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Quality and trade with many countries and industries[☆]

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ABSTRACT

This paper investigates a trade model with horizontal and vertical product differentiations, many goods and many countries. It studies the impact of productivity, population changes and trade costs on the quality composition of exports. The analysis embeds within the same tractable model a series of empirical results, including Linder hypothesis and high-income countries' specialization in high quality good production. It also shows that high-quality goods exhibiting a high degree of differentiation are traded only by high-income countries.

1. Introduction

In the past decade, researchers have highlighted important patterns in the quality of traded goods:[2] countries import more high-quality goods from higher productivity exporters; wealthier nations import a higher share of high-quality goods, specialize more in the production and exportation of high-quality goods; higher-quality goods are exported to more distant countries.[3] Those findings have naturally called for a theoretical foundation that explains the quality of traded goods in the context of many countries, goods and quality standards.

Those findings on the role of quality in trade patterns have spurred a recent theoretical literature. One strand, with a focus on intra-industry trade (Krugman, 1981), has employed horizontal differentiation models and, for the purpose of discussing product quality, has augmented them with idiosyncratic demand shifters.[4] A more recent strand has introduced features of vertical

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2 Relevant examples are Piveteau and Smagghue (2019), Fontagné et al. (2018), Roberts et al. (2018), Gervais (2015), Fan et al. (2015), Khandelwal et al. (2013), Hallak and Sivadasan (2013), Kugler and Verhoogen (2012), among others.

3 See Auer et al. (2018), Manova and Zhang (2012), Crozet et al. (2012), Fieler (2011), Hallak (2010), Choi et al. (2009), Hallak (2006), Hummels and Klenow (2005) and Schott (2004), *inter alia.*

4 See e.g. Fieler et al. (2018), Picard (2015), Jaimovich and Merella (2012, 2015), Di Comite et al. (2014), Baldwin and Harrigan (2011), Anderson and De Palma (2020), among others.

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differentiation in horizontal differentiation models in order to analyze the effect of income on the quality of traded goods.[5] It is noticeable that this literature strongly differs from the vertical differentiation framework introduced by Gabszewicz and Thisse (1979), Shaked and Sutton (1982) and Helpman and Flam (1987) who discuss the idea that producers supply *more than one* quality versions of the *same* good so that rich and poor consumers end up purchasing different quality versions of this same good. In their seminal frameworks, all consumers rank the quality versions of this good in the same way but they end up purchasing different quality versions of it because of their different income situations. This idea seems relevant and well designed to discuss how countries produce the same goods at very different qualities for other countries with different incomes (see De Lucio Fernández et al. 2016 and Fontaine et al. 2020, among others).

The aim of the present paper is thus to make a further link between quality and trade in a vertical differentiation context that is close to seminal frameworks presented above. Our first research question is about how to apply the above vertical differentiation framework to a trade context with many goods. Our second question is to about to check whether the trade properties of such a framework are in accordance with the main empirical results that we cite above. Finally, we want to explore new results that may shed light on unexplored empirical trade patterns.

We propose an analytically tractable model with many goods and countries, where all industries are perfectly competitive and vertically differentiated. This approach permits to discuss the presence of different product qualities in several markets, and the resulting price dispersion over each variety. Each country produces a continuous set of goods with high and low-quality versions. The same good can thus be exported with a high quality to some countries and a low quality to others, depending on the importer's income. On the demand side, consumers are endowed with non-homothetic preferences and purchase a single version of every good from every country. While higher quality versions give higher utility, they are more costly to produce. For each variety, consumers then compare the prices of each quality version with their marginal utility.

We focus on non-homotheticity along the "vertical" dimension: at a same price, all consumers prefer to acquire the higher quality good over the lower one, because they rank intrinsic quality in the same way. In contrast, we abstract from horizontal differentiation, whereby different consumers value different types of the same variety differently. Here, varieties are not substitutable with other ones. Non-homotheticity with respect to quality is particularly relevant because income levels or willingness to pay play a central role, while they are not strictly necessary in the analysis of horizontally differentiated goods.

The paper firstly brings a methodological innovation by proposing a class of quality and cost profiles that makes consumer expenditures linear in the consumer's inverse marginal utility. As a result, the trade equilibrium is governed by a set of linear equations that can readily be solved and discussed.[6] To single out the effect of quality margins, we close off the extensive margin from the analysis by fixing the number of varieties purchased by consumers. This restriction is necessary to evaluate the purchasing decisions in terms of quality. Accordingly, productivity differences, population discrepancies and trade costs affect only the quality margins of traded goods. Then, quality margins move in the same way as intensive margins do in the Armington (1969) model that allows only for such adjustments. A major difference is that each country produces a set of differentiated goods with heterogeneous costs and demands, which is consistent with trade data.[7]

The paper secondly brings a confirmation of the match between the theoretical framework and empirical evidence. Our theoretical results indeed encompass all the empirical patterns we mentioned above. Average import prices are higher to countries with larger per capita income and import prices are also higher for the goods shipped from more productive exporters. The explanation is here simple. In markets with vertically differentiated goods, individuals in more productive countries have more income to purchase higher quality imports from each country. Also, since workers are also more productive, they manufacture high quality goods at lower relative costs, which makes their high quality goods more attractive to foreign consumers. Following those results and explanations, richer countries trade more numerous high-quality goods with each other, as argued by Linder (1961). In addition to explaining the empirical evidence, the presence of sectors with different degrees of vertical product differentiation allows to verify the Linder hypothesis based on the industry composition of a country.[8] We find that goods with stronger vertical product differentiation are more traded by high income countries.

Our framework is also suited to analyze how the quality of traded goods change with country productivity and population size. We find that an increase in a country's productivity entices this country to specialize in high-quality goods. This is because the country is able to manufacture its high quality goods at lower relative prices. Productivity increases have different effects than population increases. Indeed, a bigger population leads to wider consumption of local, high-quality goods, but it may lead to a narrower range of high-quality imports. With a fixed number of varieties, local population growth increases labor supply for each local good and therefore pushes wages down. Local consumers become poorer and substitute high-quality imports for high-quality domestic goods. Interestingly, these results are consistent with those models with a focus on the extensive margin.

The model is consistent with the empirical effects of trade costs and distance. A fall in *ad-valorem* (iceberg) trade cost entices countries to substitute domestic for high-quality foreign goods (Fan et al., 2015). It boosts exports of high-quality goods, increases cif prices and finally raises utility everywhere. The model also leads to a gravity equation whose terms are consistent with the literature.

[5] This literature has developed random utility models where unit-purchase choices combine in continuous aggregates. Relevant examples are Verhoogen (2008), Fajgelbaum et al. (2011, 2015) and Dingel (2017), who study two-country (North-South) models with a single good (or industry) per country and a single quality version per good.

[6] This restriction may reduce the generality of the model but it does not less than the usual preferences and cost assumptions used in the literature.

[7] The extensive margins will be then reintroduced in Section 5.

[8] A similar investigation has been developed by Fieler (2011), but assuming away product quality.

Finally, the present paper extends Picard and Tampieri's (2021) analysis of two (almost) symmetric countries subject to infinitely small productivity and population shocks. These authors study the properties of marginal changes in the trade equilibrium by linearizing the equilibrium conditions around a symmetric trade configuration. This paper uses the same preference and cost structures, but it relaxes the restriction of two symmetric countries by evaluating the trade patterns of many asymmetric countries. In the presence of many countries, our model fits the empirically relevant situation where exporters may sell different qualities of the same variety to different trading partners.

Before proceeding further, it is essential to highlight how this paper departs from the existing trade theory literature on product quality.

Related literature. The starting point of the analysis is to embed a model of vertical differentiation in the spirit of Mussa and Rosen (1978) and Gabszewicz and Thisse (1979) and others into a trade framework. The paper is firstly linked to the general equilibrium studies of trade under vertical differentiation. Early papers discuss the endogenous quality spectrum of a single good, which makes them unsuitable to discuss intra-industry trade (Flam and Helpman, 1987; Stokey, 1991). By contrast, this paper considers a continuous set of goods with two quality levels, which permits the study of intra-industry trade. Besides, these papers explore vertical differentiation in a North-South setting where one country is endowed with a stronger productivity advantage (Matsuyama, 2000). Instead, we study trade between a large number of not too asymmetric countries.

In contrast to these research lines, the present paper discusses trade properties using a novel and unexplored setting of costs and preferences. We include a set of horizontally differentiated varieties produced in several quality versions, following the seminal vertical differentiation literature initiated by Mussa and Rosen (1978) and Gabszewicz and Thisse (1979). Thus, our model elaborates on preferences close to those discussed in Tarasov (2009, 2012) and Fieler (2012). In particular, Tarasov studies a continuous set of varieties versioned in one quality each and sold under monopolistic competition. Conversely, we investigate the same set of varieties versioned in two quality levels and, for the sake of simplicity, produced in perfectly competitive markets.

Many theoretical studies of quality in trade explain the empirical findings in micro-data with a focus on divisible goods, either linear-quadratic or CES utility functions, and quality modeled as a demand shifter. In these frameworks though, consumption is proportional to income (homothetic preferences) so that richer individuals and/or countries do not have higher consumption share of high-quality products.[9] The research on product quality and trade has become a search for a set of preferences that best reflects observed patterns. Jaimovich and Merella (2012, 2015) propose an upper-tier homothetic CES preferences utility together with a lower-tier subutility function that combines the log of quantity and quality levels. Like in the present analysis, richer countries consume higher quality goods. Eaton and Fieler (2017) study two-tier CES preferences nesting horizontal and vertical dimensions of goods. Their modeling differs from this paper as countries produce goods with a single quality level and goods are divisible. Others have studied CES subclass of Hannoch preferences implying heterogeneous income elasticities (Matsuyama, 2019). Jaimovich et al. (2023) extend Matsuyama (2019) to a framework that allows for vertically differentiated varieties. The present paper discusses trade properties using another novel and unexplored setting of costs and preferences.

Finally, the present paper is related to the literature on competition and trade with demand based on non-homothetic preferences, which does not necessarily refers to product quality issues, e.g. Fieler (2011), Behrens and Murata (2012), Simonovska (2015), Foellmi et al. (2018), Bertoletti and Etro (2016, 2017) and Bertoletti et al. (2018). In particular, Bertoletti and Etro (2017) study monopolistic competition with consumers endowed with additively separable indirect utilities. Quality is modeled by a shifter in (sub-)utility functions which has the property that demand elasticity is invariant to quality. As in our analysis, they reproduce the Linder hypothesis and show that higher local productivity pushes specialization in higher quality goods. However, in contrast to our analysis, each variety cannot be produced and sold in many quality versions. The punchline of this paper is to study economies where same goods can be sold at different quality levels as discussed in the seminal literature initiated by Mussa and Rosen (1978) and Gabszewicz and Thisse (1979).

The remainder of the paper is organized as follows. Section 2 describes the model of vertical differentiation with many goods and countries and presents the role of linear real expenditure. The trade equilibrium and its properties are examined in Sections 2 and 3, respectively. Section 4 discusses the model with ad-valorem trade costs and elaborates on the gravity equation resulting from this model. Section 5 studies the conditions for the existence of an Alchian Allen effect, while Section 6 concludes. Appendices include mathematical details.

