

Observability Investigation for Rotational Calibration of (Global-pose aided) VIO under Straight Line Motion

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Abstract—Online extrinsic calibration is crucial for building “power-on-and-go” moving platforms, like robots and AR devices. However, blindly performing online calibration for unobservable parameter may lead to unpredictable results. In the literature, extensive studies have been conducted on the extrinsic calibration between IMU and camera, from theory to practice. It is well-known that the observability of extrinsic parameter can be guaranteed under sufficient motion excitation. Furthermore, the impacts of degenerate motions are also investigated. Despite these successful analyses, we identify an issue with respect to the existing observability conclusion. This paper focuses on the observability investigation for straight line motion, which is a common-seen and fundamental degenerate motion in applications. We analytically prove that pure translational straight line motion can lead to the unobservability of the rotational extrinsic parameter between IMU and camera (at least one degree of freedom). By correcting the existing observability conclusion, our novel theoretical finding disseminates more precise principle to the research community and provides explainable calibration guideline for practitioners. Our analysis is validated by rigorous theory and experiments.

Index Terms—Visual inertial odometry, observability analysis, self-calibration

I. INTRODUCTION

In the last two decades, visual-inertial navigation systems (VINS) have gained great popularity thanks to their ability to provide real-time and precise 6 degree-of-freedom (DoF) motion tracking in unknown GPS-denied or GPS-degraded environments, through the usage of low-cost, low-power, and complementary visual-inertial sensor rigs [1], [2], [3]. An inertial sensor, IMU, provides high-frequency linear acceleration and local angular velocity measurements of the moving platform, with bias and noise. Therefore, integrating only the IMU measurements to obtain motion prediction inevitably suffers from drift. While visual sensors can estimate IMU bias and reduce the drift of pose estimation by perceiving static visual features from the surrounding environment.

To improve the accuracy, efficiency, robustness or consistency of pose estimation, numerous tightly-coupled visual-inertial odometry (VIO) algorithms have been proposed in the literature. These algorithms can be broadly divided into two categories: optimization-based methods and filter-based methods. Optimization-based methods include OKVIS [4], VINS-Mono [5], and ORB-SLAM3 [6]. Filter-based methods include ROVIO [7], Multi-State Constraint Kalman Filter (MSCKF) [8], [9], and SchurVINS [10].

Before running, VIO algorithm needs to know the extrinsic parameter between IMU and camera, including 3DoF

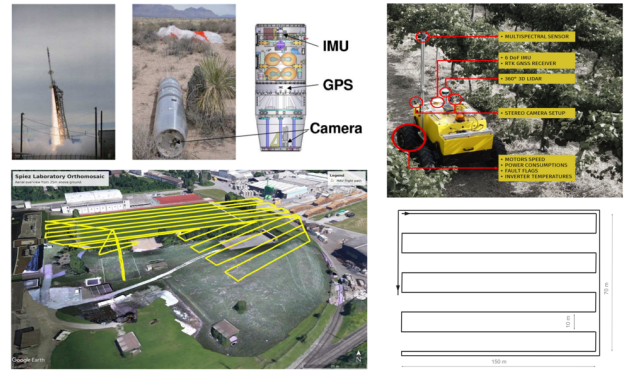


Fig. 1: Various straight line movements. Top left: Spacecraft entry, descent, and landing [1]. Bottom left: MAV flight path [11]. Top right: Agrobot movement in a vineyard field [12]. Bottom right: Survey followed by Girona 1000 AUV [13].

translational part and 3DoF rotational part, which is a bridge to link measurements from different sensors. This extrinsic parameter is also critical for other visual perception applications, for example loop correction [5], dense map [14], and tracking [15], [16]. These visual perception results are represented in camera frame. To transfer these results to the body frame (IMU frame) of the robot or vehicle, accurate extrinsic parameter is desired. A small misalignment in the extrinsic parameter could generate a large drift and error.

The extrinsic parameter is usually assumed to be rigid and constant, however, this may be not the case in practice. Considering that replacement and maintenance of sensors, and non-rigid deformation caused by mechanical vibration and varying temperature may lead to the alternation of extrinsic parameter, some researchers propose to add extrinsic parameter to state vector to perform *online calibration* [17], [18], [19]. If the extrinsic parameter is an observable state variable, online calibration can be resilient with poor prior calibration and converge to true value, which means robustness to the initial value. This feature helps to build “power-on-and-go” moving platforms without the need for repetitive, tedious, manual *offline calibration*.

The success of online extrinsic parameter calibration depends on the observability. Remarkable works have studied the observability of extrinsic parameter between IMU and camera. With the help of artificial visual features on the calibration target board, [17] conclude that extrinsic parameter is observable if the moving platform undergoes at least 2DoF rotational excitation. An interesting corollary from [17] is that the observability of extrinsic parameter is independent

of translational excitation. However, the conclusion of [17] is limited by the usage of calibration board, and cannot be applied to real operating environments without calibration board. [18] further extend the calibration of extrinsic parameter with target-less approach, and the conclusion is updated. The moving platform should undergo at least 2DoF motion excitation for both rotation and translation, to ensure the observability of extrinsic parameter.

The above-mentioned observability studies miss the analysis of degenerate motion profiles, which could be occurred and unavoidable in practice. As a supplement, [19] thoroughly explore the possible degenerate motion primitives and analyze the impact of degenerate motion on the observability of calibration parameters. We note that the rotational extrinsic parameter is summarized as observable for all identified degenerate motions (see Table I in [19]), except for no motion. However, by observing the top subplot of Fig. 2a in [19], we found that the rotational calibration results exhibit unexpected large RMSE (greater than 1 degree) for the case of pure translational straight line motion, which is clearly different from other motion cases. Actually, this distinct curve is an indicator for unobservability.

