Measuring Emotions in Mathematics:

The Achievement Emotions Questionnaire – Mathematics (AEQ-M)

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Abstract

Understanding the structure, antecedents, and outcomes of students' emotions has become a topic of major interest in research on mathematics education. Much of this work is based on the Achievement Emotions Questionnaire – Mathematics (AEQ-M), a self-report instrument assessing students' mathematics-related emotions. The AEQ-M measures seven emotions (enjoyment, pride, anger, anxiety, shame, hopelessness, boredom) across class, learning, and test contexts (internal structure). Based on control-value theory, it is assumed that these emotions are evoked by control and value appraisals, and that they influence students' motivation, learning strategies, and performance (external relations). Despite the popularity and frequent use of the AEQ-M, the research leading to its development has never been published, creating uncertainty about the validity of the proposed internal structure and external relations. We close this gap in Study 1 (N = 781 students, Grade 5-10, mean age 14.1 years, 53.5% female) by demonstrating that emotions are organized across contexts and linked to their proposed antecedents and outcomes. Study 2 (N = 699 students, Grade 7 and 9, mean age 14.0 years, 56.9% female) addresses another deficit in research on the AEQ-M, the lack of evidence regarding the assumption that emotions represent sets of interrelated affective, cognitive, motivational, and physiological/expressive components. We close this gap by evaluating extended AEQ-M scales systematically assessing these components for five core mathematics emotions (enjoyment, anger, anxiety, hopelessness, boredom). Our work provides solid grounds for future research using the AEQ-M to assess emotions and their components in the domain of mathematics.

Keywords: Achievement Emotions Questionnaire Mathematics (AEQ-M), Control-Value Theory (CVT), emotion components, assessment

1 The Achievement Emotions Questionnaire – Mathematics (AEQ-M)

Emotions play a pivotal role in academic settings and therefore constitute a major research topic in educational psychology (Pekrun & Linnenbrink-Garcia, 2014). Mathematics is an important domain of investigation in this regard (Schukajlow et al., 2017) for several reasons. First, it is paramount to investigate students' emotions regarding mathematics because it is a core subject and taught around the world. Second, mathematics is a domain to which students commonly attach rather high levels of perceived value (Goetz et al., 2014), which provides the basis for experiencing high levels of both negative (e.g., anxiety before a difficult test) and positive emotions (e.g., pride about receiving a good grade). Third, the domain of mathematics is characterized by gender differences in various psychosocial variables, including the levels, antecedents, and outcomes of emotions (Frenzel, Pekrun, & Goetz, 2007; Goetz et al., 2013). Lastly, mathematics anxiety is an oft researched topic with a considerable amount of scientific literature examining the nomological network of anxiety within the context of mathematics learning (see, e.g., Ashcraft, 2002; Hembree, 1990). This established research tradition of examining mathematics anxiety might be fruitfully expanded by assessing other discrete emotions within the same learning context of mathematics.

However, there is still a lack of instruments assessing the core discrete emotions in the domain of mathematics. One exception is the Achievement Emotions Questionnaire – Mathematics (AEQ-M; Pekrun et al., 2005), a frequently used instrument that allows researchers to assess several emotions with a single instrument. The AEQ-M comprises a set of 60 self-report items, each presenting a statement about one of seven mathematics-related emotions (two positive emotions: enjoyment, pride; five negative emotions: anger, anxiety, shame, hopelessness, boredom) and asking students to indicate the degree to which that statement applies to them personally. The items are organized in scales that cover emotions experienced during mathematics classes (e.g., "I enjoy my mathematics

class" is a sample item for class-related enjoyment), while learning for mathematics by oneself (e.g., "My mathematics math homework bores me to death" is a sample item for learning-related boredom), or while taking tests in mathematics (e.g., "When I have an upcoming mathematics test, I get sick to my stomach" is a sample item for test-related anxiety). Students' answers to the items pertaining to each emotion (e.g., ten items measuring enjoyment across class, learning, and test contexts) can be aggregated into composite scores and linked to various constructs of interest in research on mathematics education. For instance, Frenzel et al. (2007) showed that students' AEQ-M scores for anxiety, anger, and shame were more strongly associated with academic achievement and parental expectations in China than in Germany, shedding light on important cultural differences.