2. Model

2.1. The framework

We consider an economy with N trading countries $j \in \{1, \dots, N\}$ populated by a mass M_j of individuals who are each endowed with s_j labor units (skill), which can be interpreted as country productivity. The share of country j's population in the world is denoted as $m_j = M_j/M$ where $M = \sum_j M_j$. Each country j produces a set of differentiated goods/varieties $z_j \in [0, 1]$. Each variety can be produced only in one country, and the world number of varieties is equal to N. The key assumption of this paper is that each good can be versioned with high or low-quality, denoted by $k \in \{H, L\}$.

[9] See references in footnote 3.

Production Technologies are the same across countries and described by the labor input schedules for high and low quality varieties, a_H and $a_L : [0,1] \to \mathbb{R}_0^+$. Hence, following Armington (1969), a variety z_j produced in country j requires $a_H(z_j)$ and $a_L(z_j)$ labor units for the high and low-quality version, respectively. Under perfect competition and in the absence of trade cost, the price of variety z_j produced in country j and sold in country i is equal to its unit cost:

$$p_{ijk}\left(z_j\right) = a_k(z_j)w_j,\ k \in \{H, L\}, \tag{1}$$

where w_j is the wage per labor unit in the production country j. Although varieties are perfectly differentiated within and between countries, their production functions and quality profiles are the same in every country for the sake of simplicity.

Demands In country i, an individual earns the income $w_i s_i$ where s_i is her endowment of labor unit and w_i is the price (wage) of a labor unit. A variety z_j yields a utility level $b_H\left(z_j\right) > 0$ for its high-quality version and $b_L\left(z_j\right) > 0$ for its low-quality version. For conciseness, we may refer to $b_k\left(z_j\right)$ also as product quality. Every individual consumes a unit of every variety z_j produced in every country j. An individual in country i maximizes her utility

$$U_i = \sum_{j=1}^{N} \int_0^1 \left(\sum_{k=H,L} b_k\left(z_j\right) x_{ijk}\left(z_j\right) \right) \mathrm{d}z_j,$$

subject to her budget constraint

$$\sum_{j=1}^{N} \int_1^0 \left(\sum_{k=H,L} p_{ijk}\left(z_j\right) x_{ijk}\left(z_j\right) \right) \mathrm{d}z_j = w_i s_i,$$

where $p_{ijk}\left(z_j\right) > 0$ is the (destination) consumer prices and $x_{ijk}\left(z_j\right) \in \{0,1\}$ the unitary consumption decision of variety z_j. Note that, additive utility would imply perfect substitutable goods in the context of divisible goods. However, in the present context of indivisible unit demand for each variety, varieties are seen as independent from each other.

Replacing the prices by their values in (1), there exits a positive scalar μ_i such that the individual i buys the high-quality version H of a variety z_j if

$$b_H\left(z_j\right) - \frac{1}{\mu_i} a_H(z_j)w_j \geq b_L\left(z_j\right) - \frac{1}{\mu_i} a_L(z_j)w_j, \tag{2}$$

and the low-quality L otherwise. The scalar μ_i measures the inverse of the marginal utility of income and is equal to the inverse of the Lagrange multiplier of the budget constraint.

By (2), the set of high-quality varieties produced in country j and consumed in country i is given by

$$\mathcal{H}\left(\frac{\mu_i}{w_j}\right) \equiv \left\{ z_j : \frac{\mu_i}{w_j} \geq \ell(z_j) \right\}, \tag{3}$$

where μ_i / w_j is the marginal utility of income, and

$$\ell(z_j) \equiv \frac{a_H\left(z_j\right) - a_L\left(z_j\right)}{b_H\left(z_j\right) - b_L\left(z_j\right)}, \tag{4}$$

denotes the per-quality-unit labor input of upgrading variety z_j. For the sake of brevity, we shall call this the "per-quality input". The sets of the purchased low-quality varieties is defined as $\mathcal{L}\left(\mu_i/w_j\right) = [0,1] \backslash \mathcal{H}\left(\mu_i/w_j\right)$.

The above demand system requires to impose three conditions along this paper. First, we assume $\ell'(z_j) > 0$. This is done without loss of generality as one can always re-order the varieties z_j from low to high values of $\ell(z_j)$. The strict inequality guarantees that the identity $\mu_i/w_j = \ell(z_j)$ has a unique solution. Second, we require that *all consumers buy a mix of high and low qualities*, which is fulfilled if and only if

$$\frac{\mu_i}{w_j} \in (\ell(0), \ell(1)),\ \forall i, j. \tag{A1}$$

This condition allows us to discuss the identity $\mu_i/w_j = \ell(z_j)$ at its a interior solution. Finally, we haved assumed that *all consumers purchase all varieties in either high or low quality version* ($x_{ijH} + x_{ijL} = 1$). This corresponds to the "full market coverage" condition in the industrial organization literature on vertical differentiation. It implies the absence of zero consumption, which is a standard assumption in the trade literature based on the Armington model.[10] This assumption further implies that the set of consumed varieties is exogenous so that there are no extensive margin effects. Shutting down extensive margins allows us to highlight the role of quality margin.[11] Consumers purchase all varieties if the input per quality schedule ℓ lies respectively above the schedules a_L/b_L or a_H/b_H when they buy the low or the high quality varieties. This is fulfilled if either

$$\frac{\mu_i}{w_j} \geq \frac{a_H(z_j)}{b_H(z_j)} \text{ if } \frac{\mu_i}{w_j} \geq \ell(z_j), \tag{A2.a}$$

[10] A large bunch of the trade literature is based on Cobb–Douglas and CES preferences that always induce a positive demand for each good. The literature also discusses other preferences that lead to product demands with choke prices that trigger zero consumption (e.g. quadratic or exponential utility functions). Those preferences are however often combined with the assumption of high enough incomes so that choke prices are sufficiently high to induce a positive consumption for each good. The literature includes few papers where consumers may not buy all available goods. For instance, Tarasov (2009) and Foellmi et al. (2018) consider "0-1 preferences", where consumers purchase one or zero units of each good. They, however, do not model and discuss quality.

[11] Section 5 extends the baseline analysis with changes in the extensive margin.

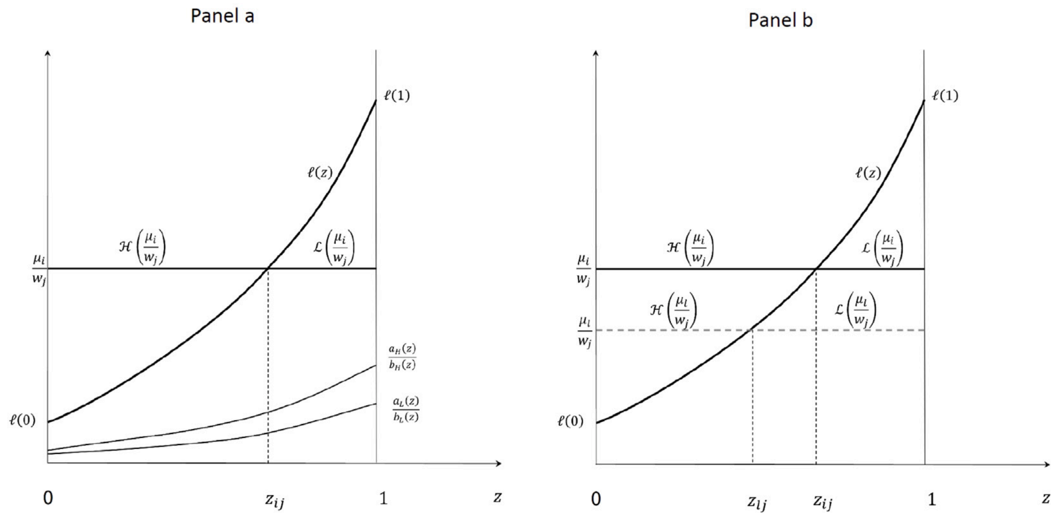

Fig. 1. Country i's individual demand for high- and low-quality varieties from country j.

or,

$$\frac{\mu_i}{w_j} > \frac{a_L(z_j)}{b_L(z_j)} \text{ if } \frac{\mu_i}{w_j} < \ell(z_j) \tag{A2.b}$$

for all i, j. As μ_i/w_j will be shown to be positively related to income, conditions A2(a) expresses that consumers should have a high enough income to purchase the high-quality varieties when they wish to do so. Condition A2(b) states that their income should be high enough to buy the low-quality varieties if they do not prefer the high-quality ones. A sufficient requirement for condition A2(a) is $\ell(z_j) \geq a_H(z_j)/b_H(z_j)$.

In the present model, where consumers purchase several varieties versioned in different qualities, the difference in quality margin is in the number of high-quality goods purchased from home and imported. From the above definition, it is apparent that μ_i/w_i is a sufficient statistic for the mass of consumers' purchases of local high-quality varieties $\mathcal{H}\left(\mu_i/w_i\right)$.

Panel a of Fig. 1 presents the schedule of per-quality input of varieties $\ell(z_j)$, $z_j \in [0,1]$. Consumptions of high- and low-quality varieties produced in a country j can readily be inferred for a consumer in another country i. This consumer has an inverse marginal utility μ_i and purchases the sets of high- and low-quality varieties from j, $\mathcal{H}\left(\mu_i/w_j\right) = [0, z_{ij}]$ and $\mathcal{L}\left(\mu_i/w_j\right) = (z_{ij}, 1]$. The high quality varieties have lower per-quality input and therefore lie to the left of the figure; that is, upgrading such varieties to high quality implies lower cost increases or higher utility increases. Condition A1 imposes the equilibrium to lie within the graph of ℓ (i.e. $(\ell(0), \ell(1))$ while Conditions A2 constrain the equilibrium to lie above the curve $a_H(z_j)/b_H(z_j)$ or $a_L(z_j)/b_L(z_j)$ according to whether consumers purchase high or low quality varieties.

Panel b of Fig. 1 presents the consumption of the goods produced in country j by the consumers in a higher income country i and a lower income one l. The set of high quality goods purchased by country i is again given by $\mathcal{H}\left(\mu_i/w_j\right) = [0, z_{ij}]$ while the set bought by country l is $\mathcal{H}\left(\mu_l/w_j\right) = [0, z_{lj}]$. So, in this model, there exists a set of goods $z \in [z_{lj}, z_{ij}]$ that are produced in country j at high quality for country i and low quality for country l. This contrasts to many models that assume that a good is produced at the same quality level for all countries.

Real expenditure Using the definition of the budget constraint, we can denote the expenditure on the set of varieties produced in country j and consumed by an individual in country i as $w_j E\left(\mu_i/w_j\right)$ where we define the function

$$E(y) \equiv \int_{\mathcal{H}(y)} a_H(z_j)\mathrm{d}z_j + \int_{\mathcal{L}(y)} a_L(z_j)\mathrm{d}z_j, \tag{5}$$

that represents a consumer's real expenditure on those varieties in terms of producing country's wage when y is evaluated at μ_i/w_j. This expression measures the labor content in this set of varieties and is a function of the simple statistics μ_i/w_j.

Trade balance To close the model, we express the trade balance condition for each country i, which equates the values of its imports and exports:

$$\sum_{l\neq i} m_i w_l E\left(\frac{\mu_i}{w_l}\right) = \sum_{l\neq i} m_l w_i E\left(\frac{\mu_l}{w_i}\right). \tag{6}$$

Trade statistics We conclude the section by establishing three measures of interest for the sequel discussion. First, the average price of imports is given by

$$\overline{p}_{ij} \equiv \int_{\mathcal{H}(\mu_i/w_j)} w_j a_H(z_j)\mathrm{d}z_j + \int_{\mathcal{L}(\mu_i/w_j)} w_j a_L(z_j)\mathrm{d}z_j = w_j E\left(\frac{\mu_i}{w_j}\right). \tag{7}$$

Table 1
Assumptions and functional forms.

