathbf{x}_s + \mathbf{n}_s)}{\beta} - \mathbf{x}_s \right\|_2^2 \right\}, \\ \text{s.t.} & \frac{1}{K_G} \text{Tr} \{ \mathbf{P}^H \mathbf{\Upsilon}_{\text{sg}}^{\text{int}} \mathbf{P} \} \leq I_{\text{thr}}, \\ & \text{Tr} \{ \mathbf{P} \mathbf{P}^H \} \leq P_T. \end{aligned} \quad (14)$$

Herein,  $P_T$  denotes the antenna power budget and  $\mathbf{\Psi}_s = \text{diag} \{ e^{j\psi_1}, \dots, e^{j\psi_{K_s}} \}$  represents the phase compensation for delay and Doppler shifts at the receiver with  $\psi_k = 2\pi (t v_k^{\text{sat}} - f \tau_k^{\text{min}})$ . Compared to the difficulty of phase compensation at the transmitter, the receiver can accurately estimate phase errors based on downlink pilot signals to assist with compensation [21]. To simplify the problem, we employ the penalty function method to mitigate interference [22]. Specifically, we introduce a penalty term  $\bar{\varsigma} \text{Tr} \{ \mathbf{P}^H \mathbf{\Upsilon}_{\text{sg}}^{\text{int}} \mathbf{P} \}$  into the objective function, transforming the problem into a convex optimization problem subject to power budget. Here,  $\bar{\varsigma}$  represents the penalty factor, and for the sake of derivation, we set  $\bar{\varsigma} = \frac{\varsigma}{\beta^2}$  and rewrite the optimization problem as

$$\begin{aligned} \min_{\mathbf{P}, \beta} & \mathbb{E}_{\mathbf{H}_{\text{ss}}, \mathbf{x}_s, \mathbf{n}_s} \left\{ \left\| \frac{\mathbf{\Psi}_s (\mathbf{H}_{\text{ss}}^H \mathbf{P} \mathbf{x}_s + \mathbf{n}_s)}{\beta} - \mathbf{x}_s \right\|_2^2 \right\} + \bar{\varsigma} \text{Tr} \{ \mathbf{P}^H \mathbf{\Upsilon}_{\text{sg}}^{\text{int}} \mathbf{P} \}, \\ \text{s.t.} & \text{Tr} \{ \mathbf{P} \mathbf{P}^H \} \leq P_T. \end{aligned} \quad (15)$$

**Proposition 1:** The following solution achieves the optimum of problem (15), where

$$\mathbf{P}^* = \beta \left( \mathbf{\Upsilon}_{\text{ss}} + \varsigma \mathbf{\Upsilon}_{\text{sg}}^{\text{int}} + \frac{K_s \sigma_s^2}{P_T} \mathbf{I} \right)^{-1} \bar{\mathbf{H}}_{\text{ss}}. \quad (16)$$

Herein, we assume  $\sigma_k^2 = \sigma_s^2$  for all UTs and  $\beta$  normalizes the power. Donote  $\mathbf{\Upsilon}_{\text{ss}} = \mathbb{E} \{ \mathbf{H}_{\text{ss}} \mathbf{H}_{\text{ss}}^H \} = \sum_{k=1}^{K_s} (\gamma_k^{\text{ss}})^2 \mathbf{v}_k^{\text{ss}} \mathbf{v}_k^{\text{ss}H}$  and  $\bar{\mathbf{H}}_{\text{ss}} = \mathbb{E} \{ \mathbf{H}_{\text{ss}} \mathbf{\Psi}_s^H \} = \sum_{k=1}^{K_s} \bar{\gamma}_k^{\text{ss}} \mathbf{v}_k^{\text{ss}}$ .

**Proof:** See Appendix A.

#### B. Closed-Form Beamforming with Interference Threshold

The selection of  $\varsigma$  is crucial, as it effectively represents the trade-off between MMSE performance and interference management capability. Specifically, the relationship is summarized in Proposition 2.