The inconsistency between the observability conclusion and the calibration results motivates the following research question as the main purpose of this work:

Is the rotational extrinsic parameter of (global-pose aided) VIO observable under pure translational straight line motion?

Straight line motions are quite common and fundamental in vehicle driving [20], agriculture [12], coverage survey [11], [13], and planetary exploration [21] (see Fig. 1 and Fig. 2). According to [19], if the rotational extrinsic parameter is observable, it is expected that practitioners would straightforward add this parameter to the state vector to perform online calibration, which has been integrated in numerous open-sourced VIO frameworks, like OKVIS [4], VINS-Mono [5], ROVIO [7], and Open-VINS [9].

However, according to our novel finding (see Tab. I), the rotational extrinsic parameter has at least one unobservable DoF when the moving platform undergoes pure translational straight line motion. This implies that performing online rotational calibration is risky, as unobservability can lead to unpredictable and incorrect calibration results. Meanwhile, the misleading observability conclusion in [19] may have adverse effect on future research. For example, Table III in [22] is directly inherited from [19]. Therefore, it is vital to convey more precise principle to the community, otherwise incorrect conclusion would continue to mislead researchers. Next, we will verify our observability investigation through rigorous theory and solid experiments.

II. NOTATION

The main purpose of this paper is to investigate the observability of rotational extrinsic parameter between IMU and camera presented in [19]. When the moving platform follows a pure translational straight line motion (no rotation), our observability conclusion regarding this rotational extrinsic

parameter is different from [19]. Like [19], we consider online calibration of rotational extrinsic parameter (rotational calibration) with two configurations, one is **pure VIO** and the other is **global-pose aided VIO**. In the following sections, we will directly analyze the observability matrix in [19]. As for the construction details of system model, measurement model, and observability matrix, interested readers are advised to refer to [19], [23].

The state vector considered in this paper is

$$x = [{}^I_G q^T \quad b_g^T \quad {}^G v_I^T \quad b_a^T \quad {}^G p_I^T \quad {}^C_I q^T \quad {}^G p_f^T]^T \quad (1)$$

where ${}^I_G q$ represents the orientation of IMU frame $\{I\}$ with respect to global frame $\{G\}$, and its corresponding rotation matrix is given by ${}^I_G R$. ${}^G v_I$ and ${}^G p_I$ refer to the velocity and position of IMU in frame $\{G\}$. b_g and b_a represent the gyroscope and accelerometer biases. ${}^G p_f$ is augmented feature, or SLAM feature [9].

${}^C_I q$ is rotational calibration parameter, and its corresponding rotation matrix is ${}^C_I R$. Compared to equation (1) in [19], x does not include ${}^C p_I$ and t_d , as the online calibration of translational extrinsic parameter, as well as the time offset between IMU and camera, are not the focus of this paper. Our analysis is independent of ${}^C p_I$ and t_d .

In following sections, $[\bullet]_\times$ is denoted as the skew symmetric matrix corresponding to a three-dimensional vector. To simplify the description, the hat symbol ($\hat{\bullet}$) is omitted, which does not affect observability analysis. Other notations are consistent with [19]. By assuming that the direction of straight line is denoted as d in the IMU frame $\{I\}$, we are ready for observability investigation now.

III. OBSERVABILITY INVESTIGATION FOR PURE VIO

Referring to equation (21) of [19], in the configuration of pure VIO, the observability matrix is

$$M_k = \Xi_k \Xi_{\Gamma_k} \\ \Xi_{\Gamma_k} = [\Gamma_1 \quad \Gamma_2 \quad -I_3 \delta t_k \quad \Gamma_3 \quad -I_3 \quad \Gamma_4 \quad I_3] \quad (2)$$

Compared to equation (21) of [19], the element ${}^G_{I_k} R_C^I R$ corresponding to ${}^C p_I$, and the element Γ_5 corresponding to t_d , have been removed in Ξ_{Γ_k} . The expressions of $\Gamma_1 \sim \Gamma_4$ in Ξ_{Γ_k} are

$$\begin{aligned} \Gamma_1 &= [{}^G p_f - {}^G p_{I_1} - {}^G v_{I_1} \delta t_k + \frac{1}{2} {}^G g \delta t_k^2]_\times {}^G_{I_1} R \\ \Gamma_2 &= [{}^G p_f - {}^G p_{I_k}]_\times {}^G_{I_k} R \Phi_{I12} - \Phi_{I52} \\ \Gamma_3 &= -\Phi_{I54} \\ \Gamma_4 &= [{}^G p_f - {}^G p_{I_k}]_\times {}^G_{I_k} R_C^I R \end{aligned} \quad (3)$$

The expression of Γ_1 in [19], equation (22), has small typos. We have corrected it by referring to equation (53) in [23].

In the context of pure translational motion, i.e. no rotation, the orientation of the moving platform does not change at any time. Therefore, ${}^G_{I(\bullet)} R$ can be directly represented by ${}^G_I R$ (constant). Referring to equation (114) in [23]

$$\begin{aligned} \Gamma_3 &= -\Phi_{I54} = \int_{t_1}^{t_k} \int_{t_1}^s {}^G_{I_r} R d\tau ds \\ &= ({}^G_I R) \int_{t_1}^{t_k} \int_{t_1}^s (1) d\tau ds = \frac{1}{2} {}^G_I R \delta t_k^2 \end{aligned} \quad (4)$$

TABLE I: Observability Investigation for Rotational Calibration of (Global-pose aided) VIO under Straight Line Motion.