Assumptions:		Resulting functional forms:
Proportionate cost upgrades:	$\frac{a_H}{a_L} = \frac{\alpha}{\alpha-1}$	$\ell = \frac{\beta}{\alpha}\frac{a_H}{b_H} = \frac{\beta-1}{\alpha-1}\frac{a_L}{b_L}$
Proportionate utility upgrades:	$\frac{b_H}{b_L} = \frac{\beta}{\beta-1}$	$r = \alpha\ell(0) - (\alpha-1)\ell(1)$
Linear real expenditure:	$E(y) = y - r$	$\ell(z_j) = \frac{a_H(0)-a_L(0)}{b_0} + \int_0^{z_j}\left(a_H(z) - a_L(z)\right)\mathrm{d}z$

Second, the share of high-quality purchases in imported goods is equal to

$$\int_{\mathcal{H}(\mu_i/w_j)} \mathrm{d}z_j \equiv \int_0^{\ell^{-1}(\mu_i/w_j)} \mathrm{d}z_j = \ell^{-1}\left(\mu_i/w_j\right).$$

where ℓ^{-1} is the inverse function of ℓ (i.e. $\ell^{-1}(\ell(z_j)) = z_j$). Finally, the indirect utility writes as $V_i = \sum_{j=1}^N V\left(\mu_i/w_j\right)$ where we define

$$V(y) \equiv \int_{\mathcal{H}(y)} b_H(z_j)\mathrm{d}z_j + \int_{\mathcal{L}(y)} b_L(z_j)\mathrm{d}z_j = \int_0^{\ell^{-1}(y)} b_H(z_j)\mathrm{d}z_j + \int_{\ell^{-1}(y)}^1 b_L(z_j)\mathrm{d}z_j. \tag{8}$$

As a result, the ratios μ_i/w_j are also sufficient statistics for utility of imports in country i from country j.

The above model builds on four classes of general functions for the utility and cost of high and low-quality goods (a_H, a_L, b_H, b_L), which yields high parameter dimensionality. By contrast, the literature usually builds on specific utility and cost distribution functions that ease analytical tractability and narrow the parameter dimension to a bunch of scalars.[12] We naturally follow this strategy and here propose specifications on primitives that prove to be analytically tractable. This reduces the level of generality of our analysis, but, in our opinion, it does not narrow the setting more than what the literature usually does.

2.2. A tractable specification

In this subsection, we present an analytically tractable specification of cost and utility that satisfies the above conditions about the mix of high and low qualities and simplifies the discussion of trade equilibrium properties. The application of such a specification constitutes the innovative element of our analysis. To ease the reading, Table 1 summarizes the chosen parameter values and functional forms.

Proportionate upgrades Conditions A2 are readily satisfied under the natural assumption of *proportionate cost and utility upgrades*. That is, $a_H/a_L = \alpha/(\alpha-1)$ and $b_H/b_L = \beta/(\beta-1)$ for the scalars β and α such that $\beta > \alpha > 1$. In this case, whatever the cost and utility profiles, we get $\ell = (\beta/\alpha)\left(a_H/b_H\right) = \left[(\beta-1)/(\alpha-1)\right]\left(a_L/b_L\right)$. So, the sufficient condition for A2(a) ($\ell \geq a_H/b_H$) holds. Similarly, the condition A2(b) is met if $\mu_i/w_j > \left[(\alpha-1)/(\beta-1)\right]\ell(1)$. The latter condition is satisfied for any value of β sufficiently higher than α, which we assume.

Linear expenditure We assume *linear real expenditure functions*, where the real expenditure function is a linear function of the marginal utility of income, i.e., $E'(y) = 1$. As a consequence, the general equilibrium is the solution of a set of linear conditions of inverse marginal utility, which will ease our analytical discussion of trade properties. This condition imposes a single functional restriction on the primitive functions (a_H, a_L, b_H, b_L). In particular, we show in the Appendix that this condition imposes that the per-quality input is given by

$$\ell(z_j) = \frac{a_H(0) - a_L(0)}{b_0} + \int_0^{z_j}\left(a_H(z) - a_L(z)\right)\mathrm{d}z \tag{9}$$

where $b_0 = b_H(0) - b_L(0)$is a positive constant. Then, by (4), one can recover the utility gain from quality upgrades satisfying $E'(y) = 1$ as

$$b_H\left(z_j\right) - b_L\left(z_j\right) = \frac{a_H\left(z_j\right) - a_L\left(z_j\right)}{\ell(z_j)},$$

where $\ell(z_j)$ is taken from (9).

The expression (9) is actually the per-quality input schedule that we have defined earlier. It increases in z_j and now depends only on the profile of upgrade costs. Such a primitive on utility imposes that quality upgrades should be substantial for goods that have substantial cost upgrades, which seems to be an acceptable and intuitive assumption. In this specification, the schedule $\ell(z_j)$ expresses the property of underlying cost distributions.[13]

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[12] Typically, the CES utility functions can be summarized by its single parameter of constant elasticity of substitution. Firm costs can be defined by a Pareto or Fréchet distribution functions that are summarized by a small bunch of parameters.

[13] It is shown in the appendix that a linear per-quality input schedule $\ell(z)$ reflects a uniform cost distribution across varieties while more convex schedules reflect stronger cost dispersions.

Table 2
Pareto cost distributions.

Assumptions on cost:		Resulting functional forms:
C.d.f.:	$F_k(\widetilde{a}_k) = 1 - \left(a_{0k}/\widetilde{a}_k\right)^{\kappa}$, $\kappa > 1$	$a_H(z_j) - a_L(z_j) = a_0(1 - z_j)^{-1/\kappa}$
Support:	$\widetilde{a}_k \in (a_{0k}, \infty)$	$r = a_0 \left[\frac{1}{b_0} - \frac{\alpha - 1}{1 - 1/\kappa}\right]$
Quality difference:	$a_0 = a_{0H} - a_{0L}$	$\ell(z_j) = \frac{a_0}{b_0} + a_0 \frac{\left[1 - (1 - z_j)^{1-1/\kappa}\right]}{1 - 1/\kappa}$

To understand the application of this specification, suppose that the cost profiles a_H and a_L are empirically given. Then, we can choose the constant b_0 which determines the per-quality input profile $\ell(z_j)$. We can finally freely choose the low quality utility profile b_L, which will determine the high quality utility profile b_H by the above expression.

Under this specification, it can be shown that the real expenditure function writes as $E(y) = y - r$ where

$$r = \alpha\ell(0) - (\alpha - 1)\ell(1).$$

For the sake of exposition, we assume that $r > 0$ in the sequel, although most results hold for not too negative values. It requires that the per-quality input does not differ too much across varieties. In terms of the above properties, it is shown that this requires a small enough b_0, a small enough α or a weak enough cost dispersion (see Appendix).

Those specifications restrict only three profiles out of the four utility and cost profiles (a_H, a_L, b_H, b_L) and relate them according to the scalar parameters α, β and b_0. Hence, one profile remains free. For instance this profile can be the cost profile a_H. Hence, many cost primitives satisfy those assumptions.

For the sake of example, Table 2 presents the application to Pareto cost distributions and reports the associated cumulative probability distributions and supports for the costs of each quality $k \in \{H, L\}$. Each distribution is defined by its dispersion parameter κ and lowest labor input parameters a_{0H} and a_{0L}. The second column reports the resulting functional forms for the difference between high and low quality labor input and "per-quality input" and our real expenditure intercept r. The latter is positive if and only if $b_0 \leq (1 - 1/\kappa)/(\alpha - 1)$. This confirms the above discussion as $r > 0$ requires a small enough b_0, a small enough α or, a high enough κ which implies sufficiently high concentration on low cost levels. Notice, though, that none of our following results will depend on this example.

Expenditure and balanced trade Under those specifications, the total expenditure of an individual in country i simplifies to

$$E_i = \sum_{j=1}^{N} w_j E\left(\frac{\mu_i}{w_j}\right) = N\mu_i - r\left(\sum_{j=1}^{N} w_j\right). \tag{10}$$

To balance budget, expenditure E_i should equal to incomes $s_i w_i$. Using this in the above identity for real expenditure, we have

$$\mu_i = \frac{s_i w_i}{N} + \frac{r}{N}\sum_{l=1}^{N} w_l,\ i \in \{1, \dots, N\}. \tag{11}$$

The inverse marginal utility of income μ_i reflects the consumer's incentive to purchase an upgraded quality version of the good amongst her basket of low-quality goods.

Finally, adding $m_i w_i E\left(\mu_i/w_i\right)$ on both sides of (6) and substituting (10), the trade balance condition becomes

$$\sum_{l=1}^{N} m_i\left(\mu_i - r w_l\right) = \sum_{l=1}^{N} m_l\left(\mu_l - r w_i\right),\ i \in \{1, \dots, N\}. \tag{12}$$

To sum up, our model is characterized by two sets of Eqs. (11) and (12) that are linear in w_i and μ_i, $i \in \{1, \dots, N\}$.

Application to trade statistics We can apply those specifications to the above three measures of interest. First, the average price of imports is given by the linear function

$$\overline{p}_{ij} = \mu_i - r w_j.$$

The share of high-quality purchases in imported goods is still given by $\ell^{-1}\left(\mu_i/w_j\right)$, which shape depends on the underlying cost functions. The indirect utility finally writes as $V_i = \sum_{j=1}^{N} V\left(\mu_i/w_j\right)$ where

$$V(y) = \ln y + \beta \ln \ell(0) + (\beta - 1)\ln \ell(1). \tag{13}$$

Hence, under linear real expenditures, the indirect utility is a function of the logs of the statistics μ_i/w_j plus a positive constant.

To sum up, this section specifies cost and utility primitives such that cost and utility upgrades are proportionate and real expenditure is a linear function of the inverse marginal utility. As shown in the sequel, the first property is used to verify the conditions of the existence of the general equilibrium. The second one is used to show its uniqueness and establish all the analytical trade properties. It is important to note that the linear expenditure assumption is not a knife-edge case. It embeds many examples of cost and utility specifications, like the Pareto cost distribution. To the best of our knowledge, the use of such properties is novel in the literature with non-homothetic preferences.

2.3. Equilibrium

A trade equilibrium is defined by

- the profiles of prices $p_H(z_j)$ and $p_L(z_j)$ that make firms break even Eq. (1) in every country $j \in \{1, \dots, N\}$,
- the vector of inverse marginal utility of income $\mu = (\mu_1, \dots, \mu_N)$ that matches individuals' optimal consumption choices at given prices Eq. (11),
- the vector of unit wages $w = (w_1, \dots, w_N)$ that balances trade conditions (12),
- consumers buy all varieties and a mix of qualities at the equilibrium (Conditions 1 and 2).

Since unit wages directly determine prices, it is sufficient to check the $2N$ conditions (11) and (12), which are linear in μ and w. Given demand homogeneity of degree zero and Walras law, the equilibrium is the solution of $2N-1$ equations and $2N-1$ values of w and μ. In the sequel, we concentrate on the relative unit wage and the marginal utility of income w_i/w_j and μ_i/w_j, respectively. Conditions (11) and (12) gives the following unique solution for relative wages

$$\frac{w_i}{w_j} = \frac{m_j s_j + r}{m_i s_i + r}. \tag{14}$$

The above first identity is remarkable because it is mainly expressed in terms of the countries' labor supply, $m_j s_j$. Relative unit wages between two countries w_i/w_j are inversely related to the ratio of their labor supplies. Intuitively, more abundant labor supplies push the price of labor units down.