Unlike instruments for assessing achievement emotions across school domains (e.g., the Achievement Emotions Questionnaire (AEQ); Pekrun et al., 2011), the AEQ-M and the data underlying its development have yet to be published. Instead, researchers have relied on a manual of the instrument that is available from its authors upon request (Pekrun et al., 2005). This is not a satisfactory state of affairs because it creates uncertainty about the validity of various assumptions made in research using the AEQ-M. The first assumption underlying the AEQ-M is that emotions can be organized into three different contexts, thus reflecting the internal structure of the AEQ-M. This organization pertains to the idea that emotions are context-dependent, that is, the experience of an emotion depends on whether students attend mathematics classes (class context), learn mathematics by themselves (learning context), or take tests in mathematics (test context). For instance, students might enjoy learning mathematics by themselves (i.e., high levels of learning-related enjoyment) more than attending mathematics classes and taking tests (i.e., low levels of class- and test-related enjoyment; Pekrun et al., 2002). Providing tentative support of this assumption about the internal structure of the AEQ-M, achievement emotions have been empirically shown to be

organized within these contexts in research using the domain-general AEQ (Pekrun et al., 2011). However, the extent to which these findings can be transferred to the AEQ-M is an open question, casting doubts on whether mathematics-related emotions should be measured in an context-dependent way.

The second assumption is that achievement emotions are best understood as a set of interrelated affective, cognitive, motivational, and physiological/expressive processes that represent distinct components of the overall emotional experience (e.g., Scherer, 2009). Consequently, this assumption again pertains to the internal structure of the AEQ-M as it affects the content-domain of the items. For example, a comprehensive approach to measuring anxiety during a mathematics test might require items that ask students whether they feel anxious (affect), worry about their performance (cognition), want to escape the situation (motivation), and get queasy (physiological/expressive). The AEQ-M accounts for the assumed component structure of emotions in a non-systematic manner, tapping into different components of each emotion but failing to cover all components of all emotions. Thus, it has not been possible to date to investigate whether the component structure established for achievement emotions in general (Pekrun et al., 2011), may also pertain to the mathematics-related emotions measured with the AEQ-M.

The third assumption is grounded in control-value theory (CVT; Pekrun, 2006, 2018, 2021), which proposes that achievement emotions are linked to specific antecedents and outcomes. According to CVT, control and value appraisals are important antecedents of achievement emotions. Control appraisals pertain to students' expectations to be able to initiate and perform achievement-related activities (e.g., studying for a mathematics test), expectations about whether these activities will produce desired outcomes (e.g., a good grade), and attributions regarding the controllability of the cause of outcomes that were attained (Pekrun, 2006, 2018). Appraisals of control are reflected in

students' academic self-concept (Shavelson et al., 1976) and self-efficacy (Bandura, 1977), two common measures of perceived control in empirical research (e.g., Goetz et al., 2012; Luo et al., 2016; for the relations between self-concept and self-efficacy in mathematics see Arens et al., 2022). In turn, value appraisals refer to the perceived value of academic activities and outcomes. Perceived value can relate to both extrinsic (e.g., importance of studying for attaining good grades) and intrinsic aspects of academic activities (e.g., interest in an activity). In addition, perceived value can also pertain to positive versus negative valence (e.g. importance of success vs. failure).

Regarding the outcomes associated with achievement emotions, these emotions are assumed to affect students' learning and academic performance (Pekrun, 2006; Pekrun et al., 2011). Emotions can affect intrinsic and extrinsic motivation (e.g., learning out of curiosity versus learning to obtain good grades) and facilitate the use of flexible (e.g., elaboration of learning materials) and rigid learning strategies (e.g., rehearsal of materials). Moreover, emotions can affect the balance between students' self-regulation (e.g., setting one's own goals) and external regulation (e.g., seeking help from others). Importantly, these cognitive and motivational processes are assumed to mediate the effects of emotions on academic performance. Unlike its structural validity (i.e., context-dependency and component structure), the external relations of the AEQ-M along the lines of CVT has already been investigated in numerous studies using the instrument. For instance, there is evidence for control and value appraisals as interactive determinants of achievement emotions in mathematics (e.g., Putwain et al., 2018) and for the impact of mathematics-related achievement emotions on students' learning and performance (e.g., Camacho-Morles et al., 2021). However, these relations have yet to be demonstrated with the data on which the development of the AEQ-M was based.