Motion	Pure VIO		Global-pose aided VIO	
	[19], [22]	Our novel finding	[19], [22]	Our novel finding
Pure translational straight line motion	observable	at least one unobservable DoF	observable	at least one unobservable DoF
Pure translational straight line motion with constant velocity	observable	fully unobservable	observable	at least one unobservable DoF

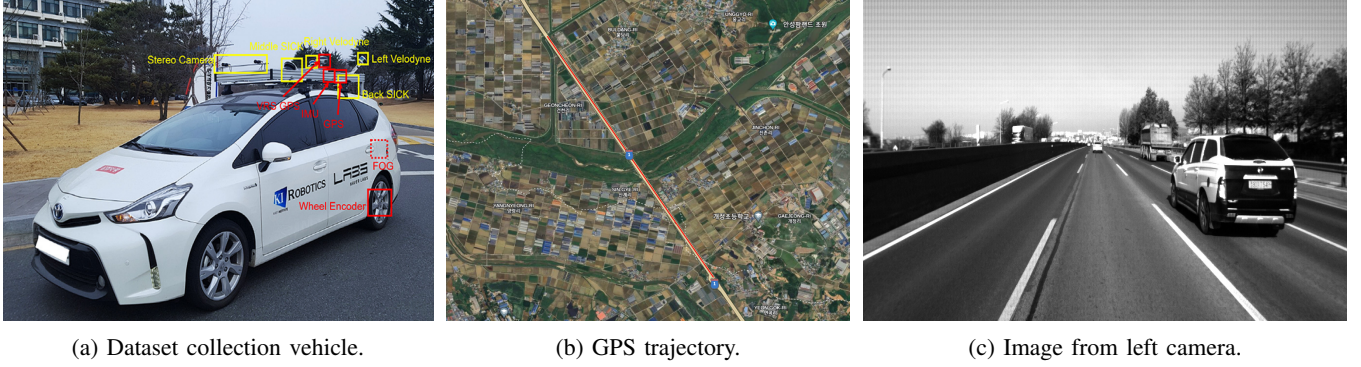


Fig. 2: Representative pure translational straight line motion from Urban22 sequence in KAIST dataset [20].

The expressions of $\Gamma_1 \sim \Gamma_4$ in Ξ_{Γ_k} become

$$\begin{aligned}\Gamma_1 &= [{}^G p_f - {}^G p_{I_1} - {}^G v_{I_1} \delta t_k + \frac{1}{2} {}^G g \delta t_k^2] \times {}^G R \\ \Gamma_2 &= [{}^G p_f - {}^G p_{I_k}] \times {}^G R \Phi_{I12} - \Phi_{I52} \\ \Gamma_3 &= \frac{1}{2} {}^G R \delta t_k^2 \\ \Gamma_4 &= [{}^G p_f - {}^G p_{I_k}] \times {}^G R {}^I R\end{aligned}\quad (5)$$

Lemma 1. *If pure VIO system undergoes pure translational straight line motion, the unobservable directions of ${}^C R$ depend on the projection of d^1 in the camera frame $\{C\}$. The corresponding right null space of M_k is*

$$N_1 = \begin{bmatrix} 0_{15 \times 1} \\ {}^C R d \\ -[{}^G p_f - {}^G p_{I_1}] \times {}^G R d \end{bmatrix} \quad (6)$$

Proof. Straight line motion indicates the following geometric constraint

$$[{}^I p_{I_k}] \times d = 0 \quad (7)$$

Given the above constraint, we first verify that N_1 belongs to the right null space of Ξ_{Γ_k} .

$$\begin{aligned}\Xi_{\Gamma_k} N_1 &= \Gamma_4 {}^C R d - [{}^G p_f - {}^G p_{I_1}] \times {}^G R d \\ &= [{}^G p_f - {}^G p_{I_k}] \times {}^G R d - [{}^G p_f - {}^G p_{I_1}] \times {}^G R d \\ &= -[{}^G p_{I_k} - {}^G p_{I_1}] \times {}^G R d\end{aligned}\quad (8)$$

One geometric relationship can be utilized

$${}^G p_{I_k} = {}^G p_{I_1} + {}^G R {}^I p_{I_k} = {}^G p_{I_1} + {}^G R {}^I p_{I_k} \quad (9)$$

Subsequently

$$\begin{aligned}\Xi_{\Gamma_k} N_1 &= -[{}^G p_{I_k} - {}^G p_{I_1}] \times {}^G R d \\ &= -[{}^G R {}^I p_{I_k}] \times {}^G R d \\ &= -{}^G R [{}^I p_{I_k}] \times {}^G R {}^T {}^G R d \\ &= -{}^G R [{}^I p_{I_k}] \times d = 0\end{aligned}\quad (10)$$

¹The definition of d is described in the last paragraph of Sec. II.

