Cogent Education

ISSN: (Print) (Online) Journal homepage: www.tandfonline.com/journals/oaed20

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To cite this article: Yves Kreis, Ben Haas, Robert Weinhandl & Zsolt Lavicza (2024) Transitioning from lectures to online flipped classrooms: enhancing pre-service teacher education in Luxembourg, Cogent Education, 11:1, 2425895, DOI: 10.1080/2331186X.2024.2425895

To link to this article: https://doi.org/10.1080/2331186X.2024.2425895

Published online: 13 Nov 2024.

COGENT EDUCATION
2024, VOL. 11, NO. 1, 2425895
https://doi.org/10.1080/2331186X.2024.2425895

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Transitioning from lectures to online flipped classrooms: enhancing pre-service teacher education in Luxembourg

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ABSTRACT

This article explores the transition from traditional to hybrid and fully online flipped classroom models in the Bachelor of Educational Sciences program at the University of Luxembourg, focusing on the mathematics education of pre-service elementary school teachers. Over eight years (2014–2022), the program evolved from teacher-centred lectures to flipped classroom approaches, emphasising self-paced, active learning. The COVID-19 pandemic accelerated the shift to online flipped classrooms, incorporating digital tools like GeoGebra and MathCityMap to facilitate interactive, real-world problem-solving in a STEAM context. Quantitative data from exam results and qualitative field notes were analysed to assess the effects of flipped classrooms on student beliefs and performance. Results revealed significant improvements in student engagement, understanding, and academic performance, particularly in collaborative learning environments. Students developed greater confidence in applying mathematical concepts and pedagogical principles while acquiring critical self-regulation skills. While the flipped classroom model has shown promise, it is not without its challenges. Varied student backgrounds in mathematics and superficial engagement with pre-class materials have underscored the need for additional support measures. This study also identified opportunities for further innovation, including personalised learning and technology integration. Overall, the flipped classroom model emerged as a flexible, effective approach that significantly enhanced student learning outcomes. This was particularly evident when the model was complemented by targeted interventions and authentic assessments. The research presented here provides valuable insights into the benefits and limitations of flipped learning for teacher education, especially in the context of diverse and rapidly changing educational environments.

ARTICLE HISTORY
Received 26 April 2024
Revised 20 October 2024
Accepted 31 October 2024

KEYWORDS
Flipped classroom; hybrid learning; mathematics education; online courses; STEAM education

SUBJECTS
Mathematics Education; Teachers & Teacher Education; Higher Education; Open & Distance Education and eLearning

Introduction

At the beginning of their studies, pre-service elementary school teachers delve into the unfamiliar world of teacher education and discover didactical principles in mathematics (e.g. how to develop skills in number theory or teach geometric properties). For the first time in their scholarship, they are taught how and what to teach in mathematics. Pre-service elementary school teachers experience a new understanding of mathematical contents often different to the teaching they previously experienced. According to Michaluk, et al.(2018), pre-service teachers refer to their own experiences in mathematics, and transitioning to new teaching models is one major challenge in training. Furthermore, in the literature, we identified the essential factor of self-regulated learning in teacher education (Corrigan & Taylor, 2004), which should be increased further during the initial teacher training.

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Overview of the transition

In their first year, these topics were delivered through traditional lectures with low inputs from pre-service teachers. The lecturer presented didactical approaches to teaching content and process skills on campus based on a predefined syllabus and national curriculum (MENFP, 2011). The course was based on a mixture of mathematical content (e.g. patterns in algorithms or basic calculus) and didactical content (e.g. development of counting in early childhood or developing process skills in mathematical thinking). These traditional lectures were widely teacher-centred, and pre-service elementary school teachers depended entirely on the time management of the lectures on campus. In the following study years, seminars in smaller groups offered the possibility to deepen the concepts and explore the impact in schools of the different methodologies.

Students were given optional assignments corresponding to the topic of each lecture to prepare for their end-of-semester assessment. However, they were left alone regarding the resolution, and generally, no solution was distributed. Finally, pre-service elementary school teachers were graded using an exam at the end of the semester. It was composed of open questions focusing on didactical situations (e.g. how to illustrate the sum of a fraction to children) or on mathematical content (e.g. explain why a square fulfils the definition of a parallelogram). This assessment did not consider individual learning paths, paces or further didactical reflections. Traditionally, we evaluated the presented topics from our courses but could not evaluate the whole learning process. Thus, the exam outcome could only reflect this particular aspect, and we needed to rethink different assessment forms. Over time, our pre-service elementary school mathematics education teachings required profound changes to better adapt to individual learning processes (Kreis et al., 2020). We obtained high student attendance in traditional lectures but with relatively low interactions or unspectacular student developments. Teaching did not take into account students' learning paces and backgrounds. Consequently, only a few students participated actively in class, and as lecturers, we could not reach every pre-service teacher. Furthermore, discussions between lecturers and pre-service teachers to render mathematics education courses more suitable for each student's needs were made more challenging by increasing student numbers and recently by the COVID-19 pandemic.

Facing higher participant numbers whilst improving individual contact, we decided eight years ago to switch our teaching methodology in mathematics education courses in pre-service elementary school teachers' initial training to a flipped classroom setting. This alteration was needed since, after several years of teacher-centred instruction of pre-service teachers in Luxembourg, it was observed that students achieved low learning outcomes while only spending four teaching units on the campus at their teacher training courses. We decided to create in-class learning moments, where a strong focus was on vital didactical exchanges and out-of-class learning moments, where students could learn at their own pace.

After working for some years in the flipped classroom setting, the subsequent transference was driven by the COVID-19 confinement, where we needed to switch to an entirely online flipped classroom with peer learning and outdoor activities (to respect the sanitary conditions) for the assessments.

Research questions

The following research questions guided the process of changing teaching and learning methods:

1. What effects on students' beliefs and performances are likely to observe with flipped classroom method approaches compared to traditional in-class lectures?
2. What are the limitations and opportunities within the different approaches of a flipped classroom?