**Proposition 2:** For the closed-form solution of  $\mathbf{P}$  in (16), the derivative of the total interference power  $I_{\text{sg}}$  with respect to  $\varsigma$  is non-positive.

$$\nabla_{\varsigma} I_{\text{sg}}(\varsigma) = \frac{\partial I_{\text{sg}}(\varsigma)}{\partial \varsigma} = \frac{\partial \text{Tr} \{ \mathbf{P}^H(\varsigma) \mathbf{\Upsilon}_{\text{sg}}^{\text{int}} \mathbf{P}(\varsigma) \}}{\partial \varsigma} \leq 0. \quad (17)$$

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**Algorithm 1** Closed-Form Satellite IA Beamforming
 

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- 1: **Input:**  $\{\mathbf{v}_k^{ss}, \gamma_k^{ss}, \tilde{\gamma}_k^{ss}\}_{k=1}^{K_S}$ ,  $\mathbf{\Upsilon}_{sg}^{\text{int}}$ ,  $I_{\text{thr}}$ ,  $P_T$ ,  $\sigma_s^2$ .
  - 2: Find  $\varsigma$  for given  $I_{\text{thr}}$  and SNR via the bisection method.
  - 3: Calculate  $\mathbf{P}^* = \beta \left( \mathbf{\Upsilon}_{ss} + \varsigma \mathbf{\Upsilon}_{sg}^{\text{int}} + \frac{K_S \sigma_s^2}{P_T} \mathbf{I} \right)^{-1} \bar{\mathbf{H}}_{ss}$ .
  - 4: **Output:**  $\mathbf{P}^*$ .
- 

**Proof:** See Appendix B.

According to the above Proposition, the value of  $\varsigma$  for given  $I_{\text{thr}}$  and signal-to-noise ratio (SNR) can be determined via the bisection method. The continuous-time variation of  $\varsigma$  can be modeled and predicted, significantly reducing the frequency of bisection executions. Although the bisection method requires iterative computation of  $\mathbf{P}$ , with this approach for  $\varsigma$ , Algorithm 1 can be regarded as a closed-form computation.

#### IV. SATELLITE-TO-TERRESTRIAL INTERFERENCE APPROXIMATION IN BEAMFORMING DESIGN

In the proposed scheme, IA beamforming depends on the integral term  $\mathbf{\Upsilon}_{sg}^{\text{int}}$ , which is computationally intensive and requires real-time terrestrial user distribution, posing practical challenges. Due to the limited coverage of terrestrial BSs compared to satellite beams [15], we approximate the satellite-to-UT interference channel using the satellite-to-BS channel, eliminating the need for user distribution integral.

We define  $\bar{\mathbf{H}}_{sg}$  to represent the channel from the satellite to the terrestrial BSs and the approximation of  $\mathbf{\Upsilon}_{sg}^{\text{int}}$  can be expressed as  $\tilde{\mathbf{\Upsilon}}_{sg} = \bar{K}_G \mathbb{E} \left\{ \bar{\mathbf{H}}_{sg} \bar{\mathbf{H}}_{sg}^H \right\}$  where  $\bar{K}_G$  is the number of UTs per terrestrial BS. The original constraint of average interference power can be approximated as

$$\begin{aligned} & \frac{1}{\bar{K}_G} \text{Tr} \left\{ \mathbf{P}^H \bar{K}_G \mathbb{E} \left\{ \bar{\mathbf{H}}_{sg} \bar{\mathbf{H}}_{sg}^H \right\} \mathbf{P} \right\} \\ &= \frac{1}{\bar{K}_G} \text{Tr} \left\{ \mathbf{P}^H \tilde{\mathbf{\Upsilon}}_{sg} \mathbf{P} \right\} \leq I_{\text{thr}}. \end{aligned} \quad (18)$$

The other steps in Section III remain unchanged, except for the formula in (16), which needs to be modified as

$$\mathbf{P}_{\text{MMSEIA-PA}} = \beta \left( \mathbf{\Upsilon}_{ss} + \varsigma \tilde{\mathbf{\Upsilon}}_{sg} + \frac{K_S \sigma_s^2}{P_T} \mathbf{I} \right)^{-1} \bar{\mathbf{H}}_{ss}. \quad (19)$$

**Remark 1:** Approximating  $\mathbf{\Upsilon}_{sg}^{\text{int}}$  with  $\tilde{\mathbf{\Upsilon}}_{sg}$  significantly reduces computational complexity, lowering the order from

$\mathcal{O}(M_S^2 N_G N_r N_\phi)$  to  $\mathcal{O}(M_S^2 N_G)$ , where  $N_r N_\phi$  represents the integral sampling points.