Therefore, the AEQ-M is based on several assumptions about the internal structure and external relations of mathematics-related emotions. It is difficult for readers to evaluate these

assumptions as neither the AEQ-M itself nor the data used for its development have been published thus far. Moreover, data permitting a systematic investigation of the proposed component structure of emotions are currently not available. This creates uncertainty about the psychometric properties and validity of the AEQ-M and might impede the progress of research on the role of emotions in mathematics education. We aim to close these gaps by demonstrating the validity of the assumptions behind the AEQ-M in two studies, based on the data used for developing the AEQ-M (Study 1) and novel data with extended AEQ-M scales for enjoyment, anger, anxiety, boredom, and hopelessness (Study 2) through the following:

1. Examine the assumed context-dependency of the emotions in Studies 1 and 2 – that is, the assumption that discrete emotions (i.e., enjoyment, pride, anger, anxiety, shame, hopelessness, and boredom) differ between academic contexts (i.e., attending class, studying, and taking tests).
2. Introduce extended AEQ-M scales to examine the assumed component structure of emotions In Study 2 – that is, the assumption that emotions represent sets of interrelated affective, cognitive, motivational, and physiological/expressive processes. These extended AEQ-M scales comprise 127 items measuring all four components of enjoyment, anger, and anxiety in class, learning, and test contexts, boredom in class and learning contexts, and hopelessness in test contexts.
3. Finally, in Study 1 we also establish the external validity of the AEQ-M by investigating the relationship between emotions and their proposed core antecedents (control, value) and outcomes (motivation, learning strategies, achievement).

2 Study 1

Study 1 is based on the data used for developing the AEQ-M. While scattered results from analyses of these data have been reported elsewhere (Goetz, 2004; Pekrun et al., 2005), a comprehensive and systematic analysis of the psychometric properties of the AEQ-M and its internal and external validity has not been conducted yet. To investigate the external validity of the AEQ-M, we followed the approach taken in the development of the domain-general AEQ (Pekrun et al., 2011) and assessed various measures of control and value appraisals (i.e., academic self-concept, self-efficacy, value of achievement, and interest), motivation (i.e., intrinsic motivation, achievement motivation, and effort), learning strategies (i.e., elaboration, rehearsal, self-regulation, and external regulation), and academic performance (i.e., grades).

2.1 Methods

2.1.1 Sample

This study draws upon a sample of 781 German secondary school students (53.5% female, 46.5% male) from Grades 5 to 10 (5: n = 177, 6: n = 103; 7: n = 140; 8: n = 149; 9: n = 110; 10: n = 102) with a mean age of M = 14.1 years (SD = 1.92). Students attended three different tracks referred to as Hauptschule (lowest track; n = 205 from 10 classrooms), Realschule (middle track; n = 270 from 10 classrooms), and Gymnasium (highest track; n = 306 from 12 classrooms).

2.1.2 Missing Data

A total of 0.93% of data were missing, stemming from 279 incomplete records. The percentage of missing values across all variables ranged from 0.00% to 2.69%. Full information maximum likelihood (FIML) was used to deal with missing data (see Enders, 2010).

2.1.3 Measures

All questionnaires were administered in a paper-and-pencil format and students provided their answers on 5-point Likert Scales (1 = *not true at all*, 2 = *hardly true*, 3 = *somewhat true*, 4 = *largely true*, 5 = *exactly true*).

Achievement Emotions. Achievement emotions were assessed with the Achievement Emotions Questionnaire – Mathematics (AEQ-M; Pekrun et al., 2005). The AEQ-M comprises 60 items (see Appendix 1 for the instructions and Appendix 2 for the items) that measure seven discrete achievement emotions in the domain of mathematics: two positive emotions (enjoyment and pride) and five negative emotions (anger, anxiety, shame, hopelessness, and boredom). Emotions are measured across three different contexts (i.e., class, learning, and test), with respect to three different points in time (i.e., before, during, and after the corresponding situation), and in terms of four components (i.e., affective, cognitive, motivational, and physiological/expressive). It should be noted, however, the component structure of emotions is not systematically covered in the AEQ-M and several components are not represented (e.g., no items tap into the affective component of test-related anger and the cognitive component of learning-related boredom). Also, the AEQ-M does not cover all emotions in all contexts (boredom is only assessed in the class and learning context, while hopelessness is only assessed in the test context) and across all points in time.

Antecedents of Achievement Emotions. Students' academic self-concept, self-efficacy, performance-related valence, and interest were measured as antecedents of achievement emotions.

Academic Self-Concept. Three items were used to measure student's academic self-concept in the domain of mathematics (e.g., "Mathematics is one of my best subjects;" Goetz, 2004; Marsh, 1990). Cronbach's alpha of this scale was .87.

Self-Efficacy. Self-efficacy was measured with four items (e.g., "I am confident that I can master the skills taught in mathematics"; adapted from Kunter et al., 2002, to the domain of mathematics). Cronbach's alpha of this scale was .86.