This article will describe the learning methodology, opportunities, and concerns in switching to full flipped classroom teaching. Further, we will detail the possible effects on student learning and upcoming investigations. We will examine the shift from on-campus to online teaching of pre-service teacher education courses in Luxembourg during the past four years. In our study, we aimed to evaluate both our training program and students' transition to the new online teacher education program by collecting various data.

Theoretical background and literature review

Actively constructing mathematical competencies (Bozkurt, 2017) and individually expanding one's own body of knowledge and competencies (Jazim, Anwar, & Rahmawati 2017) are potential success factors for learning and teaching mathematics. According to Niss and Højgaard (2019), we consider mathematical competencies to reveal students' willingness to react appropriately to certain types of mathematical problems and tasks in a given situation. Actively constructing mathematical competencies and individually expanding one's own body of knowledge also implies that teaching and learning approaches must be adjusted. Flipped approaches to learning and teaching mathematics are one way to support students in actively doing mathematics and thereby expanding their own mathematical competencies. We decided to use flipped approaches to education because many experts (Cevikbas & Kaiser, 2020; Cronhjort et al., 2018; Esperanza et al., 2016; Maciejewski, 2015; Menon & Azam, 2021; Schallert et al., 2022) have demonstrated in studies that learning mathematics using flipped approaches has a positive effect on students' performance, attitudes and enjoyment. Although flipped approaches to learning and teaching have been discussed intensively at the scientific level in recent years (O'Flaherty & Phillips, 2015; Prodromou, 2017), there is still no uniform definition of this teaching and learning methodology (Enfield, 2016; Wasserman et al., 2017). Despite the lack of a uniform definition, some elements can be found in most descriptions of learning and teaching following Flipped Classroom. A characteristic of Flipped Classroom is that direct instruction and passive learning should occur outside the classroom. The resulting in-class time should then be used for student-centred and active learning. Therefore, in-class time is at the heart of learning and teaching in flipped classrooms (Galway et al., 2014; Wasserman et al., 2017). According to Wagoner et al. (2016), this implies that basic learning activities, such as remembering in Bloom's taxonomy, take place outside the classroom, and higher-level learning activities, such as creating, are performed in the classroom. Learning by creating implies, among other things, that higher-level and real-world problems and tasks are also dealt with in the classroom. Gainsburg (2008) states that real-world problems and tasks characterise high-quality mathematics learning and teaching. Tackling higher-level or real-world problems and tasks should also result in students producing actual learning products that are equivalent to those produced in the flipped classroom (Galway et al., 2014; Herreid & Schiller 2013). Thereby it is shown that active and self-developing students are a connecting element of flipped classroom and learning and teaching mathematics.

Another typical element of modern flipped classrooms is that educational technologies and media are used (Esperanza et al., 2016; Fischer & Spannagel, 2012). In particular, the increasingly simple production of learning or explanatory videos and also the increasing number of online available learning or explanatory videos have made flipped classrooms popular (Blair et al., 2016; Leo & Puzio, 2016). However, a flipped classroom is much more than just video learning and instructional videos. Educational technologies and media should not be used in flipped classrooms for their own sake but should always support students in learning or serve an educational goal (Lemmer, 2013; Tolks et al., 2016). Limiting flipped classrooms to educational or explanatory videos could also cause problems, as Weidlich and Spannagel (2014) demonstrated that videos are often viewed only superficially by students. In addition to videos, other activities or media such as quizzes or (multiple-choice) questions (Bouwmeester et al., 2016; Morin et al., 2013) and eBooks (Enfield, 2016) could improve learning in a flipped classroom. The importance of using technologies in the flipped classroom in a pedagogically targeted way is a different way to learn and teach mathematics. According to a meta-analysis by Li and Ma (2010), using technologies and media in teaching and learning mathematics has a long tradition. However, it is not only a long tradition that technologies have in learning and teaching mathematics. According to Kim et al. (2014), online or technology-enhanced mathematics learning can positively impact students' overall learning performance.

Although modern flipped classroom teaching is a relatively new phenomenon, there are already further developments in this educational approach. One such development is learning and teaching using flipped learning approaches (Flipped Learning Network, 2014). While the flipped classroom mainly distinguished between learning and teaching in pre-class and in-class phases, according to the Flipped Learning Network (2014), using flipped learning leads to a distinction between learning in individual or group learning spaces. Knowledge and competencies should be acquired in individual learning spaces, while in group learning spaces, learners should use new knowledge and competencies to solve tasks and problems. Switching between individual and group learning spaces is determined by the students themselves when using flipped learning. This leads to even higher self-determination, individualisation and design of the learning process by the learners. In our study the principles of flipped learning were mainly applied in the second phase of our study.

Individual and active learning of mathematics when using technologies also requires teachers to provide an appropriate learning environment and be trained for such approaches to learning and teaching. According to the TPACK model by Mishra and Koehler (2006), teachers of flipped learning need knowledge from the fields of technology, pedagogy, and mathematical content and especially knowledge and competencies when these three areas are combined. According to Weinhandl et al. (2020), doing, sharing and enhancing knowledge and competencies are promising learning activities in professional mathematics and STEM teacher development for flipped approaches.

Methodological approaches

The approach

To address the problems identified in the introduction, we created a first holistic approach to mathematics education, where we wanted, on the one hand, pre-service teachers to understand and foster mathematical concepts and process skills and experience teaching mathematics to elementary school pupils (aged 3 to 12) on the other hand. Hence, in mathematics courses, as mentioned earlier, they learned methodologies, didactical principles, contents and process skills in mathematics. The next step was to take mathematics closer to the expectations from the national curriculum and state-of-the-art research. Hence, we wanted to come closer to real-world settings and problem applications in addition to textbooks in courses. Thus, we started to present frameworks on mathematical modelling (Blum, 2013; Selter & Zannetin, 2021), process skills (Bossé et al., 2010; NCTM, 1999) and content skills (MENFP, 2011) within classroom tasks and settings to the pre-service elementary school teachers.