#### V. NUMERICAL RESULTS

In this section, we employ the Monte Carlo method to evaluate the proposed schemes. We consider a wideband LEO satellite communication system, the channel Rician factor is set to  $\kappa_k = \kappa_s$  for all UTs [11], and the radius of the terrestrial BS is 500 m. We set  $K_S = 24$ ,  $N_G = 24$ , and  $\bar{K}_G = 10$  and calculate  $\text{SNR} = P_T - 10 \log K_S + G_T + 10 \log M_S - \text{PL} + G_R - 10 \log(k_{\text{bol}}[T + (F - 1)T_0]B)$  where  $k_{\text{bol}}$ ,  $T_0$  and  $B$  are the Boltzmann constant, standard temperature, and system bandwidth. The noise power  $\sigma_s^2$  varies with the SNR. The remaining system configurations are shown in Tab. I.

TABLE I  
SATELLITE SYSTEM PARAMETERS [18], [19]

Parameter	Value
Orbit Altitude $h_{\text{sat}}$	600 km
Satellite Coverage Radius $R_{\text{sat}}$	630 km
Carrier Frequency $f$	2 GHz
Number of Antennas $M_S$	$16 \times 16$
Rician Factor $\kappa_s$	10 dB
Satellite Transmit Power $P_T$	25 dBW
Per-Antenna Gain $G_T, G_R$	6 dBi, 0 dBi
Noise Figure $F$	9 dB
Noise Temperature $T$	290 K

We validate the effectiveness of our satellite IA beamforming algorithm and compare the following schemes:

- ‘MRT’/‘ZF’/‘MMSE’: Three kinds of conventional linear beamforming [7]–[9].
- ‘MMSEIA’: MMSE-based satellite interference avoidance beamforming in Algorithm 1.
- ‘MMSEIA-PA’: Algorithm 1 with position-aided (PA) interference approximation in (19).

In Fig. 2, we present the beam patterns, showing that the interference power at the positions of terrestrial UTs (marked by blue circles) is lower with both ‘MMSEIA’ and ‘MMSEIA-PA’ compared to ‘MMSE’, demonstrating the intuitive effect of interference avoidance. In Fig. 3, the ‘MMSEIA’ and ‘MMSEIA-PA’ schemes show sum rate performances close to that of the ‘MMSE’ scheme, indicating its effectiveness

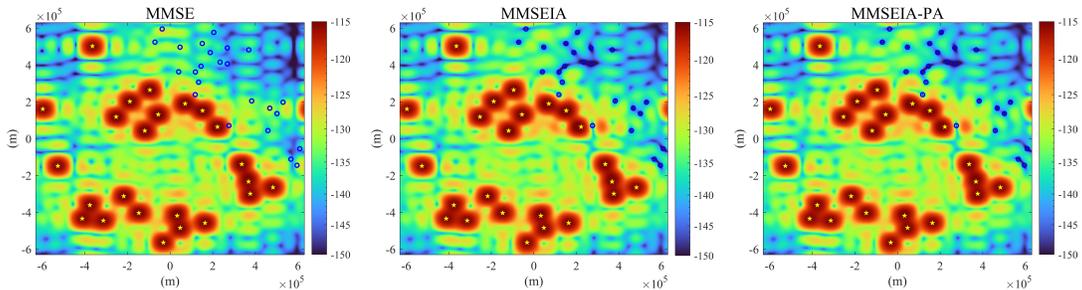


Fig. 2. Satellite beam patterns (yellow stars represent satellite UTs and blue circles represent terrestrial BS positions),  $I_{\text{thr}} = -160$  dBW, SNR = 10 dB.

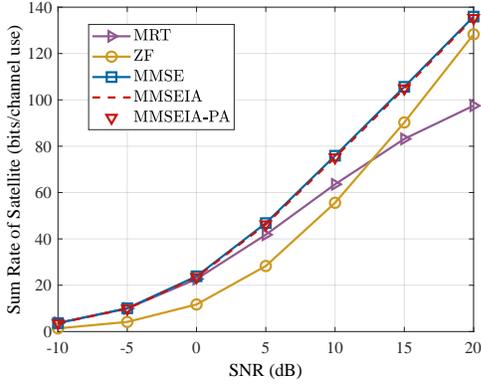


Fig. 3. Satellite sum rate vs SNR,  $I_{\text{thr}} = -160$  dBW.