Positive Value of Achievement. The positive value of achievement was measured with five items capturing the value of success in mathematics (e.g., "It is very important for me to get a good grade in mathematics;" Goetz, 2004). Cronbach's alpha of this scale was .85.

Interest. Interest was assessed with eight items capturing the intrinsic value of activities in mathematics (e.g., "Engaging in mathematics is one of my favorite activities;" Goetz, 2004). Cronbach's alpha of this scale was .90.

Outcomes of Achievement Emotions. We measured students' motivation, learning strategies, and self-regulation and external regulation of learning as outcomes of achievement emotions.

Intrinsic and Achievement Motivation and Effort Regulation. Intrinsic motivation was assessed with three items (e.g., "In mathematics I do my homework because I like this subject") and achievement motivation with two items (e.g., "I study for mathematics because I don't want to get bad grades"; Goetz, 2004). Cronbach's alpha of these scales was .89 and .75, respectively. Moreover, effort regulation was assessed with nine items from the German Learning Strategies of University Students questionnaire (e.g., "I work hard to do well in mathematics classes even if I don't like what we are doing.;" Wild & Schiefele, 1994). Cronbach's alpha of this scale was .79.

Learning Strategies. Elaboration and rehearsal were measured with 9 items (e.g., "When I study for mathematics, I try to connect the material to things I've already learned in other subjects") and 4 items (e.g., "When I study for mathematics, I practice by reciting formulas over and over"; items

adapted from Baumert et al., 1997; Kunter et al., 2002), respectively. Cronbach's alpha of the two scales was .86 and .75, respectively.

Self- and Other-Regulated Learning. Self-regulated learning was assessed with nine items (e.g., "When studying for mathematics, I set my own goals that I want to achieve"; scale modified from Goetz, 2004) and external regulation of learning was measured with six items (e.g., "In the way I solve my mathematics problems, I follow my teacher's recommendations exactly;" scale modified from Goetz, 2004). Cronbach's alpha of these scales was .83 and .74, respectively.

Academic Achievement. Students reported their last midterm mathematics grade according to their report card. Grades range from 1 (*very good*) to 6 (*insufficient*). Grades were inverted for ease of interpretation, so that higher values corresponded to better achievement.

2.1.4 Analytic Strategy

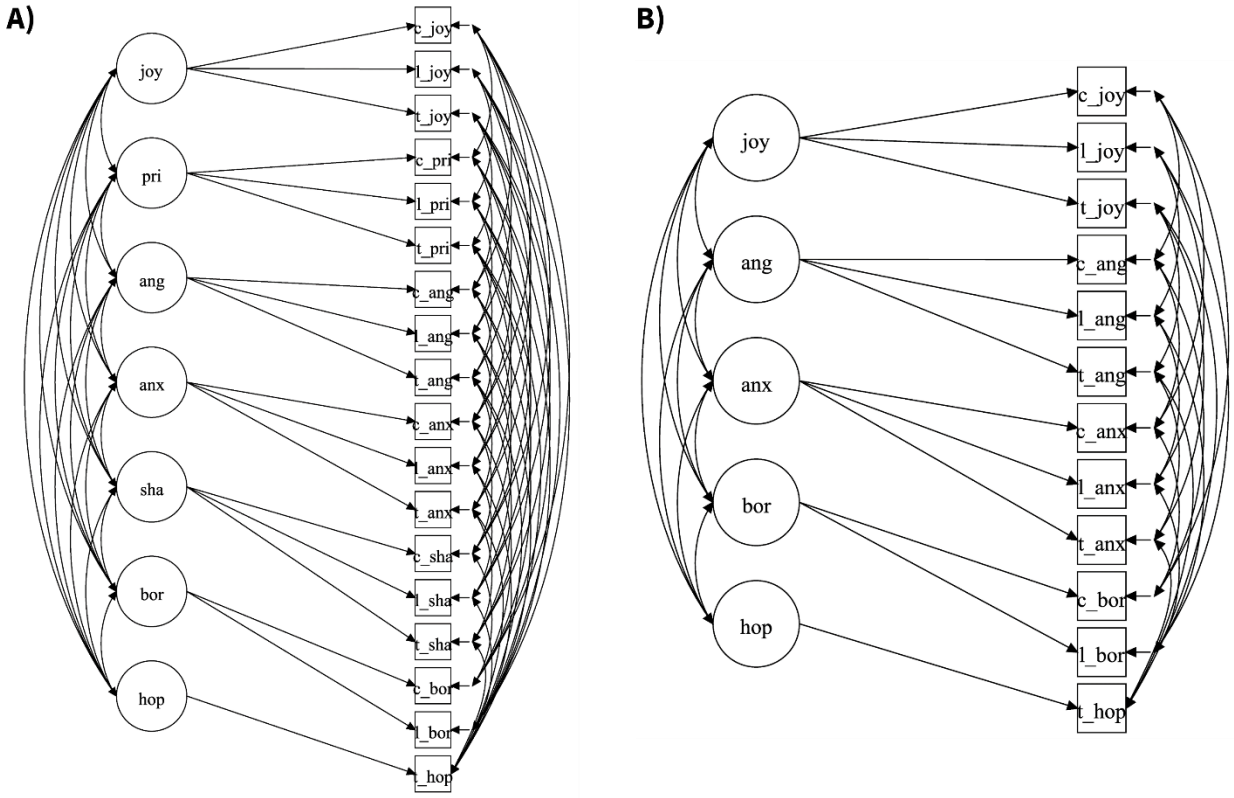
A series of confirmatory factor analyses (Brown, 2015) was conducted to investigate the structural relationships between emotions. First, a total of four CFA models representing different hypotheses about these relationships were estimated (see Pekrun et al., 2011, for an analogous approach using the domain-general AEQ): A model with one general bipolar factor across all contexts and emotions (M1), a model with seven factors representing each emotion (M2), a model with three factors representing each context (M3), and a model with seven factors representing each emotion and correlated uniqueness within settings (M4; see Figure 1). Second, we computed latent correlations of emotions with control and value appraisals, motivation, strategies, and performance based on single indicator models with model-based corrections for unreliability (see Cole & Preacher, 2014). Measurement models were evaluated using the fit indices CFI, TLI, RMSEA, and SRMR based on common cut-off criteria (CFI and TLI ≥ .95, SRMR ≤ .08, RMSEA ≤ .05; see Kline, 2016). In addition,