Moreover, pre-service elementary school teachers applied mathematical modelling and didactical approaches in active participation (e.g. playing mathematical games on number theory or manipulating learning materials), mathematical concepts and process skills. Regularly these activities are reflected in the mentioned theoretical frameworks and deepened in research work with students.

Further, over the past years, we integrated educational technology software which has shown to be promising in improving the understanding of mathematics (Gassner & Hohenwarter, 2012; Haas et al., 2020b; Kreis & Dording, 2009; Lavicza, et al., 2020). Some of the technologies have been developed based on national research projects, e.g. educational technology software MathemaTIC (SCRIPT, 2021), or international open-access software, e.g. GeoGebra 3D Graphing Calculator (Hohenwarter et al., 2022) or MathCityMap (Ludwig et al., 2021). In addition to our courses, pre-service elementary school teachers applied the learned teaching skills in internships in elementary school classes (aged 3 to 12) and special needs classes. During their internships, they were supported by a practitioner from a school and a tutor from university.

Participants

The participants in this study were first-year students enrolled in the Bachelor en Sciences de l'éducation program at the University of Luxembourg between 2014 and 2022 (see Table 1). This program prepares pre-service elementary school teachers, equipping them with theoretical knowledge and practical skills for teaching in diverse and multilingual educational environments.

Each cohort of participants was composed of students entering the program with varying levels of prior experience in education. However, all shared a common goal of becoming qualified primary school teachers. These students, aged approximately 18 to 25, brought with them a rich tapestry of diverse cultural and linguistic backgrounds, reflecting the multicultural setting of Luxembourg.

Over the course of the study period, which spans eight academic years, these participants demonstrated remarkable resilience as they experienced significant shifts in their learning environments.

Table 1. Overview of the number of participants in each cohort.

Year	1st Semester	2nd Semester
'2014/2015'	76	
'2015/2016'	56	
2016/2017	62	
2017/2018	61	62
2018/2019	88	78
2019/2020	99	96
2020/2021	105	103
2021/2022	97	93

The transition from traditional in-person courses to hybrid and fully online modalities, driven by institutional decisions and external factors such as the COVID-19 pandemic, necessitated flexible and adaptive learning approaches. This diverse and evolving learning context provides valuable insights into how future educators adjust to and engage with changing modes of instructional delivery.

Data collection

Our data collection methods incorporated quantitative and qualitative strategies to explore the impact of the transition from traditional to hybrid and online learning for pre-service elementary school teachers, mainly through flipped classroom approaches. This blended approach allowed us to capture a holistic view of students' beliefs and performances and the limitations and opportunities of flipped classroom models.

Quantitative data

Students' performance records were gathered through exam grades from flipped versus traditional classroom formats. The goal was to quantify any correlation between teaching methods and academic success.

Qualitative data

Field notes were used to document informal interactions, in-class behaviours, and contextual factors that may have influenced student experiences with different teaching approaches. The primary aim of field notes was to provide a deeper, qualitative understanding of the learning environment and to record the subtleties of student engagement in both traditional and flipped classroom settings as well as during their internship.

Data analyses

In our study, we employed a mixed-methods approach to analyse the quantitative and qualitative data collected, aiming to provide a comprehensive understanding of the transition from traditional to hybrid and online courses for pre-service elementary school teachers, focusing on flipped classroom methods.

Quantitative data analysis

The primary source of quantitative data was students' performance records, specifically exam grades from both flipped classroom and traditional in-class formats. To analyse this data, we used the statistical software R (R Core Team, 2022) using the packages tidyverse (Wickham et al., 2019), ggpubr (Kassambara, 2022a) and rstatix (Kassambara, 2022b). Type I error was set to $\alpha = .05$ for all analyses. We visualised the data using box plots and performed a test for equal means followed by multiple pairwise comparisons as post-hoc analysis. These statistical analyses helped us to identify patterns and potential trends in how teaching methods impact academic success among pre-service teachers.

Qualitative data analysis

The qualitative data comprised field notes documenting informal student interactions, in-class behaviour, and other contextual factors that may have influenced the learning experience. Qualitative data analysis was used to support and detail the findings of the quantitative analysis. This led to a more nuanced understanding of the effects of the transition to flipped, hybrid, and online teaching methods on both student performance and beliefs.

Intervention and data Analysis

In the following section, we will present the different interventions, as well as collected and analysed data which led to each transition. Overall, we experienced 4 phases (from 2016 and ongoing) and consequently compared the changed teaching methodology to exam results from different phases. Each phase will be described and followed by a discussion.

Phase 1 (2016/2017 till 2018/2019)

Description of phase 1

Our first step in 2016/2017 was to switch the lectures to flipped classrooms and re-use the time of presence of the students to do mathematics learning activities together in the auditorium during the first semester while keeping the end-of-semester exam unchanged. In the second semester, we planned seminars with small groups to rework the same topics and linked these closely to schoolbooks and material usage in schools. This split setup was mainly due to organisational reasons. Initially, we planned a three-year phase of minor adaptations ending with an in-depth evaluation of the new setting and followed by further adjustments, if needed. The evaluation was based on students' assessment results and observations on students' development progress in the course and during their internships.

Using a flipped classroom methodology allowed us to base our activities on shared knowledge of the topic acquired beforehand. The students prepared the topic before the course, mainly using readings (e.g. articles, book chapters, webpages) and videos (e.g. recorded lectures, explanation videos) in an asynchronous setting at their own pace. The online availability of high-quality resources facilitated the switch. We guided the learning outside the classroom through the Moodle^{TM} platform of the University, where the digital resources were made available online.

