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Population Ageing, Economic Growth and the Composition of Government Expenditure

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Abstract

This paper investigates how population ageing affects economic growth by altering the composition of government expenditure. We develop and test an original political economy model in which an aging population shifts the preferences of the median voter, leading to increased elderly spending at the expense of private investment, thus reducing growth. The model yields three predictions: population ageing (i) raises elderly spending (as a share of output); (ii) does not significantly affect productive expenditure; and (iii) lowers economic growth. Using OECD data from 2007-2018 and both OLS and IV regression analyses, we find strong support for prediction (i): population ageing significantly increases spending on "old age" and "hospital services." Consistent with (ii), there is no significant impact on "tertiary education," "transport," "communication," or "R&D." Finally, using GMM-based estimation with a broader sample of 178 countries, we confirm prediction (iii): healthcare expenditure negatively affects growth.

Keywords: ageing, demographics, endogenous economic growth, generalised method of moments, government spending, median voter.

JEL Codes: D72, E62, H40, J10, O40

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1 Introduction

Most economies in the 21*st* century have experienced population ageing, i. e., older individuals have become a proportionally larger fraction of the total population (Weil, 2008). This trend is predicted to last (Lutz et al., 2008). Indeed, recent UN estimates suggest that the global share of the population aged 65 and above was 10.21 percent in 2024 and will increase to 16.33 percent by 2050 (United Nations, 2025). Population ageing is predicted to especially affect industrialised economies.

This paper develops and empirically tests an original political economy model in which population ageing leads to an older median voter who, under majority voting, influences the composition of government expenditure so that it crowds out private investment. The model generates three testable predictions, which we evaluate using data: (i) population ageing increases public elderly spending (as a share of output); (ii) it has no significant effect on productive expenditure (as a share of output); and (iii) it reduces the rate of economic growth. The empirical analysis provides strong support for all three predictions.

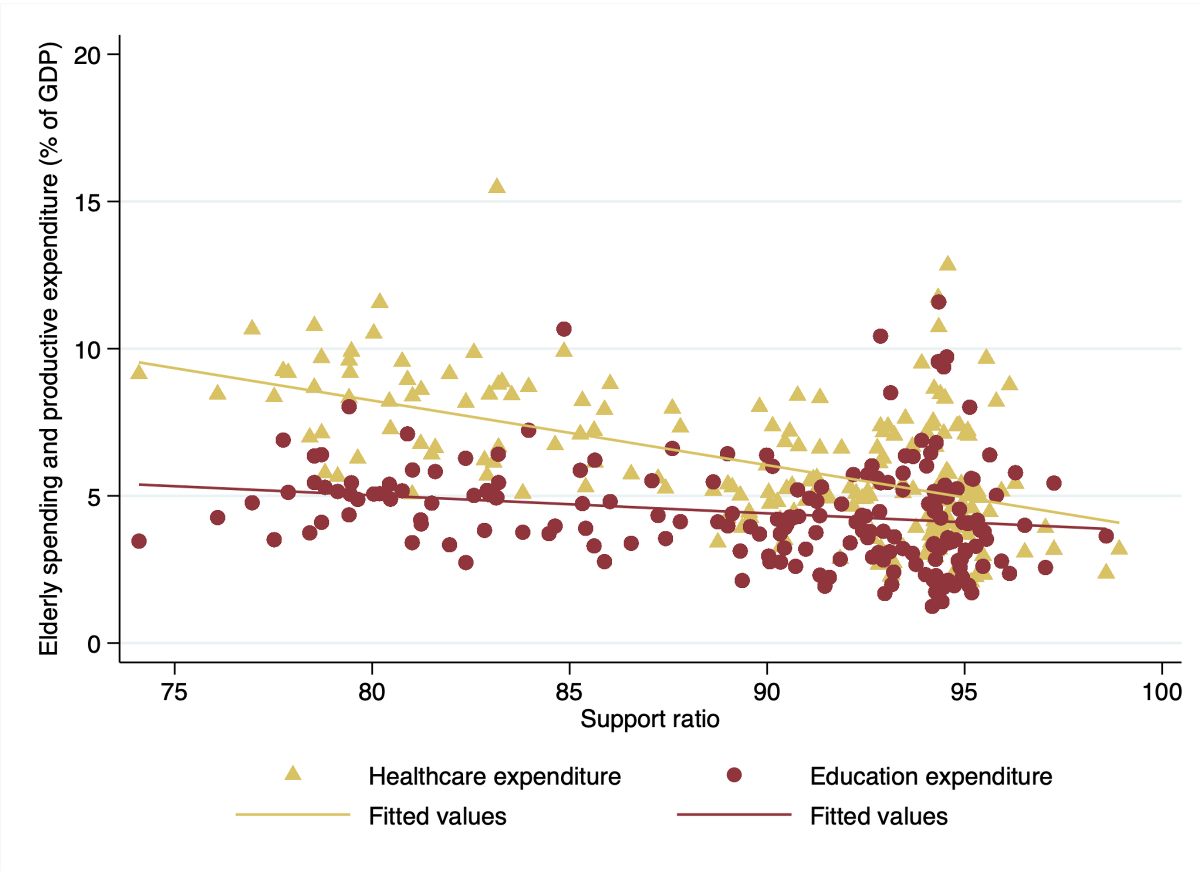
We capture a country's age structure through its support ratio, defined as the share of the population aged 15-64 relative to the population aged 15 and over. A declining support ratio indicates population ageing.[1] This measure has a natural counterpart in our theoretical analysis.

In the empirical analysis of the composition of government expenditure, we focus on two key categories: *elderly spending* and *productive expenditure*. The former directly benefits older individuals, for example through healthcare services, while the latter includes investments such as education that enhance the economy's overall productivity.

Figure 1 shows the association between population ageing, elderly spending, and productive expenditure across a sample of 181 countries over the period 2000-2018. We use healthcare expenditure as a proxy for elderly spending and education expenditure as a proxy for productive expenditure. Consistent with predictions (i) and (ii), the figure reveals a positive association between population ageing and elderly spending, while the link between population ageing and productive expenditure appears relatively weak.

[1] To fix ideas, consider the actual and the predicted support ratios for Germany in 2022 and 2050. According to United Nations (2023), these ratios are 0.74 and 0.65, respectively. Roughly, this implies that in 2022 three working members of the population had to support one economically dependent elderly whereas in 2050, two workers are expected to bear the same burden.

Figure 1: Elderly spending (healthcare expenditure) versus productive expenditure (education expenditure) and population ageing across 181 countries (time averages 2000-2018)



Notes: The horizontal axis shows the support ratio, i. e., the share of the population aged 15-64 in the population aged 15 or over. A declining support ratio implies population ageing. The vertical axis depicts elderly spending exemplified by healthcare expenditure as well as productive expenditure exemplified by education expenditure, both as a share in GDP. The two fitted lines suggest that population ageing (the support ratio) is positively (negatively) associated with elderly spending in GDP. In contrast, the association with productive expenditure is weak.

We examine the three theoretical predictions empirically using regression analysis. First, we focus on the association between population ageing and the composition of government expenditure. For a sample of 30 (OECD) countries over the 2007-2018 period we find that population ageing indeed increases elderly spending. By contrast, we find no support for an effect of population ageing on productive expenditure.

Second, we investigate the impact of elderly spending and productive expenditure on economic growth by employing various generalized method of moments (GMM) estimators on a baseline sample of 178 countries over the 2000-2018 period. Our findings suggest that healthcare expenditure, a proxy for elderly spending, reduces economic growth. This further corroborates the predictions of the theoretical model.

The present paper contributes to at least three strands of the literature. The first concerns the relationship between population aging and the composition of public expenditure. This literature is equivocal. For instance, Razin et al. (2002), Jäger and Schmidt (2016) and Tamai and Wang (2025) find that under democratic regimes, population ageing reduces public investment and welfare expenditure. In contrast, Disney (2007), Sanz and Velázquez (2007) and Kühnel (2011) report contradicting results. We depart from existing contributions by identifying two categories of public expenditure, that is elderly spending and productive expenditure. Moreover, for these categories, we explore a large set of subcategories. As to elderly spending, these include "old age" and "hospital services". As to productive expenditure, these encompass "tertiary education", "transport", "communication" and "R&D".

The second strand studies the role of government expenditure for endogenous economic growth. For example, building on the seminal work of Barro (1990), Angelopoulos et al. (2007) and Felice (2016) find that productive expenditure is positively associated with economic growth.[2] Examining specifically the role of population ageing, In contrast to this literature our analysis emphasizes the role of different government spending categories and their relation to the age distribution among voters.

Finally, we contribute to the literature on the relationship between population ageing and economic growth. Here, Lee and Shin (2019) show using panel data analysis that there is a negative and nonlinear effect of population ageing on economic growth. Studying population ageing in China, Liu et al. (2023) also demonstrate that ageing impedes growth by negatively impacting industrial structure upgrading.

Against these findings, Acemoglu and Restrepo (2017) argue that there is no direct relationship between population ageing and slow economic growth which they explain by an endogenous response of technology. Using panel data analysis, Bloom et al. (2010) find a modest negative effect of population ageing on economic growth. These authors also suggest that ageing-related demographic changes result in policy updates including increasing the legal age of retirement and behavioural amendments such as higher female labour participation to mitigate the growth impact of ageing.

[2] See Irmen and Kühnel (2009) for a survey of the literature on productive expenditure and economic growth in the spirit of Barro (1990).

Distinguishing between the underlying causes of ageing (i. e., lower fertility rates and higher longevity) Prettner (2013) and Iong (2019) reveal that higher longevity is growth-inducing whilst lower fertility is growth-impeding. Irmen (2021) shows that not only the source of aging but also the time horizon determine the qualitative effect of population aging on economic growth. Other studies including Fougère and Mérette 1999, Boucekkine et al. 2002, and Choi and Shin 2015 posit that ageing promotes economic growth through human capital accumulation and upskilling. In contrast to this strand of the literature we emphasise the role of the composition of government expenditure for economic growth.

The remainder of this paper is organized as follows. Section 2 describes the economic framework and determines the economic as well as the political-economic equilibrium. The key predictions of the model are summarized in Corollary 1: (i) population ageing increases the share of elderly spending in total output, (ii) population ageing keeps the share of productive expenditure in total output unchanged, and (iii) population ageing reduces the economy's growth rate. Section 3 presents the empirical analysis. It describes the data, outlines the empirical methodology, and discusses the results. The empirical findings support predictions (i) - (iii). Section 4 concludes. Section 5 is the Appendix. It contains proofs, theoretical extensions, and supplementary information for the empirical analysis.

2 Theory - Population Ageing, Economic Growth, and the Composition of Government Expenditure

Consider a closed economy in continuous time, i. e., $t \in [0, \infty)$.[3] The economy is populated by a continuum of infinitely-lived household-producers of mass 1 and a government.

Each household-producer is represented by a unique real number $i \in [0, 1]$ and comprises $N^i > 0$ members, of which $L^i > 0$ are working young and the remaining $N^i - L^i$ are economically-dependent elderly. We use the support ratio $\phi^i \equiv L^i/N^i \in (0, 1]$ to capture i's age structure. Both, N^i and L^i, are time-invariant. Hence, the age structure of each household-producer is a given constant. This assumption seems particularly acceptable in our setup as we are not interested in the demographic evolution of individual household-producers over time but rather in shifts of the entire population age distribution in response to population ageing.

Household-producers behave competitively and produce one good that can be consumed or invested. At all t, prices are expressed in units of the contemporaneous output of this good. While all household members derive utility from private consumption, the elderly additionally benefit from public spending, e. g., on health care. The government taxes household-producers' income to finance the utility-enhancing public good as well as productive expenditure.

[3]We shall often suppress the time argument in our notation whenever this does not cause confusion.

2.1 Production, Preferences, and Government Policy

Production Technology At each t, household-producer i has access to the following production function

$$Y^i(t) = AK^i(t)^\alpha \left(G^i(t)L^i(t)\right)^{1-\alpha}. \tag{2.1}$$

Here, $Y^i(t)$ denotes i's output at t, $K^i(t)$ its private capital stock, $G^i(t)$ the flow of services it derives from total productive expenditure, and $L^i(t)$ its working young members. Moreover, $A > 0$ is the time-invariant total factor productivity, and $0 < \alpha < 1$ the output elasticity of capital.

The key feature of the production function (2.1) is constant returns to scale in private capital and public productive services. Thus, if K^i grows at the same rate as G^i then diminishing returns to the accumulation of private capital will be offset by growth in (labor-augmenting) public services. For this reason, the economy will exhibit endogenous steady-state growth.