Bayesian information criterion (BIC) was used to select among competing models, where a lower BIC value indicates a better trade-off between model fit and model complexity.

Models were estimated with Mplus 8.4 (Muthén & Muthén, 1998-2017) using robust maximum likelihood estimation method (MLR) with chi-square test statistic and standard errors taking into account non-independence of observations due to students nested in classrooms.

Figure 1

Model 4 with seven (A; Study 1) and five factors (B; Study 2) representing discrete achievement emotions and correlated uniqueness within settings



Note. joy = enjoyment, pri = pride, ang = anger, anx = anxiety, sha = shame, bor = boredom, hop = hopelessness. The prefixes c, l, and t denote class, learning, and test contexts, respectively.

2.2 Results and Discussion

Descriptive statistics and manifest correlations of the AEQ-M scales are presented in Table 1. We observed higher means for positive than for negative emotions, sufficient variation in item scores, and low levels of skewness and kurtosis. All seven scales displayed good or very good reliability, .84 ≤ α ≤ .91. The positive emotions of enjoyment and pride were positively correlated, r = .78, and the negative emotions of anger, anxiety, shame, hopelessness, and boredom were also positively correlated, .25 ≤ r ≤ .86. Positive and negative emotions were negatively correlated, −.14 ≤ r ≤ −.62.

Table 1

Descriptive Statistics and Zero-Order Correlations of the AEQ-M Scales

								Correlations					
Scale	Items	*M*	*SD*	Skew	Kurt	$\bar{r}_{i(t-i)}$	α	1	2	3	4	5	6
1 Enjoyment	10	2.76	0.88	0.32	−0.45	.64	0.90						
2 Pride	6	2.77	1.02	0.27	−0.63	.67	0.87	.78					
3 Anger	9	2.45	0.97	0.51	−0.50	.62	0.88	−.62	−.43				
4 Anxiety	15	2.29	0.85	0.63	−0.15	.61	0.91	−.45	−.34	.70			
5 Shame	8	1.90	0.80	1.09	0.92	.57	0.84	−.26	−.14	.52	.72		
6 Hopelessness	6	2.19	1.04	0.83	−0.11	.70	0.89	−.50	−.41	.70	.86	.68	
7 Boredom	6	2.50	1.11	0.56	−0.58	.72	0.89	−.59	−.39	.69	.35	.25	.40

Note. $\bar{r}_{i(t-i)}$ denotes the average part-whole corrected item-total correlation. Items were answered on 5-point Likert Scales (1 = *not true at all,* 5 = *exactly true*). All correlation coefficients were statistically significant at α = .05.