Analysis and discussion of phase 1

In Bloom's Taxonomy, we covered the first two levels, 'Remembering' and 'Understanding'. In general, this change in teaching and learning made it possible that students could prepare themselves for upcoming teaching sessions and gather questions. Thus, pre-service teachers participated more actively and frequently used specific mathematical and pedagogical languages. We wanted to address this significant change, as practitioners reported that during the internships, pre-service teachers showed, in many cases, low skills in correctly using mathematical and pedagogical languages.

Furthermore, during the on-campus learning phase, where the presence was optional, the topics were consolidated. The remaining time was used to work together on the tasks that were left as unattended exercises for the students during the 'traditional teaching and learning' (as described beforehand). This collaborative problem-solving had the advantage that the solving process (including error and aberration) became visible to the students and thus led to more profound insight. This pool of tasks was made available to the students beginning in 2017/2018 through the Moodle^{TM} platform. They could thus better prepare for the final assessment as they already had access to the exercises during their learning phase outside the classroom.

While reworking the same topics during the second semester, we could observe a basic knowledge of the topics from pre-service elementary school teachers. Nevertheless, we had to remediate some basic content, which was a loss of time to some extent. Although that offered knowledge and training allowed pre-service teachers to learn and work on different topics, they had different backgrounds in mathematics skills and teaching mathematics. Some were at a novice level, and some already had a deeper understanding. However, the grades in the seminar work vastly surpassed the ones in the exam (see Tables 2 and 3).

We analysed the data in the statistical software R. The box plots of the exam results (Figure 1) clearly revealed major differences between the grades of '2014/2015' and the other years. No extreme outliers (3 times the interquartile range below Q1 or above Q3) were detected. The inspection of QQ plots led us to assume a reasonable normal distribution of the data. However, homogeneity of variance could not be observed for all conditions. We performed a test for equal means, which revealed that grades were highly significant (Welch Anova, $F(5, 191.42) = 17.25$, $p < .0001$, $n = 442$) between years. Games-Howell post hoc analyses revealed, as expected, that the decreases from '2014/2015' to '2015/2016' (−2.9, 95% CI $[-4.45, -1.36]$), to 2016/2017 (−3.8, 95% CI $[-5.34, -2.25]$), to 2017/2018 (−3.17, 95% CI $[-5.1, -1.25]$), to 2018/2019 (−3.93, 95% CI $[-5.52, -2.34]$), and to 2019/2020 (−3.85, 95% CI $[-5.24, -2.46]$) were highly significant ($p < .0001$) and no other statistically significant differences between the years were discovered.

Table 2. Means, standard deviations, and quartiles of first-semester exams.[1]

Year	n	M	SD	Min	Q1	Q2	Q3	Max
'2014/2015'	76	14.7	3.05	6.29	13.0	15.2	17.1	19.3
'2015/2016'	56	11.8	3.01	4.75	10.1	11.7	14.3	17.8
2016/2017	62	10.9	3.19	5	8.5	10.5	12.5	18.5
2017/2018	61	11.5	4.39	0.5	8.5	12	15.5	18.5
2018/2019	88	10.8	4.01	0	7.88	11.5	13.5	18.5
2019/2020	99	10.9	3.31	3	8.5	11	13.2	18
2020/2021	105	16.2	1.43	12	15	16.5	17	19
2021/2022	97	16.7	1.41	13.5	15.5	16.5	17.5	19.5

Table 3. Means, standard deviations, and quartiles of second-semester exams.

Year	n	M	SD	Min	Q1	Q2	Q3	Max
2017/2018	62	13.4	3.14	0	11	13	16	18.5
2018/2019	78	13.1	2.98	0	11.5	13	15	18
	77	13.3	2.59	5				
2019/2020	96	17.0	1.41	13	16	17	18	19.5
2020/2021	103	16.7	1.41	12.5	15.5	17	17.5	19.5
2021/2022	93	16.8	1.23	13.5	16	16.5	18	19.5

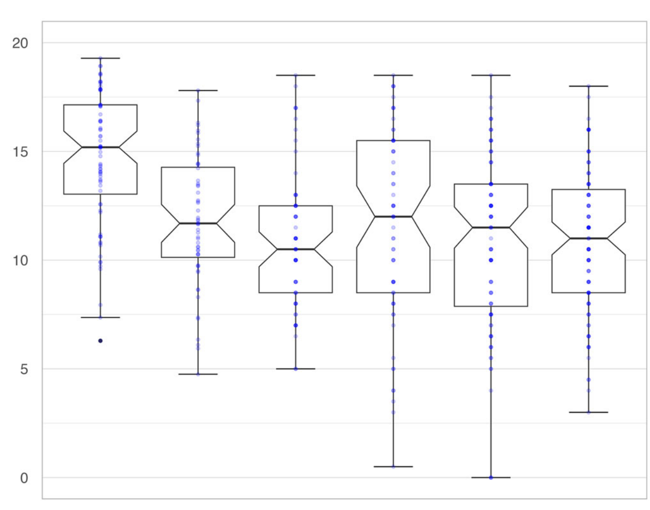

Figure 1. First-semester exam results over the years (part 1).[1]

The drop in performance from '2014/2015' to '2015/2016' triggered a rethinking of the first-year mathematics education course for pre-service elementary school teachers. 2016/2017 was the first year with flipped classroom methodology. Everything was new, and some adaptations needed to be made for the upcoming years. Further, the methodology was implemented with a design-based research approach (Amiel & Reeves, 2008), and we experimented with different settings (e.g. different interactions in tasks in Moodle™ or blended learning approaches) in the courses. 2019/2020 was the first year with intertwined flipped classroom lectures and hands-on seminars (see phase 2). Further, the course assessment changed to multi-choice multiple-response exam questions. However, the written exam with questions about the lectures did not focus on 'applying mathematics in teaching and learning situations', which was promoted starting in 2016/2017. The results of 2020/2021 and 2021/2022 will be discussed in phase 4.

