ISCS Symposium

Towards assimilating InSAR data into a model of a highly anisotropic aquifer system

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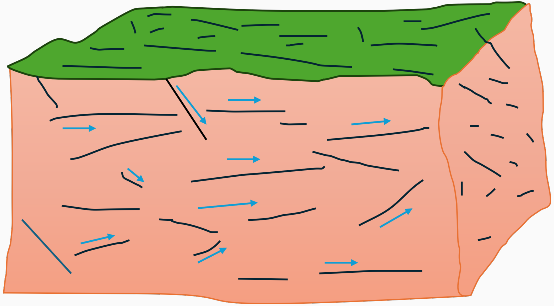
Groundwater flow in fractured aquifers

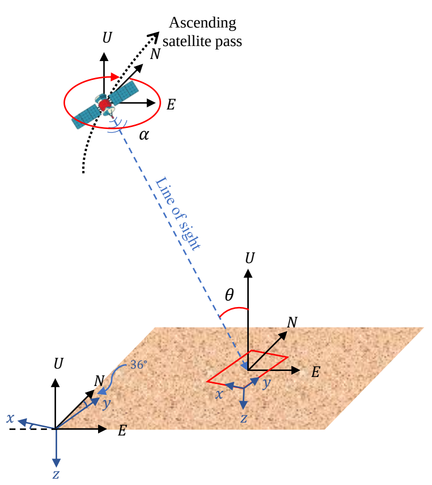
- Water flow in many aquifers is highly influenced by **anisotropy**, often caused by features like fractures and faults.
- Most existing Bayesian inference methods assume **isotropic** prior models, which may oversimplify real subsurface conditions.



Fractured aquifer

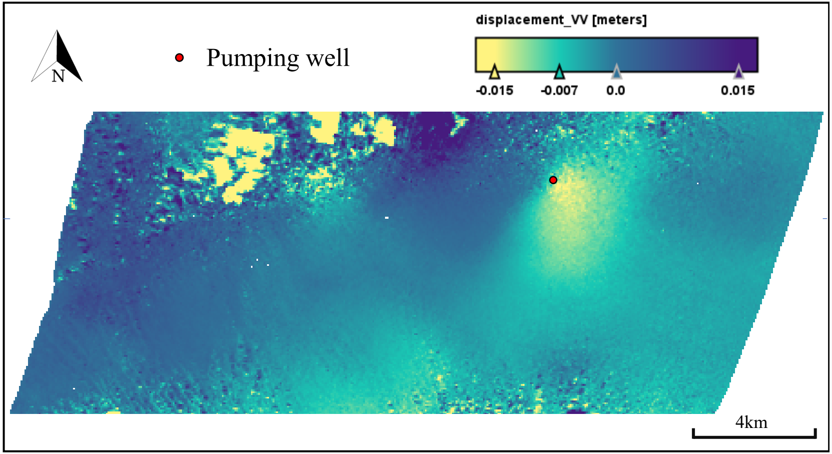
Using InSAR to Reveal Aquifer Secrets

- What is InSAR?
- Recent work (Alghamdi et al., 2024; Salehian Ghamsari et al., 2025) has proposed **InSAR** (Interferometric Synthetic Aperture Radar) data to improve aquifer property estimation.



from (Salehian Ghamsari, van Dam, & Hale, 2025)

Leveraging InSAR to uncover subsurface anisotropy



InSAR surface displacement of Nevada aquifer pumping test (Burbey, Warner, Blewitt, Bell, & Hill, 2006)

Research objective

- **Goal:** Develop a mathematical framework to assimilate InSAR-derived surface displacement data into a groundwater model for estimating **anisotropic hydraulic conductivity (AHC)**.
- **Forward Modeling:** Use a poroelastic finite element model that incorporates AHC to simulate line-of-sight (LOS) surface displacements detectable by InSAR (Salehian Ghamsari et al., 2025).
- **Prior Modeling:** We develop a probabilistic model for describing anisotropy in aquifer systems that can incorporate prior information from complex, potentially multi-modal, structural geological data.