Finally

$$\Rightarrow M_k N_1 = \Xi_k \Xi_{\Gamma_k} N_1 = 0 \quad (11)$$

Hence, N_1 belongs to the right null space of M_k . N_1 indicates that the unobservable directions of ${}^C R$ are dependent on the non-zero components of ${}^C R d$. \square

Lemma 2. *If pure VIO system undergoes pure translational straight line motion with constant velocity, the 3DoF of ${}^C R$ are all unobservable. The corresponding right null space of M_k is*

$$N_2 = \begin{bmatrix} {}^I R \\ 0_3 \\ 0_3 \\ -{}^I R [{}^G g] \times \\ 0_3 \\ -{}^C R {}^I R \\ 0_3 \end{bmatrix} \quad (12)$$

Proof. Straight line motion with constant velocity indicates the following geometric constraint

$${}^G p_{I_k} = {}^G p_{I_1} + {}^G v_{I_1} \delta t_k \quad (13)$$

Given the above constraint, we first verify that N_2 belongs to the right null space of Ξ_{Γ_k} .

$$\begin{aligned}\Xi_{\Gamma_k} N_2 &= \Gamma_1 {}^I R - \Gamma_3 {}^I R [{}^G g] \times - \Gamma_4 {}^C R {}^I R \\ &= [{}^G p_f - {}^G p_{I_1} - {}^G v_{I_1} \delta t_k + \frac{1}{2} {}^G g \delta t_k^2] \times \\ &\quad - \frac{1}{2} [{}^G g] \times \delta t_k^2 - [{}^G p_f - {}^G p_{I_k}] \times \\ &= [{}^G p_{I_k} - {}^G p_{I_1} - {}^G v_{I_1} \delta t_k] \times = 0\end{aligned}\quad (14)$$

Finally

$$\Rightarrow M_k N_2 = \Xi_k \Xi_{\Gamma_k} N_2 = 0 \quad (15)$$

Hence, N_2 belongs to the right null space of M_k . N_2 indicates that the 3DoF of ${}^C R$ are all unobservable. \square

Remark. We note that the rotational extrinsic parameter ${}^C_I R$ has at least one degree of freedom that is unobservable when the platform undergoes pure translational straight line motion. More specifically, when moving with constant velocity, the 3 degrees of freedom of ${}^C_I R$ are completely unobservable. When moving with variable velocity, at least one degree of freedom is unobservable as $\|{}_I^C R d\| \neq 0$.

IV. OBSERVABILITY INVESTIGATION FOR GLOBAL-POSE AIDED VIO

Like [19], the observability of rotational extrinsic parameter is also discussed in the configuration of global-pose aided VIO. Our conclusion is different from [19]. Referring to equation (40) of [19], the observability matrix is

$$M_k^{(g)} = \Xi_k^{(g)} \Xi_{\Gamma_k}^{(g)} \quad (16)$$

$$\Xi_{\Gamma_k}^{(g)} = \begin{bmatrix} \Gamma_1 & \Gamma_2 & -I_3 \delta t_k & \Gamma_3 & -I_3 & \Gamma_4 & I_3 \\ \Phi_{I11} & \Phi_{I12} & 0_3 & 0_3 & 0_3 & 0_3 & 0_3 \\ \Phi_{I51} & \Phi_{I52} & \Phi_{I53} & \Phi_{I54} & I_3 & 0_3 & 0_3 \end{bmatrix}$$

The last two rows of $\Xi_{\Gamma_k}^{(g)}$ in [19] is incorrect. We have corrected it by multiplying the measurement Jacobian matrix with the state transition matrix. Detailed derivations are provided in Sec. VII of supplementary material [24].

Lemma 3. If global-pose aided VIO system undergoes pure translational straight line motion, the unobservable directions of ${}^C_I R$ depend on the projection of d in the camera frame $\{C\}$. The corresponding right null space of $M_k^{(g)}$ is N_1 .

Proof. A naive way of finding the corresponding right null space for $M_k^{(g)}$ is to test the product of $\Xi_{\Gamma_k}^{(g)}$ and N_1

$$\Xi_{\Gamma_k}^{(g)} N_1 = \begin{bmatrix} \Gamma_4 {}^C_I R d - [{}^G p_f - {}^G p_{I1}]_{\times} {}^G_I R d \\ 0_{3 \times 1} \\ 0_{3 \times 1} \end{bmatrix} \quad (17)$$

According to Lemma 1

$$\Xi_{\Gamma_k}^{(g)} N_1 = 0 \quad (18)$$

Finally

$$\Rightarrow M_k^{(g)} N_1 = \Xi_k^{(g)} \Xi_{\Gamma_k}^{(g)} N_1 = 0 \quad (19)$$

Hence, N_1 belongs to the right null space of $M_k^{(g)}$. N_1 indicates that the unobservable directions of ${}^C_I R$ are dependent on the non-zero components of ${}^C_I R d$. \square

Lemma 4. If global-pose aided VIO system undergoes pure translational straight line motion with constant velocity, the unobservable directions of ${}^C_I R$ depend on the projection of d in the camera frame $\{C\}$. The corresponding right null space of $M_k^{(g)}$ is still N_1 .

Proof. A naive way of finding the corresponding right null space for $M_k^{(g)}$ is to test the product of $\Xi_{\Gamma_k}^{(g)}$ and N_2

$$\Xi_{\Gamma_k}^{(g)} N_2 = \begin{bmatrix} \Gamma_1 {}^I_G R - \Gamma_3 {}^I_G R [{}^G g]_{\times} - \Gamma_4 {}^C_I R {}^I_G R \\ \Phi_{I11} {}^I_G R \\ \Phi_{I51} {}^I_G R - \Phi_{I54} {}^I_G R [{}^G g]_{\times} \end{bmatrix} \quad (20)$$

According to Lemma 2

$$\Xi_{\Gamma_k}^{(g)} N_2 = \begin{bmatrix} 0_3 \\ \Phi_{I11} {}^I_G R \\ \Phi_{I51} {}^I_G R - \Phi_{I54} {}^I_G R [{}^G g]_{\times} \end{bmatrix} \quad (21)$$

Referring to equation (46) in [23], $\Phi_{I11} \neq 0$, it is clear that $\Xi_{\Gamma_k}^{(g)} N_2 \neq 0$. Therefore, the unobservable direction N_2 is no longer hold due to the inclusion of global pose measurement.