Phase 2 (2019/2020 till 12 March 2020)

Description of phase 2

After analysing the collected data, we concluded that intertwining flipped classroom lectures with hands-on seminars would benefit pre-service elementary school teachers' learning processes. Moreover, a closer follow-up with pre-service elementary school teachers was possible in this setting. Besides this organisational change, we also decided to switch from open questions, in the course assessments, to multiple-choice, multiple-response questions in response to the rising number of students. To prepare the students for this new assignment, we used Kahoot! (Kahoot!, 2021) during our lectures. Another goal of playing Kahoot! games was to detect weaknesses that could be addressed immediately at the beginning of the onsite learning phase. This regular Kahoot! application provided performance feedback for the lecturers and these moments were important to familiarise with content and process skills for pre-service teachers. Moreover, pre-service teachers learned to use technologies that could be reinvested in their future teachings.

Thus, in between lectures, pre-service elementary school teachers joined seminars to apply the learned contents and skills in teaching settings.

Analysis and discussion of phase 2

Peers and lecturers, composed of a mathematical content expert and a more general didactics expert, supported students during these seminars. We observed how pre-service teachers improved during peer exchanges and created new questions and inputs regarding mathematics education. According to Britton and Anderson (2010), peer coaching is likely to increase performance and, in our case, the understanding and application of teaching skills. Furthermore, lecturers aimed to foster reflections and anticipations of learning opportunities and difficulties within these settings. These seminars were intense as students and lecturers had many interactions, discussions and reflections. However, some pre-service elementary school teachers were overwhelmed by the seminars and could not delve into the learning situations. As mentioned earlier, some pre-service teachers' autobiography in mathematics, foremost among first-semester students, was not strong in mathematics. This could be related to unsatisfactory learning experiences or accumulated knowledge and skills gaps. Hence, we observed how crucial learning at one's own pace is in pre-service teacher training, as it goes beyond simple content acquisition. Sometimes, we offered low-achieving students a remediation course to acquire the required level for the courses.

The exam results in the first semester remained constant (see Figure 1). However, we need to note – something that we discovered only lately – that since phase 1, we did not manage to assess the new competencies students develop with these new learning settings. Let us discover in the next chapter how the online courses that started in March 2020 changed our perception and allowed us to make a huge step towards an online integrated mathematics education course with an adapted evaluation to develop pre-service teachers' competencies best.

Phase 3 (13 March 2020 till 2019/2020)

Description of phase 3

With the arrival of COVID-19, sanitary measures and the closure of on-campus teaching starting 13 March 2020, we needed to present on short notice a viable teaching setting to the pre-service elementary school teachers in mathematics (Kreis et al., 2020). Teaching could only happen through online learning settings and activities that would not compromise students' and lecturers' health. Based on the given circumstances, we changed our course profoundly. On the one hand, we extended our flipped classroom to an entirely online flipped classroom, and on the other hand, we increased the range of the course from mathematics to STEAM (Science – Technology – Engineering – Arts – Mathematics). Introducing a new concept for assessing the course using the educational technology software MathCityMap (Ludwig et al., 2021) made this shift necessary. MathCityMap is a GPS-based software that allows educators to create outdoor trails with tasks within STEAM-integrated approaches (Kelley & Knowles, 2016). Pre-service elementary school teachers integrated mathematical tasks (e.g. calculate the volume of a given raised bed) in real-world situations (e.g. estimate or measure the dimensions of the raised bed).

This shift to mathematics in real-world situations required pre-service elementary school teachers to consider topics other than mathematics. Hence, some tasks were related to arts (e.g. mathematical modelling of a monument), engineering (e.g. planning of additional rails for a train road), technology (e.g. application of technology in solving problems) or science (e.g. scientific reasoning). However, teaching with this broader approach required changes in our course. Thus, in the first online course, pre-service elementary school teachers received an online presentation of MathCityMap, including a demo outdoor trail. After this course, they created outdoor mathematical trails close to local elementary schools in groups of two to three pre-service teachers in 2019/2020 and individually from 2020/2021. As lecturers, we offered remote support through several online tutorials while elaborating on the trails. The finalised outdoor trails were uploaded on the Moodle™ platform and prepared for assessment. Instead of letting the lecturers grade the trails alone, pre-service teachers performed a peer review of three trails and a self-evaluation of their own trail. We estimated that a peer-reviewed assessment would allow pre-service elementary school teachers to develop their trails further and encounter new best practices, similar to experiences researchers gain in peer-review processes when publishing. Furthermore, a peer-review process with 100 students should support a broader quality assurance than two lecturers.

Analysis and discussion of phase 3

We used the same procedure as beforehand to analyse the data in the statistical software R. The box plots of the seminar results (see Figure 2) revealed a major increase in the grades in 2019/2020 and continuing in the following years. One extreme outlier (3 times the interquartile range below Q1 or above Q3) was detected for 2018/2019, but it only influenced the results marginally (see below). The inspection of QQ plots led us to assume reasonable normality distribution of the data. However, homogeneity of variance could not be observed for all conditions. We performed a test for equal means, which revealed that grades were highly significant (Welch ANOVA, $F(4, 190.27) = 44.33$, $p < .0001$, $n = 432$; Welch Anova, $F(4, 190.2) = 48.75$, $p < .0001$, $n = 431$) between years. Games-Howell post hoc analyses revealed, as expected, that the increases from 2017/2018 to 2019/2020 (3.57, 95% CI $[2.38, 4.75]$), to 2020/2021 (3.25, 95% CI $[2.07, 4.42]$), and to 2021/2022 (3.39, 95% CI $[2.22, 4.56]$) as well as from 2018/2019 to 2019/2020 (3.88, 95% CI $[2.86, 4.89]$; 3.71, 95% CI $[2.79, 4.62]$), to 2020/2021 (3.56, 95% CI $[2.54, 4.57]$; 3.39, 95% CI $[2.48, 4.29]$), and to 2021/2022 (3.70, 95% CI $[2.69, 4.70]$; 3.53, 95% CI $[2.63, 4.42]$) were highly significant ($p < .0001$) and no other statistically significant differences between the years were discovered.