Forward model

Three-field Biot equations with AHC

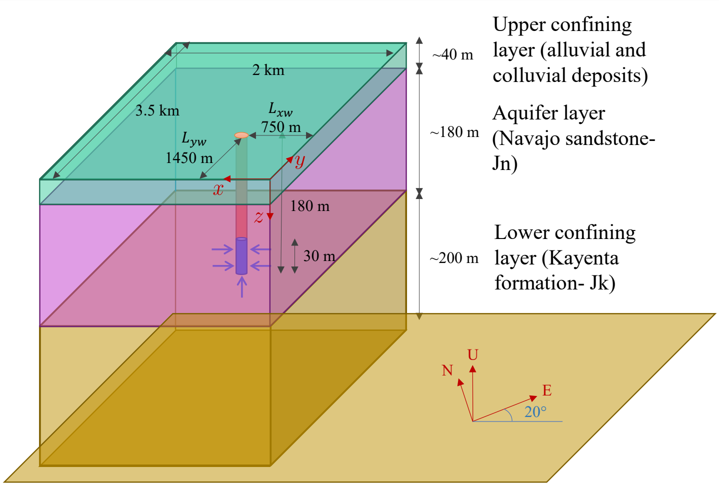
Find the fluid-pore pressure $p : \Omega \times (0, T] \to \mathbb{R}$, deformation $u : \Omega \times (0, T] \to \mathbb{R}^3$ and fluid flux $q : \Omega \times (0, T] \to \mathbb{R}^3$ such that:

$$
\begin{aligned}
(S_\epsilon p + \alpha \nabla \cdot u)_t + \nabla \cdot q = f_p \text{ on } \Omega \times (0, T], \\
-\nabla \cdot \bar{\sigma}(u, p) = f_u \text{ on } \Omega \times (0, T], \\
q + k \nabla p = 0 \text{ on } \Omega \times (0, T],
\end{aligned}
$$

k is the anisotropic hydraulic conductivity (AHC) tensor

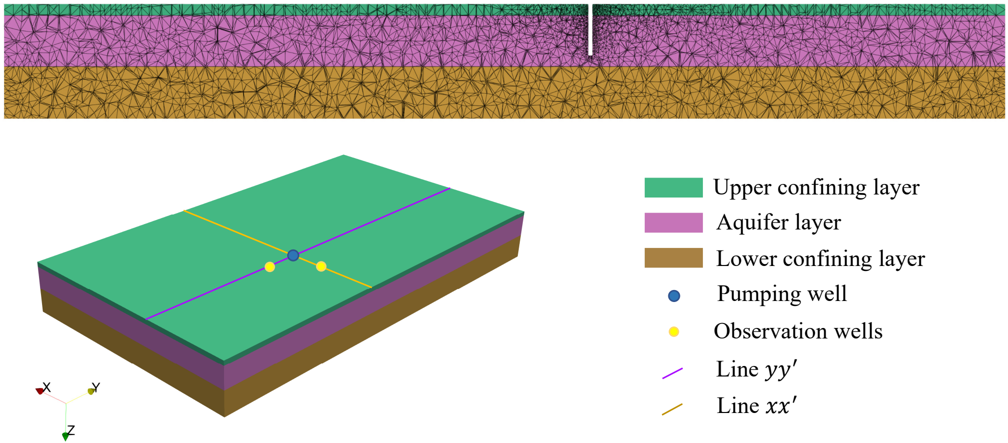
$$
k = \left[\begin{array}{cc|c} k_{xx} & k_{xy} & 0 \\ k_{yx} & k_{yy} & 0 \\ \hline 0 & 0 & k_{zz} \end{array}\right],
$$

Simplified conceptual model of the Anderson Junction aquifer system

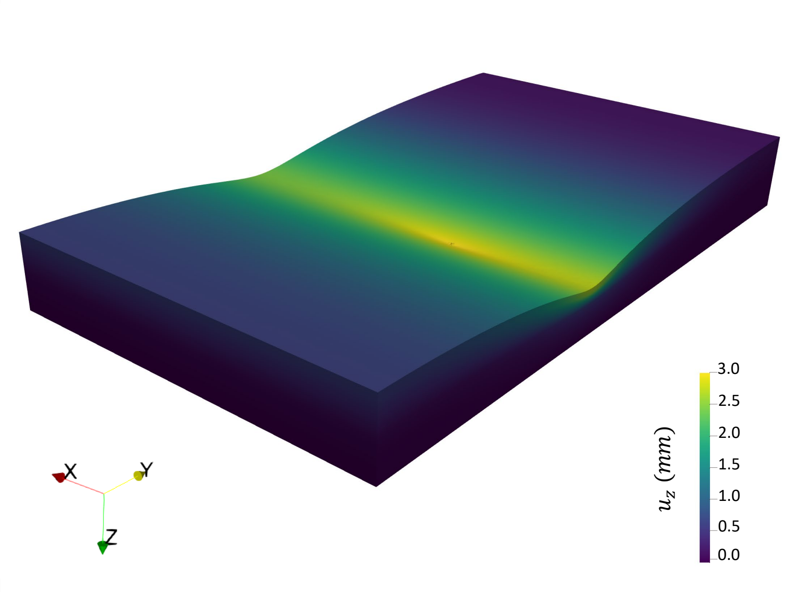


from (Salehian Ghamsari et al., 2025)