Given the above, one gets the relative inverse marginal utility of income:

$$\frac{\mu_i}{w_j} = \frac{1}{N}\left(\frac{w_i}{w_j} s_i + r \sum_{l=1}^{N} \frac{w_l}{w_j}\right). \tag{15}$$

Thus, the incentive to purchase high-quality goods in country i from j, μ_i/w_j, increases with the individual's productivity s_i and relative unit wages w_i/w_j between countries i and j. The last identity can be re-written as a function of the exogenous variables as

$$\frac{\mu_i}{w_j} = \frac{1}{N}\left(\frac{m_j s_j + r}{m_i s_i + r} s_i + r \sum_{l=1}^{N} \frac{m_j s_j + r}{m_l s_l + r}\right). \tag{16}$$

Hence, if it exists, the equilibrium is unique. The restrictions for the existence are Conditions A1 and A2. For readability, we focus on the existence of a trade equilibrium with symmetric countries where $m_i = m$ and $s_i = s$. We remind that, under proportional utility upgrades, β measures the utility upgrade that every variety brings: $\beta = 1/\left(1 - b_L/b_H\right)$.

Proposition 1. *Suppose countries with symmetric populations and productivities and suppose sufficiently high utility upgrades: that is, $(\beta-1) > (\alpha-1)\,\ell(1)/\ell(0)$. Then, a trade equilibrium exists and is unique for $\int_0^1 a_L(z)\,dz < s/N < (\alpha/(\alpha-1)) \int_0^1 a_L(z)\,dz$.*

Proof. At the symmetric equilibrium, we have $\mu_i/w_j \equiv \mu = s/N + r$ by (16). Conditions A1 and A2 impose the two conditions $\mu \in (\ell(0), \ell(1))$ and $\mu > \left[(\alpha-1)/(\beta-1)\right]\ell(1)$. The RHS of the second condition is lower than $\ell(0)$ if and only if $(\beta-1) > (\alpha-1)\,\ell(1)/\ell(0)$. Under this requirement, the second condition does not bind when the first one holds. Then, a trade equilibrium exists and is unique for $s/N + r \in [\ell(0), \ell(1)]$. Using the value of r, we get the condition: $s/[N(\ell(1)-\ell(0))] \in (\alpha-1, \alpha)$. Using the definition of ℓ, we further get the condition in the proposition. For example, in the case of Pareto cost distributions, the equilibrium satisfy conditions A1 and A2 if $a_{0L}(1-1/\kappa) < s/N < (\alpha/(\alpha-1))a_{0L}(1-1/\kappa)$. ■

The symmetric country trade equilibrium exists for a non-zero measure of productivity levels. However, an individual's productivity and, in turn, income must rise with the number of countries because, in this Armington model, consumers are required to purchase all varieties from each country. This assumption contrasts to usual models with divisible goods. Finally, by continuity, trade equilibria exist for not too asymmetric country productivities. Notice that this assumption is loosened in Section 5, where the extensive margins of trade are active.

In what follows, we assume a set of parameters such that a trade equilibrium exists. We now turn to the discussion of the properties of trade equilibria.

3. Country characteristics

In this section, we analyze the equilibrium properties. We first consider the trade properties between country pairs because of their application in empirical studies. We then focus on the effect of changes in the countries' productivity and population sizes.

3.1. Country pair properties

In this subsection, we compare the trade patterns of two countries with respect to a third trade partner. Such an approach is often used in econometric works to isolate the effects of each country's factors from the rest of the world. First note that, by (14), *a higher productivity s_i in country i reduces its unit wage relative to any other country.* This effect occurs because its labor supply rises while the mass of local variety does not change.

3.1.1. Exports from the same origin

Take two countries, i and j, importing from the same exporting country l ($l \neq i \neq j$). Then, by (15), we can write

$$\frac{\mu_i}{w_l} - \frac{\mu_j}{w_l} = \frac{1}{N}\frac{w_j s_j}{w_l}\left(\frac{w_i s_i}{w_j s_j} - 1\right), \tag{17}$$

so that

$$\frac{\mu_i}{w_l} \geq \frac{\mu_j}{w_l} \iff \frac{w_i s_i}{w_j s_j} \geq 1.$$

Therefore, given that μ_i/w_l is a sufficient statistic for the larger share of high-quality varieties and its associated utility, the last condition states that *a country with larger per capita income imports a larger share of high-quality varieties from country l and gets a more substantial utility from its imports from country l*. The intuition is simple: in markets with vertically differentiated goods, individuals working in more productive countries generate more value and spend it on the higher quality imports. This result is corroborated by some recente empirical evidence, see Auer et al. (2018), who also explain it theoretically using a vertical differentiated model in a partial equilibrium analysis. Here, we show that this finding applies also with a general equilibrium framework. By (7), it can further be shown that average import prices rank such as

$$\bar{p}_{il} \geq \bar{p}_{jl} \iff \frac{\mu_i}{w_l} \geq \frac{\mu_j}{w_l}.$$

Therefore, *the average import price is higher to the country with larger per capita income.* Empirically, one should find a positive correlation between import prices and importer income per capita. In addition, by (14), the ratio of income per capita can be related to exogenous productivity parameters as

$$\frac{w_i s_i}{w_j s_j} = \frac{s_i/\left(m_i s_i + r\right)}{s_j/\left(m_j s_j + r\right)} \tag{18}$$

This relationship implies that more productive countries import a larger share of high-quality goods and have higher average import prices.

Finally, consider the case where the total income of the importing countries is the same, but per capita income and population size differ. This implies $m_i s_i = m_j s_j$, which holds for many combinations of$m$and s. Substituting $m_i s_i = m_j s_j$into the ratio of income per capita (18) one gets

$$\frac{w_i s_i}{w_j s_j} = \frac{s_i/\left(m_i s_i + r\right)}{s_j/\left(m_i s_i + r\right)} \iff w_i = w_j.$$

When total income is the same but per capita income and population differ, equilibrium requires that nominal wages are the same in the two importing countries.

3.1.2. Imports from different origins

Take a country l that imports from two different exporting countries i and j ($l \neq i \neq j$). Then, by (15),

$$\frac{\mu_l}{w_i} - \frac{\mu_l}{w_j} = \frac{1}{N}\left(\frac{1}{w_i} - \frac{1}{w_j}\right)\left(w_l s_l + r\sum_{k=1}^{N} w_k\right).$$

So, we have

$$\frac{\mu_l}{w_i} \geq \frac{\mu_l}{w_j} \iff \frac{w_i}{w_j} \leq 1 \iff \frac{m_i s_i}{m_j s_j} \geq 1.$$

Therefore, country l imports a larger share of high-quality products from the country with a higher labor supply. Controlling for exporter sizes, *country l imports a larger share of high-quality varieties and thus have higher expenditures for the varieties manufactured by the more productive exporters*. Intuitively, high-quality goods are manufactured at lower relative costs in more productive countries and therefore more attractive to importers. Thus, lower relative prices also make the low-quality goods less attractive compared to high-quality ones.

Using (7), one shows that average import prices rank such as

$$\bar{p}_{li} \geq \bar{p}_{lj} \iff w_i \leq w_j$$

Therefore, *the average import price to country l is higher for the goods shipped from more productive exporters*. Empirically, this leads to a positive correlation between exporter income per capita and unit price, as shown in Eaton and Fieler (2017).

3.1.3. Linder hypothesis

The above arguments concur to the Linder's 1961 hypothesis according to which richer countries trade more numerous high-quality goods with each other than poorer ones. Indeed, from Section 3.1.1, two rich countries (i, j) export more high-quality goods between each other than to a third poorer country l if their incomes per capita are larger than the poorer country l' one. From Section 3.1.2, controlling for population sizes, they also import more high-quality goods between each other than from the third country if they are more productive than the poorer country l. Hence, more productive and therefore richer countries trade more high-quality goods between each others. *The high-income countries therefore specialize in the production of higher quality goods and trade more of those*, which confirms Linder (1961).

In addition, our model allows us to evaluate the Linder hypothesis over goods with various degrees of vertical product differentiation. To fix ideas, consider two goods z_j and z'_j ($z_j < z'_j$) such that the technology gap between quality versions is smaller in the former: $a_H(z_j) - a_L(z_j) < a_H(z'_j) - a_L(z'_j)$. Hence, those goods exhibit increasing vertical product differentiation between their high and low quality versions. Then, by the above argument, this model predicts the more differentiated good z'_j is more likely to be imported by a high income country than by a low income nation. In other words, the Linder hypothesis is strengthened by the degree of vertical product differentiation. This result relates to Fieler (2011) who finds that high income countries trade more of the highly differentiated goods in a model with horizontal differentiation.

We now study the effects of productivity and population size on the changes on the consumption of high-quality varieties.

3.2. Productivity changes

Consider an increase in country i's individuals number of labor productivity units s_i. Then, country's labor supply $m_i s_i$ rises, and its unit wage falls relative to other countries as we compute

$$\frac{\mathrm{d}\left(w_i/w_j\right)}{\mathrm{d}s_i} = -\frac{m_i\left(m_j s_j + r\right)}{\left(m_i s_i + r\right)^2} < 0. \tag{19}$$

This effect depresses its relative prices and makes the country more competitive in international markets. Although those individuals obtain a smaller price for their labor units, they increase their labor endowment, which rises their income relative to other countries:

$$\frac{\mathrm{d}}{\mathrm{d}s_i}\left(\frac{s_i w_i}{s_j w_j}\right) = \frac{r}{s_j}\frac{m_j s_j + r}{\left(m_i s_i + r\right)^2} > 0.$$

As a result of (19), every other country $j \neq i$ imports more numerous high-quality goods from country i, substituting for the trade of high-quality goods with third countries $l \neq j \neq i$. Indeed, one can compute the changes in high-quality imports into country j from countries i and $l \neq i$ as

$$\frac{\mathrm{d}\left(\mu_j/w_i\right)}{\mathrm{d}s_i} = m_i \frac{s_j + r\sum_{l=1,l\neq i}^{N}\frac{r+m_j s_j}{m_l s_l + r}}{N\left(r + m_j s_j\right)} > 0 \text{ and } \frac{\mathrm{d}\left(\mu_j/w_l\right)}{\mathrm{d}s_i} = -\frac{r\left(m_l s_l + r\right)}{N\left(m_i s_i + r\right)^2} < 0.$$

At a given wage, country i's workers benefit from larger incomes and from cheaper production of local high-quality goods. But, although their relative unit wage falls and import prices become higher relative to their incomes, they import a wider range of high-quality goods as indeed,

$$\frac{\mathrm{d}\left(\mu_i/w_j\right)}{\mathrm{d}s_i} = r\frac{\left(1-m_i\right)\left(r+m_j s_j\right)}{N\left(r+m_i s_i\right)^2} > 0.$$

They, however, purchase a broader range of local high-variety goods as

$$\frac{\mathrm{d}\left(\mu_i/w_i\right)}{\mathrm{d}s_i} = \frac{1}{N}\left(1 + r\sum_{l=1,l\neq i}^{N}\frac{m_i s_i + r}{m_l s_l + r}\right) > 0.$$

Proposition 2. *In the equilibrium of trade network with N countries, a rise in productivity of country i entices this country to specialize in high-quality varieties. Consumers from country i purchase a wider range of local and imported high-quality varieties. Other countries import more high-quality varieties from country i and less from each other.*

One consequence of the proposition is that the average quality of home imports increases when home productivity rises. The result supports Jaimovich and Merella (2012).