For simplicity, private capital does not depreciate. The economy's total output at t, $Y(t)$, obtains from aggregation over all firms, i. e., $Y(t) = \int_0^1 Y^i(t)di$.

The production function (2.1) delivers household-producer i's output per worker at t as

$$y^i(t) \equiv \frac{Y^i(t)}{L^i} = Ak^i(t)^\alpha G^i(t)^{1-\alpha}, \tag{2.2}$$

where $k^i(t) \equiv K^i(t)/L^i$ denotes i's capital stock per worker.

The level of productive government services that household-producer i enjoys from aggregate productive expenditure at t, $G(t)$, is given by

$$G^i(t) \equiv G(t)\,\frac{y^i(t)}{y(t)}, \tag{2.3}$$

where $y(t) \equiv Y(t)/L$ is the economy's aggregate output per worker and $L \equiv \int_0^1 L^i di$ the economy's aggregate labor supply. Equation (2.3) describes a situation of relative congestion (see, e. g., Barro and Sala-í-Martin, 1992, or Turnovsky, 1996), i. e., the level of services household-producer i derives from the public good G at t depends on her own usage, represented by her own output per worker, relative to aggregate usage, represented by the economy's aggregate output per worker.[4]

[4]The specification of congestion using per-worker magnitudes eliminates an undesirable scale effect in the accumulation path of household-producers. If the level of services derived by an individual household-producer depended on her own total output relative to the economy's aggregate output, i. e., if $G^i = GY^i/Y$, then household-producers with more workers would accumulate at a faster rate since the marginal product of private capital would positively depend on L^i.

In economic terms, (2.3) means that household-producers take the ratio of the aggregates, $G(t)/y(t)$, as given. Moreover, they understand that an expansion of $y^i(t)$ increases $G^i(t)$ and, hence, the productivity of both private inputs. The working young, $L^i(t)$, benefit from the labor-augmenting nature of $G^i(t)$, and private capital, $K^i(t)$, becomes more productive since both private inputs are complements. These features explain the difference between (2.2) and the following expression of output per worker as perceived by each household-producer i. Indeed, combining (2.2) and (2.3) gives

$$y^i(t) = A^{\frac{1}{\alpha}} \left(\frac{G(t)}{y(t)} \right)^{\frac{1-\alpha}{\alpha}} k^i(t). \tag{2.4}$$

Hence, for all i, production per worker has constant returns to the private input k^i as long as the government maintains a given state of congestion, i. e., as the long as the ratio G/y is constant. Moreover, the greater this ratio the greater is $y^i(t)$.

Consumer Preferences Household-producer i seeks to maximize her overall intertemporal utility given by

$$\begin{aligned} U^i(0) &\equiv \int_0^\infty \left[N^i \ln c^i(t) + (N^i - L^i) b \ln H(t) \right] e^{-\rho t}\, dt \\ &= N^i \int_0^\infty \left[\ln c^i(t) + (1-\phi^i) b \ln H(t) \right] e^{-\rho t}\, dt; \end{aligned} \tag{2.5}$$

here, $c^i(t)$ denotes private consumption per household member at t, $H(t)$ aggregate public spending for the elderly at t, $b > 0$ measures the weight the elderly assign to public relative to private consumption goods, and $\rho > 0$ is the constant instantaneous rate of time preference. Observe that working members do not derive utility from the public consumption good. This assumption highlights the intergenerational conflict. The key point here is that the old derive considerably greater benefit from spending on health and care services than the young.[5]

Household-producer i may use her after-tax income either for consumption or investment in private capital. However, when $N^i > L^i$ and $\phi^i < 1$, then consumption per household member at t is only a fraction of after-tax output per worker net of investment per worker:

$$c^i = \phi^i \left[(1-\tau) y^i - \dot{k}^i \right], \tag{2.6}$$

where $\tau \geq 0$ denotes a non-discriminatory income tax rate.

[5]Two further remarks are in order. First, if all members of household-producer i work, i. e., if $\phi^i = 1$, then (2.5) reduces to the standard utility function $U^i(0) = \int_0^\infty N^i e^{-\rho t} \ln c^i(t)\, dt$. Second, we have chosen an additively separable utility specification. However, our key predictions stated in Corollary 1 below do not change if we use a similar utility function with non-separable preferences between private and public consumption. For details see Appendix 5.2.1.

Government Policy In each period t, the government taxes household-producers' income at rate $\tau \equiv \tau_G + \tau_H$. Revenues collected from household-producers fund productive expenditure (the component corresponding to τ_G) as well as elderly spending (the component corresponding to τ_H). Thus, a balanced government budget requires

$$\tau Y = G + H = \tau_G Y + \tau_H Y. \tag{2.7}$$

Note that $\tau_G = G/Y \in [0,1]$ and $\tau_H = H/Y \in [0,1]$ also represent the ratio of the respective spending component to aggregate output. When we turn to the determination of government policy in Section 2.3, voting will be on the policy mix (τ_G, τ_H). This policy mix then automatically yields the overall income tax rate τ.

2.2 Economic Equilibrium

The economic equilibrium is the decentralized competitive equilibrium of the economy for an exogenously given, time-invariant government policy (τ_G, τ_H). We will show in Section 2.3 that the tax rates τ_G and τ_H will indeed be time-invariant in the political equilibrium.

The optimization problem for each household-producer i consists in choosing the paths $c^i(t)$ and $k^i(t)$ that maximize (2.5), subject to (2.4), (2.6), and an initial capital stock per worker $k^i(0) = k_0 > 0$.[6]

When making her consumption-savings decision each household-producer takes the paths of G, H, y, and τ as given and disregards the possible impact of her investment decision on the amount of public services provided. Then, the intertemporal optimization problem leads to the following Euler condition

$$\frac{\dot{c}^i(t)}{c^i(t)} = (1 - \tau_G - \tau_H) A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}} - \rho \equiv \gamma(\tau_G, \tau_H), \quad \text{for all } i. \tag{2.8}$$

Hence, irrespective of their support ratio all household-producers accumulate at the same rate in equilibrium. In addition, the equilibrium requires the following transversality condition to be met

$$\lim_{t \to \infty} \left[\lambda^i(t) k^i(t) \right] = 0, \tag{2.9}$$

where λ^i denotes the present-value shadow price of household-producer i's capital stock.

Then, the following proposition holds.

[6] We assume that all households have the same initial capital stock per worker, i.e., $k^i(0) = k_0 > 0$. This assumption implies that households with fewer working members have a lower initial capital holding. Alternatively, one could suppose that the economy starts off with an equal distribution of capital. This assumption does not affect our qualitative results. For details see Appendix 5.2.2.

Proposition 1 *(Economic Equilibrium)*

Consider a given, time-invariant policy mix, (τ_G, τ_H)*.*

1. *There exists a unique steady-state growth path along which all variables grow at the same constant rate* $\gamma(\tau_G, \tau_H)$*.*
2. *For any admissible set of initial conditions the economy immediately jumps to this steady-state path.*

According to Proposition 1 the rate at which all household-producers accumulate is given by $\gamma(\tau_G, \tau_H)$. The latter is also the growth rate of economy-wide (aggregate) variables as well as of government expenditure. The intuition is straightforward. The ratio of productive government spending per unit of the economy's output per worker consistent with condition (2.7) is

$$\frac{G}{y} = \tau_G L \tag{2.10}$$

since $y = Y/L$. Hence, for all t the average product of capital is constant and equal to $A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}}$.

Since all households have the same initial capital stock per worker, the common growth rate of all household-producers implies that the capital stock per worker and output per worker is also the same for all i and t. However, each household's demographic composition determines her instantaneous level of total income

$$Y^i(t) = y^i(0) L^i e^{\gamma(\cdot)t} = A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}} k_0 L^i e^{\gamma t} \tag{2.11}$$

[from (2.4) and (2.10)], and of consumption per capita

$$c^i(t) \quad = \quad c^i(0) e^{\gamma(\cdot)t} = \phi^i \rho k_0 e^{\gamma t}, \tag{2.12}$$

[from (2.4), (2.6), and (2.8)] where the argument of γ is (τ_G, τ_H). Intuitively, the smaller a household's labor force, the smaller is her aggregate income at each t. Similarly, the smaller the share of working members in total members, i. e., the smaller the support ratio, the lower is the level of consumption per capita at each t.

As all households accumulate at the same rate and the labor supply of each household is constant, it is not surprising that the growth rate γ also applies to the economy's aggregate variables, which obtain from aggregation over all households. For instance, the economy's aggregate output is given by

$$Y(t) = \int_0^1 Y^i(t) di = A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}} k_0 L e^{\gamma t}. \tag{2.13}$$

Finally, as G and H are proportional to aggregate income, they also grow at rate γ.

The steady-state growth rate depends on the public policy parameters τ_G and τ_H. There is a negative relationship between the government's expenditure ratio for services that benefit the elderly and the steady-state growth rate, i. e., $\partial\gamma(\tau_G, \tau_H)/\partial\tau_H < 0$. The reason is that each household-producer in her optimization problem disregards that her choice of k^i via aggregate output, Y, affects the aggregate amount of public spending for the elderly, H, and thus the household's overall per-period utility. Thus, τ_H only affects the steady-state growth rate by reducing each household's net income.

In contrast, a rise in τ_G has two opposing effects on $\gamma(\tau_G, \tau_H)$. According to (2.10) a greater τ_G increases the provision of G and, thus, the private marginal product of private capital increases. At the same time, it reduces the after-tax value of the private marginal product of private capital due to the distortionary tax financing of government expenditure.[7]

2.3 The Political-Economic Equilibrium

This section endogenizes government policy. For this purpose, we first characterize each household's policy preferences and then determine the policy mix that will be implemented by the government under pure majority voting.

Policy Preferences Let (τ_G^i, τ_H^i) denote household-producer i's most preferred policy mix. This mix maximizes i's overall intertemporal utility in an economic equilibrium where the tax rates τ_G^i and τ_H^i apply to all household-producers. In other words, household-producer i's most preferred policy mix is the solution to

$$\begin{aligned}
\max_{\tau_G,\tau_H} U^i(0) &= N^i \int_0^\infty \left[\ln c^i(t) + (1-\phi^i) b \ln H(t)\right] e^{-\rho t}\, dt \\
\text{s.t.} & \\
c^i(t) &= \phi^i \rho k_0 e^{\gamma(\tau_G,\tau_H)t} \\
H(t) &= \tau_H A^{\frac{1}{\alpha}} \left(\tau_G L\right)^{\frac{1-\alpha}{\alpha}} k_0 L e^{\gamma(\tau_G,\tau_H)t},
\end{aligned}$$

where $H(t)$ follows from (2.7) and (2.13). The constraints make clear how the choice of policy affects household i's indirect utility. First, a rise in τ_G has two effects on U^i. On the one hand, a higher τ_G increases utility by raising aggregate production today and thus today's provision of H. On the other hand, a change in τ_G affects U^i by altering the steady-state growth rate. The direction of this effect depends on the size of τ_G compared to its growth-maximizing size $(1-\alpha)(1-\tau_H)$ (see Footnote 7). A greater growth rate

[7] For a given τ_H, the steady-state growth rate $\gamma(\tau_G, \tau_H)$ is maximized at $\tau_G = (1-\alpha)(1-\tau_H)$. Overall, maximum growth is obtained at $\tau_H = 0$ and $\tau_G = 1-\alpha$. Observe also that the steady-state growth rate depends on the economy's aggregate labor supply, L, i. e., there is a scale effect. The latter occurs since a greater labor supply increases aggregate household income and, thus, the tax base form which productive expenditure is financed.

is utility-enhancing because it increases future private as well as public consumption possibilities. Second, a rise in τ_H positively affects households' well-being by directly increasing the provision of H but impinges on U^i by reducing the steady-state growth rate.

Henceforth, we make the following assumption, which will be motivated below:

Assumption 1 *It holds that* $\rho \leq \frac{1+b}{b} A^{\frac{1}{\alpha}} \left[(1-\alpha)L\right]^{\frac{1-\alpha}{\alpha}}$.

Then, the following proposition holds.

Proposition 2 *(Most-Preferred Policy Mix)*

For each household-producer i there is a unique most-preferred policy mix, (τ_G^i, τ_H^i), given by

$$\tau_G^i = 1-\alpha \quad and \quad \tau_H^i = \frac{(1-\phi^i)b\rho}{\left[1+(1-\phi^i)b\right] A^{\frac{1}{\alpha}} \left[(1-\alpha)L\right]^{\frac{1-\alpha}{\alpha}}}. \tag{2.14}$$

Since time does not appear in these expressions, we find our conjecture confirmed that the actual policy mix, will involve time-invariant tax rates. Thus, a behavior of household-producers based on time-invariant tax rates, τ_G and τ_H, is fully consistent with the actual equilibrium outcome. Moreover, Assumption 1, which is easily met for a small ρ or a large A, assures that $\tau_H^i \leq 1$ for any ϕ^i.