2.2.1 Structural Relationships

In order to examine structural relationships between emotions, four CFA models were estimated (see Table 2). Results showed that the model representing the two-facet structure of the instrument (i.e., seven emotions nested within three contexts; M4) showed an acceptable model fit ($\chi^2(70) = 244.78$, CFI = .978, TLI = 0.951, RMSEA = 0.057, SRMR = 0.041) according to all fit indices, as well as the smallest BIC. This indicates the best trade-of between model fit and model complexity among the four competing models. This finding is in line with CVT, showing that several discrete

achievement emotions can be distinguished and that they are context-dependent (i.e., nested in class, learning, and test settings).

Table 2

Confirmatory Factor Analysis: Model Comparison

Model	χ^2	df	CFI	TLI	RMSEA	SRMR	BIC
M1: One-Emotion Factor Model	3736.49	135	.541	0.480	0.185	0.138	35582.28
M2: Seven-Emotion Factor Model	992.08	115	.888	0.851	0.099	0.058	32756.47
M3: Three-Context Factor Model	3464.42	132	.575	0.508	0.180	0.144	35255.77
M4: Seven-Emotion x Context Factor Model	**244.78**	**70**	**.978**	**0.951**	**0.057**	**0.041**	**32135.69**

Note. *N* = 781. Seven emotions are enjoyment, pride, anger, anxiety, shame, hopelessness, and boredom. Three contexts are class, learning, and test. Model selected by the BIC is shown in boldface.

2.2.2 Correlations with External Criteria

The AEQ-M scales and external measures of antecedents and outcomes of achievement emotions in mathematics can be found in Table 3. As expected and in line with CVT (Pekrun, 2006), the positive emotions of enjoyment and pride were positively associated with all included measures, indicating that higher levels of positive emotions are related to higher levels of control and value appraisals, higher levels of motivation, more frequent use of learning strategies, and better academic performance. The negative emotions of anger, anxiety, shame, hopelessness, and boredom, on the other hand, were in general negatively associated with control and value appraisals, motivation, and performance. This pattern of results is in line with research on the domain-general AEQ (Pekrun et al., 2011) and previous work investigating the AEQ-M (e.g., Frenzel, Pekrun, & Goetz, 2007; Putwain et al., 2018).

However, there are noteworthy exceptions from this general pattern. Anxiety, shame, and hopelessness correlated positively with declarative repetition, and anxiety and shame correlated positively with external regulation of learning. These negative emotions might prompt students to use more rigid study strategies and to seek help in order to prevent failure. Anxiety may stimulate learning and performance by promoting extrinsic motivation (e.g., Bieleke et al., 2022), but hamper self-regulation and performance by overtaxing cognitive resources (e.g., through processing worry cognitions; Roos, Goetz, Krannich, et al., 2021), resulting in variable associations with performance (Pekrun, 2018). Interestingly, anger and boredom were negatively associated with perceived positive value of achievement, whereas anxiety, shame, and hopelessness did not relate to positive achievement value. The negative link between value and boredom is in line with CVT propositions (i.e., boredom is generally linked to low levels of value; Pekrun, 2006).

Table 3

Latent Correlations of Emotions with Appraisals, Motivation, Strategies, and Performance.

Emotion	Control and Value Appraisals				Motivation			Learning Strategies				Performance
	Academic self-concept	Self-efficacy	Positive Value of Achievement	Interest	Intrinsic motivation	Achievement motivation	Effort	Elaboration	Rehearsal	Self-regulation	External regulation	Grade
Enjoyment	**.73**	**.70**	**.46**	**.91**	**.93**	**.40**	**.62**	**.63**	**.26**	**.71**	**.49**	**.46**
Pride	**.67**	**.66**	**.44**	**.80**	**.78**	**.40**	**.57**	**.62**	**.30**	**.69**	**.49**	**.43**
Anger	**-.56**	**-.56**	**-.27**	**-.57**	**-.60**	**-.25**	**-.39**	**-.33**	.04	**-.43**	**-.16**	**-.38**
Anxiety	**-.60**	**-.60**	.05	**-.34**	**-.39**	-.05	**-.11**	**-.12**	**.27**	**-.26**	**.11**	**-.45**
Shame	**-.41**	**-.45**	.05	**-.15**	**-.22**	-.05	**-.12**	-.02	**.28**	**-.21**	**.16**	**-.35**
Hopelessness	**-.65**	**-.64**	-.09	**-.36**	**-.43**	**-.17**	**-.25**	**-.18**	**.19**	**-.33**	.01	**-.49**
Boredom	**-.37**	**-.36**	**-.40**	**-.61**	**-.64**	**-.35**	**-.56**	**-.44**	**-.21**	**-.42**	**-.32**	**-.17**

Note. N = 781. Higher numbers in grade indicate better performance; statistically significant coefficient at α = .05 are shown in boldface.