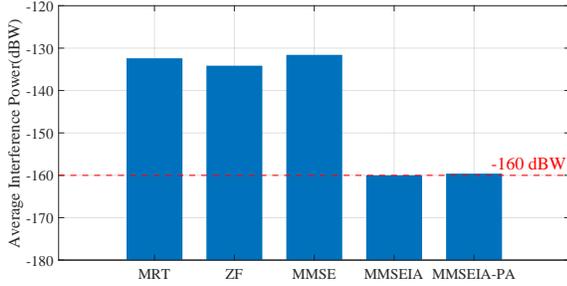


Fig. 4. Average interference power,  $I_{\text{thr}} = -160$  dBW, SNR = 10 dB.

in sustaining capacity. In Fig. 4, the two IA beamforming schemes, ‘MMSEIA’ and ‘MMSEIA-PA’, successfully achieve the interference threshold of  $-160$  dBW at SNR = 10 dB, illustrating their effectiveness in interference management.

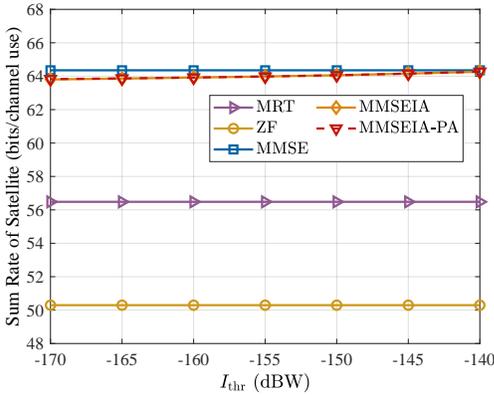


Fig. 5. Satellite sum rate vs  $I_{\text{thr}}$ , SNR = 10 dB.

We further study the effect of the interference threshold. In Fig. 5, the satellite sum rate performances for the ‘MRT’, ‘ZF’, and ‘MMSE’ schemes remain unchanged, while those of ‘MMSEIA’ and ‘MMSEIA-PA’ schemes decrease slightly as the interference threshold  $I_{\text{thr}}$  is reduced. In Fig. 6, the average interference power remains constant for ‘MRT’, ‘ZF’, and ‘MMSE’ but decreases adaptively with ‘MMSEIA’ and ‘MMSEIA-PA’ as the interference threshold  $I_{\text{thr}}$  is lowered.

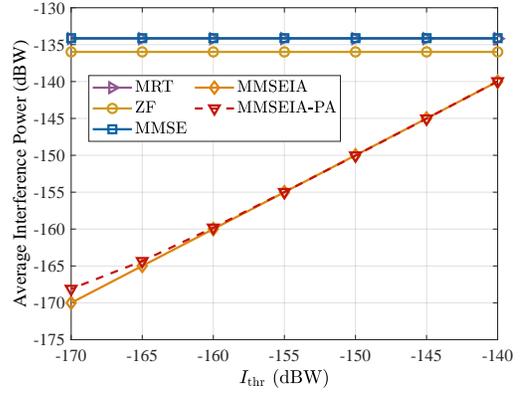


Fig. 6. Average interference vs  $I_{\text{thr}}$ , SNR = 10 dB.

TABLE II  
COMPLEXITY COMPARISON

Scheme	Complexity Order
MRT	$\mathcal{O}(M_S^2 K_S)$
ZF	$\mathcal{O}(M_S^2 K_S + M_S^3)$
MMSE	$\mathcal{O}(M_S^2 K_S + M_S^3)$
MMSEIA	$\mathcal{O}(M_S^2 K_S + M_S^2 N_G N_r N_\phi + M_S^3)$
MMSEIA-PA	$\mathcal{O}(M_S^2 K_S + M_S^2 N_G + M_S^3)$

In Tab. II, we compare the computational complexity of the considered schemes.

## VI. CONCLUSION

This paper investigates robust IA beamforming based on sCSI against satellite-to-terrestrial UT interference in the ITSN. We establish an integral-form interference model and design robust beamforming schemes under power and interference constraints. A closed-form solution for the MMSE criterion is derived and a bisection method is applied to meet interference constraints. To reduce reliance on complex integral calculations, we propose an approximation scheme based on terrestrial BS positions. Simulation results verify the effectiveness of our proposed schemes.