According to Proposition 2, for all households the ideal share of productive expenditure, τ_G^i, is equal to $1-\alpha$. The latter is the output elasticity of productive expenditure, which is the same for all households. As productive expenditure affects all household-producers in the same way, it is intuitive that the preferred expenditure ratio is independent of the households' demographic composition, i. e., $\partial \tau_G^i / \partial \phi^i = 0$.[8]

By contrast, equation (2.14) reveals that household-producer i's preferred spending ratio for services that benefit the elderly, τ_H^i, depends on ϕ^i. Thus, households with different support ratios prefer different tax rates. Since τ_H^i affects the steady-state growth rate, this difference also translates into the preferred growth rate. Assuming that i's most preferred policy mix is the one implemented by the government, one readily establishes that

$$\frac{d\tau_H^i}{d\phi^i} < 0 \quad \text{and} \quad \frac{d\gamma(\tau_G^i, \tau_H^i)}{d\phi^i} > 0. \tag{2.15}$$

[8] It is noteworthy that at τ_G^i productive expenditure satisfies the so-called natural condition of productive efficiency, i. e., the marginal contribution of government expenditure to aggregate output is one (see, e. g., Barro, 1990). In the present context, as aggregate equilibrium output can be written as $Y = AK^\alpha (GL)^{1-\alpha}$, we have $dY/dG = (1-\alpha)(Y/G) = (1-\alpha)/\tau_G^i = 1$.

Intuitively, households with a greater share of elderly members (i. e., a lower ϕ^i) are willing to pay higher taxes for the provision of public services that benefit these members and to accept lower growth rates of private consumption.

Policy Choice under Majority Voting Let's turn to the policy mix that will be implemented by the government under a pure majority rule. In particular, we will show that the median voter theorem can be applied to this voting problem.

For all household-producers the optimal policy mix involves $\tau_G = 1 - \alpha$. Thus, voters only differ in their preferences for τ_H and the voting problem becomes one-dimensional. Moreover, each voter's preferences for τ_H are single-peaked because the indirect utility function U^i is strictly concave in τ_H^i for $\tau_G = 1 - \alpha$. In addition, there exists a monotonic relationship between household i's ideal tax rate τ_H^i and her support ratio ϕ^i. Thus, the median voter theorem can be applied to this voting problem and the share of public spending for the elderly that the government implements coincides with the one preferred by the median voter. The following proposition summarizes the political equilibrium, i. e., the actual choice of policy under majority rule.

Proposition 3 *(Political-Economic Equilibrium)*

The actual policy mix involves

$$\tau_G^* = 1 - \alpha \quad \textit{and} \quad \tau_H^* = \frac{(1-\phi^m)b\rho}{\left[1+(1-\phi^m)b\right] A^{\frac{1}{\alpha}} \left[(1-\alpha)L\right]^{\frac{1-\alpha}{\alpha}}}, \tag{2.16}$$

where ϕ^m denotes the support ratio of the median household. The corresponding steady-state growth rate of household and economy-wide variables is

$$\gamma^* = \alpha A^{\frac{1}{\alpha}} \left[(1-\alpha)L\right]^{\frac{1-\alpha}{\alpha}} - \frac{(1-\phi^m)b\rho}{1+(1-\phi^m)b} - \rho. \tag{2.17}$$

Implicitly, we have assumed that taxes are voted on and implemented with full commitment at time zero. However, due to the infinite time horizon and exponential discounting, this policy choice is time-consistent (see Laibson, 2003). Thus, it has to coincide with the solution that would be obtained if the government could not commit itself to future policies. Intuitively, as households only differ in their support ratio (which does not affect the accumulation path) and as the identity of the median voter does not change over time, strategic intertemporal voting cannot occur.[9]

[9] Equation (2.16) reveals that $\tau_H^* > 0$ for any $\phi^m < 1$. Thus, as long as the median household is not solely composed of working members, majority voting cannot yield the economy's maximum growth rate (which requires $\tau_H = 0$).

2.4 Implications of Population Ageing

How does population ageing affect actual government spending and long-run economic growth? Population ageing corresponds to an (exogenous) change in the distribution of households such that there are more households with a large fraction of elderly members and the median household has a lower support ratio. The following Corollary follows from Proposition 3 and equation (2.15).

Corollary 1 *(Population Ageing, Government Spending, and Growth)*

It holds that

$$\frac{d\tau_H^*}{d\phi^m} < 0, \quad \frac{d\tau_G^*}{d\phi^m} = 0, \quad \textit{and} \quad \frac{d\gamma^*}{d\phi^m} > 0. \tag{2.18}$$

Corollary 1 reveals that a fall in the median voter's support ratio, ϕ^m, involves a higher τ_H^*, an unchanged τ_G^*, and a lower γ^*. Thus, our theory predicts that population ageing increases public spending for the elderly (as a share of output), does not affect productive expenditure (as a share of output), increases the overall tax burden (because $\tau = \tau_G + \tau_H$), and lowers the economy's growth rate.[10] The following section confronts these predictions with the data.

3 Empirics - Population Ageing, Economic Growth, and the Composition of Government Expenditure

The following section confronts the predictions of Corollary 1 with the data.

3.1 Data

The support ratio measures the share of the working-age population (ages 15 to 64) relative to the total adult population (ages 15 and above). We construct country-level support ratios using demographic data from the World Bank Development Indicators (World Bank, 2024). These support ratios serve as our independent variable of interest.

To examine the impact of population ageing on the composition of government expenditure, we use a sample of 30 countries from the Organisation for Economic Co-operation

[10] One readily verifies that the political-economic equilibrium implies for $t = 0$ that $Y(0) = A^{\frac{1}{\alpha}} \left((1-\alpha) L\right)^{\frac{1-\alpha}{\alpha}} K_0$, $C(0) = \rho K_0$, $G(0) = (1-\alpha) Y(0)$, $\dot{K}(0) = \gamma\left(1-\alpha, \tau_H(\phi^m)\right) K_0$, and $H(0) = \tau_H(\phi^m) Y(0)$. Hence, a decline in ϕ^m increases τ_H^* which crowds our private investment in physical capital one to one, i. e., $-d\dot{K}(0)/d\phi^m = dH(0)/d\phi^m$.

and Development (OECD) over the 2007−2018 period. The list of sampled countries is presented in Appendix 5.3.1.

This sample was selected for two main reasons. First, the OECD imposes a *sine qua non* condition of pluralist democratic regimes to its entering members (OECD, 2018) which aligns with our theoretical framework that studies the impact of ageing on public spending through a majority voting mechanism. Second, the OECD follows a standardised system for categorising government expenditure according to the Classification of the Functions of Government (COFOG). This classification provides consistently measured data across all OECD countries, facilitating cross-country comparisons and enabling a comprehensive analysis of different public expenditure subcategories.

We associate the following two subcategories with elderly spending: "old age" and "hospital services," hereafter referred to as *Old age* and *Hospitals*, respectively. *Old age* encompasses both in-kind and cash benefits directed specifically towards the elderly. In-kind benefits refer to elderly-targeting services such as lodging and the provision of special services including in-home help and allowances paid to persons looking after an elderly. Cash benefits include old-age pensions paid upon reaching the standard retirement age and other one-time or lump-sum payments made on account of old age. We use hospital services as a measure of elderly spending since the elderly are a highly vulnerable demographic group with a high demand for healthcare services (Fong, 2019). This subcategory pertains to services of general and specialist hospitals in addition to those of convalescent and nursing homes.

For productive expenditure, we select the following four subcategories "tertiary education", "transport", "communication" and "R&D", and denote them by *Education*, *Transport*, *Communication*, and *R&D*. *Education* refers to the provision of tertiary education. The selected subcategory includes spending on universities and other tertiary educational institutions as well as scholarships and other grants paid to students. *Transport* relates to costs incurred for the administration, maintenance and construction of transportation infrastructure and services. *Communication* concerns the expenditure associated with the administration of services for the construction as well as the maintenance of communication systems, including postal, telephone and wireless communication systems. Finally, R&D denotes spending on programs aimed at acquiring knowledge with the purpose of generating novel materials and products as well as to establish new systems, services and processes. In practice, this spending subcategory includes funds paid to governmental and non-governmental agencies that are engaged in applied research.

In line with our theoretical framework, the subcategories on elderly spending and on productive expenditure are recorded as a percentage of annual gross domestic product.

For the regression analyses shown in Section 3.2.1 below, we include the following country-level control variables: population density, unemployment rates, inflation rates, government effectiveness as well as total government expenses and current account balance (CAB), both as shares of GDP. In the regression tables, these are respectively labeled *Population density*, *Unemployment*, *Inflation*, *Government effectiveness*, *Expenses* and *CAB*.

Population density defines the average number of people living within one square kilometer. The latter may affect demand for public goods and services, particularly infrastructure. More densely populated countries may require higher spending to maintain public services. *Unemployment* measures the proportion of the workforce that is unemployed. Higher unemployment rates could strain fiscal resources and influence allocations toward social services or subsidies, thus impacting public expenditure. *Inflation* accounts for broader economic conditions that may influence government budget allocation decisions. *Expenses* measures total government expenses as a share of GDP and reflects the overall fiscal capacity of a government. *CAB* controls for exports net of imports; it captures possible trade deficits that may results in fiscal adjustments. Lastly, *Government Effectiveness,* sourced from the V-Dem dataset (Coppedge et al., 2021), is a measure of the credibility of governments in delivering public services. Data for all the other control variables are obtained from the World Bank Development Indicators.

Table 1 presents the summary statistics for a balanced panel comprising the 30 countries mentioned above over the 2007-2018 period. The panel has 360 observations except for R&D expenditure. Data on Bulgaria is missing for the entire period, i .e., 2007 to 2018 and data for Lithuania is only available for the years 2007, 2008 and 2009.

Countries in the sample have a mean support ratio of 79.57, approximately corresponding to an average of 4 workers per 1 economically-dependent old person. Italy is the country with the lowest support ratio, estimated at 73.74 in 2018; this indicates high population ageing. By contrast, Ireland in 2007 has the highest support ratio of 86.6.

For elderly spending, the variable Old age averages 8.54 percent of GDP across the sample. Greece, for example, records the highest spending on old age, estimated at 16 percent of GDP in 2016. Conversely, Iceland has the lowest Old age spending, estimated at just 1.9 percent in 2007 and 2008. The Hospitals subcategory also exhibits substantial variability. Denmark had the highest share of GDP allocated to hospital services, reaching 6.3 percent in 2009 against only 1.3 percent for Switzerland in the years 2007 and 2008.

There is considerable variation in the sample regarding productive expenditure subcategories. Tertiary education expenditure averages 0.96 percent of GDP, with the lowest value recorded in Luxembourg at 0.2 in 2007 and 2008, and the highest in Finland at 2.1 percent in 2010. Transport spending averages 2.67 percent of GDP, with a minimum value of 0.6 percent recorded in Cyprus in 2014, 2015, and 2016, and a maximum value of 6.1 percent in Croatia in 2008 and 2015. Communication expenditure is relatively low across the panel of countries considered, averaging 0.04 percent of GDP. The minimum value of 0 is observed across various countries like Luxembourg, Switzerland, and Austria and over multiple years. Slovenia reports the highest share at 0.3 in 2008. R&D spending has an average of 0.03 percent of GDP and shows significant cross-country variation. The minimum R&D expenditure is 0 percent; it is also observed across multiple years and countries such as Lithuania and Cyprus; the maximum of 0.5 percent of GDP is recorded in Romania in 2008.