3 Study 2

In Study 2, we developed extended scales for the assessment of enjoyment, anger, anxiety, hopelessness, and boredom that systematically covered all four components (i.e., affective, cognitive, motivational, and physiological/expressive). We focused on these five emotions because they play a pivotal role in research on emotions in mathematics. This allowed us to establish the overall structural validity of the AEQ-M by examining the robustness of the confirmatory factor analyses conducted in Study 1. More importantly, it enabled us to further investigate aspects of the structural validity of each emotion scale. In line with research on the domain-general AEQ (Pekrun et al., 2011), we expected that models with four correlated components (i.e., four-component models) and models with four second-order components governed by a higher-order factor representing the emotion (i.e., hierarchical models) would demonstrate better fit to the data than models with a single factor representing the emotion (i.e., single-factor models). The former two models represent the idea that emotions are sets of interrelated affective, cognitive, motivational, and physiological/expressive components, one of the assumptions underlying the AEQ-M (e.g., Lange & Zickfeld, 2021; Scherer, 2009). The latter model represents the idea that emotions are unitary constructs with no distinguishable components.

3.1 Methods

3.1.1 Sample

This study draws upon a sample of 699 German secondary school students (56.9% female, 41.1% male) from Grades 7 and 9 (7: n = 83, 9: n = 616) with a mean age of M = 14.0 years (SD = 0.9). Students attended three different tracks referred to as Hauptschule (lowest track; n = 205 in Grade 9), Realschule (middle track; n = 83 in Grade 7, n = 203 in Grade 9), and Gymnasium (highest track; n = 208 in Grade 9).

3.1.2 Measures

The construction of items for the extended AEQ-M scales was based on the same qualitative interviews and pilot studies that have been used to construct the AEQ-M (Goetz, 2004; Molfenter, 1999; Titz, 2001). All questionnaires were administered in a paper-and-pencil format and students provided their answers on 5-point Likert Scales (1 = *not true at all*, 2 = *hardly true*, 3 = *somewhat true*, 4 = *largely true*, 5 = *exactly true*).

3.1.3 Missing Data

A total of 0.27% of data were missing, stemming from 141 incomplete records. The percentage of missing values across all variables ranged from 0.00% to 1.14%. Full information maximum likelihood (FIML) was used to deal with missing data (see Enders, 2010).

Achievement Emotions. The achievement emotions of enjoyment, anger, anxiety, boredom, and hopelessness were assessed with a total of 125 items. Specifically, the corresponding scales of the Achievement Emotions Questionnaire – Mathematics (AEQ-M; Pekrun et al., 2005) were supplemented by several newly developed items to represent each emotion in terms of all four components (e.g., the affective, cognitive, motivational, and physiological/expressive components of class-related boredom). As in the AEQ-M, boredom was measured only in class and learning contexts and hopelessness was measured only in test contexts. An overview of all items of the extended AEQ-M scales is provided in Appendix 3. We refer to these scales as the extended AEQ-M scales to avoid confusion with the original AEQ-M scales.

3.1.4 Analytic Strategy

In line with Study 1, a series of confirmatory factor analyses (Brown, 2015) was conducted to investigate the structural relationships between emotions. First, the same set of four CFA models as in Study 1 were estimated. Second, the component structure was investigated for each of the five

emotions and three contexts by estimating three CFA models for each combination of emotion and context: A model with one general factor across all components (M1), a model with four factors representing each component, and a second-order factor model based on the four factors representing each component (M3). This two-step approach facilitates comparisons between Studies 1 and 2 as well as with previous research on the validation of the domain-general AEQ (e.g., Bieleke et al., 2021; Pekrun et al., 2011), which has taken an analogous approach to examine the structural validity of the AEQ. Measurement models were evaluated using the fit indices CFI, TLI, RMSEA, and SRMR based on common cut-off criteria (see Kline, 2016). In addition, Bayesian Information criterion (BIC) was used to select among competing models, where a lower BIC value indicates a better trade-of between model fit and complexity.

Models were estimated with Mplus 8.4 (Muthén & Muthén, 1998-2017) utilizing robust maximum likelihood estimation (MLR), as the chi-square test statistic and standard errors needed to take into account the non-independence of observations due to students nested in classrooms.