## APPENDIX A PROOF OF PROPOSITION 1

The Lagrangian function is derived as

$$\begin{aligned} \mathcal{L}(\mathbf{P}, \beta, \lambda) = & \frac{\text{Tr}\{\mathbf{P}^H \boldsymbol{\Upsilon}_{\text{ss}} \mathbf{P}\}}{\beta^2} - \frac{\text{Tr}\{\bar{\mathbf{H}}_{\text{ss}}^H \mathbf{P}\}}{\beta} - \frac{\text{Tr}\{\mathbf{P}^H \bar{\mathbf{H}}_{\text{ss}}\}}{\beta} \\ & + K_S + \frac{K_S \sigma_s^2}{\beta^2} + \frac{\varsigma}{\beta^2} \text{Tr}\{\mathbf{P}^H \boldsymbol{\Upsilon}_{\text{sg}} \mathbf{P}\} + \lambda (\text{Tr}\{\mathbf{P} \mathbf{P}^H\} - P_T). \end{aligned} \quad (20)$$

Then, we set the gradient of the Lagrange function to zero

$$\nabla_{\mathbf{P}^*} \mathcal{L} = \frac{\boldsymbol{\Upsilon}_{\text{ss}} \mathbf{P}}{\beta^2} - \frac{\bar{\mathbf{H}}_{\text{ss}}}{\beta} + \frac{\varsigma}{\beta^2} \boldsymbol{\Upsilon}_{\text{sg}} \mathbf{P} + \lambda \mathbf{P} = \mathbf{0}, \quad (21)$$

$$\mathbf{P} = \beta (\boldsymbol{\Upsilon}_{\text{ss}} + \varsigma \boldsymbol{\Upsilon}_{\text{sg}} + \lambda \beta^2 \mathbf{I})^{-1} \bar{\mathbf{H}}_{\text{ss}}. \quad (22)$$

We define  $\zeta = \lambda\beta^2$ ,  $\mathbf{A} = \mathbf{\Upsilon}_{ss} + \varsigma\mathbf{\Upsilon}_{sg} + \zeta\mathbf{I}$ ,  $\mathbf{P} = \beta\mathbf{A}^{-1}\bar{\mathbf{H}}_{ss}$  and  $\tilde{\mathbf{\Upsilon}}_{ss} = \bar{\mathbf{H}}_{ss}\bar{\mathbf{H}}_{ss}^H$ . By normalizing,  $\beta = \sqrt{\frac{P_T}{\text{Tr}\{\mathbf{A}^{-2}\tilde{\mathbf{\Upsilon}}_{ss}\}}}$ , and the problem can be transformed into an unconstrained optimization problem as  $\min_{\zeta} f(\mathbf{P}(\zeta), \beta(\zeta))$ , where the objective function is

$$f(\zeta) = \text{Tr}\{\mathbf{A}^{-1}\mathbf{\Upsilon}_{ss}\mathbf{A}^{-1}\tilde{\mathbf{\Upsilon}}_{ss}\} - 2\text{Tr}\{\mathbf{A}^{-1}\tilde{\mathbf{\Upsilon}}_{ss}\} + K_S + \frac{K_S\sigma_s^2}{P_T}\text{Tr}\{\mathbf{A}^{-2}\tilde{\mathbf{\Upsilon}}_{ss}\} + \varsigma\text{Tr}\{\mathbf{\Upsilon}_{sg}\mathbf{A}^{-1}\tilde{\mathbf{\Upsilon}}_{ss}\mathbf{A}^{-1}\}. \quad (23)$$

We calculate the gradient of (20) with respect to  $\zeta$  as  $\nabla_{\zeta}f = 2\text{Tr}\left\{\left(\zeta - \frac{K_S\sigma_s^2}{P_T}\right)\left(\mathbf{A}^{-3}\tilde{\mathbf{\Upsilon}}_{ss}\right)\right\}$ , and set it to zero. Therefore, we have  $\zeta = \lambda\beta^2 = \frac{K_S\sigma_s^2}{P_T}$ . Substituting this into  $\mathbf{P}$  and  $\beta$  gives the closed-form solution

$$\mathbf{P}^* = \beta\left(\mathbf{\Upsilon}_{ss} + \varsigma\mathbf{\Upsilon}_{sg} + \frac{K_S\sigma_s^2}{P_T}\mathbf{I}\right)^{-1}\bar{\mathbf{H}}_{ss}. \quad (24)$$