Following the derivation of the baseline results, we implement an instrumental variable (IV) analysis to address potential endogeneity in the relationship between population ageing and government expenditure composition. Without the use of an IV, our regression results may be biased due to reverse causality or omitted variable bias. For instance, government spending decisions – particularly in areas such as healthcare or family support policies – could influence demographic outcomes, potentially affecting the support ratio. Additionally, unobserved factors, such as long-term structural policies, may be correlated with both the support ratio and government spending. To mitigate these potential biases, we instrument the support ratio using forecasts from the UN World Population Projections of 1992 (United Nations, 1992). The instrumental variable is referred to as *Support Ratio Forecasts.*

Table 1: Summary statistics

Variable	Description	Mean	SD	Min	Max	Count
Main independent variable						
Support ratio	Working population aged 15-64 (% of adult population aged 15 and above)	79.57	2.86	73.75	86.62	360
Government expenditure						
Hospitals	General government spending on hospital services (% of GDP)	3.11	0.99	1.30	6.30	360
Old Age	General government spending on old age (% of GDP)	8.54	2.75	1.90	16.00	360
Education	General government spending on tertiary education (% of GDP)	0.96	0.34	0.20	2.10	360
Transport	General government spending on transportation (% of GDP)	2.67	0.95	0.60	6.10	360
Communication	General government spending on communication (% of GDP)	0.04	0.05	0.00	0.30	360
R&D	General government spending on general public services (% of GDP)	0.03	0.08	0.00	0.50	339
Control variables						
Population density	Population density (people per sq. km of land area)	163.40	244.36	3.11	1514.47	360
Unemployment	Unemployment, total (% of total labor force)	8.52	4.56	2.24	27.47	360
Inflation	Inflation, GDP deflator (annual %)	2.05	2.68	-9.73	20.07	360
Expense (% of GDP)	Government expense on goods and services (% of GDP)	29.29	8.17	10.09	91.50	360
CAP (% of GDP)	Current account balance (% of GDP)	0.50	6.05	-25.76	15.77	360
Government effectiveness	Government effectiveness	1.17	0.60	-0.36	2.35	360

Summary: This table presents summary statistics for the variables used in the analysis examining the effect of population ageing on elderly spending and productive expenditure. For each variable, we show the mean, standard deviation, minimum and maximum values as well as the number of observations.

Finally, to evaluate our theoretical prediction that population ageing lowers the economy's growth rate, we construct a panel of 178 countries over the period 2000−2018; the countries sampled are listed in Appendix 5.3.2. The expansion of the sample to include a larger cross-section of countries (with $N \gg T$) allows for the sound application of the generalised method of moments (GMM) estimation procedure. GMM is advantageous as it addresses potential endogeneity and unobserved heterogeneity, making it a robust approach for analysing the relationship between population ageing, government expenditure and economic growth.

In this analysis, we use the natural logarithm of GDP per capita in constant 2017 international dollars as our outcome variable; data are sourced from the World Bank Development Indicators. Similar to our theoretical model, we continue employing the support ratio as our measure of population ageing. For this expanded sample, we adopt proxies for elderly spending and productive expenditure that are more widely available across countries. Specifically, we use healthcare expenditure as a percentage of GDP, referred to as *Healthcare expenditure*, as a proxy for elderly spending. For productive expenditure, we use total government expenditure on education as a percentage of GDP; the latter is referred to as *Education expenditure*. Incorporating the support ratio as well as proxies of elderly spending and productive expenditure in the model allows us to distinguish between the fiscal and the demographic effects of population ageing.

To control for other factors that may affect economic growth, we include a set of time-varying, country-specific covariates. *Government effectiveness* captures the regulatory quality at the country level and across time. The latter has been found to affect economic growth (see, e.g., Nawaz 2015; Rodrik et al. 2004). Human capital is also an important determinant of long-term economic growth by improving productivity (see, e.g., Barro 1991). We use the rates of secondary school enrollment, referred to as *Secondary school enrollment*, to capture this effect. Furthermore, we control for infant mortality rates (*Infant mortality*) as an additional control for human capital. *Gross capital formation*, representing investment in physical capital such as infrastructure and equipment, is also included. Furthermore, we control for *Trade*, measured as the share of trade in GDP, to capture the degree of integration into the global economy. Greater openness is generally associated with higher economic growth (e.g., Frankel and Romer, 1999; Karras, 2003). We also incorporate *Population density* in the set of our control variables.

All data pertaining to the growth analyses are obtained from the World Bank Development Indicators except data for *Government effectiveness* which are sourced from the V-DEM dataset (Coppedge et al., 2021). Descriptive statistics for the variables used in the growth regressions are provided in Table 5 in Section 5.3.4.

3.2 Empirical Analysis

3.2.1 Population Ageing and Government Spending Categories

To test the first two predictions of Corollary 1, i. e, the differential impact of population ageing on the share of spending for the elderly in total output, τ_H, and the share of productive expenditure in total output, τ_G, we estimate the following equations distinguishing the two subcategories of elderly spending and the four subcategories of productive government spending measured relative to total output:

$$(\tau_H)_{jit} = \alpha_j + \beta_{Hj}\phi^m_{it} + \mathbf{X}_{it}\chi_j + \mu_t + \lambda_i + \epsilon_{jit}, \tag{3.1}$$

$$(\tau_G)_{jit} = \alpha_j + \beta_{Gj}\phi^m_{it} + \mathbf{X}_{it}\chi_j + \mu_t + \lambda_i + \epsilon_{jit}. \tag{3.2}$$

Here, $(\tau_H)_{jit}$ and $(\tau_G)_{jit}$ refer to the respective share of category j of elderly spending and of productive expenditure in GDP for country i in year t. ϕ^m_{it} denotes the support ratio for country i at year t. $\mathbf{X}_{it}$ is the vector of control variables. μ_t and λ_i capture year and country fixed effects, respectively, to control for unobserved heterogeneity at the country and year level. Finally, ϵ_{jit} is the country- and time-specific error term of category j.

In equations (3.1) and (3.2), β_{Hj} and β_{Gj} are our coefficients of interest; they capture the differential impact of population ageing on government expenditure categories; elderly spending and productive expenditure. In light of Corollary 1, we expect β_{Hj} to be negative and significant corresponding to a positive effect of population ageing on elderly spending (share of output). In contrast, we expect β_{Gj} to be negative and/or insignificant. Equations (3.1) and (3.2) are initially estimated using ordinary least squares (OLS) which may suffer from endogeneity issues. Our main concern relates to reverse causality as elderly spending, especially on hospital services, may increase life expectancy and alter the support ratio. In that case, the estimated OLS coefficient may be inconsistent and biased, making the regressions results spurious.

To address these potential shortcomings, we apply a two-stage least squares (2SLS) approach. This procedure starts by regressing the independent variable of interest (i. e., the support ratio) on an instrumental variable. We choose the 1992 United Nations' forecasts of the support ratio constructed using population projections.[11] We include all other country-level covariates in addition to country and year fixed effects. This yields the fitted values for the support ratio which are employed as an explanatory variable in the second stage along with all other country-level controls and fixed effects.

[11] Using 1992 population projections implies the loss of 4 countries in the sample, namely, Czechia, Croatia, Slovenia and Slovakia, which were either part of Yugoslavia or Czechoslovakia when the statistics were produced by the United Nations.

3.2.2 Population Ageing and Economic Growth

To test the third prediction of Corollary 1, i. e., the impact of population ageing on the economy's growth rate, we estimate the following model:

$$\ln y_{it} = \alpha ln y_{i,t-1} + \beta_1 \phi^m_{it} + \beta_2 (\tau_H)_{it} + \beta_3 (\tau_G)_{it} + \mathbf{X}_{it}\theta + \mu_t + \lambda_i + \epsilon_{it} \tag{3.3}$$

In equation (3.3), $\ln y_{it}$ denotes the natural logarithm of GDP per capita for country i in year t, $\ln y_{i,t-1}$ represents its lagged value to capture persistence over time. ϕ^m_{it} is the country-level support ratio in year t. $(\tau_H)_{it}$ is Healthcare expenditure; it captures elderly spending and measures healthcare expenditure as a share of GDP in country i at year t. By contrast, $(\tau_G)_{it}$, denoting Education expenditure, serves as a proxy for productive expenditure which is measured by education expenditure relative to GDP.

Country-level control variables are represented by $\mathbf{X}_{it}$. These include: Government effectiveness, Secondary school enrollment, Gross capital formation, Infant mortality, Trade and Population density as detailed in the previous section.

We begin by estimating the model using Pooled Ordinary Least Squares (POLS), which provides a preliminary estimate of the relationships between growth, population ageing and government expenditure. This method assumes that unobserved country-specific characteristics and year-specific effects are uncorrelated with the explanatory variables. If this assumption is not satisfied, the results of the POLS regression will be biased (Wooldridge, 2010). To account for unobserved heterogeneity, we then explicitly include country and year fixed effects; at both the country and year levels, we refer to the latter approach as a Fixed Effects (FE) model.

We also estimate the model using difference Generalised Method of Moments (GMM) (Arellano and Bond, 1991) and system Generalised Method of Moments (Arellano and Bover 1995 and Blundell and Bond 1998).

Difference GMM eliminates country-level fixed effects by first-differencing the model and uses past observations of the regressors as instruments. In this paper, we specifically use a two-step difference GMM estimation which accounts for the variance-covariance structure of the differenced disturbance term. This method employs a robust weighting matrix in the second step, thus providing standard errors that are robust to panel-specific autocorrelation and heteroskedasticity (Erickson and Whited, 2002).

We complete our analysis with a two-step system GMM estimation to address possible shortcomings associated with difference GMM. The latter method performs poorly when considering variables that are highly persistent over time since most of the variation is eliminated through first-differencing which may result in weak identification. System GMM combines difference equations with those in levels and uses the lagged first differences of the explanatory variables as instruments. This method has been shown to yield the smallest bias among the class of GMM estimators (Bun and Windmeijer, 2010).

3.3 Empirical Findings

3.3.1 Results on Ageing and Spending Categories

Ordinary Least Squares Regressions

As per the theoretical predictions in Corollary 1, we anticipate a positive effect of population ageing on elderly spending categories and no significant effect on productive spending categories.

Columns (1) and (2) of Table 2 report the estimated effects pertaining to elderly spending subcategories, i. e., *Old age* and *Hospitals*. In both regressions, the coefficients on the support ratio are negative and statistically significant at the 1 percent level, indicating a strong relationship between population ageing and increased spending in these categories. The results imply that a 1 unit decrease in the support ratio is associated with a 0.076 and a 0.360 percentage point increase in spending on hospital services and old age, respectively.

In line with Corollary 1, the results for productive expenditure categories − presented in columns (3), (4), (5), and (6) − show no significant relationship with the support ratio at the 1 percent level. Albeit, the coefficient estimate on the support ratio when considering education expenditure is positive but small, estimated at 0.023, and only significant at the 5 percent significance level. Whereas, the coefficient estimate on the support ratio is −0.004 and only significant at the 10 percent significance level.

Overall, these results support the theoretical prediction that population ageing is associated with increased elderly spending and has no sizable association with productive expenditure. Nonetheless, the obtained OLS estimates are susceptible to possible endogeneity issues especially those pertaining to reverse causality. Elderly spending, especially on hospital services, may increase life expectancy and alter the support ratio. In that case, the estimated OLS coefficient are inconsistent and biased, making the regressions results spurious. We address potential endogeneity concerns in subsequent analyses through a two-stage least squares (2SLS) approach.

Table 2: OLS estimates of the effect of population ageing on elderly spending and productive expenditure – OECD countries

	Spending for the elderly		Productive spending			
	(1)	(2)	(3)	(4)	(5)	(6)
	Hospitals	Old Age	Education	Transport	Communication	R&D
Support ratio	-0.076***	-0.360***	0.023**	0.020	0.001	-0.004*
	(0.020)	(0.078)	(0.011)	(0.040)	(0.004)	(0.002)
Population density	0.001	-0.009***	0.002***	0.004*	-0.000***	-0.000
	(0.001)	(0.002)	(0.001)	(0.002)	(0.000)	(0.000)
Unemployment	-0.018**	0.106***	0.004	-0.016	0.001	0.002*
	(0.008)	(0.017)	(0.003)	(0.011)	(0.001)	(0.001)
Inflation	-0.010	-0.070***	-0.004	-0.049***	0.001	0.003*
	(0.009)	(0.018)	(0.004)	(0.012)	(0.001)	(0.002)
Government effectiveness	-0.027	-0.701**	-0.088	-0.464**	0.007	-0.008
	(0.109)	(0.310)	(0.059)	(0.186)	(0.020)	(0.014)
Expenses	0.012**	0.034***	0.001	0.021	-0.001	0.000
	(0.005)	(0.011)	(0.002)	(0.014)	(0.000)	(0.000)
CAB	-0.007	-0.013	-0.009***	-0.035***	-0.002***	-0.000
	(0.006)	(0.018)	(0.002)	(0.008)	(0.001)	(0.001)
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	360	360	360	360	360	339
Adjusted R-squared	0.942	0.959	0.905	0.776	0.511	0.894

Summary: This table presents OLS estimates of the relationship between population ageing and different types of government spending. It illustrates that population ageing (support ratio) is associated with an increase (decrease) in public expenditure for the elderly given by spending on hospital services and spending on old age. By contrast, it shows that population ageing does not have a sizable association with productive spending subcategories, namely education, transport, communication and R&D.
Notes: (i) Support ratio is the ratio of the working population (aged between 15 and 64) to the total adult population (aged 15 and above) (ii) standard errors are clustered at country and year level; robust and clustered standard errors are reported in parentheses; (iii) *** denotes statistical significance at the 1 percent level ($p < 0.01$), ** at the 5 percent level ($p < 0.05$), and * at the 10 percent level ($p < 0.10$), all for two-sided hypothesis tests.