It is worth noting that Lemma 4 is a special case of Lemma 3. Hence, N_1 still belongs to the right null space of $M_k^{(g)}$. N_1 indicates that the unobservable directions of ${}^C_I R$ are dependent on the non-zero components of ${}^C_I R d$. \square

Remark. We note that the rotational extrinsic parameter ${}^C_I R$ has at least one degree of freedom that is unobservable when the platform undergoes pure translational straight line motion, regardless of variable velocity or constant velocity. In the case of constant velocity, the unobservable directions can be decreased with the aides of global pose measurement, compared to the pure VIO configuration. More specifically, in the global-pose aided VIO configuration, the worst case is three degrees of freedom are unobservable, while the best case is only one degree of freedom is unobservable. The difference between our conclusion and [19] is marked in Tab. I.

V. RESULTS

We conduct verification experiments based on Open-VINS [9]. As this paper focuses on the observability investigation of the rotational extrinsic parameter, we only perform online calibration for the rotational extrinsic parameter and set the translational extrinsic parameter and time offset as true values, referring to our state vector (Eq. (1)).

A. Comments on results in [19]

Table I from [19] show that the rotational extrinsic parameter is observable for pure translational motion. However, we find that [19] actually did not perform theoretical analysis on the rotational calibration (${}^C_I R$). Besides that, it can be seen from the top subplot of Fig. 2a in [19], if the simulation trajectory is a pure translational straight line motion with constant velocity, the calibration result of the rotational extrinsic parameter shows large RMSE (greater than 1 degree). Regarding the inconsistency between observability assertion and simulation result, no ablation experiments were conducted, by calibrating the rotational extrinsic parameter only and turning off the calibration of the translational extrinsic parameter and time offset. Moreover, Section VI of [19] did not validate the convergence consistency with different initial ${}^C_I R$. Section VII of [19] missed the verification of pure translational straight line motion in real-world experiments.

B. Numerical Study

Employing the Open-VINS simulator and importing the desired 6DoF trajectory, realistic multi-sensor data are generated for experiments under two different configurations. For the pure VIO configuration, we generate IMU measurements

TABLE II: Final calibration results of the rotational extrinsic parameter for pure VIO system undergoes pure translational straight line motion with variable velocity. The absolute errors of roll, pitch, and yaw at 60s, are recorded with different perturbations.

Perturbations of (roll, pitch, yaw)	Case-1			Case-2			Case-3		
	Roll	Pitch	Yaw	Roll	Pitch	Yaw	Roll	Pitch	Yaw
(2, -4, -5)	11.31	0.04	0.03	5.28	2.17	0.05	4.67	1.83	2.21
(-4, 3, 3)	0.37	0.05	0.02	2.16	0.91	0.03	4.04	1.69	1.69
(5, -2, -1)	13.38	0.04	0.02	7.02	2.89	0.09	1.01	0.41	0.53
(-1, -5, -3)	8.96	0.03	0.02	2.67	1.10	0.01	4.99	1.94	2.23
(3, 0, 1)	9.55	0.04	0.01	4.60	1.89	0.06	0.40	0.19	0.20
(1, 2, -4)	7.34	0.06	0.04	3.52	1.44	0.05	5.98	2.44	2.63
(0, 5, 2)	2.51	0.05	0.02	1.60	0.64	0.07	1.22	0.57	0.52
(-3, 4, 0)	1.58	0.05	0.03	0.63	0.28	0.03	4.82	1.98	2.08
(-5, 1, 4)	0.44	0.04	0.01	3.20	1.35	0.02	6.09	2.50	2.53
(4, -1, 5)	9.20	0.02	0.01	4.49	1.84	0.05	1.12	0.39	0.51
(-2, -3, -2)	7.36	0.04	0.02	0.77	0.30	0.01	5.89	2.34	2.56
Avg	6.55	0.04	0.02	3.27	1.35	0.04	3.66	1.48	1.61