#### APPENDIX B PROOF OF PROPOSITION 2

From (16) and (17), we simplify  $\mathbf{P}(\varsigma) = \beta\mathbf{A}^{-1}\bar{\mathbf{H}}_{ss}$ ,  $\alpha = \text{Tr}\{\mathbf{A}^{-2}\tilde{\mathbf{\Upsilon}}_{ss}\}$ , and  $\tilde{\mathbf{\Upsilon}}_{ss} = \bar{\mathbf{H}}_{ss}\bar{\mathbf{H}}_{ss}^H$ . The gradient is

$$\nabla_{\varsigma}J_{sg} = \frac{2\beta^2}{\alpha}\text{Tr}\left\{\tilde{\mathbf{\Upsilon}}_{ss}\mathbf{A}^{-1}\mathbf{\Upsilon}_{sg}^{\text{int}}\mathbf{A}^{-2}\left(\mathbf{A}^{-1}\mathbf{\Upsilon}_{sg}^{\text{int}}\tilde{\mathbf{\Upsilon}}_{ss}\mathbf{A}^{-1} - \alpha\mathbf{\Upsilon}_{sg}^{\text{int}}\right)\right\}, \quad (25)$$

where both the complex matrix  $\mathbf{M} = \frac{2\beta^2}{\alpha}\tilde{\mathbf{\Upsilon}}_{ss}\mathbf{A}^{-1}\mathbf{\Upsilon}_{sg}^{\text{int}}\mathbf{A}^{-2}$  and matrix  $\mathbf{N} = \mathbf{A}^{-1}\mathbf{\Upsilon}_{sg}^{\text{int}}\tilde{\mathbf{\Upsilon}}_{ss}\mathbf{A}^{-1} - \alpha\mathbf{\Upsilon}_{sg}^{\text{int}}$  are Hermitian matrices. From *Von Neumann Trace Inequality*, we derive that

$$\text{Tr}\{\mathbf{MN}\} = \sum_{i=1}^{M_S}\lambda_i(\mathbf{MN}) \leq \sum_{i=1}^{M_S}\lambda_i(\mathbf{M})\lambda_i(\mathbf{N}), \quad (26)$$

where  $\lambda(\cdot)$  represents the eigenvalues. From the *Cauchy-Buniakowsky-Schwarz Inequality*,

$$\begin{aligned} \sum_{i=1}^{M_S}\lambda_i(\mathbf{M})\lambda_i(\mathbf{N}) &\leq \sqrt{\sum_{i=1}^{M_S}\lambda_i(\mathbf{M})^2\sum_{i=1}^{M_S}\lambda_i(\mathbf{N})^2} \\ &\leq \sum_{i=1}^{M_S}\lambda_i(\mathbf{M})\sum_{i=1}^{M_S}\lambda_i(\mathbf{N}) = \text{Tr}\{\mathbf{M}\}\text{Tr}\{\mathbf{N}\}, \end{aligned} \quad (27)$$

i.e.,  $\text{Tr}\{\mathbf{MN}\} \leq \text{Tr}\{\mathbf{M}\}\text{Tr}\{\mathbf{N}\}$ . Similarly, we define  $\mathbf{T} = \tilde{\mathbf{\Upsilon}}_{ss}\mathbf{A}^{-2} - \alpha$  and  $\mathbf{S} = \mathbf{\Upsilon}_{sg}^{\text{int}}$  and we have  $\text{Tr}\{\mathbf{TS}\} \leq \text{Tr}\{\mathbf{T}\}\text{Tr}\{\mathbf{S}\}$ . Substituting these inequalities into (25),

$$\begin{aligned} \nabla_{\varsigma}J_{sg} &\leq \text{Tr}\left\{\frac{2\beta^2}{\alpha}\tilde{\mathbf{\Upsilon}}_{ss}\mathbf{A}^{-1}\mathbf{\Upsilon}_{sg}^{\text{int}}\mathbf{A}^{-2}\right\} \\ &\quad \times \text{Tr}\left\{\mathbf{A}^{-1}\mathbf{\Upsilon}_{sg}^{\text{int}}\tilde{\mathbf{\Upsilon}}_{ss}\mathbf{A}^{-1} - \alpha\mathbf{\Upsilon}_{sg}^{\text{int}}\right\} \leq 0. \end{aligned} \quad (28)$$

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