The one key element for undergraduate research is. . .

It is not that uncommon for university students or even high school students to perform research, especially when properly mentored. In mathematics, it is crucial to have an accessible problem. For example, for a thesis project, the student needs to quickly master the necessary techniques and then apply them to the problem at hand—not all mathematical topics can be mastered in such a short period of time. But even if the theory has been learned, it often requires a senior mathematician to find a suitable problem. A true expert can often judge, with a sort of intuition, whether a problem is impossible (say, expected not to be solved for decades) or whether it is rather a research exercise—something that one basically only needs to write down with some care.

In a broader sense of the word "research," including various forms of original output, university students and even high school students can contribute to a good cause with a useful output. They can write or edit Wikipedia pages, write scientific texts aimed at the general public, teachers, or pupils. They can invent exhibits, conduct experiments, produce images, create 3D prints…

In my experience, not only the cleverest students can achieve much. It is also the skill of the mentor to identify the students' qualities and build on them, compensating for what is missing.

I have published with high school students, Bachelor's students, Master's students, my PhD candidates, and PhD candidates not under my supervision (and, of course, with senior mathematicians as well). More precisely, we have achieved research work in number theory, geometry, graph theory, and recreational mathematics. We have published texts for didacticians, texts aimed at teachers, and websites aimed at the general public. The number of my junior coauthors keeps growing every year.

After all that I have learned from experience, I would complete the title without any hesitation as follows: a good topic (accessible, original, interesting, and doable). Indeed, a good topic leads to success, and a bad topic leads to failure. With a good topic, success is pre-planned because some outcome will be there, and the mentor can guarantee it. In a nutshell, the mentor should always offer a project that seems very promising and that the mentor would be able to carry out themselves. All the rest comes naturally—at least, that has been my experience.