Generated 3D mesh of the aquifer system



Magnified visualization of aquifer displacement

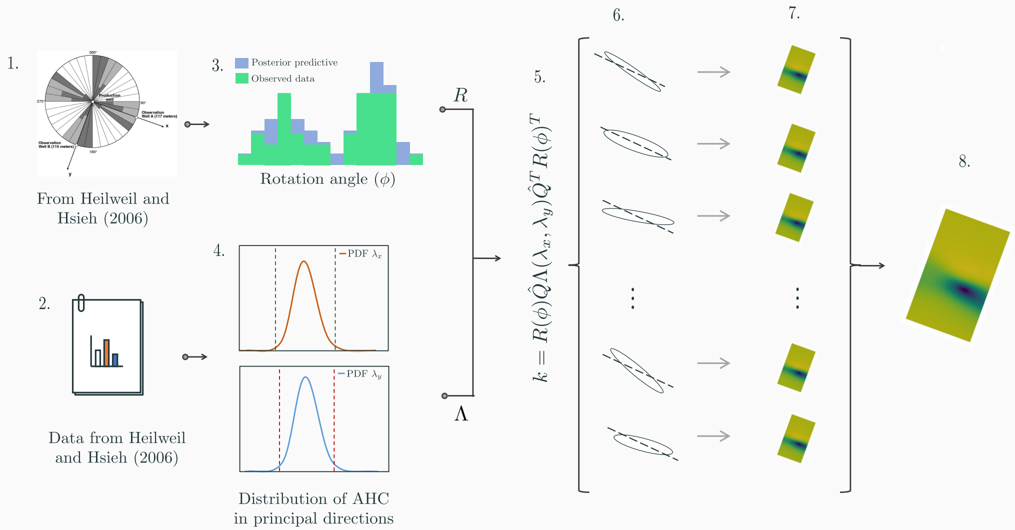


Stochastic extension

AHC Magnitude and Direction

- The poroelastic PDE model requires the anisotropic hydraulic conductivity (AHC) tensor to be **symmetric positive definite (SPD)** to ensure mathematical well-posedness (Aris, 2012).
- (Shivanand, Rosić, & Matthies, 2024) recently proposed a Lie group approach for constructing symmetric positive definite matrices.
- Building on this, we develop a **stochastic prior model** for the AHC tensor that allows independent control of both:
  - **Magnitude**
  - **Direction**

Methodology



Methodology

- It is well known that any k can be decomposed into a tensor of eigenvalues Λ and a tensor of eigenvectors Q

$$k = Q\Lambda Q^T.$$

- We can further rotate the eigenvectors Q by applying a rotation tensor R

$$k = (RQ)\Lambda(RQ)^T,$$

- Using the rotation angle ϕ about the z-axis, we construct a rotation tensor W as

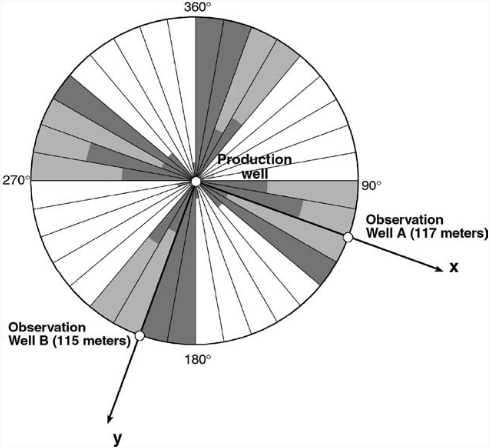
$$W = \phi \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