The significant increase in grades starting with the 2019/2020 exam probably is since, for the first time, the focus of the assessment was on 'doing mathematics', i.e. creating a mathematical trail for elementary school students. The pre-service elementary school teachers were highly motivated, and some groups even reworked their trail to release it publicly.

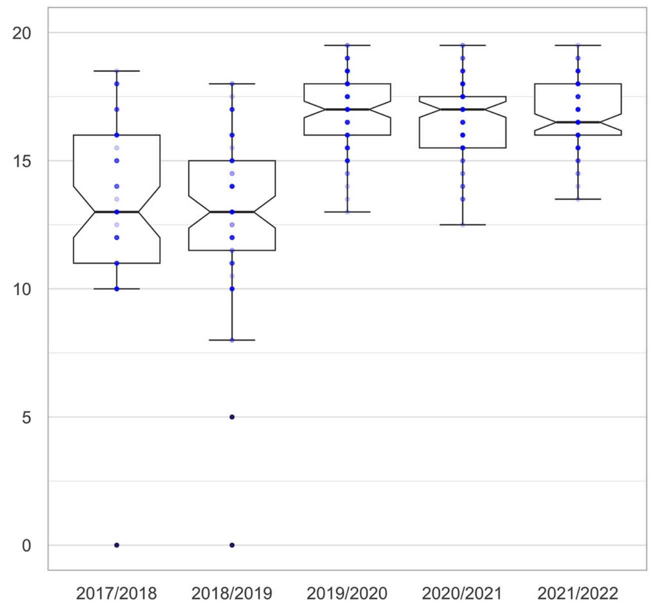

Figure 2. Second-semester seminar results over the years.

Figure 3. Reversible plates and thousand book.

Phase 4 (2020/2021 and ongoing)

Description of phase 4

During 2020/2021, only 40 students were allowed in a large auditorium with a capacity of 240 places (ratio 1:6 for auditorium), while 17 students were allowed in a large seminar room with a capacity of 50 places (ratio 1:3 for seminar room). With 110 pre-service teachers in our mathematics course and the limited number of large auditoriums available, these restrictions required thinking out of the box. Based on our ongoing studies combining real-world situations (Haas et al., 2020a), we wanted to continue teaching and supporting pre-service elementary school teachers to 'do mathematics' and actively participate in their own learning process. Thus, we went entirely online with flipped classroom methodology and offered no on-campus teaching and learning settings except the first launch event, held three times in small groups with 37 pre-service elementary school teachers each.

The flipped classroom structure stayed the same, with a mixture of lectures, seminars and self-learning in peer groups. In one week, pre-service elementary school teachers participated in an online lecture, which was prepared through flipped classroom and in the next week, they exchanged their self-learning activities. The required learning materials were either handed out (i.e. 100 reversible plates and 1 thousand book; see Figure 3) to the pre-service elementary school teachers during their onsite lecture or deposited (i.e. board games or iPads) in the University SciTeach Center, where all teachers can borrow materials. The exploration of these learning materials was done before the online sessions using instructions on the Moodle™ platform requiring content reflection (i.e. text or image) from the pre-service elementary school teachers.

For the online sessions, the University provides Cisco Webex, one of several excellent solutions for remote teaching. Sessions with 110 students are easily manageable and through the break-out room function, group work, which is currently not possible for on-campus courses, is perfectly practicable with an easy possibility for the teacher to join groups separately. Kahoot! is still being used as it gives a quick insight into the pre-service elementary school teachers' preparation work and still guides the teacher in deepening knowledge and skills.

Onsite courses were allowed again without restrictions starting with the summer semester of 2021/2022. However, we stayed mainly online with the course as the quality of the group work performed by the students online in the break-out rooms vastly exceeded the possibilities available in an auditorium. This fact was increased as part of the groups met physically, for example, in the learning centre. Nevertheless, the seminars using board games or iPads were organised onsite to simplify the organisation.

Analysis and discussion of phase 4

Regarding the assessment, after the highly positive experience, we decided to keep elaborating a MathCityMap trail with minor modifications. The pre-service elementary school teachers will start to elaborate a trail individually during the winter semester (based on the worked course topics) and finalise it during the summer semester. Furthermore, we will integrate technology more often and work with pre-service elementary school teachers on digital and physical modelling with GeoGebra 3D Graphing Calculator (Hohenwarter et al., 2022). Augmented reality and 3D printing have been shown to develop new innovative approaches among pre-service teachers (Haas et al., 2020a). A self-evaluation and a peer review using pre-known criteria will help the pre-service elementary school teachers progress. Optional testing of their trail during their internship is encouraged.

We used the same procedure as beforehand to analyse the data in the statistical software R. The box plots of the exam results (Figure 4) revealed the same major increase as for the seminars in the grades in 2020/2021 and continuing the following year. No extreme outliers (3 times the interquartile range below Q1 or above Q3) were detected. The inspection of QQ plots led us to assume a reasonable normal distribution of the data. However, homogeneity of variance could not be observed for all conditions. We performed a test for equal means, which revealed that grades were highly significant (Welch

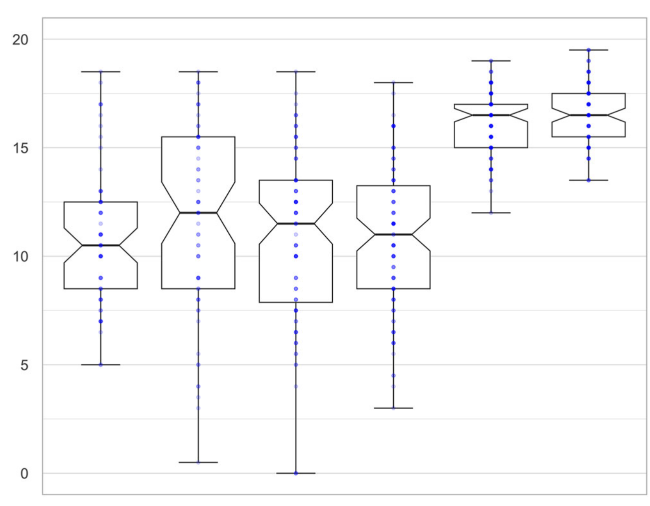

Figure 4. First-semester exam results over the years (part 2).