Instrumental Variable Regressions

To address potential endogeneity concerns in our analysis, we instrument the support ratio using forecasts from the 1992 United Nations World Population Projections, denoted as *Support Ratio Forecasts*].[12] These projections were produced 15 years before the start of our sample period. They were constructed based on historical trends in fertility and mortality, ensuring a plausible correlation with the support ratio during the sample period. Furthermore, these projections rely solely on demographic data available before 1992. As a result, they are likely exogenous to economic shocks, fiscal policies, or other unobserved factors that may influence government expenditure or growth during the period studied.

The first stage results, shown in Table 6 in Section 5.3 of the Appendix, indicate that the *Support Ratio Forecasts* serves as a strong and relevant instrument for population ageing. The coefficient on the instrumental variable is estimated at 0.620 and is significant at the 1 percent level for Sample A, which includes observations available for all dependent variables except R&D expenditure. In Sample B, limited to observations available for R&D, the coefficient on the instrument is 0.634 and significant at the 1 percent level. The high R-squared values from both first-stage estimations support the relevance of the instrument.

Table 3 presents the second stage results of our instrumental variable regression analysis. Under IV estimation, the coefficients on the predicted values of the support ratio are large and significant for elderly spending subcategories, i. e., *Hospitals* and *Old age.* The estimated effect of the predicted support ratio on hospital services spending is -0.234, while the effect on old age spending is -0.637; both are significant at the 1 percent level. These estimates suggest that a 1 unit decrease in the support ratio, corresponding to a 1 percent reduction in the share of the working population relative to the total adult population, results in a 0.234 percent growth in spending on hospital services relative to total GDP. A similar change in the support ratio causes a 0.637 percentage point increase in old age spending. This finding further corroborates our theoretical prediction that population ageing increases elderly spending.

The coefficient estimate on the predicted values of the support ratio is only significant for the education subcategory. However, the coefficient estimated at -0.067 is considerably smaller in magnitude than those observed for elderly spending subcategories and only significant at the 5 percent significance level. Moreover, consistent with our expectations, the IV estimation does not indicate a significant causal effect of population ageing on the remaining productive expenditure categories: *Transport*, *Communication* and *R&D.*

In summary, the IV results confirm a causal link between population ageing and increased spending for the elderly, while the effects on productive expenditure remain mostly insignificant except for the education expenditure subcategory. These findings support the theoretical predictions outlined in Corollary 1, demonstrating that the primary impact of population ageing is concentrated within elderly-related spending, with only marginal

[12]The earliest available forecasts from the United Nations were produced in 1992.

and insignificant effects on productive expenditure.

Table 3: IV estimates of the effect of population ageing on elderly spending and productive expenditure – OECD countries

	(1) Hospital	(2) Old Age	(3) Education	(4) Transport	(5) Communication	(6) R&D
Support ratio (predicted)	-0.234***	-0.637***	-0.067**	-0.065	-0.006	-0.007
	(0.053)	(0.179)	(0.029)	(0.081)	(0.007)	(0.008)
Population density	-0.000	-0.012***	0.001*	0.003	-0.000***	-0.000
	(0.001)	(0.002)	(0.001)	(0.002)	(0.000)	(0.000)
Unemployment	-0.017**	0.116***	0.006**	-0.011	0.001	0.002*
	(0.007)	(0.017)	(0.003)	(0.011)	(0.001)	(0.001)
Inflation	-0.012	-0.073***	-0.006	-0.052***	0.001	0.004*
	(0.009)	(0.020)	(0.004)	(0.012)	(0.001)	(0.002)
Government effectiveness	0.008	-0.534	-0.046	-0.493**	0.004	-0.008
	(0.115)	(0.326)	(0.057)	(0.193)	(0.020)	(0.014)
Expenses	0.013**	0.036**	0.001	0.022	-0.001*	0.000
	(0.006)	(0.015)	(0.002)	(0.015)	(0.000)	(0.000)
CAB	-0.012**	-0.016	-0.012***	-0.033***	-0.002***	-0.000
	(0.006)	(0.018)	(0.002)	(0.008)	(0.001)	(0.001)
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	312	312	312	312	312	291
Adjusted R-squared	0.946	0.959	0.910	0.796	0.527	0.902

Summary: This table presents the second-stage IV estimates of the relationship between population ageing and different types of government spending. It illustrates that population ageing (support ratio) increases (decreases) public expenditure for the elderly given by spending on hospital services and spending on old age. By contrast, it shows that population ageing does not have a sizable impact on productive spending subcategories, namely education, transport, communication and R&D.
Notes: (i) The support ratio is the share of the working-age population (aged between 15 and 65) relative to the total adult population (aged 15 and above). (ii) Standard errors are clustered at the country and year levels; robust standard errors are reported in parentheses. (iii) *** denotes statistical significance at the 1 percent level ($p < 0.01$), ** at the 5 percent level ($p < 0.05$), and * at the 10 percent level ($p < 0.10$), all for two-sided hypothesis tests.

3.3.2 Results on Population Ageing and Economic Growth

In Section 3.3.1, we found that population ageing increases elderly spending and has no sizable effect on productive expenditure. In this section, we investigate the third prediction of Corollary 1 which stipulates that population ageing lowers the economy's growth rate. We differentiate between the demographic and the fiscal effect of population ageing by incorporating separate measures of population ageing, elderly spending and productive expenditure. As described in Section 3.1, population ageing is captured using the *Support ratio*, elderly spending and productive expenditure are measured using *Healthcare expenditure* and *Education expenditure*, respectively. Economic growth is measured using the natural logarithm of GDP per capita.

We investigate the effect of population ageing and the two categories of public expenditure using regression analysis. In Table 4, Column (1) shows the results of the Pooled Ordinary Least Squares analysis; it only includes the variables *Support ratio*, *Healthcare expenditure* and *Education expenditure*. Column (2) features the results of the POLS estimation including all controls. Column (3) reports the results of the Fixed Effects (FE) estimation. Columns (4) and (5) display the results of two-step Difference and System GMM estimations, respectively.

The support ratio exhibits a significant and negative association with the natural logarithm of GDP per capita under both the POLS and Fixed Effects estimations. In the baseline POLS regression in Column (1), a one unit increase in the support ratio is associated with a 0.001 percent decrease in GDP per capita. The coefficient estimate is significant at the 1 percent significance level. The inclusion of all the control variables in Column (2) leaves this coefficient estimate unchanged but the latter is only significant at the 5 percent significance level. In the Fixed Effects model shown in Column (3), the magnitude of the effect increases, with a coefficient estimated at -0.004. Surprisingly, the estimation of the POLS and Fixed Effects models suggest that population ageing, reflected by a declining support ratio, has a positive association with economic growth.

The estimates of both the POLS and the Fixed Effects models show that healthcare expenditure has negative and significant association with economic growth. In the baseline POLS estimation displayed in Column (1), a 1 percent increase in healthcare expenditure as a share of GDP is associated with a 0.003 percent decrease in GDP per capita. The inclusion of additional controls in Column (2) reduces the magnitude of the coefficient to -0.002, while the Fixed Effects model in Column (3) shows a larger absolute effect of -0.005. The results from the POLS and the Fixed Effects model suggest that elderly spending, measured by Healthcare expenditure, is associated with lower economic growth.

Education expenditure shows a negative relationship with GDP per capita only according to the POLS estimations. In the baseline POLS model shown in Column (1), the coefficient estimate is -0.001 and significant at the 5 percent significance level. Including the set of control variables in Column (2) increases the magnitude of the coefficient to -0.004 and is significant at the 1 percent significance level. However, in the Fixed Effects model in Column (3), the coefficient estimate declines in absolute terms to -0.002 and is no longer statistically significant.

Column (4) presents the results of the two-step Difference GMM estimation. The coefficient on the support ratio is estimated at -0.026, but it is not statistically significant. This suggests that once endogeneity is addressed, the direct demographic impact of population ageing on economic growth cannot be established.

Under the Difference GMM estimation, the coefficient estimate on Healthcare expenditure is -0.046 and is statistically significant at the 5 percent level. The magnitude of the coefficient estimate is considerably larger than those obtained using POLS and Fixed Effects models. By contrast, the coefficient estimate on Education expenditure is not statistically significant. This suggest that the estimates obtained under POLS and Fixed Effects models may be driven by unobserved confounders.

Column (5) of Table 4 reports the findings of the two-steep System GMM; a method which has been shown to yield the smallest bias amongst the class of GMM estimators (Bun and Windmeijer, 2007). Overall, the results of the two-step System GMM estimation are consistent with those of the two-step Difference GMM estimation.

The coefficient on the support ratio is estimated at -0.028 but is not statistically significant. This further demonstrates that, once endogeneity is accounted for, population ageing does not have a direct impact on economic growth.

Under the System GMM estimation, the coefficient estimate on Healthcare expenditure is negative and significant at the 1 percent significance level. It shows that a 1 percent increase in healthcare expenditure as a share of GDP reduces GDP per capita by 0.047 percent. The coefficient estimate on Education expenditure is not found to be statistically significant.

Employing both two-step Difference and System GMM estimation methods reveals that Healthcare expenditure reduces economic growth. Support ratio and Education expenditure, in contrast, exhibit no significant impact

For the GMM estimates to be consistent, the instruments must fulfill the exogeneity condition. This can be examined using the J-statistic of the Hansen test which assumes the exogeneity of the instruments under the null hypothesis. According to Roodman (2009), a failure to reject the null entails that the p-value associated with this test does not fall bellow a threshold of 0.1.

The validity of the instruments can also be further evaluated using the Arellano-Bond test for AR(2) serial correlation in first differences. A rejection of the null hypothesis indicates serial correlation in the residuals, thus requiring the use of higher order lags in the instruments.

A final potential issue to consider when implementing a GMM estimation technique is that of *too many instrumental variables* which results in an overfit of the endogenous variables. Although there is no gold standard, Roodman (2009) advised that the number of instruments must not exceed that of the panels (i. e., countries) in the sample examined. To limit the proliferation of instruments in this analysis, we use only up to two lags and we also employ the "collapse" function in Stata 14, which is a common technique in the literature (Ribeiro et al., 2020).

In both the difference and the system GMM specifications, the exogenous variables are represented by the year dummies whereas all the other independent variables are treated as endogenous. Reported in Table 4, the p-values associated with the Hansen and the Arellano and Bond tests are well above the 0.1 threshold for both the Difference GMM and the System GMM. The results of these tests indicate a non-rejection of the respective null hypotheses and lend credibility to the validity of the instruments employed.

Overall, the results of the two-step Difference and System GMM estimation reveal that population ageing and productive expenditure, respectively measured by Support ratio and Education expenditure, have no effect on GDP per capita. By contrast, Healthcare expenditure, capturing elderly spending, is found to have a negative and statistically significant effect. This, combined with our findings that population ageing increases elderly spending (see Section 3.3.1) demonstrates population ageing on economic growth. that there is a negative fiscal effect of population ageing on economic growth.

Table 4: Estimates of the effect of population ageing, elderly spending and productive expenditure on economic growth – All countries

	(1)	(2)	(3)	(4)	(5)
Support ratio	-0.001***	-0.001**	-0.004***	-0.026	-0.028
	(0.000)	(0.000)	(0.001)	(0.059)	(0.026)
Healthcare expenditure	-0.003***	-0.002***	-0.005***	-0.046**	-0.047***
	(0.001)	(0.001)	(0.002)	(0.023)	(0.014)
Education expenditure	-0.001**	-0.004***	-0.002	0.011	-0.004
	(0.001)	(0.001)	(0.002)	(0.011)	(0.009)
Government effectivenesss		0.005**	0.018***	0.093	0.029
		(0.002)	(0.006)	(0.109)	(0.046)
Secondary school enrollment		0.000***	0.000	0.003	0.000
		(0.000)	(0.000)	(0.004)	(0.001)
Gross capital formation		0.001***	0.002***	0.003	0.003**
		(0.000)	(0.000)	(0.003)	(0.001)
Infant mortality		-0.000	-0.001**	0.010	-0.001
		(0.000)	(0.000)	(0.016)	(0.011)
Trade		0.000***	0.000***	0.000	0.000
		(0.000)	(0.000)	(0.001)	(0.000)
Population density		-0.000***	0.000**	0.004	0.003*
		(0.000)	(0.000)	(0.005)	(0.002)
Log GDP per capita, lag	0.994***	0.986***	0.900***	0.788***	0.925***
	(0.001)	(0.002)	(0.013)	(0.196)	(0.023)
Observations	1955	1349	1349	821	1001
Number of countries	165	138	138	107	120
Instruments	-	-	-	45	56
AR(2) p-value	-	-	-	0.894	0.541
Hansen p-value	-	-	-	0.933	0.942

Summary: This table presents the regression results of economic growth, measured by the natural logarithm of GDP per capita on population ageing, elderly spending and productive expenditure; respectively measured using the support ratio, healthcare expenditure and education expenditure. Columns (1), (2) and (3) show the results of the POLS and Fixed Effects estimation. Columns (4) and (5) report the findings of the two-step difference GMM and the two-step system GMM analyses, respectively. The GMM regression findings establish the presence of a negative and significant effect of elderly spending on economic growth and does not reveal a direct effect of population ageing and productive expenditure on economic growth.