$$R = \exp(W).$$

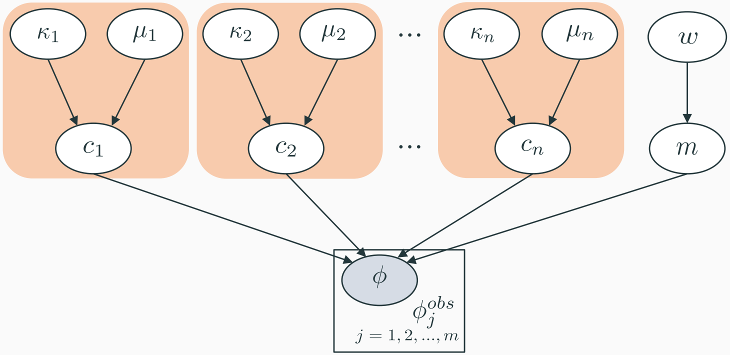
Prior information

Source of uncertainty in anisotropic hydraulic conductivity (AHC) tensor

- **Magnitude (Λ):** the reported uncertainty of hydraulic conductivity magnitude in regional study
- **Direction (R):** fracture outcrop data



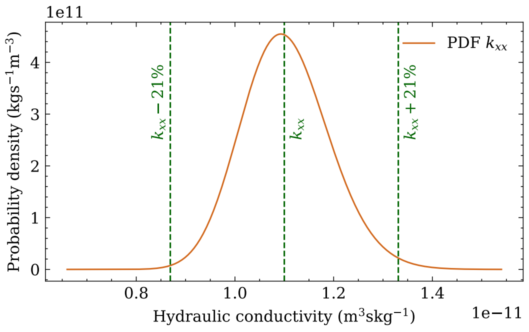
Rotation angle model



$\mu_i \sim$ VonMises
$\kappa_i \sim$ Gamma
$c_i \sim$ VonMises
$w \sim$ Dirichlet
$m \sim$ Categorical
$\phi \sim$ Mixture

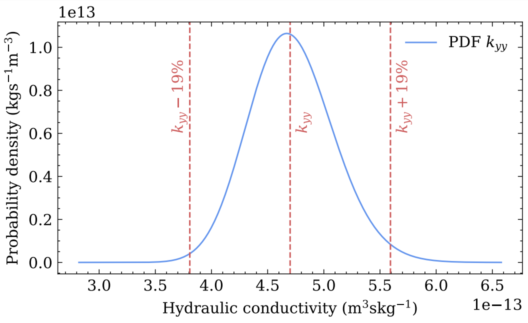
Eigenvalue model

$$\lambda_x \sim \text{lognorm}(\hat{\mu}_x, \hat{\sigma}_x^2)$$



Distribution of AHC magnitude in the major axis.

$$\lambda_y \sim \text{lognorm}(\hat{\mu}_y, \hat{\sigma}_y^2)$$



Distribution of AHC magnitude in the minor axis.

Anisotropic hydraulic conductivity model

- AHC tensors with randomness in both scaling and rotation

$$k(\omega) = R(\phi)\widehat{Q}\Lambda(\lambda_x, \lambda_y)\widehat{Q}^T R(\phi)^T.$$

- AHC tensors with randomness only in scaling (ϕ is known)

$$k_s(\omega) = \widehat{Q}\Lambda(\lambda_x, \lambda_y)\widehat{Q}^T.$$

- AHC tensors with randomness only in rotation (both λ_x and λ_y are known)

$$k_r(\omega) = R(\phi)\widehat{Q}\widehat{\Lambda}(\hat{\lambda}_x, \hat{\lambda}_y)\widehat{Q}^T R(\phi)^T.$$

Forward uncertainty propagation

The mean of the LOS displacement is then computed as

$$\mu(u_{\mathrm{LOS}}(x,t)) := \frac{1}{N}\sum_{i=1}^{N} u_{\mathrm{LOS}}(x,t,k_i),$$

and the unbiased estimation of standard deviation is calculated as

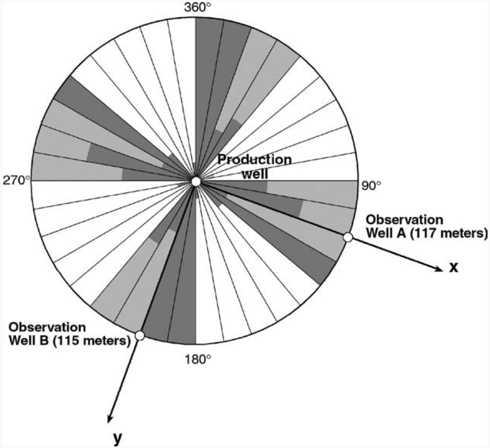
$$\mathsf{Std}(u_{\mathrm{LOS}}(x,t)) := \left[\frac{1}{N-1}\sum_{i=1}^{N}\left[u_{\mathrm{LOS}}(x,t,k_i) - \mu(u_{\mathrm{LOS}}(x,t))\right]^2\right]^{1/2}.$$