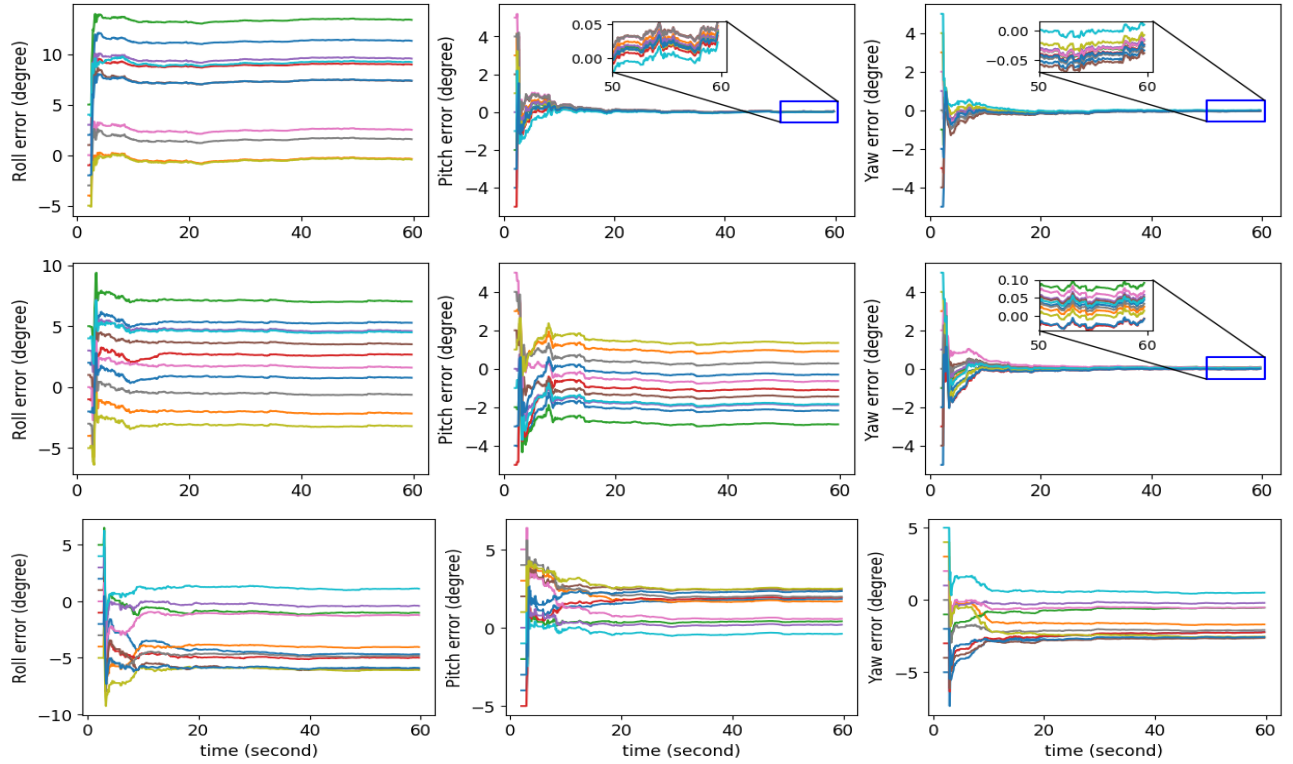


Fig. 3: Calibration results for pure VIO system undergoes pure translational straight line motion with variable velocity. y -axis represents errors of the rotational calibration parameter over time respect to different initial guesses. x -axis represents time in seconds. Top to bottom corresponds to Case-1 to Case-3 in Sec. V-B.

at 400 Hz and image measurements at 10 Hz. For the global-pose aided VIO configuration, additional 10 Hz global-pose measurements are generated. The global-pose measurement noises are defined as

$$\begin{aligned} n_p &\sim \mathcal{N}(0_{3 \times 1}, \sigma_p^2 I_3), \sigma_p = 0.1m \\ n_\theta &\sim \mathcal{N}(0_{3 \times 1}, \sigma_\theta^2 I_3), \sigma_\theta = 0.1rad \end{aligned} \quad (22)$$

where n_p and n_θ represent Gaussian noises for global position and orientation measurement, respectively.

This paper focuses on pure translational straight line motion, therefore the orientation of the input trajectory, ${}^I_G R$, is

set as I_3 . To validate the observability assertion summarized in Tab. I, two types of straight line motion with different velocity profiles are designed as

- Trajectory-1: ${}^G p_I = [2 \cos(\frac{\pi}{5}t) \ 0 \ 0]^T$.
- Trajectory-2: ${}^G p_I = [0.5t \ 0 \ 0]^T$.

Trajectory-1 corresponds to variable velocity motion, while Trajectory-2 corresponds to constant velocity motion. The direction vector corresponding to both these two trajectories is $d = [1 \ 0 \ 0]^T$. As the unobservable directions of ${}^C_I R$ may depend on the non-zero components of ${}^C_I R d$,

three types of groundtruth ${}^C_I R$ are designed as

- Case-1:

$${}^C_I R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, {}^C_I R d = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$
- Case-2:

$${}^C_I R = \begin{bmatrix} 0.707 & 0.707 & 0 \\ -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}, {}^C_I R d = \begin{bmatrix} 0.707 \\ -0.707 \\ 0 \end{bmatrix}.$$
- Case-3:

$${}^C_I R = \begin{bmatrix} 0.5 & 0.707 & -0.5 \\ -0.5 & 0.707 & 0.5 \\ 0.707 & 0 & 0.707 \end{bmatrix}, {}^C_I R d = \begin{bmatrix} 0.5 \\ -0.5 \\ 0.707 \end{bmatrix}.$$

For each case, we initialize ${}^C_I R$ by adding different perturbations to the three degrees of freedom of ${}^C_I R$ (roll, pitch, and yaw), and collect calibration error with respect to groundtruth ${}^C_I R$. The range of perturbation is $[-5.0^\circ, 5.0^\circ]$. If a certain degree of freedom is observable, it should be robust to different perturbations, namely, the calibration error should consistently converge to 0. On the contrary, if it is unobservable, the calibration error can not converge to 0 and is expected to be sensitive to the initial value.

Firstly, we analyze the calibration results for Case-1 of Trajectory-1 in the pure VIO configuration, as shown in the Tab. II. Pitch and yaw exhibit observable characteristic, while roll not. This is because the non-zero component of ${}^C_I R d$ corresponds to roll. For Case-2, yaw exhibits observable characteristic, while roll and pitch not. This is because non-zero components of ${}^C_I R d$ correspond to roll and pitch. For Case-3, roll, pitch, and yaw all exhibit unobservable characteristic. This is because none of the three components of ${}^C_I R d$ are zero. The calibration results over time are shown in the Fig. 3. Similar analysis also applies to different combinations of configurations and trajectories, please refer to Tab. III and Sec. VIII of supplementary material [24] for other results. These results successfully validate that our novel observability conclusions are correct. Overall, observable degree of freedom shows deterministic behavior, i.e. converging to groundtruth over time, while unobservable degree of freedom exhibits unpredictable behavior.