Notes: (i) The support ratio is the share of the working-age population (aged between 15 and 65) relative to the total adult population (aged 15 and above). (ii) For POLS estimation, the standard errors are clustered at the country and year levels; robust standard errors are reported in parentheses. (iii) *** denotes statistical significance at the 1 percent level ($p < 0.01$), ** at the 5 percent level ($p < 0.05$), and * at the 10 percent level ($p < 0.10$), all for two-sided hypothesis tests. (iv) Year dummies are treated as strictly exogenous under both the difference and the system GMM. (v) Under the difference and system GMM specifications, the first and second lags of the endogenous variables were used as instruments for the endogenous variables. (vi) The null hypothesis under the Hansen test establishes that the instruments are uncorrelated with the residuals. (viii) The null hypothesis under the Arellano-Bond test for AR(2) posits that in first differences, the errors are not serially correlated of order 2.

4 Concluding Remarks

This paper develops a political-economic model with endogenous economic growth and households that are heterogeneous in their age composition. This framework is used to analyse how population ageing via a democratic voting process endogenously affects the composition of government spending and economic growth. The model predicts that population aging (i) induces a higher level of elderly spending, (ii) leaves productive spending unaffected, and (iii) slows down economic growth.

Applying regression analysis on a sample of 30 OECD countries over the 2007-2018 period delivers supporting evidence for predictions (i) and (ii). To evaluate the tenets of prediction (iii), we use dynamic regression analysis on a baseline sample of 178 countries from 2000 to 2018. The obtained results considerably support this prediction. Hence, this paper provides a theoretical rationale and empirical evidence for the conjecture that population ageing reduces economic growth through its effect on the composition of government expenditure.

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5 Appendix

5.1 Proofs

This appendix contains the proofs of Proposition 1 - 3 and of Corollary 1.

5.1.1 Proof of Proposition 1

Claim 1 Household-producer i's intertemporal optimization problem gives rise to the following present-value Hamiltonian

$$\mathcal{H}^i \equiv N^i \left[\ln c^i + (1-\phi^i) b \ln H\right] e^{-\rho t} + \lambda^i \left[(1-\tau) A^{\frac{1}{\alpha}} \left(\frac{G}{y}\right)^{\frac{1-\alpha}{\alpha}} k^i - \frac{c^i}{\phi^i}\right], \qquad (5.1)$$

where λ^i denotes the present-value shadow price of household-producer i's capital stock. In performing the optimization, the individual household-producer takes τ, y, G, and H as given. Then, the necessary and sufficient optimality conditions are[13]

$$\frac{e^{-\rho t}}{c^i} = \frac{\lambda^i}{L^i} \qquad (5.2)$$

$$\dot{\lambda}^i = -\lambda^i (1-\tau) A^{\frac{1}{\alpha}} \left(\frac{G}{y}\right)^{\frac{1-\alpha}{\alpha}} \qquad (5.3)$$

$$0 = \lim_{t\to\infty} \left[\lambda^i k^i\right]. \qquad (5.4)$$

Since L^i is constant, $G/y = \tau_G L$, and $\tau = \tau_G + \tau_H$ we may combine (5.2) and (5.3) to obtain (2.8).

From the flow budget constraint (2.6) we know that

$$\frac{c^i}{\phi^i k^i} = (1-\tau) A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}} - \frac{\dot{k}^i}{k^i}. \qquad (5.5)$$

In a steady state the growth rate of the household-producer's capital stock per worker has to be constant. Therefore, for constant tax rates, τ_G and τ_H, the right-hand side of (5.5) is constant. Consequently, $c^i/(\phi^i k^i)$ is constant. Moreover, for a constant ϕ^i, the growth rate of the household-producer's capital stock per worker equals the growth rate of consumption per household member. Hence, in the steady state output per worker grows at the same rate as k^i and c^i.

[13] The Hamiltonian H^i is the sum of a concave function of c^i and a linear function of (k^i, c^i). Therefore, it is concave in (k^i, c^i). Moreover, it is strictly concave in c^i. Thus, the paths of c^i and k^i implied by (5.2)-(5.4) deliver a unique global maximum.

Since all households have the same initial capital-labor share $k^i(0) = k_0$ and $y^i(t) = A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}} k^i(t)$ it is the case that all household-producers at all t have the same output and the same capital per worker. Accordingly, in equilibrium individual and economy-wide variables per worker coincide, i. e., $k = \int_0^1 k^i di = k^i$ and $y = \int_0^1 y^i di = y^i$. Each household-producer's instantaneous level of aggregate capital, output, and consumption depends on her respective (constant) labor supply and is proportional to k^i:

$$K^i(t) = k^i(t)L^i \tag{5.6}$$

$$Y^i(t) = A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}} k^i(t)L^i \tag{5.7}$$

$$C^i(t) = \rho k^i(t)L^i. \tag{5.8}$$

Thus, these three variables grow at the same rate as k^i. Finally, the economy-wide aggregate variables are given by

$$K(t) = \int_0^1 K^i(t)di = k^i(t) \int_0^1 L^i di = k^i(t)L \tag{5.9}$$

$$Y(t) = \int_0^1 Y^i(t)di = A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}} k^i(t)L \tag{5.10}$$

$$C(t) = \int_0^1 C^i(t)di = \rho k^i(t)L. \tag{5.11}$$

Hence, for a constant aggregate labor supply (L), K, Y, and C have to grow at the same rate as k^i. Finally, as G and H are proportional to aggregate output, these variables also have to grow at this rate. Thus, we have indeed established the existence of a steady-state growth path along which all variables, including the respective levels of public services, grow the same constant rate

$$\gamma(\tau_G, \tau_H) = \frac{\dot{c}^i}{c^i} = \frac{\dot{k}^i}{k^i}. \tag{5.12}$$

Moreover, using (5.2) and (5.3) to evaluate (5.4) one readily verifies that the transversality condition holds for any parameter constellation.

Claim 2 It is straightforward to show that the economy immediately jumps onto the steady-state path. The proof of this mirrors the one of a standard AK model. ■

5.1.2 Proof of Proposition 2

Substituting for $c^i(t)$ and $H(t)$ in household-producer i's utility function (2.5) and solving the integral gives

$$
\begin{aligned}
U^i(0) &= \frac{N^i}{\rho}\left[\ln(\phi^i \rho k_0) + (1-\phi^i)b \ln\left(\tau_H \left(\tau_G\right)^{\frac{1-\alpha}{\alpha}} k_0 \left(AL\right)^{\frac{1}{\alpha}}\right) + \frac{(1+(1-\phi^i)b)\gamma(\cdot)}{\rho}\right] \\
&\equiv U^i\left(\tau_G^i, \tau_H^i\right).
\end{aligned} \tag{5.13}
$$

Then, the optimization problem of household i reduces to choosing $\tau_G \in [0,1]$ and $\tau_H \in [0,1]$ to maximize (5.13) with γ given by (2.8). To determine the global maximum of U^i in the square $0 \leq \tau_G \leq 1$ and $0 \leq \tau_H \leq 1$ we proceed in two steps. First, we show that there exists a unique local maximum in the interior of the square. Second, we verify that this policy mix represents the global maximum in the square by comparing its implied utility level with the utility obtained at the local extrema on the boundary of the square and at corner points.

1. Derivation of the unique local maximum in the interior of the square:

   The above optimization problem delivers the following pair of necessary first-order conditions for an interior optimum

$$
\frac{\partial U^i}{\partial \tau_G} = \frac{(1-\phi^i)b(1-\alpha)}{\alpha \tau_G} + \frac{1+(1-\phi^i)b}{\rho}\frac{\partial \gamma(\cdot)}{\partial \tau_G} = 0 \tag{5.14}
$$

$$
\frac{\partial U^i}{\partial \tau_H} = \frac{(1-\phi^i)b}{\tau_H} + \frac{1+(1-\phi^i)b}{\rho}\frac{\partial \gamma(\cdot)}{\partial \tau_H} = 0, \tag{5.15}
$$

   where

$$
\frac{\partial \gamma}{\partial \tau_G} = \frac{A^{\frac{1}{\alpha}}\left(\tau_G L\right)^{\frac{1-\alpha}{\alpha}}}{\alpha \tau_G}\left[(1-\alpha)(1-\tau_H) - \tau_G\right] \tag{5.16}
$$

$$
\frac{\partial \gamma}{\partial \tau_H} = -A^{\frac{1}{\alpha}}\left(\tau_G L\right)^{\frac{1-\alpha}{\alpha}} < 0. \tag{5.17}
$$

   Rewriting conditions (5.14) and (5.15), household i's most preferred expenditure shares τ_G^i and τ_H^i are implicitly determined by

$$
\frac{1+(1-\phi^i)b}{(1-\phi^i)b\rho} = -\frac{(1-\alpha)}{\alpha \tau_G^i \left(\frac{\partial \gamma(\cdot)}{\partial \tau_G}\right)\Big|_{\tau_G^i,\tau_H^i}} \tag{5.18}
$$

$$
\frac{1+(1-\phi^i)b}{(1-\phi^i)b\rho} = -\frac{1}{\tau_H^i \left(\frac{\partial \gamma(\cdot)}{\partial \tau_H}\right)\Big|_{\tau_G^i,\tau_H^i}}, \tag{5.19}
$$

respectively. Then, combining (5.18) and (5.19) and taking (5.16) and (5.17) into account yields

$$-\frac{1-\alpha}{(1-\tau_H^i)(1-\alpha)-\tau_G^i}=\frac{1}{\tau_H^i}$$

and thus $\tau_G^i = 1-\alpha$.

Then, substituting $\tau_G^i = 1-\alpha$ and (5.17) in (5.15) and rearranging yields

$$\tau_H^i = \frac{(1-\phi^i)b\rho}{\left[1+(1-\phi^i)b\right] A^{\frac{1}{\alpha}} \left[(1-\alpha)L\right]^{\frac{1-\alpha}{\alpha}}}, \tag{5.20}$$

which is equation (2.14).