First scenario

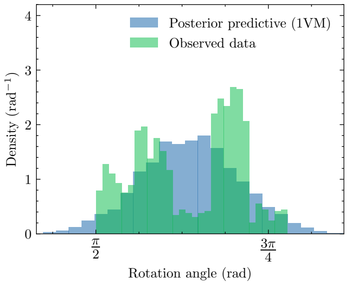
Prior information (first scenario)

Source of uncertainty in anisotropic hydraulic conductivity (AHC) tensor **with** (Heilweil & Hsieh, 2006) results

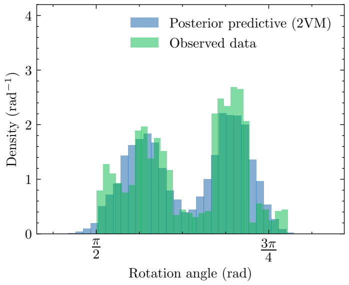
- **Magnitude (Λ):** $\approx 20\%$ uncertainty in magnitude of principal directions.
- **Direction (R):** Multimodal uncertainty in fracture outcrop data. (uncertainty only in x direction)



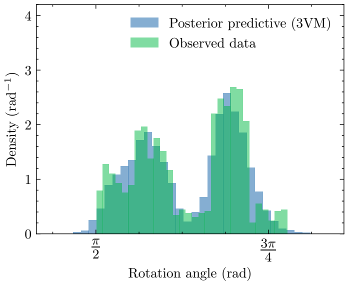
Random rotation angle: model selection



Posterior predictive distribution of 1VM.



Posterior predictive distribution of 2VM.



Posterior predictive distribution of 3VM.

Random rotation angle: model selection

Model	elpd_loo	p_loo	elpd_diff	SE
3VM	276.80	23.41	0.00	20.63
2VM	260.81	4.95	15.99	21.52
1VM	44.48	1.41	232.32	13.98

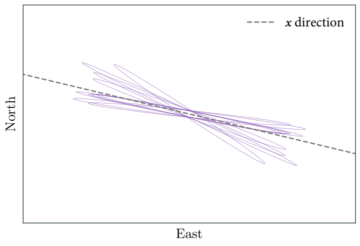
Table 1: Model selection results.

- elpd_loo: Expected log pointwise predictive density.
- p_loo: Estimated effective number of parameters.
- elpd_diff: The difference in ELPD between models, computed relative to the top-ranked model.
- SE: Standard error of the ELPD estimate.

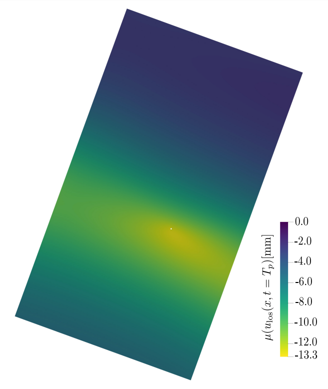
Forward uncertainty analysis

- Calculating random hydraulic conductivity tensors
- Using FEniCSx on ULHPC to run the poroelastic finite element model
- Running in parallel with thousands of random tensors
- Statistical analysis of the outputs

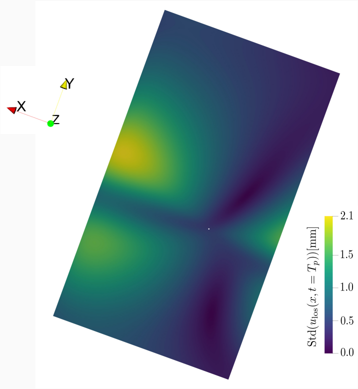
Forward uncertainty analysis: random scaling and rotation AHC



Elliptical representation of random AHC tensors.



Mean of LOS displacement.