3.3. Population changes

Consider an infinitesimal increase in country i's population size, $\mathrm{d}M_i$. Keeping constant other countries' populations, this impacts the population ratios of all countries as follows:

$$\mathrm{d}m_i = \frac{M_i + \mathrm{d}M_i}{M + \mathrm{d}M_i} - \frac{M_i}{M} \simeq \left(1 - m_i\right)\frac{\mathrm{d}M_i}{M},$$

$$\mathrm{d}m_j = \frac{M_j}{M + \mathrm{d}M_i} - \frac{M_j}{M} \simeq -m_j\frac{\mathrm{d}M_i}{M}.$$

It increases country i's population ratio m_i and decreases other countries' m_j, $j \neq i$, in proportion to global population changes $\mathrm{d}M_i/M$ and initial population distributions. Combining this with the effects of population ratios on μ_i/w_j we can establish the following comparative statics properties. First, there is a decrease in wage for country i relative to other countries $j \neq i$.[14] This effect occurs because country i's population growth raises labor supply and decreases local production cost and product prices. As their local prices fall and import prices rise, individuals in country i have an incentive to augment their consumption of local, high-quality varieties. We indeed show that $\mathrm{d}\left(\mu_i/w_i\right)/\mathrm{d}M_i > 0$ while $\mathrm{d}\left(\mu_i/w_j\right)/\mathrm{d}M_i < 0$ if countries' labor supplies are close to symmetry ($s_l m_l \simeq s_j m_j$).

Proposition 3. *A rise in the population of country i in a trade network with N countries brings about*

- *a decrease in unit wage for country i relative to other countries $j \neq i$;*
- *a rise in country l's unit wage relative to country j's if l has a larger effective labor supply than j ($m_l s_l > m_j s_j$);*
- *a rise in country i's consumption of its local high-quality varieties;*
- *a decrease in the range of high-quality imports consumed by country i's consumers, if countries are sufficiently symmetric.*

The first line of Proposition 3 is intuitive. A larger domestic population increases labor supply in country i and reduces local unit wages. Therefore, the growing country incurs a fall in its unit wage compared to each other trade partner. By the same token, other countries have a rise in their unit wages relative to country i.

The terms of trade between each other countries also change: a country l has a rise in its unit wage compared to country j if it has a more abundant labor supply $m_j s_j > m_l s_l$. Moreover, the fall in wages negatively affects domestic consumers' purchasing power so that they buy fewer high-quality local goods.

The effects of a rise in country i population on high-quality imports are unclear. The first part of (27) in the appendix, is always negative, reflecting the fall in unit wage due to the increase in supply in country i. The second effect in the second part of the equation is ambiguous, and it is determined by the differences in the effective labor supplies of other countries, which affect the interplays of wages among countries. Suppose, for instance, that country j has the highest effective labor supply of the whole economy. Then, purchasing goods from country j becomes more expensive for country i consumers, who reduce the number of high-quality goods imported from j. If conversely, country j has a very low effective labor supply, the effect due by the difference in the productivity of other countries might be positive for high-quality import of country i and might also compensate the fall in unit wage.

Finally, if countries are symmetric, the increase in population depresses the range of high-quality goods purchased by country i. In this case, the effect of differences in productivity is nil, leaving the fall in purchasing power, driven by the decrease in country i wages.

4. Trade costs

In this section, we consider how the quality of traded goods change with trade costs. We focus on symmetric (iceberg) trade costs $\tau_{ij} \geq 1$ where a share $1/\tau_{ij}$ of each good arrives at destination i after shipment from country j. Trade costs are symmetric across countries and nil within countries: $\tau_{ji} = \tau_{ij}$ and $\tau_{ii} = 1$. Accordingly, the (destination) consumption price of an unit z_j imported from country j to country i is given by $p_{ijk}(z_j) = \tau_{ij} w_j a_k(z_j)$, $k = H, L$. Using the same argument as in Section 2.1, an individual in country i with inverse marginal utility μ_i purchases a high-quality variety z_j if $\mu_i/(\tau_{ij} w_j) \geq \ell(z_j)$ where $\ell(z_j)$ is the per-quality-unit input schedule defined by (4) in the absence of trade cost. The incentive to purchase a high-quality good is then given by the statistics $\mu_i/(\tau_{ij} w_j)$: the higher this is, the wider the range of consumed high-quality imports. *Ceteris paribus*, a higher τ_{ij} entices consumers to reduce their range of high-quality goods.

Expenditure and balanced trade Following the previous procedure and using the above definition of E, the expenditure of an individual in country i for goods produced in j is successively given by

$$\begin{aligned} E_{ij} &\equiv \int_{\mathcal{H}\left(\frac{\mu_i}{\tau_{ij} w_j}\right)} \tau_{ij} w_j a_H(z_j) dz_j + \int_{\mathcal{L}\left(\frac{\mu_i}{\tau_{ij} w_j}\right)} \tau_{ij} w_j a_L(z_j) dz_j \\ &= \tau_{ij} w_j E\left(\frac{\mu_i}{\tau_{ij} w_j}\right) \\ &= \tau_{ij} w_j \left(\frac{\mu_i}{\tau_{ij} w_j} - r\right) \\ &= \mu_i - r\tau_{ij} w_j. \end{aligned}$$

Income is equal to total expenditure: $w_i s_i = E_i \equiv \sum_{j=1}^{N} E_{ij}$. That is,

$$w_i s_i = N\mu_i - r\sum_{j=1}^{N} \tau_{ij} w_j.$$

This gives the incentives to purchase as a function of relative factor prices and trade costs:

$$\frac{\mu_i}{\tau_{ij} w_j} = \frac{1}{N}\frac{s_i}{\tau_{ij}}\frac{w_i}{w_j} + \frac{r}{N}\sum_{l=1}^{N}\frac{\tau_{il}}{\tau_{ij}}\frac{w_l}{w_j}. \tag{20}$$

[14] We show in the Appendix that $\mathrm{d}\left(w_i/w_j\right)/\mathrm{d}M_i < 0$.

In country i, trade balances the value of imports and exports as

$$\sum_{j\neq i}^{N} m_i \tau_{ij} w_j E\left(\frac{\mu_i}{\tau_{ij} w_j}\right) = \sum_{j\neq i}^{N} m_j \tau_{ji} w_i E\left(\frac{\mu_j}{\tau_{ji} w_i}\right),$$

Given the linear expenditure function, the balanced trade condition simplifies to

$$\sum_{j=1}^{N} m_i \left(\mu_i - r\tau_{ij} w_j\right) = \sum_{j=1}^{N} m_j \left(\mu_j - r\tau_{ji} w_i\right).$$

It is useful to denote the country i's average ad-valorem trade cost $\overline{\tau}_i \equiv 1 + \sum_{j=1}^{N} m_j \left(\tau_{ij} - 1\right)$ that measures the remoteness of the consumers of country i's goods. Hence, the relative factor prices and incentives to purchase high-quality goods simplify to

$$\frac{w_i}{w_j} = \frac{m_j s_j + r\overline{\tau}_j}{m_i s_i + r\overline{\tau}_i}, \tag{21}$$

$$\frac{\mu_i}{\tau_{ij} w_j} = \frac{1}{N\tau_{ij}}\left(\frac{m_j s_j + r\overline{\tau}_j}{m_i s_i + r\overline{\tau}_i} s_i + r\sum_{l=1}^{N} \tau_{il} \frac{m_j s_j + r\overline{\tau}_j}{m_l s_l + r\overline{\tau}_l}\right). \tag{22}$$

Those expressions compare to the ones without trade costs.

Trade statistics We finally recall our three measures of interest. The share of high-quality purchases in imported goods is given by

$$\int_{\mathcal{H}(\mu_i/\tau_{ij} w_j)} \mathrm{d}z_j = \ell^{-1}\left(\frac{\mu_i}{\tau_{ij} w_j}\right)$$

and the indirect utility in country i simplifies to

$$V_i = \sum_{j=1}^{N} V\left(\frac{\mu_i}{\tau_{ij} w_j}\right).$$

where the functions ℓ^{-1} and V have been defined in Section 2.2 As a result, the ratios $\mu_i/\tau_{ij} w_j$ are also sufficient statistics for the share of high-quality goods and the utility from imports. Because of trade costs, the average import prices must be distinguished by whether they are evaluated at origin or destination. Following international trade terminology, freight on board (fob) prices do not include trade costs while cost, insurance & freight (cif) prices include them. Exports are most generally reported in fob values at the borders of exporting countries and imports are denominated in cif prices at the gates of importing countries. As a result, we extend our earlier definition of average prices as

$$\overline{p}_{ij}^{\mathrm{fob}} = w_j E\left(\frac{\mu_i}{\tau_{ij} w_j}\right) = \frac{1}{\tau_{ij}}\left(\mu_i - r\tau_{ij} w_j\right), \tag{23}$$

$$\overline{p}_{ij}^{\mathrm{cif}} = \tau_{ij}\overline{p}_{ij}^{\mathrm{fob}} = \tau_{ij} w_j E\left(\frac{\mu_i}{\tau_{ij} w_j}\right) = \mu_i - r\tau_{ij} w_j. \tag{24}$$

4.1. Symmetric countries

To highlight the impact of trade costs, we firstly consider the case of symmetric countries and trade costs where $s_i = s$, $m_i = 1/N$, $\tau_{ij} = \tau$, $i \neq j$ while $\overline{\tau}_i \equiv \overline{\tau} = 1 + (\tau - 1)(N - 1)/N$. Then, the equilibrium conditions simplify as

$$\frac{w_i}{w_j} = 1, \ \frac{\mu_i}{w_i} = \frac{1}{N}\left[s + r + r\tau(N-1)\right] \text{ and } \frac{\mu_i}{\tau w_j} = \frac{1}{N}\left[\frac{s+r}{\tau} + r(N-1)\right]. \tag{25}$$

It can be shown that a unique equilibrium exists for large enough β and not too high trade cost τ (see Appendix). If the latter condition does not hold, import prices are too large and consumers have incentives to purchase no foreign high-quality varieties.

The identities in (25) imply that a global fall in ad-valorem trade cost (lower τ)induces workers to consume fewer local high-quality goods (μ_i/w_i falls) and a larger share of high-quality imports ($\mu_i/\left(\tau_{ij} w_j\right)$ rises). Denoting unit wages by w, the average fob and cif prices compute as

$$\overline{p}_{ij}^{\mathrm{fob}} = \frac{w}{N}\left(\frac{s+r}{\tau} - r\right) \text{ and } \overline{p}_{ij}^{\mathrm{cif}} = \tau\overline{p}_{ij}^{\mathrm{fob}} = \frac{w}{N}(s + r - r\tau).$$

So, both average prices rise with the fall in trade cost. Lower trade costs indeed induce consumers to import a larger share of high-quality goods, which pushes up the average fob price. Interestingly, the average cif price rises. Consumers increase more their expenditure on import than what they save on trade cost. This effect occurs because they reduce their purchases of local high-quality goods. The country utility successively computes as

$$\begin{aligned} V_i &= N\ln\frac{\mu_i}{w_i} - (N-1)\ln\tau + \text{constant} \\ &= N\ln\left[s + r + r\tau(N-1)\right] - (N-1)\ln\tau + \text{constant'}. \end{aligned}$$

The first and second terms express the impact of local consumption and the effect of trade cost on imports. It can be shown that the utility falls with τ when the trade equilibrium exists. By a continuity argument, the same properties apply for not too dissimilar countries.