The sufficient condition for (τ_G^i, τ_H^i) to be a local maximum is

$$D\left(\tau_G^i, \tau_H^i\right) \equiv \left.\frac{\partial^2 U^i}{\partial \tau_G^2}\right|_{\tau_G^i,\tau_H^i} \times \left.\frac{\partial^2 U^i}{\partial \tau_H^2}\right|_{\tau_G^i,\tau_H^i} - \left(\left.\frac{\partial^2 U^i}{\partial \tau_G \partial \tau_H}\right|_{\tau_G^i,\tau_H^i}\right)^2 > 0, \tag{5.21}$$

where

$$\frac{\partial^2 U^i}{\partial \tau_H^2} = -\frac{N^i}{\rho}\frac{(1-\phi^i)b}{(\tau_H)^2}, \tag{5.22}$$

$$\frac{\partial^2 U^i}{\partial \tau_G \partial \tau_H} = -\frac{N^i}{\rho}\frac{1-\alpha}{\alpha}\frac{1+(1-\phi^i)b}{\rho}\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}(\tau_G)^{\frac{1-\alpha}{\alpha}-1}, \tag{5.23}$$

$$\frac{\partial^2 U^i}{\partial \tau_G^2} = \frac{N^i}{\rho}\left[-\frac{(1-\phi^i)b\,(1-\alpha)}{\alpha\,(\tau_G)^2}+\frac{1+(1-\phi^i)b}{\rho}\frac{\partial^2\gamma}{\partial \tau_G^2}\right] \quad \text{with} \tag{5.24}$$

$$\frac{\partial^2 \gamma}{\partial \tau_G^2} = -\frac{1-\alpha}{\alpha}\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}(\tau_G)^{\frac{1-\alpha}{\alpha}-2}\left[\frac{\tau_G}{\alpha}+\left(1-\frac{1-\alpha}{\alpha}\right)(1-\tau_H)\right].$$

Evaluating equations (5.22)-(5.24) at $\tau_G^i = 1-\alpha$ and at τ_H^i as given by (5.20), substituting the resulting expressions in (5.21) and rearranging yields

$$D\left(\tau_G^i, \tau_H^i\right) = \frac{\left[1+(1-\phi^i)b\right]\left(AL^{1-\alpha}\right)^{\frac{3}{\alpha}}(1-\alpha)^{\frac{3(1-\alpha)}{\alpha}}}{(1-\phi^i)b\rho^3} > 0.$$

Thus, the policy mix (τ_G^i, τ_H^i) is a local maximum in the interior of the square. The utility level associated with this policy mix is

$$\begin{aligned} U^i\left(\tau_G^i,\tau_H^i\right) &= \frac{N^i}{\rho}\left[\ln(\phi^i\rho k_0)+(1-\phi^i)b\ln\left(\frac{(1-\phi^i)b\rho}{1+(1-\phi^i)b}k_0L\right)\right] \\ &+ \frac{N^i(1+(1-\phi^i)b)}{\rho^2}\alpha\left(A(1-\alpha)^{1-\alpha}L^{1-\alpha}\right)^{\frac{1}{\alpha}} \\ &- \frac{N^i}{\rho}\left[(1-\phi^i)b+(1+(1-\phi^i)b)\right]. \end{aligned} \tag{5.25}$$

2. Comparison to local maxima on the boundary of the square and to corner points:

   The boundary of the square consists of 4 parts. On the first two sides with either $\tau_G = 0$ or $\tau_H = 0$ no relative maximum exists as U^i tends to $-\infty$ if one of the tax rates approaches zero. Side 3 is $\tau_G = 1$ and $\tau_H \in [0,1]$. On this side, we have

$$U^i\left(1,\tau_H\right) = \frac{N^i}{\rho}\left[\ln(\phi^i\rho k_0)+(1-\phi^i)b\ln\left(\tau_H k_0\left(AL\right)^{\frac{1}{\alpha}}\right)+\frac{(1+(1-\phi^i)b)}{\rho}\left(-\tau_H\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}-\rho\right)\right]$$

   and $\frac{\partial U^i(1,\tau_H)}{\partial\tau_H} = 0$ delivers the relative extremum

$$\hat{\tau}_H = \frac{(1-\phi^i)b\rho}{\left[1+(1-\phi^i)b\right]\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}}.$$

   Evaluating U^i at this critical point gives

$$\begin{aligned} U^i\left(1,\hat{\tau}_H\right) &= \frac{N^i}{\rho}\left[\ln(\phi^i\rho k_0)+(1-\phi^i)b\ln\left(\frac{(1-\phi^i)b\rho}{1+(1-\phi^i)b}k_0L\right)\right] \\ &- \frac{N^i}{\rho}\left[(1-\phi^i)b+(1+(1-\phi^i)b)\right], \end{aligned}$$

   which is strictly smaller than $U^i\left(\tau_G^i,\tau_H^i\right)$ given by (5.25).

   Side 4 is $\tau_H = 1$ and $\tau_G \in [0,1]$. On this side, we have

$$U^i\left(\tau_G,1\right) = \frac{N^i}{\rho}\left[\ln(\phi^i\rho k_0)+(1-\phi^i)b\ln\left(\left(\tau_G\right)^{\frac{1-\alpha}{\alpha}}k_0\left(AL\right)^{\frac{1}{\alpha}}\right)-\frac{(1+(1-\phi^i)b)\left(\tau_G AL^{1-\alpha}\right)^{\frac{1}{\alpha}}}{\rho}\right]$$

   and $\frac{\partial U^i(1,\tau_H)}{\partial\tau_G} = 0$ delivers the relative extremum

$$\bar{\tau}_G = \left[\frac{(1-\phi^i)\rho b\left(1-\alpha\right)}{\left[1+(1-\phi^i)b\right]\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}}\right]^{\alpha}.$$

Evaluating U^i at this critical point then gives

$$\begin{aligned} U^i\left(\bar{\tau}_G,1\right) &= \frac{N^i}{\rho}\left[\ln(\phi^i\rho k_0)+(1-\phi^i)b\ln\left(\left[\frac{(1-\phi^i)\rho b\,(1-\alpha)}{\left[1+(1-\phi^i)b\right]\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}}\right]^{1-\alpha}k_0\left(AL\right)^{\frac{1}{\alpha}}\right)\right] \\ &- \frac{N^i}{\rho}\left[(1-\phi^i)b\,(1-\alpha)+(1+(1-\phi^i)b)\right], \end{aligned}$$

which under Assumption 1 can be shown to be strictly smaller than (5.25).

The only candidate for a corner solution is $\tau_G=\tau_H=1$. In this case we obtain

$$U^i\left(1,1\right)=\frac{N^i}{\rho}\left[\ln(\phi^i\rho k_0)+(1-\phi^i)b\ln\left(k_0\left(AL\right)^{\frac{1}{\alpha}}\right)-\frac{(1+(1-\phi^i)b)\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}}{\rho}-(1+(1-\phi^i)b)\right],$$

which is strictly smaller than (5.25) because

$$\left[1+\ln\left(\frac{\left[1+(1-\phi^i)b\right]\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}}{(1-\phi^i)b\rho}\right)\right] < \frac{(1+(1-\phi^i)b)\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}}{(1-\phi^i)b\rho}\left[1+\alpha\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}}\right].$$

Thus, we have shown that all corner points and local extrema on the boundary of the square yield a lower utility than the interior local maximum at (τ_G^i,τ_H^i). Thus, this policy mix is the global maximum in the square. ■

5.1.3 Proof of Proposition 3

Follows with Proposition 2 and the arguments given in the main text. ■

5.1.4 Proof of Corollary 1

Follows immediately from Propositon 3 and and (2.15). ■

5.2 Extensions

5.2.1 Non-Separable Preferences

To gauge the sensitivity of our results this section considers an alternative specification of the utility function with non-separable preferences between private and public consumption.

In particular, we assume that household i's intertemporal utility is given by

$$U^i(0) = \int_0^\infty \frac{\left((C^i(t))^{\phi^i} H(t)^{1-\phi^i}\right)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \tag{5.26}$$

where $C^i(t) = c^i(t)N^i$ denotes household i's aggregate private consumption and σ is the reciprocal of the intertemporal elasticity of substitution for consumption. We assume $1-\sigma < 1$ such that the instantaneous utility function is strictly concave in its arguments. The share of private consumption in household i's utility relative to public consumption is given by the support ratio ϕ^i. The greater the share of elderly members, i. e., the smaller ϕ^i, the more important is the public consumption good for overall household utility. For simplicity, we normalize each households labor supply to unity, i. e., $L^i = 1$, such that in equilibrium $L = 1$. As the size of the household and her labor supply are assumed to be constant, this assumption does not affect our qualitative results, but simply eliminates the scale effect in the steady-state growth rate.

Economic Equilibrium

In this case, the optimization problem for each household i is to choose $c^i(t)$ and $k^i(t)$ to maximize (5.26), subject to (2.4), (2.6), and an initial capital stock per worker $k^i(0) = k_0 > 0$, taking G, H, and $\tau = \tau_G + \tau_H$ as given. The corresponding present-value Hamiltonian is

$$\mathcal{H}^i \equiv \frac{\left((c^i/\phi^i)^{\phi^i} H^{1-\phi^i}\right)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} + \lambda^i \left[(1-\tau_G-\tau_H)A^{\frac{1}{\alpha}} \left(\frac{G}{y}\right)^{\frac{1-\alpha}{\alpha}} k^i - \frac{c^i}{\phi^i}.\right], \tag{5.27}$$

where λ^i denotes the present-value shadow price of household-producer i's capital stock. The necessary and sufficient first-order conditions of this optimization problem yield

$$\left[(1-\sigma)\phi^i - 1\right] \frac{\dot{c}^i}{c^i} + (1-\phi^i)(1-\sigma)\frac{\dot{H}}{H} - \rho = \frac{\dot{\lambda}^i}{\lambda^i}, \tag{5.28}$$

$$\left[(1-\tau_G-\tau_H)A^{\frac{1}{\alpha}} \left(\frac{G}{y}\right)^{\frac{1-\alpha}{\alpha}}\right] = -\frac{\dot{\lambda}^i}{\lambda^i}, \tag{5.29}$$

$$\lim_{t\to\infty} \left[\lambda^i k^i\right] = 0. \tag{5.30}$$

Equation (2.7) implies for a time-invariant τ_H that $\dot{H}/H = \dot{Y}/Y$. Moreover, on a balanced growth path with a stationary population all variables have to grow at the same rate, i. e., $\gamma \equiv \dot{c}^i/c^i = \dot{H}/H$. Taking this into account and combining (5.28) and (5.29) with (2.10) for $L = 1$ yields the steady-state growth rate as

$$\gamma(\tau_G, \tau_H) = \frac{\dot{\tilde{c}}^i}{\tilde{c}^i} = \frac{1}{\sigma}\left[(1 - \tau_G - \tau_H)\, A^{\frac{1}{\alpha}} \left(\tau_G\right)^{\frac{1-\alpha}{\alpha}} - \rho\right], \tag{5.31}$$

which generalizes equation (2.8) to $\sigma \neq 1$. As before, the economy has no transitional dynamics and is always in a position at which all variables at the household and the economy-wide level as well as government expenditure grow at the rate γ. For utility to be bounded $\rho > \gamma(1 - \sigma)$ has to hold.

Given a starting amount of capital, $k^i(0)$, the levels of all variables are again determined. In particular, the initial quantity of consumption is

$$c^i(0) = \phi^i \left[(1 - \tau_G - \tau_H)\, A^{\frac{1}{\alpha}} \left(\tau_G\right)^{\frac{1-\alpha}{\alpha}} - \gamma\right] k^i(0) \tag{5.32}$$

and the initial level of the public consumption good is

$$H\left(0\right) = \tau_H A^{\frac{1}{\alpha}} \left(\tau_G\right)^{\frac{1-\alpha}{\alpha}} k^i\left(0\right). \tag{5.33}$$

Also note that equations (5.31) and (5.32) imply that $c^i(0)$ can be written as

$$c^i\left(0\right) \;=\; \phi^i \left[\rho - \gamma\left(1 - \sigma\right)\right] k^i\left(0\right). \tag{5.34}$$

Political-Economic Equilibrium

In the following we use the above results to determine household i's most preferred policy mix. The relevant optimization problem is

$$\max_{\tau_G, \tau_H} \int_0^\infty \frac{\left(\left(c^i(t)/\phi^i\right)^{\phi^i} H(t)^{1-\phi^i}\right)^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t}\, dt \quad \text{s.t.}$$

$$c^i(t) = c^i(0) e^{\gamma(\tau_G, \tau_H)t} \quad \text{and} \quad H(t) = H(0) e^{\gamma(\tau_G, \tau_H)t}.$$

For a constant γ the integral in the above equation can be simplified to yield (aside from a constant)

$$U^i(\tau_G, \tau_H) \;=\; \frac{\left(c^i\left(0\right)/\phi^i\right)^{\phi^i(1-\sigma)} H\left(0\right)^{\left(1-\phi^i\right)(1-\sigma)}}{(1 - \sigma)\left[\rho - \gamma\left(1 - \sigma\right)\right]}. \tag{5.35}$$

Then, using equations (5.33)-(5.34) in (5.35) gives i's indirect utility function as

$$U^i \;=\; \frac{k^i\left(0\right)^{1-\sigma}}{1 - \sigma} \left(\rho - \gamma(\cdot)\left(1 - \sigma\right)\right)^{\phi^i(1-\sigma)-1} \left(\tau_H A^{\frac{1}{\alpha}} \left(\tau_G\right)^{\frac{1-\alpha}{\alpha}}\right)^{\left(1-\phi^i\right)(1-\sigma)}. \tag{5.36}$$