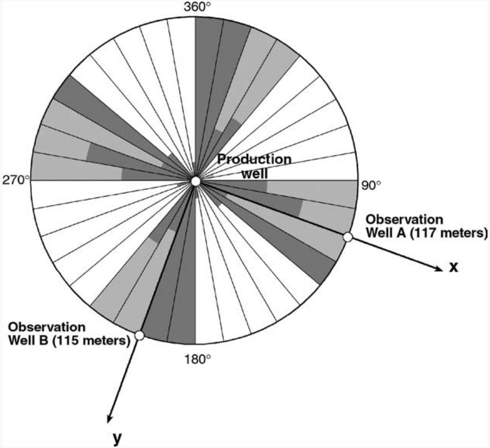
Standard deviation of LOS displacement.

Second scenario

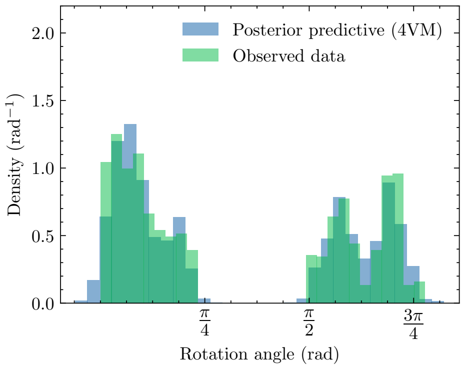
Prior information (second scenario)

Source of uncertainty in anisotropic hydraulic conductivity (AHC) tensor **without** (Heilweil & Hsieh, 2006) results

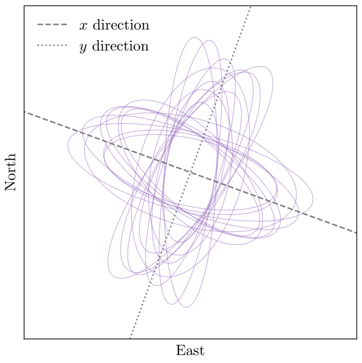
- **Magnitude (Λ):** no information available
- **Direction (R):** Multimodal uncertainty in fracture outcrop data. (uncertainity in both x and y directions)



Random rotation angle and random AHC

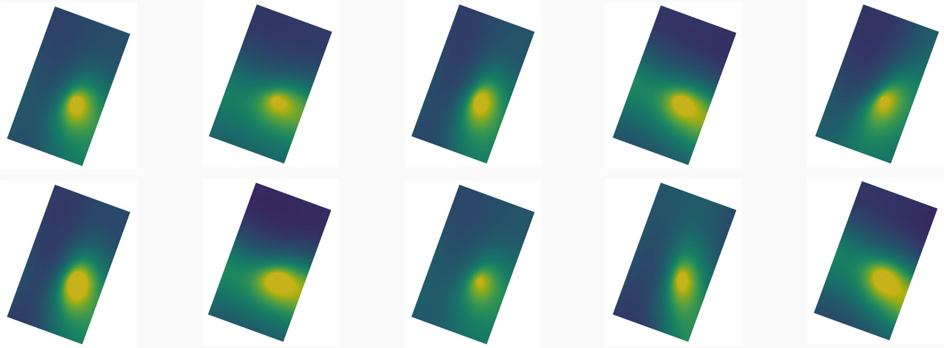


Posterior predictive distribution of mixture of four von Mises model for the major principal direction.



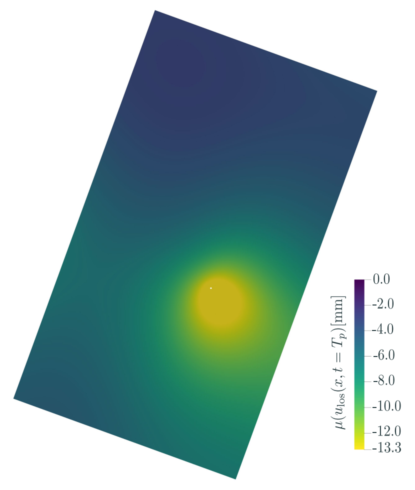
Elliptical representation of random AHC tensors.

Forward uncertainty analysis

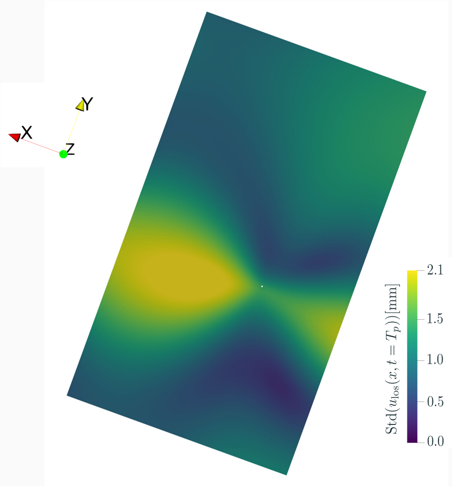


Samples of LOS displacement.