The following proposition outlines the effects of a variation in symmetric trade costs.

Proposition 4. *A fall in trade cost induces each country to consume a smaller share of high-quality varieties from home and a larger one from abroad. It boosts exports of high-quality varieties, increases both average* fob *and* cif *prices, and raises utility everywhere.*

This proposition confirms the existence of gains from trade in the general equilibrium context of vertical differentiation and many goods. It further highlights the trade-off between quality and trade cost for fixed number and quantity of goods consumed. Thus, it complements the trade literature about the trade-offs between trade costs, intensive and extensive margins of trade. This result is in line with recent evidence showing that a tariff decrease pushes the country's producers to increase the quality of their exports (Fan et al., 2015).

Whereas the above text discusses the effect of a uniform bilateral trade cost, we now study the effect of discrepancies in such cost. Hence, we consider the same country pairs as in Section 3.1 but add idiosyncratic bilateral trade costs.

4.2. Exports from the same origin

Take two countries i and j importing from the same exporter l ($l \neq i \neq j$). We know that high-quality import shares and utility from those imports depend on the incentives to buy high-quality goods $\mu_i / (\tau_{il} w_l)$ and $\mu_j / (\tau_{jl} w_l)$. Interestingly, the comparison of average fob import prices also depend on those ratios since, using (23), one gets

$$\overline{p}_{il}^{\text{fob}} \geq \overline{p}_{jl}^{\text{fob}} \iff \frac{\mu_i}{\tau_{il} w_l} \geq \frac{\mu_j}{\tau_{jl} w_l}.$$

Then, cross-country comparisons between high-quality import shares, utility and average fob import prices can be studied with the differences in incentives to buy high-quality goods. By (20), the incentives to purchase high quality goods are ranked such that

$$\frac{\mu_i}{\tau_{il} w_l} \geq \frac{\mu_j}{\tau_{jl} w_l} \iff \frac{s_i w_i}{\tau_{il}} + r\frac{\sum_{h=1}^{N} \tau_{ih} w_h}{\tau_{il}} \geq \frac{s_j w_j}{\tau_{jl}} + r\frac{\sum_{h=1}^{N} \tau_{jh} w_h}{\tau_{jl}},$$

which reduces to (17) in the absence of trade cost. In this expression, the term $\sum_{h=1}^{N} \tau_{ih} w_h$ reflects the average trade cost of importer i to its trade partners and has the same function as Anderson and Van Wincoop's 2003 "multilateral resistance". The comparisons between high-quality import shares and average fob import prices then depend on incomes, bilateral trade cost and average trade costs expressed in the last inequality. For the sake of exposition, let us focus on the share of high quality imports. On the one hand, it is larger in country i when the first term on the LHS is larger than the one on its RHS. This occurs if that country has higher per-capita income $s_i w_i$ as already discussed above, and also now, if the country has lower bilateral trade cost τ_{li}. So, a lower bilateral trade cost has a first effect to augment the share of high quality imports because it decreases prices and entices consumer to purchase higher quality goods. On the other hand, the share of high quality import is also larger when the second term on the LHS outweighs the one on the RHS. That is, when the bilateral trade cost gets small relatively to the average trade costs. When trade costs are interpreted as geographical distance, this means that the country imports higher quality goods from the exporters that are relatively closer in their trade network. Hummels and Skiba (2004, p.1387), Manova and Zhang (2012, Table 2), Crozet et al. (2012, pp 630–631 and Section 5) and others empirically verify similar effects of distance and remoteness.

4.3. Imports from different origins

Now, consider a country l that imports from two different exporters i and j ($l \neq i \neq j$). Using (23), we obtain the following conditions on the ranking of average cif price:

$$\overline{p}_{li}^{\text{cif}} \geq \overline{p}_{lj}^{\text{cif}} \iff \tau_{li} \leq \tau_{lj}.$$

Therefore, the average cif price is higher from an exporting country with a lower bilateral trade barrier. The prices of low and high quality goods are lower but consumers have an incentive to upgrade the set of goods imported from that country at the expense of the goods imported from other countries.

To compare high-quality shares and utility contributions of imports from various countries, we can establish the following inequalities:

$$\frac{\mu_l}{\tau_{li} w_i} \geq \frac{\mu_l}{\tau_{lj} w_j} \iff \frac{w_i \tau_{li}}{w_j \tau_{lj}} \leq 1 \iff \frac{m_i s_i}{\tau_{li}} + r\frac{\overline{\tau}_i}{\tau_{li}} \geq \frac{m_j s_j}{\tau_{lj}} + r\frac{\overline{\tau}_j}{\tau_{lj}},$$

which collapses to the comparison under no trade cost if $\tau_{ij} = 1 \ \forall i, j$. The trade cost adds two effects that correspond to the two terms on each side of the last inequality. First, it mitigates the productivity advantage of the exporting country s_i: a higher productivity exporter sells higher quality goods as long as it is at a close distance from the importing country (first term). Second, it reduces its exports if its bilateral distance τ_{li} is relatively larger than the average distance to its trade partner $\overline{\tau}_i$ (second term). In geographical terms, the country ships higher quality goods to the countries that are more central to the geographical center of its trade network.

4.4. Gravity

We end up with the discussion of the traditional gravity equation that expresses trade values as functions of local incomes and distances. Country j's export to country i is captured by the expenditure and number of high-quality variety, which increases with the statistics $\mu_i/\left(\tau_{ji}w_j\right)$. The (nominal) expenditure on import from j to i (at cif prices) is given by

$$E_{ij}^{\text{cif}} = \tau_{ij} w_j E\left(\frac{\mu_i}{\tau_{ij} w_j}\right) = \frac{1}{N} s_i w_i - r\tau_{ij} w_j + \frac{r}{N}\sum_{l=1}^{N} \tau_{il} w_l.$$

From this expression, it comes that trade expenditure rises with importer's higher income per capita $s_i w_i$, higher exporter's unit wage w_j, lower bilateral trade cost τ_{ij} and higher remoteness, here measured by $\sum_{l=1}^{N} \tau_{il} w_l$.

The above gravity equation includes exporter's unit wage rather than income. We can substitute unit wage by income using the following procedure. Assuming that trade cost is paid in exporting country's labor, we note the national income is sequentially given by

$$\begin{aligned} Y_j &= \sum_{h=1}^{N} m_h E_{hj}^{\text{cif}} \\ &= \frac{1}{N}\sum_{h=1}^{N} m_h s_h w_h - r w_j \sum_{h=1}^{N} m_h \tau_{hj} + \frac{r}{N}\sum_{l=1}^{N}\sum_{h=1}^{N} m_h \tau_{hl} w_l \\ &= \frac{1}{N}\sum_{h=1}^{N} m_h s_h w_h - r w_j \overline{\tau}_j + \frac{r}{N}\sum_{l=1}^{N} \overline{\tau}_l w_l. \end{aligned}$$

So, we can extract the unit wage as

$$w_j = \frac{1}{r\overline{\tau}_j}\left(-Y_j + \overline{Y} + \frac{r}{N}\sum_{l=1}^{N} \overline{\tau}_l w_l\right)$$

where $\overline{Y} \equiv \frac{1}{N}\sum_{h=1}^{N} Y_h = \frac{1}{N}\sum_{h=1}^{N} m_h s_h w_h$ is the average world income.

We finally plug this back to the gravity equation, which gives

$$E_{ij}^{\text{cif}} = \frac{1}{N} s_i w_i + \frac{\tau_{ij}}{\overline{\tau}_j}\left[Y_j - \overline{Y} - \frac{r}{N}\sum_{l=1}^{N} \overline{\tau}_l w_l\right] + \frac{r}{N}\sum_{l=1}^{N} \tau_{il} w_l.$$

The per-capita import expenditure rises with higher importer's per-capita income $s_i w_i$ and now with higher exporter's national income Y_j. Hence, the size of the exporter matters. Note that the squared bracket term is negative if exporter j has a national income no higher than average or/and countries are close to symmetry. In that case, the import expenditure falls with bilateral trade cost τ_{ij} and increases with the remoteness indicators $\overline{\tau}_j$ of exporter j. The last term $\sum_{l=1}^{N} \tau_{il} w_l$ is the remoteness indicator associated to importer i, which is pinned down as the "inward" multilateral resistance in Anderson and Van Wincoop's 2003 and reflects a rise in expenditure due to importer's remoteness to its trade partners.

5. Alchian Allen conjecture

The previous section discussed the role of ad-valorem trade cost in the quality composition of traded goods. Such trade costs do not explain the Alchian and Allen (1964) effect according to which exports are biased towards high-quality goods for more distant trading partners. The effect is apparent in the trade data where fob export prices rise with distance from the U.S. to its trade partners. Hummels and Skiba (2004) empirically highlight this effect and provide an explanation through the existence of *unit* trade costs that accrue on each good independently on their value. When unit trade costs increase, consumers are enticed not only to purchase fewer goods in total but also to consume relatively fewer low-quality goods. This effect occurs because a rise in unit trade cost has a relatively stronger impact on the low-cost low-quality version of a good than on its corresponding high-cost high-quality version.

To encompass the Alchian and Allen conjecture, we must change our model by allowing endogenous extensive margins consumers so that consumers may not purchase all varieties from each producing country. To fix ideas, we focus on Hummels and Skiba's 2004 partial equilibrium analysis by fixing relative prices w_j and inverse marginal utility μ_i. For the sake of generality, we resume to the model with general primitives $a_k(z_j)$ and $b_k(z_j)$, $k = L, H$, and assume an identical unit trade cost t so that consumer prices become $p_{ijk}(z_j) = \left(a_k\left(z_j\right) + t\right) w_j$. Like for ad valorem trade costs, the per-quality input $\ell(z_j)$ can be shown to be independent of the unit trade cost t. The choice for high-quality over low-quality, therefore, is driven by the same condition as before: $\mu_i/w_j \geq \ell(z_j)$.

In this subsection, we are interested in the situation where consumers purchase only a subset of the low-quality goods. We keep on assuming that high-quality goods are purchased when they are preferred over low-quality ones. That is,

$$\ell(z_j) > \frac{\mu_i}{w_j} \Rightarrow \ell(z_j) \geq \frac{a_H(z_j) + t}{b_H(z_j)}.$$

However we assume that some goods are not purchased even in their low quality version. The full market coverage condition becomes binding at the address $z_j = n$ that corresponds to the last low-quality good purchased by consumers:

$$\frac{\mu_i}{w_j} = \frac{a_L(n) + t}{b_L(n)}.$$

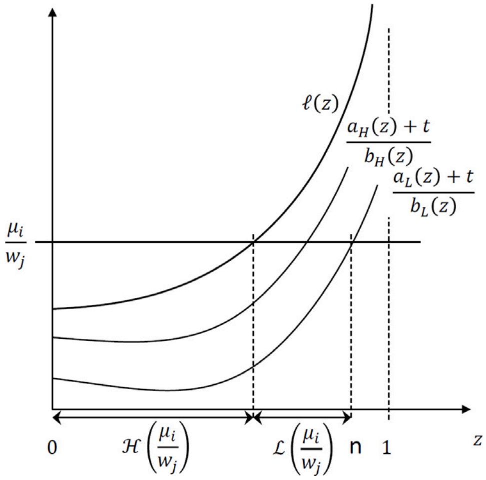

Fig. 2. Country's individual demands when not all goods are consumed.