C. Real-world Dataset

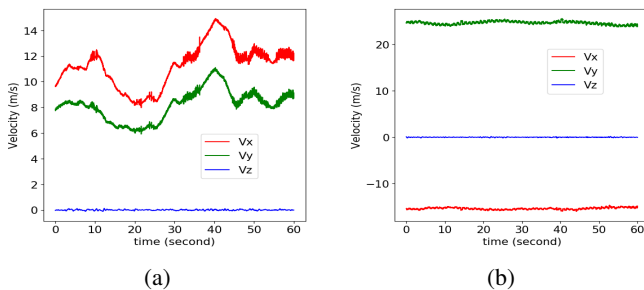


Fig. 4: Velocity profiles of Urban34 (a) and Urban22 (b).

Straight line motions are quite common in real-world scenarios. On one hand, straight line cruise is the most efficient and energy-saving trajectory for most robot applications. On the other hand, substantial artificial scenarios have specific

constraints on motion, such as applications in agriculture, warehousing, logistics, and transportation.

The KAIST urban dataset [20] contains the driving scenario on the highway, as shown in the Fig. 2. Urban34 and Urban22 from this dataset are leveraged to confirm our observability finding, as these two sequences represent variable velocity motion and constant velocity motion, respectively. The vehicle used to collect data follows the same lane during driving, so its trajectory can be regarded as a pure translational straight line. Corresponding ${}^C_I R d$ is

$${}^C_I R d = \begin{bmatrix} -0.00413 \\ -0.01966 \\ 0.99980 \end{bmatrix}$$

The velocity curve of Urban34 sequence (see Fig. 4a) is variable over time. Fig. 5 shows the calibration results with the pure VIO configuration and the global-pose aided VIO configuration. Roll and pitch exhibit observable characteristic, while yaw not. This is because the non-zero component of ${}^C_I R d$ is dominated by the yaw component (0.99980). The velocity curve of Urban22 sequence (see Fig. 4b) is approximately constant. Fig. 6 shows the calibration results of Urban22. In the pure VIO configuration, roll, pitch, and yaw all exhibit unobservable characteristic due to constant velocity motion. In the global-pose aided VIO configuration, unobservable degrees of freedom are reduced from 3 to 1 (yaw). Interestingly, the convergence error of pitch is larger than that of roll, which can be attributed to the fact that the absolute value of pitch component (0.01966) is larger than that of roll (0.00413).

Furthermore, we evaluate the localization accuracy with calibration (w. calib) and without calibration (wo. calib), under different perturbations on the rotational extrinsic parameter. Since real-world data is more sensitive than simulation data, the perturbation amplitude is reduced to half of its value listed in the Tab. II. The Absolute Trajectory Error (ATE) results are reported in Tab. IV and Tab. V.

In the pure VIO configuration (Tab. IV), calibration significantly improves the localization accuracy for Urban34, as model error from two degrees of freedom (roll and pitch) of the rotational calibration parameter can be corrected to near 0, thanks to online calibration (see top of Fig. 5). Urban22 exhibits large localization error as scale becomes unobservable under constant velocity motion [21]. And it is observed that performing calibration further degrades the localization due to the fully unobservable property of the rotational calibration parameter (see top of Fig. 6). In the global-pose aided VIO configuration (Tab. V), the localization accuracy is mainly dominated by global pose measurements, thus the calibration of rotational extrinsic parameter has negligible impact on the accuracy.

Remark. If the calibration parameter is observable, online calibration typically brings positive benefits to localization [25]. However, if it is unobservable, the impact of calibration on localization is unpredictable (negative, no impact or positive). In other words, we cannot determine the observability of the calibration parameter from localization accuracy.

TABLE III: Numerical Study Results for Rotational Calibration of (Global-pose aided) VIO under Straight Line Motion.

Motion	Pure VIO		Global-pose aided VIO	
	calibration results	conclusion	calibration results	conclusion
Trajectory-1 in Sec. V-B	Tab. II and Fig. 3	at least one unobservable DoF	Tab. VI and Fig. 7 in supplementary material [24]	at least one unobservable DoF
Trajectory-2 in Sec. V-B	Tab. VII and Fig. 8 in supplementary material [24]	fully unobservable	Tab. VIII and Fig. 9 in supplementary material [24]	at least one unobservable DoF