ANOVA, $F(5, 204.5) = 115.75$, $p < .0001$, $n = 512$) between years. Games-Howell post hoc analyses revealed, as expected, that the increases to 2020/2021 from 2016/2017 (5.31, 95% CI [4.06, 6.56]), from 2017/2018 (4.69, 95% CI [2.99, 6.39]), from 2018/2019 (5.44, 95% CI [4.14, 6.75]), and from 2019/2020 (5.36, 95% CI [4.32, 6.40]) as well as to 2021/2022 from 2016/2017 (5.76, 95% CI [4.50, 7.01]), from 2017/2018 (5.14, 95% CI [3.43, 6.84]), from 2018/2019 (5.89, 95% CI [4.58, 7.20]), and from 2019/2020 (5.81, 95% CI [4.76, 6.86]) were highly significant ($p < .0001$) and no other statistically significant differences between the years were discovered.

The significant increase in grades starting with the 2020/2021 exam probably has the same explanation as beforehand, i.e. the focus of the assessment was on 'doing mathematics'. The detailed analysis of the assessment (e.g. peer review of three trails and self-evaluation of the own trail), as well as its impact on the summer semester grades, is beyond the scope of this article and will be published separately.

Discussion and conclusions

The shift from traditional in-class lectures to flipped classroom methods at the University of Luxembourg's Bachelor in Educational Sciences program has demonstrated substantial changes in pre-service elementary school teachers' engagement and performance, particularly in mathematics education. Initially, traditional methods were characterized by low interactivity and limited individualization, with passive learning prevailing. Over time, a flipped classroom model was adopted, providing opportunities for pre-service teachers to prepare learning materials outside the classroom and engage in active, collaborative learning during in-class sessions. This transition allowed students to process foundational knowledge beforehand and use classroom time to solve problems, reflect, and apply concepts interactively.

Further changes brought about by the COVID-19 pandemic led to a fully online flipped classroom, which expanded into STEAM education and integrated real-world tasks using digital tools like MathCityMap. This holistic approach enriched students' learning experiences by promoting self-paced learning (Menon & Azam, 2021), peer collaboration, and innovative teaching methodologies. These adaptations were facilitated by educational technologies such as GeoGebra and Kahoot!, and the new model fostered both mathematical competencies and process skills, ultimately leading to significant improvements in student performance as reflected in exam results.

Research question 1

What effects on students' beliefs and performances are likely to be observed with flipped classroom method approaches compared to traditional in-class lectures?

The effects of the flipped classroom on students' beliefs and performances are multi-faceted, involving changes not only in academic outcomes but also in students' attitudes towards learning and teaching mathematics (Campillo-Ferrer & Miralles-Martínez, 2021).

One of the most prominent effects observed was the shift in students' perception of their role in the learning process. The flipped classroom model fostered a more active and autonomous learning experience, encouraging students to take ownership of their learning (Aburayash, 2021). The requirement to engage with learning materials before class and the focus on collaborative problem-solving during in-class sessions empowered students to approach mathematics education more confidently (Ledezma et al., 2024). Qualitative data from field notes and observations indicated that students became more comfortable using mathematical and pedagogical terminology, suggesting a deeper engagement with the material. This shift in attitude also influenced students' beliefs about their ability to teach mathematics. Many pre-service teachers reported feeling better prepared to apply the concepts they had learned in real classroom settings, as they had more opportunities to practice and reflect on these concepts through in-class activities and peer discussions (Limin 2017).

The flipped classroom approach also significantly improved students' academic performance, as evidenced by the quantitative data collected over several years. Statistical analysis revealed a steady increase in exam scores following the introduction of the flipped classroom model. This was particularly noticeable in the final phases of the study, where online learning and STEAM integration further enhanced the learning experience (Chen, 2021). Besides, the evaluation aligned with the competencies developed in the course, creating a mathematical trail for elementary school students using MathCityMap (Fredriksen, 2021).

However, the effects on performance were not uniform across all students. Those with weaker mathematical backgrounds initially needed help with the increased responsibility for their own learning, particularly in the pre-class phase, where they were expected to engage with challenging content independently. This suggests that while the flipped classroom model can improve learning outcomes, it also requires careful scaffolding to support students needing additional guidance in developing self-regulated learning skills (Sun et al., 2016).

Research question 2

What are the limitations and opportunities within the different approaches of a flipped classroom?

The flipped classroom model presents several opportunities for enhancing the learning experience in teacher education, but it also has its limitations (Akçayır & Akçayır, 2018). One of the key opportunities is the ability to personalise the learning process. In a traditional lecture setting, all students are expected to progress at the same pace, which can be problematic for those who need more time to grasp complex concepts. The flipped classroom allows students to engage with learning materials at their own pace before class, allowing them to revisit complex topics as needed. This is particularly beneficial in a diverse cohort where students have varying prior knowledge and experience levels (Goedhart et al., 2019).

Another opportunity is the increased use of class time for active learning. Instead of spending valuable class time on direct instruction, where students passively receive information, the flipped classroom approach uses this time for interactive activities that promote critical thinking, problem-solving, and peer collaboration. This aligns with best practices in teaching mathematics, where active engagement with real-world problems is known to enhance learning. Using digital tools such as GeoGebra and MathCityMap further enriches the learning experience by providing students with practical, hands-on tasks that connect mathematical concepts to real-life applications (Fredriksen, 2021).

However, the flipped classroom also has limitations, particularly regarding its increased demands on students' self-regulation. The success of the flipped model depends mainly on students' ability to engage with pre-class materials independently. This can be a significant challenge for students who struggle with time management or who lack foundational solid skills (Ng, 2018).