We denote this solution by the function $n\left(\mu_i/w_j,t\right)$. To ensure that some low-quality goods are not purchased, we impose $n\left(\mu_i/w_j,t\right) < 1$, or equivalently, $\mu_i/w_j < \left(a_L(1)+t\right)/b_L(1)$. A natural assumption is that the number of purchased goods falls with unit trade cost t; that is,

$$n_t \equiv \frac{\partial n}{\partial t} = \frac{1}{\left(\mu_i/w_j\right) b'_L(n) - a'_L(n)} < 0.$$

Then, the sets of high and low-quality purchases is given by $\mathcal{H}(\mu_i/w_j) = [0, \ell^{-1}(\mu_i/w_j)]$ and $\mathcal{L}(\mu_i/w_j) = (\ell^{-1}(\mu_i/w_j), n\left(\mu_i/w_j,t\right)]$. Fig. 2 depicts this situation where the consumer does not purchase all goods.

We are now equipped to verify the existence of the Alchian and Allen conjecture according to which the average fob price increases with larger t. The fob price of good z_j is given by $p_{ijk}^{\text{fob}}\left(z_j\right) \equiv a_k(z_j)w_j$, $k = L, H$, while the average fob price is equal to

$$\overline{p}_{ij}^{\text{fob}} = \frac{1}{n\left(\mu_i/w_j,t\right)}\left[\int_0^{\ell^{-1}(\mu_i/w_j)} a_H(z_j)w_j\mathrm{d}z_j + \int_{\ell^{-1}(\mu_i/w_j)}^{n(\mu_i/w_j,t)} a_L(z_j)w_j\mathrm{d}z_j\right].$$

Since the function ℓ and its inverse ℓ^{-1} are independent of t, we get

$$\frac{\mathrm{d}\overline{p}_{ij}^{\text{fob}}}{\mathrm{d}t} = \left[a_L(n)w_j - \overline{p}_{ij}^{\text{fob}}\right]\frac{n_t}{n}.$$

As a result, using $p_{ijL}^{\text{fob}}(n) = a_L(n)w_j$ and $n_t < 0$, a rise in unit trade cost increases the average fob price if and only if

$$p_{ijL}^{\text{fob}}(n) \le \overline{p}_{ij}^{\text{fob}}.$$

That is, the first variety dropped by consumers ($z_j = n$) has a low quality price lower than the average price of the basket.

Proposition 5. *Suppose consumers purchase only a subset of the low-quality varieties and the number of purchased varieties falls with higher unit trade cost t. Then, the average fob price of a variety increases with larger t if and only if $p_{ijL}^{\text{fob}}(n) \le \overline{p}_{ij}^{\text{fob}}$. This holds true for decreasing profiles a_L.*

Proof. Given $a_H > a_L$, we successively have that $\overline{p}_{ij}^{\text{fob}} = \frac{w_j}{n}\left[\int_0^{\ell^{-1}} a_H(z_j)\mathrm{d}z_j + \int_{\ell^{-1}}^{n} a_L(z_j)\mathrm{d}z_j\right] > \frac{w_j}{n}\left[\int_0^{n} a_L(z_j)\mathrm{d}z_j\right]$ where n is evaluated at $\left(\mu_i/w_j,t\right)$. The last term is lower than $w_j a_L(n)$ if a_L is a decreasing function of z_j. ■

Hence, a sufficient condition is that the lowest quality goods dropped by consumers have low prices.

6. Concluding remarks

We have analyzed a trade model with many countries, many goods, each versioned in two quality versions, and non-homothetic preferences. Once we derived the equilibrium, we have first examined the effects of differences in productivity among countries.

We have shown that a rise in the productivity of one country implies a fall in domestic wage relative to other countries. Richest countries demand more high-quality varieties from abroad. Between two countries of the same size, the more productive specializes in exporting goods of higher quality. High-income countries specialize in the production of high-quality goods and trade more of those, as suggested by the Linder hypothesis (1961). Using several vertically differentiated industries with heterogeneous technology, we are able to examine how the level of product differentiation explains the volumes of trade quality. High-quality goods exhibiting a high degree of differentiation are traded only by high-income countries.

We have then investigated the effects of changes in population and productivity in one country. An increase in population induces a decrease in relative prices and, subsequently, in the consumption of high-quality goods. A rise in productivity favors the consumption of local high-quality goods only if the relative size of the country is sufficiently small, while high-quality exports decrease. Our theoretical framework may help explaining important empirical regularities in the trade literature.

CRediT authorship contribution statement

P.M. Picard: Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis, Conceptualization. **A. Tampieri:** Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

Proportionate upgrades and linear real expenditures

Defining $a(z_j) = a_H(z_j) - a_L(z_j)$ and $b(z_j) = b_H(z_j) - b_L(z_j)$, the assumption of proportionate upgrades imply $a_H = \alpha a$ and $a_L = (\alpha - 1)a$ and $b_H = \beta b$ and $b_L = (\beta - 1)b$.

The real expenditure successively writes as

$$\begin{aligned} E(y) &= \int_{\mathcal{H}(y)} a_H(z_j)\mathrm{d}z_j + \int_{\mathcal{L}(y)} a_L(z_j)\mathrm{d}z_j \\ &= \int_0^{\ell^{-1}(y)} a(z_j)\mathrm{d}z_j + \int_0^1 a_L(z_j)\mathrm{d}z_j \\ &= \int_{\ell(0)}^{y} \frac{a(\ell^{-1}(y))}{\ell'(\ell^{-1}(y))}\mathrm{d}y + \int_{\ell(0)}^{\ell(1)} \frac{a_L(\ell^{-1}(y))}{\ell'(\ell^{-1}(y))}\mathrm{d}z_j. \end{aligned}$$

where we substitute z_j by $\ell^{-1}(y)$. The assumption $E'(y) = 1$ imposes that the term within the first integral is equal to 1 and therefore

$$\ell'(z_j) = a(z_j). \tag{26}$$

Differentiating $\ell(z_j) = a(z_j)/b(z_j)$ and plugging in this expression gives the differential equation

$$\frac{a'(z_j)}{a(z_j)} = \frac{b'(z_j)}{b(z_j)} + b(z_j),$$

which accepts the solution

$$b(z_j) = \frac{a(z_j)}{c + \int_0^{z_j} a(\zeta)d\zeta}$$

where c is a constant. Note that $b'(z_j) \le 0 \iff a'\left(z_j\right)\ell(z_j) \le a\left(z_j\right)^2 \iff \ell''(z_j)\ell(z_j) \le \left[\ell'(z_j)\right]^2$. This occurs for not too convex function ℓ. Using (26) and proportionate upgrade cost the real expenditure successively writes as

$$\begin{aligned} E(y) &= \int_{\ell(0)}^{y} \mathrm{d}y + \int_{\ell(0)}^{\ell(1)} \frac{a_L(\ell^{-1}(y))}{a(\ell^{-1}(y))}\mathrm{d}z_j \\ &= \int_{\ell(0)}^{y} \mathrm{d}y + \int_{\ell(0)}^{\ell(1)} (\alpha - 1)\,\mathrm{d}z_j \\ &= (y - \ell(0)) + (\alpha - 1)(\ell(1) - \ell(0)). \end{aligned}$$

The indirect utility is successively given by

$$\begin{aligned} V(y) &= \int_0^{\ell^{-1}(y)} b(z_j)\mathrm{d}z_j + \int_0^1 b_L(z_j)\mathrm{d}z_j \\ &= \int_{\ell(0)}^{y} \frac{b(\ell^{-1}(y))}{\ell'(\ell^{-1}(y))}\mathrm{d}y + \int_{\ell(0)}^{\ell(1)} \frac{b_L(\ell^{-1}(y))}{\ell'(\ell^{-1}(y))}\mathrm{d}z_j \\ &= \int_{\ell(0)}^{y} \frac{b(\ell^{-1}(y))}{a(\ell^{-1}(y))}\mathrm{d}y + \int_{\ell(0)}^{\ell(1)} \frac{b_L(\ell^{-1}(y))}{a(\ell^{-1}(y))}\mathrm{d}z_j \end{aligned}$$

$$
\begin{aligned}
&= \int_{\ell(0)}^{y} \frac{1}{\ell(\ell^{-1}(y))} \mathrm{d}y + \int_{\ell(0)}^{\ell(1)} \frac{b_L(\ell^{-1}(y))}{b(\ell^{-1}(y))} \frac{1}{\ell(\ell^{-1}(y))} \mathrm{d}z_j \\
&= \int_{\ell(0)}^{y} \frac{1}{y} \mathrm{d}y + \int_{\ell(0)}^{\ell(1)} \frac{b_L(\ell^{-1}(y))}{b(\ell^{-1}(y))} \frac{1}{y} \mathrm{d}z_j \\
&= \int_{\ell(0)}^{y} \frac{1}{y} \mathrm{d}y + \int_{\ell(0)}^{\ell(1)} (\beta - 1) \frac{1}{y} \mathrm{d}z_j \\
&= (\ln y - \ln \ell(0)) + (\beta - 1) \ln (\ln \ell(1) - \ln \ell(0))
\end{aligned}
$$

where we substitute z_j by $\ell^{-1}(y)$ in the second equality, substitute $\ell'(z_j)$ by $a(z_j)$ in the third one, use $\ell(\ell^{-1}(y)) = y$ in the fifth one, and use $b_L = (\beta - 1)b$ in the sixth one.

Under proportional cost upgrades, upgrade costs are proportional to average costs of high and low quality varieties so that the above interpretation holds for both types of cost. Finally, the intercept of the real expenditure function is then equal to $r = a(0)/b_0 - (\alpha - 1)\,\mathrm{E}[a]$, which is positive for small enough b_0 and low enough $\mathrm{E}[a]$. That is, $r > 0$ iff $b_0 < \left[a_H(0) - a_L(0)\right]/(\alpha - 1)/[\ell(1) - \ell(0)]$ or equivalently, $b_0 < (1/(\alpha - 1)) * a_H(0)/\mathbf{E}(a_H)$, where $\mathbf{E}(a_H)$ is the expected value of a_H and $\mathbf{E}(a_H)/a_H(0)$ is a measure of cost dispersion. This requires a small enough b_0, a small enough α or a weak enough cost dispersion.

The Pareto cost distributions have the form $F_k(a_k) = 1 - \left(a_{0k}/a_k\right)^{\kappa}$ with $\kappa > 1$ and $k = L, H$. That is, $F_H(a_H) = 1 - \left(a_{0H}/a_H\right)^{\kappa}$ and $F_L(a_L) = 1 - \left(a_{0L}/a_L\right)^{\kappa}$ with $\kappa > 1$, $a_H > a_{0H}$, $a_L > a_{0L}$ while $a_{0H} = \beta/(\beta - 1)a_{0L} > 0$. This implies that the upgrade cost $a = a_H - a_L$ is distributed as $F(a) = 1 - \left(a_0/a\right)^{\kappa}$ where $a_0 \equiv a_{0L}/(\beta - 1)$. Inverting this function gives the upgrade cost profile $a(z_j) = a_0(1 - z_j)^{-1/\kappa}$ where $a_0 = a_{0H} - a_{0L}$. Choosing $b_0 > 0$, we get the increasing schedule $\ell(z_j) = a_0/b_0 + a_0 \left[1 - (1 - z_j)^{1-1/\kappa}\right]/(1 - 1/\kappa)$. The inverse schedule is $\ell^{-1}(y) = 1 - \left[1 - \frac{\kappa}{\kappa-1}\left(\frac{y}{a_0} - \frac{1}{b_0}\right)\right]^{\frac{\kappa}{\kappa-1}}$. The utility upgrade is given by $b\left(z_j\right) = (1 - 1/\kappa)(1 - z_j)^{-1/\kappa}/\left[1/b_0 + 1 - (1 - z_j)^{1-1/\kappa}\right]$, which then yields the utility profiles $b_H = \beta\left(b_H - b_L\right)$ and $b_L = (\beta - 1)\left(b_H - b_L\right)$. The real expenditure intercept is equal to $r = a_0\left[1/b_0 - (\alpha - 1)/(1 - 1/\kappa)\right]$, which is positive iff $b_0 \leq (1 - 1/\kappa)/(\alpha - 1)$. The indirect utility is given by $V(y) = \ln y + \beta \ln\left(a_0/b_0\right) + (\beta - 1)\ln\left[a_0/b_0 + a_0/(1 - 1/\kappa)\right]$.