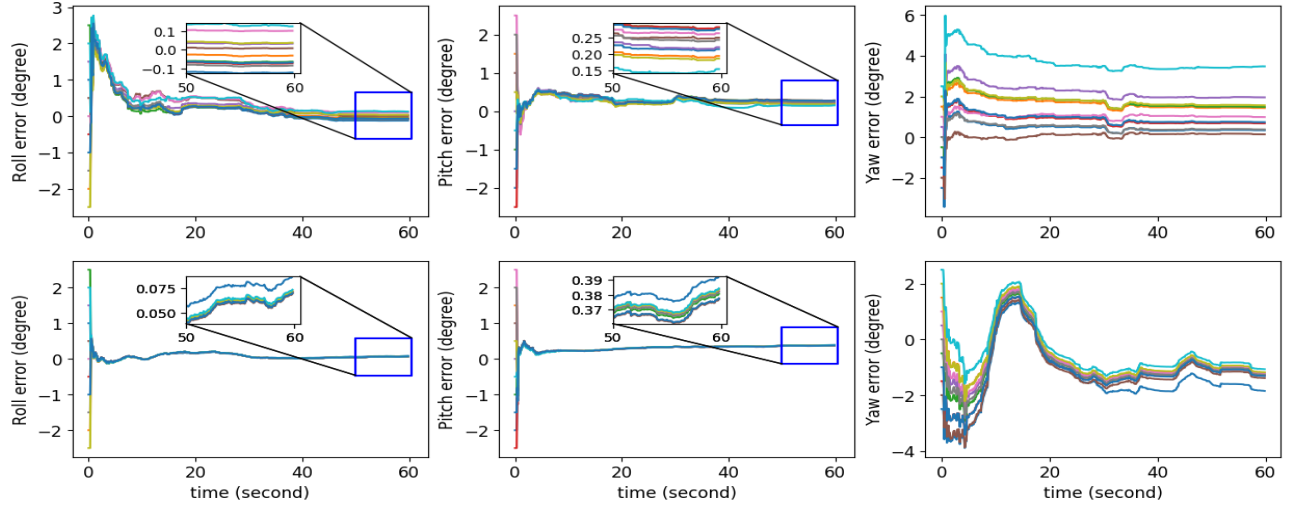


Fig. 5: Calibration results for Urban34. Top: Results for pure VIO system. Bottom: Results for global-pose aided VIO system. y -axis represents errors of the rotational calibration parameter over time respect to different initial guesses. x -axis represents time in seconds.

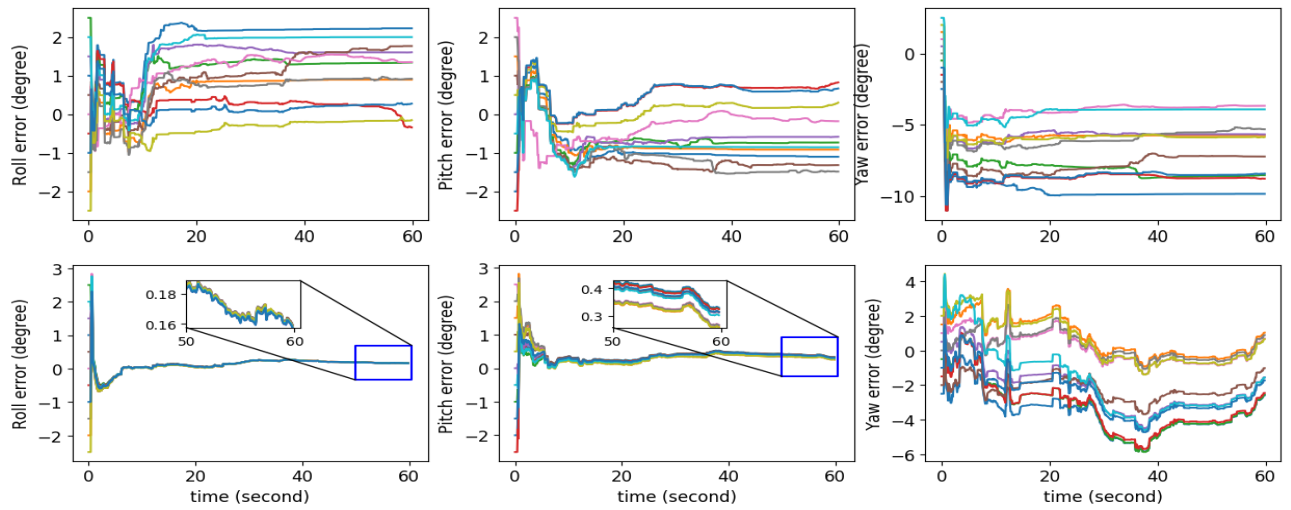


Fig. 6: Calibration results for Urban22. Top: Results for pure VIO system. Bottom: Results for global-pose aided VIO system. y -axis represents errors of the rotational calibration parameter over time respect to different initial guesses. x -axis represents time in seconds.

TABLE IV: ATE (meter) Comparison for Pure VIO.

Perturbations	Urban34		Urban22	
	w. calib	wo. calib	w. calib	wo. calib
(1.0, -2.0, -2.5)	2.49	13.27	257.52	212.16
(-2.0, 1.5, 1.5)	4.24	108.88	258.47	88.39
(2.5, -1.0, -0.5)	2.90	106.31	268.02	308.66
(-0.5, -2.5, -1.5)	2.38	53.36	125.40	90.07
(1.5, 0.0, 0.5)	10.23	36.76	282.57	119.22
(0.5, 1.0, -2.0)	9.78	5.31	238.12	102.42
(0.0, 2.5, 1.0)	7.65	49.58	266.51	128.76
(-1.5, 2.0, 0.0)	2.30	80.54	198.29	54.08
(-2.5, 0.5, 2.0)	5.54	112.60	78.57	97.87
(2.0, -0.5, 2.5)	39.81	127.98	248.98	335.19
(-1.0, -1.5, -1.0)	2.37	45.99	94.43	56.92
Avg	8.15	67.33	210.63	144.89

TABLE V: ATE (meter) Comparison for Global-pose aided VIO.

Perturbations	Urban34		Urban22	
	w. calib	wo. calib	w. calib	wo. calib
Avg	0.41	0.42	0.19	0.19

VI. CONCLUSION

We investigate the observability from [19], [22], and prove that the common-seen pure translational straight line motion can lead to the unobservability of the rotational extrinsic parameter between IMU and camera (at least one degree of freedom). Our novel finding is carefully verified through rigorous theory, numerical study, and real-world experiment. This finding makes up for the shortcomings of the existing research conclusions. When the observability conclusion is inconsistent with the numerical study results (see our comments in Sec. V-A), we recommend:

- Perform ablation experiments to eliminate the influence of other calibration parameters.
- Try different initial values to test the convergence consistency of the interested calibration parameter.

Mathematical derivations of this paper and [19] require delicate search for the null space of the observability matrix. And this process is case by case, which prompts us a research question for future work. Is there an automatic and natural way to find degenerate motion and corresponding unobservable degrees of freedom, thus avoiding potential manual missing or mistake?

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