Maximizing (5.36) with respect to τ_G and τ_H yields the following pair of first-order conditions

$$\begin{aligned}
&\frac{(1-\alpha)\left(1-\phi^i\right)}{\alpha\frac{\partial\gamma}{\partial\tau_G}}\left(\left[\rho-\gamma\left(1-\sigma\right)\right]\right)^{\phi^i(1-\sigma)-1}\left(\tau_H A^{\frac{1}{\alpha}}\right)^{\left(1-\phi^i\right)(1-\sigma)}\left(\tau_G\right)^{\frac{(1-\alpha)(1-\sigma)\left(1-\phi^i\right)}{\alpha}-1} \\
= & -\left(1-\phi^i\left(1-\sigma\right)\right)\left(\left[\rho-\gamma\left(1-\sigma\right)\right]\right)^{\phi^i(1-\sigma)-2}\left(\tau_H A^{\frac{1}{\alpha}}\left(\tau_G\right)^{\frac{1-\alpha}{\alpha}}\right)^{\left(1-\phi^i\right)(1-\sigma)}, \qquad (5.37)
\end{aligned}$$

$$\begin{aligned}
&\frac{1-\phi^i}{\frac{\partial\gamma}{\partial\tau_H}}\left(\left[\rho-\gamma\left(1-\sigma\right)\right]\right)^{\phi^i(1-\sigma)-1}\left(\tau_H\right)^{\left(1-\phi^i\right)(1-\sigma)-1}\left(A\left(\tau_G\right)^{1-\alpha}\right)^{\frac{(1-\sigma)\left(1-\phi^i\right)}{\alpha}} \\
= & -\left(1-\phi^i\left(1-\sigma\right)\right)\left(\left[\rho-\gamma\left(1-\sigma\right)\right]\right)^{\phi^i(1-\sigma)-2}\left(\tau_H A^{\frac{1}{\alpha}}\left(\tau_G\right)^{\frac{1-\alpha}{\alpha}}\right)^{\left(1-\phi^i\right)(1-\sigma)}, \qquad (5.38)
\end{aligned}$$

where

$$\frac{\partial\gamma}{\partial\tau_G} = \frac{A^{\frac{1}{\alpha}}\left(\tau_G\right)^{\frac{1-\alpha}{\alpha}}}{\sigma\alpha\tau_G}\left[(1-\alpha)(1-\tau_H)-\tau_G\right], \qquad (5.39)$$

$$\frac{\partial\gamma}{\partial\tau_H} = -\frac{1}{\sigma}A^{\frac{1}{\alpha}}\left(\tau_G\right)^{\frac{1-\alpha}{\alpha}} < 0. \qquad (5.40)$$

Combining conditions (5.37) and (5.38) and taking into account equations (5.39) and (5.40) yields

$$-\frac{1-\alpha}{(1-\tau_H^i)(1-\alpha)-\tau_G^i} = \frac{1}{\tau_H^i}$$

and thus $\tau_G^i = 1-\alpha$.

Then, substituting $\tau_G^i = 1-\alpha$ and (5.40) in (5.38) and rearranging delivers

$$\left(1-\phi^i\right)\left[\rho-\gamma\left(1-\sigma\right)\right] = \frac{1}{\sigma}A^{\frac{1}{\alpha}}\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}}\left(1-\phi^i\left(1-\sigma\right)\right)\tau_H^i. \qquad (5.41)$$

Then, using (5.31) in (5.41) yields

$$\tau_H^i = \frac{\left(1-\phi^i\right)\left[\rho-\alpha A^{\frac{1}{\alpha}}\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}}\right]}{A^{\frac{1}{\alpha}}\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}}\phi^i\sigma}.$$

As τ_H^i cannot be negative, household i's most preferred spending share is given by

$$\tau_H^i = \max\left\{0, \frac{\left(1-\phi^i\right)\left[\rho-\alpha A^{\frac{1}{\alpha}}\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}}\right]}{A^{\frac{1}{\alpha}}\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}}\phi^i\sigma}\right\}. \qquad (5.42)$$

Intuitively, if ρ is sufficiently small, i. e., if households care a lot about the future, then i prefers a high growth rate and thus $\tau_H^i = 0$. Henceforth, we assume that $\rho > \alpha A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}}$.

Equivalently to the main text, the voting problem has become one-dimensional. Moreover, preferences are single-peaked as U^i is strictly concave in τ_H^i for $\tau_G = 1-\alpha$.[14] Thus, the median voter theorem can be applied and the actual policy mix involves

$$\tau_G^* = 1-\alpha \quad \text{and} \quad \tau_H^* = \frac{(1-\phi^m)\left[\rho - \alpha A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}}\right]}{A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} \phi^m \sigma}, \tag{5.43}$$

where ϕ^m denotes the support ratio of the median household. The corresponding steady-state growth rate of household and economy-wide variables is $\gamma^* \equiv \gamma(\tau_G^*, \tau_H^*)$.

Finally, it is straightforward to verify that population ageing, i. e., a decline in the median voter's support ratio has the following steady-state effects

$$\frac{d\tau_H^*}{d\phi^m} < 0, \quad \frac{d\tau_G^*}{d\phi^m} = 0, \quad \text{and} \quad \frac{d\gamma^*}{d\phi^m} > 0,$$

thereby confirming the results of Corollary 1.

5.2.2 Economic Equilibrium with an Equal Initial Capital Distribution, i. e., $K^i(0) = K_0 > 0$ for all i.

The optimization problem for each household-producer i is

$$\max_{\{c^i(t), k^i(t)\}_{t=0}^{\infty}} \quad U^i(0) \qquad \text{s.t. (2.4), (2.6), and } K^i(0) = K_0 > 0,$$
$$\text{taking } \tau,\ G,\ H, \text{ and } y \text{ as given.} \tag{5.44}$$

This gives rise to the same necessary and sufficient optimality conditions as in Appendix 5.1.1, namely equations (5.2)-(5.4). Thus, it is straightforward to show that (equivalent to Proposition 1) along the steady-state growth path for given time-invariant expenditure ratios τ_G and τ_H all variables will grow at the same constant rate $\gamma(\tau_G, \tau_H)$ given by (2.8).

The main difference to an equal initial capital distribution occurs at the level of per-period household variables. With the same initial capital stock all households have the same initial income and produce the same output at all t. To see this, note that

$$Y^i(t) = Y^i(0) e^{\gamma t}, \tag{5.45}$$
$$\text{where} \quad Y^i(0) = A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}} K_0. \tag{5.46}$$

[14] A proof of this is available upon request.

The argument of γ is (τ_G, τ_H). Thus, all households independent of the size of their labor force produce the same output but differ in their output per worker

$$y^i(t) = \frac{Y^i(t)}{L^i} = A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}} \frac{K_0}{L^i} e^{\gamma t}. \tag{5.47}$$

This is possible because firms in this setting asymmetrically benefit from productive expenditure at each t

$$G^i = G\frac{y^i}{y} = G\frac{L}{L^i}, \tag{5.48}$$

where we have used that in equilibrium at each t

$$y = \int_0^1 y^i di = \int_0^1 \frac{Y^i}{L^i} di = Y^i \int_0^1 \frac{1}{L^i} di = \frac{Y^i}{L}. \tag{5.49}$$

Intuitively, equation (5.48) implies that the government via the provision of public productive services subsidizes the production of firms with a smaller labor force. By contrast, when households have the same initial capital stock per worker as assumed in the main text they all produce the same output per worker at each t but differ in their aggregate output according to the size of their labor force.

5.3 Supplementary Information and Tables for the Empirical Analyses

5.3.1 List of Sampled Countries in Public Expenditure Regressions

Austria	Belgium	Bulgaria	Croatia	Cyprus
Czechia	Denmark	Estonia	Finland	France
Germany	Greece	Hungary	Iceland	Ireland
Italy	Latvia	Lithuania	Luxembourg	Malta
Netherlands	Norway	Poland	Portugal	Romania
Slovakia	Slovenia	Spain	Sweden	Switzerland

5.3.2 List of Sampled Countries in Growth Regressions

Afghanistan	Albania	Algeria	Angola	Antigua and Barbuda
Argentina	Armenia	Australia	Austria	Azerbaijan
Bahamas, The	Bahrain	Bangladesh	Barbados	Belarus
Belgium	Belize	Benin	Bhutan	Botswana
Brazil	Brunei Darussalam	Bulgaria	Burkina Faso	Burundi
Cabo Verde	Cambodia	Cameroon	Canada	Central African Republic
Chad	Chile	Colombia	Comoros	Congo, Dem Rep
Congo, Rep	Costa Rica	Cote d'Ivoire	Croatia	Cyprus
Czech Republic	Denmark	Djibouti	Dominican Republic	Ecuador
Egypt, Arab Rep	El Salvador	Estonia	Eswatini	Ethiopia
Fiji	Finland	France	Gabon	Gambia, The
Georgia	Germany	Ghana	Greece	Grenada
Guatemala	Guinea	Guinea-Bissau	Guyana	Haiti
Honduras	Hungary	Iceland	India	Indonesia
Iran, Islamic Rep	Ireland	Israel	Italy	Jamaica
Japan	Jordan	Kazakhstan	Kenya	Kiribati
Korea, Rep	Kuwait	Kyrgyz Republic	Lao PDR	Latvia
Lebanon	Lesotho	Liberia	Lithuania	Luxembourg
Madagascar	Malawi	Malaysia	Maldives	Mali
Malta	Mauritania	Mauritius	Mexico	Micronesia, Fed Sts
Moldova	Mongolia	Morocco	Mozambique	Myanmar
Namibia	Nepal	Netherlands	New Zealand	Nicaragua
Niger	North Macedonia	Norway	Oman	Pakistan
Panama	Papua New Guinea	Paraguay	Peru	Philippines
Poland	Portugal	Qatar	Romania	Russian Federation
Rwanda	Samoa	Sao Tome and Principe	Saudi Arabia	Senegal
Serbia	Seychelles	Sierra Leone	Singapore	Slovak Republic
Slovenia	Solomon Islands	South Africa	Spain	Sri Lanka
St Lucia	St Vincent and the Grenadines	Sudan	Sweden	Switzerland
Tajikistan	Tanzania	Thailand	Timor-Leste	Togo
Tonga	Trinidad and Tobago	Tunisia	Turkey	Turkmenistan
Uganda	Ukraine	United Kingdom	United States	Uruguay
Uzbekistan	Vanuatu	Vietnam	Zambia	Zimbabwe

5.3.3 Summary Statistics for Growth Analysis

Table 5: Summary statistics

Variable	Description	Mean	SD	Min	Max	Count
Dependent variable						
Log of GDP per capita	Natural logarithm of GDP per capita, PPP (constant 2017 international $)	9.20	1.17	6.59	11.63	2213
Control variables						
Support ratio	Working population aged 15-64 (% of adult population aged 15 and above)	87.72	6.27	74.00	99.18	1349
Healthcare expenditure	Current health expenditure (% of GDP) 2017 international $)	6.18	2.40	1.03	24.26	2268
Education expenditure	Government expenditure on education, total (% of GDP)	4.41	1.70	0.62	14.06	2268
Government effectiveness	Government effectiveness	0.09	0.99	-2.08	2.44	1818
School enrollment	School enrollment, secondary (% gross)	79.93	29.30	6.49	163.93	1827
Gross capital formation	Gross capital formation (% of GDP)	23.98	7.90	-0.10	77.89	2111
Infant mortality rate	Mortality rate, infant (per 1,000 live births)	27.71	25.90	1.60	139.50	2268
Trade	Trade (% of GDP)	84.17	49.87	0.20	437.33	2136
Population density	Population density (people per sq. km of land area)	258.35	1307.41	1.54	19196.00	2261

Summary: This table presents summary statistics for the variables used in the analysis examining the effect of population ageing, elderly spending and productive expenditure on economic growth. For each variable, we show the mean, standard deviation, minimum and maximum values as well as the number of observations.

5.3.4 First Stage Results for Government Expenditure Instrumental Variable Regressions

Table 6: The effect of population ageing on government expenditure categories - First stage IV estimates

	(1) Sample A	(2) Sample B
Support ratio forecasts	0.620***	0.634***
	(0.067)	(0.069)
Population density	0.000	0.001
	(0.003)	(0.003)
Unemployment	0.035***	0.035***
	(0.012)	(0.013)
Inflation	-0.025**	-0.026**
	(0.012)	(0.013)
Government effectiveness	-0.737***	-0.769***
	(0.231)	(0.242)
Expenses	0.005	0.006
	(0.009)	(0.009)
CAP	-0.060***	-0.063***
	(0.010)	(0.012)
Expenses	0.005	0.006
	(0.009)	(0.009)
Year fixed effects	Yes	Yes
Country fixed effects	Yes	Yes
Observations	312	291
Adjusted R-squared	0.978	0.978

Summary: The table presents the results for the first stage of the instrumental variable regression for samples A and B, with the latter being delimited by observations available for R&D spending and the former by all other expenditure categories.

Notes: (i) Support ratio forecasts are produced by the United Nations in 1992 (ii) standard errors are clustered at country and year level; robust and clustered standard errors are reported in parentheses; (iii) *** denotes statistical significance at the 1 percent level ($p < 0.01$), ** at the 5 percent level ($p < 0.05$), and * at the 10 percent level ($p < 0.10$), all for two-sided hypothesis tests.