Population changes

Consider an absolute increase in the population size M_i of country i by $\mathrm{d}M_i$. This situation implies the simultaneous first order changes in relative population sizes

$$
\begin{aligned}
\mathrm{d}m_i &= \frac{M_i + \mathrm{d}M_i}{M + \mathrm{d}M_i} - \frac{M_i}{M} \simeq \left(1 - m_i\right)\frac{\mathrm{d}M_i}{M}, \\
\mathrm{d}m_j &= \frac{M_j}{M + \mathrm{d}M_i} - \frac{M_j}{M} \simeq -m_j \frac{\mathrm{d}M_i}{M}.
\end{aligned}
$$

Hence, for any variable X, an increase in the population size M_i implies

$$
\frac{\mathrm{d}X}{\mathrm{d}M_i} = \frac{\partial X}{\partial m_i}\frac{\mathrm{d}m_i}{\mathrm{d}M_i} + \sum_{k \neq i} \frac{\partial X}{\partial m_k}\frac{\mathrm{d}m_k}{\mathrm{d}M_i} = \frac{1}{M}\left[\left(1 - m_i\right)\frac{\partial X}{\partial m_i} - \sum_{k \neq i} m_k \frac{\partial X}{\partial m_k}\right]. \tag{27}
$$

Relative factor prices. For $i \neq j \neq l$,

$$
\frac{\partial w_i/w_j}{\partial m_i} = -\frac{s_i\left(m_j s_j + r\right)}{\left(m_i s_i + r\right)^2} < 0, \quad \frac{\partial w_j/w_i}{\partial m_i} = \frac{s_i}{m_j s_j + r} > 0 \text{ and } \frac{\partial w_l/w_j}{\partial m_i} = 0.
$$

Hence, we have

$$
\begin{aligned}
\frac{\mathrm{d}w_i/w_j}{\mathrm{d}M_i} &= \frac{1}{M}\left[\left(1 - m_i\right)\frac{\partial w_i/w_j}{\partial m_i} - \sum_{k \neq i} m_k \frac{\partial w_i/w_j}{\partial m_k}\right] \\
&= \frac{1}{M}\left[\left(1 - m_i\right)\frac{\partial w_i/w_j}{\partial m_i} - m_j \frac{\partial w_i/w_j}{\partial m_j}\right] \\
&= -\frac{1}{M}\frac{m_j s_j + r}{m_i s_i + r}\left[\frac{\left(1 - m_i\right)s_i}{\left(m_i s_i + r\right)} + \frac{m_j s_j}{\left(m_j s_j + r\right)}\right] < 0.
\end{aligned} \tag{28}
$$

So, the more populated country incurs a fall in its unit wage with respect to each other trade partner. Also,

$$
\begin{aligned}
\frac{\mathrm{d}w_j/w_i}{\mathrm{d}M_i} &= \frac{1}{M}\left[\left(1 - m_i\right)\frac{\partial w_j/w_i}{\partial m_i} - m_j \frac{\partial w_j/w_i}{\partial m_j} - \sum_{k \neq i \neq j} m_k \frac{\partial w_j/w_i}{\partial m_k}\right], \\
&= \frac{1}{M}\left[\left(1 - m_i\right)\frac{\partial w_j/w_i}{\partial m_i} - m_j \frac{\partial w_j/w_i}{\partial m_j}\right], \\
&= \frac{\left(m_i s_i + r\right)}{M\left(m_j s_j + r\right)}\left[\frac{\left(1 - m_i\right)s_i}{m_i s_i + r} + \frac{m_j s_j}{m_j s_j + r}\right] > 0.
\end{aligned} \tag{29}
$$

So, the other countries have a rise in their unit wages with respect to the more populated country. Finally,

$$\frac{\mathrm{d}w_l/w_j}{\mathrm{d}M_i} = \frac{1}{M}\left(\left(1-m_i\right)\frac{\partial w_l/w_j}{\partial m_i} - \sum_{k\neq i} m_k \frac{\partial w_l/w_j}{\partial m_k}\right),$$
$$= -\frac{1}{M}\left(m_l \frac{\partial w_l/w_j}{\partial m_l} + m_j \frac{\partial w_l/w_j}{\partial m_j}\right),$$
$$= \frac{1}{M}\frac{r\left(m_l s_l - m_j s_j\right)}{\left(m_l s_l + r\right)^2}. \tag{30}$$

This is positive for $m_l s_l > m_j s_j$. A country l has a rise in its unit wage compared to country j if it has a larger effective labor supply. In turn

$$\frac{\mathrm{d}}{\mathrm{d}M_i}\left(\sum_{l=1}^{N} w_l/w_j\right) = \sum_{l=1}^{N}\frac{\mathrm{d}w_l/w_j}{\mathrm{d}M_i},$$
$$= \frac{\mathrm{d}w_i/w_j}{\mathrm{d}M_i} + \sum_{l\neq i}^{N}\frac{\mathrm{d}w_l/w_j}{\mathrm{d}M_i}.$$

By (28) and (30), this is

$$\frac{\mathrm{d}}{\mathrm{d}M_i}\left(\sum_{l=1}^{N} w_l/w_j\right) = -\frac{1}{M}\frac{m_j s_j + r}{m_i s_i + r}\left[\frac{\left(1-m_i\right)s_i}{\left(m_i s_i + r\right)} + \frac{m_j s_j}{\left(m_j s_j + r\right)}\right] \tag{31}$$
$$+ \frac{1}{M}\sum_{l\neq i}^{N}\frac{\left(m_j s_j + r\right)m_l s_l - \left(m_l s_l + r\right)m_j s_j}{\left(m_l s_l + r\right)^2}$$
$$= -s_i\frac{m_j s_j + r}{M\left(m_i s_i + r\right)^2} + \frac{1}{M}\sum_{l}^{N}\frac{r\left(m_l s_l - m_j s_j\right)}{\left(m_l s_l + r\right)^2}$$

The first part is negative. A sufficient condition of negativity of the second part is $m_j s_j < m_l s_l$ for all $l \neq j$. The expression is also negative if countries' labor supply is close to symmetry $m_l s_l \to m_j s_j$.

Country i local consumption. By (15), the incentives to consume local high-quality goods are given by

$$\frac{\mathrm{d}\mu_i/w_i}{\mathrm{d}M_i} = \frac{r}{N}\left(\sum_{l=1}^{N}\frac{\mathrm{d}w_l/w_i}{\mathrm{d}M_i}\right) = \frac{r}{N}\left(\sum_{l\neq i}^{N}\frac{\mathrm{d}w_l/w_i}{\mathrm{d}M_i}\right),$$

which is positive by (29).

Country i imports from country j. Differentiating μ_i/w_j in (15) with respect to M_i yields:

$$\frac{\mathrm{d}\mu_i/w_j}{\mathrm{d}M_i} = \frac{1}{N}\left(s_i\frac{\mathrm{d}w_i/w_j}{\mathrm{d}M_i} + r\frac{\mathrm{d}}{\mathrm{d}M_i}\left(\sum_{l=1}^{N} w_l/w_j\right)\right).$$

By (28) and (31), the first term is negative while the second is negative if $m_j s_j < m_l s_l$ for all $l \neq j$ or if countries' labor supply are close to symmetry $m_l s_l \to m_j s_j$.

After some simplifications we get

$$\frac{\mathrm{d}\mu_i/w_j}{\mathrm{d}M_i} - \frac{\mathrm{d}\mu_i/w_k}{\mathrm{d}M_i} = \frac{1}{N}s_i\left(\frac{\mathrm{d}w_i/w_j}{\mathrm{d}M_i} - \frac{\mathrm{d}w_i/w_k}{\mathrm{d}M_i}\right) + \frac{r}{N}\left(\frac{\mathrm{d}}{\mathrm{d}M_i}\left(\sum_{l=1}^{N} w_l/w_j\right) - \frac{\mathrm{d}}{\mathrm{d}M_i}\left(\sum_{l=1}^{N} w_l/w_k\right)\right)$$
$$= -\frac{1}{MN}\left(m_j s_j - m_k s_k\right)\left[s_i\frac{2r + s_i}{\left(r + m_i s_i\right)^2} + \sum_{l}^{N}\frac{r^2}{\left(m_l s_l + r\right)^2}\right]$$

Therefore, a rise in country i's population entices this country to replace its high-quality imports from high labor supply countries by high-quality imports from low labor supply countries ($\frac{\mathrm{d}\mu_i/w_j}{\mathrm{d}M_i} - \frac{\mathrm{d}\mu_i/w_k}{\mathrm{d}M_i} > 0 \iff m_j s_j < m_k s_k$).

Country j imports from country l. Differentiating μ_j/w_l in (15) with respect to M_i yields:

$$\frac{\mathrm{d}\mu_l/w_j}{\mathrm{d}M_i} = \frac{1}{N}\left(s_l\frac{\mathrm{d}}{\mathrm{d}M_i}\frac{w_l}{w_j} + r\frac{\mathrm{d}}{\mathrm{d}M_i}\sum_{k=1\neq i\neq j}^{N}\frac{w_k}{w_j} + r\frac{\mathrm{d}}{\mathrm{d}M_i}\frac{w_i}{w_j}\right)$$

The last term is always negative. The first and second terms are negative if $m_j s_j < m_l s_l$ for all $l \neq j$ or $m_l s_l \to m_j s_j$. So, under the latter condition, the expression is negative.

Trade costs

Suppose symmetric countries with iceberg trade costs: $s_i = s$, $m_i = 1/N$, $\tau_{ij} = \tau$, $i \neq j$ while $\overline{\tau}_i \equiv \overline{\tau} = 1 + (\tau - 1)(N-1)/N$. Using the same argument as in Proposition 1, it can be shown the symmetric trade equilibrium exists and is unique for a non-zero measure of productivity levels s and high enough utility upgrades β, if, in addition, the trade cost is not too high: $\tau < \ell(1)/\ell(0)$. Indeed, Condition A2 is satisfied under the same condition on β as in Proposition 1. Condition A1 requires that μ_i/w_i and $\mu_i/(\tau w_j) \in (\ell(0), \ell(1))$. This implies $\mu_i/w_i \in (\tau\ell(0), \ell(1))$, or equivalently, $[s+r+r\tau(N-1)]/N \in (\tau\ell(0), \ell(1))$. The latter interval is not empty for not too high trade costs: $\tau < \ell(1)/\ell(0)$. If the latter condition does not hold, import prices are too large and consumers have incentives to purchase no foreign high-quality varieties.

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