Another limitation is that not all students engage deeply with pre-class materials, particularly videos, as some tend to watch them superficially or skip parts. This superficial engagement can limit the effectiveness of the flipped model, as students may come to class unprepared for deeper discussions and problem-solving (Mei, 2021). To address this issue, the course incorporated quizzes and interactive tools such as Kahoot! to ensure students engaged with the content and provide immediate feedback on their understanding (Chernov et al., 2021).

Lastly, transitioning to an entirely online flipped classroom during the COVID-19 pandemic introduced technical and motivational challenges. While digital tools and online platforms allowed for the continuation of learning, some students found it challenging to stay motivated and engaged without face-to-face interactions in a physical classroom. This highlights the importance of creating a supportive online learning environment that fosters interaction and maintains student motivation, particularly in a fully remote setting (Breda et al., 2020).

Conclusions

The transition from traditional to hybrid and eventually to fully online flipped classroom models in the University of Luxembourg's Bachelor of Educational Sciences program represents a profound evolution in the way mathematics education is delivered to pre-service elementary school teachers. This journey, spanning several years and different phases, reveals critical insights into the effectiveness of flipped classroom approaches in enhancing student learning, engagement, and performance while also highlighting some of the limitations inherent in such pedagogical innovations.

At its core, the flipped classroom model successfully addresses many of the shortcomings of traditional lecture-based education. In traditional settings, students are often passive recipients of information, with limited opportunities to engage deeply with the content during class time. The flipped classroom disrupts this dynamic by shifting the responsibility for acquiring foundational knowledge to the pre-class phase, typically through readings, videos, and other digital resources. This allows in-class time dedicated to active learning activities, where students apply what they have learned, collaborate with peers, and receive immediate instructor feedback. In this way, flipped learning fosters a more participatory, hands-on approach to education that better aligns with the active and reflective learning practices critical for future educators.

One of the most significant outcomes of this transition has been the enhancement of student performance and engagement. The data from the study reveals a marked improvement in exam scores and overall academic achievement following the introduction of the flipped classroom model. This suggests that the model facilitates a deeper understanding of mathematical concepts and helps pre-service teachers develop the skills necessary for teaching those concepts effectively in elementary school classrooms. The flipped approach has empowered students to become more active participants in their own learning process, encouraging them to take ownership of their education, improve their self-regulation skills, and engage more meaningfully with the course content.

Additionally, integrating digital tools such as GeoGebra, MathCityMap, and Kahoot! has enriched the flipped classroom experience by providing innovative ways for students to engage with mathematical problems and real-world applications. These technologies offer opportunities for interactive, multi-modal learning beyond traditional textbook approaches. They also prepare pre-service teachers to integrate technology into their future classrooms, a critical skill in today's increasingly digital and technology-driven educational environments.

The shift to fully online learning during the COVID-19 pandemic further underscored the versatility and adaptability of the flipped classroom model. Although the pandemic posed significant challenges, particularly in terms of maintaining student motivation and ensuring equitable access to technology, it also highlighted the potential of online platforms to support flipped learning at scale. The use of digital tools to create virtual learning environments allowed students to continue their education without disruption, demonstrating the model's resilience in the face of external pressures.

However, the flipped classroom is not without its limitations. The model requires students to take on more responsibility for their learning, particularly during the pre-class phase, where they are expected to engage with materials independently. This can be challenging for students who lack strong self-regulation skills or come into the program with weaker mathematical foundations. As the study revealed, some students struggled to adjust to the demands of flipped learning, particularly in the early phases of the transition. These students often required additional support, such as remediation courses or peer coaching, to help them keep up with the course content and succeed in the flipped classroom environment. Moreover, the risk of students engaging superficially with pre-class materials, particularly videos, was a noted challenge. Without careful monitoring and accountability measures, such as quizzes or interactive tools, students might not fully engage with the materials, limiting the flipped classroom's effectiveness.

The study also highlighted the importance of assessing student learning in a way that aligns with the flipped classroom's goals. Traditional assessments, which often focus on recall and basic understanding, are insufficient to evaluate the more profound learning outcomes the flipped model aims to promote. As the University of Luxembourg's experience demonstrated, adapting assessments to focus on practical application, such as creating real-world mathematical trails through MathCityMap, can significantly enhance student motivation and performance. These authentic assessments allow students to demonstrate their understanding and encourage them to think creatively and critically, skills essential for effective teaching.

The continued evolution of flipped classroom approaches will likely involve further integration of technology and personalized learning. The success of the online flipped classroom during the pandemic suggests that blended and online models may continue to play a role in the future of teacher education.

These models offer the flexibility to accommodate diverse student needs, support individualized learning paths, and foster collaboration in ways that traditional classroom settings may not. Moreover, the growing emphasis on STEAM education provides an exciting opportunity to expand the flipped classroom model to incorporate interdisciplinary learning, where students can engage with mathematics not only as an abstract subject but as a tool for solving complex, real-world problems across various fields.

In conclusion, the transition to flipped learning has been a transformative experience for the University of Luxembourg's pre-service teacher education program. It has demonstrated that flipped classroom approaches can significantly enhance both student engagement and learning outcomes, mainly when supported by innovative technologies and authentic assessments. However, the model's success depends on careful design, continuous support for students, and thoughtful integration of pedagogical tools. As the field of education continues to evolve, the flipped classroom approach offers a promising pathway toward more effective, personalized, and engaging teacher education that prepares future educators to meet the challenges of modern classrooms (Engelbrecht et al., 2023).

Note

1. The grades of 2014/2015 and 2015/2016 have been calculated proportionally using the grades from the first- and second-semester exams while only considering the exam topics treated in the upcoming years.

Ethical approval

The research project has been approved by the Ethics Review Panel of the University of Luxembourg (ERP 20-036 MCM@BScE & ERP 20-082 MathEduc@BScE). The students have given their informed consent through the Moodle™ platform.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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