Forward uncertainty analysis



Mean of LOS displacement.



Standard deviation of LOS displacement.

Conclusion

- By calibrating the model against fracture outcrop and optionally pump test data from Anderson Junction, we were able to express two conceptual states of belief about the site.
- The proposed methodology provides a flexible tool for modeling the effect of random anisotropy on InSAR-measurable surface displacements.
- The proposed stochastic model could work as a prior in a Bayesian inference setting where InSAR-derived line-of-sight data contains information about AHC.

Future work:

✈ We will solve an inverse problem using InSAR data to estimate AHC

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References

Aris, R. (2012). *Vectors, Tensors and the Basic Equations of Fluid Mechanics*. Courier Corporation.

Burbey, T. J., Warner, S. M., Blewitt, G., Bell, J. W., & Hill, E. (2006, March). Three-dimensional deformation and strain induced by municipal pumping, part 1: Analysis of field data. *Journal of Hydrology*, *319*(1), 123–142. doi: 10.1016/j.jhydrol.2005.06.028

Cha, G.-W., Moon, H., Kim, Y.-M., Hong, W.-H., Hwang, J.-H., Park, W.-J., & Kim, Y.-C. (2020, 09). Development of a prediction model for demolition waste generation using a random forest algorithm based on small datasets. *International journal of environmental research and public health*, *17*. doi: 10.3390/ijerph17196997

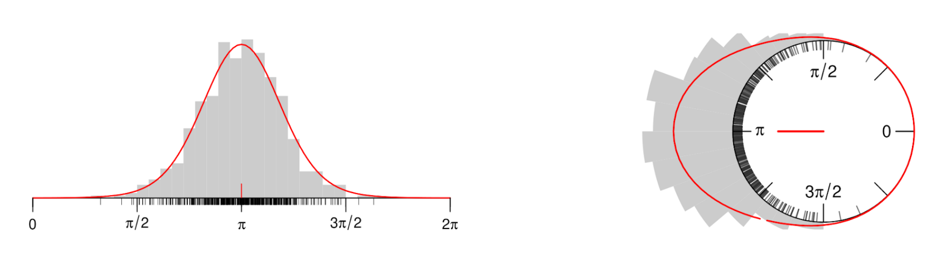
Heilweil, V. M., & Hsieh, P. A. (2006). Determining Anisotropic Transmissivity Using a Simplified Papadopulos Method. *Groundwater*, *44*(5), 749–753. doi: 10.1111/j.1745-6584.2006.00210.x

Lang, M. N., Schlosser, L., Hothorn, T., Mayr, G. J., Stauffer, R., & Zeileis, A. (2020). *Circular regression trees and forests with an application to probabilistic wind direction forecasting.* doi: 10.48550/arXiv.2001.00412

Salehian Ghamsari, S., van Dam, T., & Hale, J. (2025). Can the anisotropic hydraulic conductivity of an aquifer be determined using surface displacement data? a case study. Retrieved from 10.1016/j.acags.2025.100242

Shivanand, S. K., Rosić, B., & Matthies, H. G. (2024). Stochastic modelling of symmetric positive definite material tensors. *Journal of Computational Physics*, *505*, 112883. doi: 10.1016/j.jcp.2024.112883

The von-Mises disrtibution

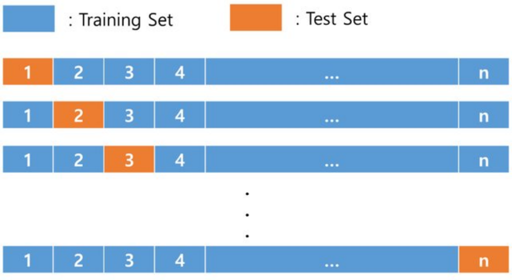


From (Lang et al., 2020)

$$f(\phi(\omega_r)|\mu,\kappa) = \frac{\exp(\kappa\cos(\phi(\omega_r)-\mu))}{2\pi I_0(\kappa)}$$

where I_0 is the modified Bessel function of order 0.

Leave-one-out cross-validation



From (Cha et al., 2020)

- $ELPD_{loo}$: expected log pointwise predictive density.
  Higher ELPD indicates higher out-of-sample predictive fit ("better" model).
- P_{loo}: Estimated effective number of parameters.