3.2 Results and Discussion

Descriptive statistics and manifest correlations of the extended AEQ-M scales are presented in Table 4. In line with the results of Study 1, we observed higher levels of positive than negative emotions, sufficient variation in item scores, and low levels of skewness and kurtosis. All seven scales displayed good to very good reliability, $.91 \leq \alpha \leq .96$. The negative emotions of anger, anxiety, hopelessness, and boredom were positively correlated, $.31 \leq r \leq .84$, and negatively correlated with enjoyment, $-.26 \leq r \leq -.61$.

Table 4

Descriptive Statistics and Zero-Order Correlations of the AEQ-M Scales

								Correlations			
Scale	Items	*M*	*SD*	Skew	Kurtosis	$\bar{r}_{i(t-i)}$	α	1	2	3	4

1. Enjoyment	30	2.44	0.67	0.47	0.04	.57	.94				
2. Anger	31	2.24	0.80	0.75	0.09	.62	.95	−.47			
3. Anxiety	33	2.15	0.72	0.76	0.23	.60	.95	−.26	.65		
4. Hopelessness	22	2.08	0.89	0.85	0.02	.68	.91	−.42	.70	.84	
5. Boredom	9	2.82	0.99	0.34	−0.75	.72	.96	−.61	.69	.31	.46

Note. $\bar{r}_{i(t-i)}$ denotes the average part-whole corrected item-total correlation. Items were answered on 5-point Likert Scales (1 = *not true at all,* 5 = *exactly true*). All correlation coefficients were statistically significant at α = .05.

3.2.1 Structural Relationships

To examine structural relationships between emotions, we estimated the same set of four CFA models as in Study 1 (Table 5). The CFA model representing the two-facet structure of the instrument (i.e., five emotions nested within three contexts, M4) again provided the best fit to our data, and the best trade-of between model fit and complexity among the four competing models. The model fit according to CFI, TLI, and SRMR was acceptable, whereas the RMSEA exceeded the threshold for acceptable model fit ($\chi^2(27) = 213.04$, CFI = .969, TLI = 0.925, RMSEA = 0.099, SRMR = 0.046).

3.2.2 Component Structure

As expected, the component factor and the hierarchical models fit our data well (Table 6) and, according to the Bayesian Information Criterion (BIC), were superior to single-factor models in all cases except learning-related anger and boredom. In these latter two cases, however, the fit of the component factor and the hierarchical models were also very good. In general, the best fitting models provided acceptable fit to the data in absolute terms with only few exceptions (e.g., enjoyment). Overall, this pattern of results suggests that achievement emotions measured with the extended AEQ-M scales capture the component structure predicted by the control-value theory.

Table 5

Confirmatory Factor Analysis: Model Comparison

Model	χ^2	df	CFI	TLI	RMSEA	SRMR	BIC
M1: One-Emotion Factor Model	2779.00	54	.552	0.452	0.269	0.146	17110.11
M2: Five-Emotion Factor Model	808.43	45	.874	0.816	0.156	0.056	15040.46
M3: Three-Context Factor Model	2761,59	51	.554	0.423	0.276	0.148	16984.23
M4: Five-Emotion x Context Factor Model	**213.04**	**27**	**.969**	**0.925**	**0.099**	**0.046**	**14491.45**

Note. *N* = 699. Five emotions are enjoyment, anger, anxiety, hopelessness, and boredom. Three contexts are class, learning, and test. Model selected by the BIC is shown in boldface.

Table 6

Emotion Component Structure of AEQ Scales: Confirmatory Factor Analysis

Emotion	Model	Class-related emotions							Learning-related emotions							Test-related emotions						
		χ^2	df	CFI	TLI	RMSEA	SRMR	BIC	χ^2	df	CFI	TLI	RMSEA	SRMR	BIC	χ^2	df	CFI	TLI	RMSEA	SRMR	BIC
Enjoyment	M1	169.63	35	.944	0.928	0.074	0.042	17237.58	413.24	35	.792	0.733	0.124	0.086	19115.42	455.91	35	.818	0.767	0.131	0.082	19575.14
	M2	89.57	29	.975	0.961	0.055	0.031	17167.59	**197.49**	**29**	**.907**	**0.856**	**0.091**	**0.054**	**18848.05**	**219.42**	**29**	**.918**	**0.873**	**0.097**	**0.058**	**19355.64**
	M3	**92.72**	**31**	**.974**	**0.963**	**0.053**	**0.031**	**17163.05**	240.93	31	.885	0.833	0.098	0.066	18902.21	247.44	31	.907	0.864	0.100	0.065	19382.49
Anger	M1	121.79	44	.966	0.957	0.050	0.030	20295.23	**120.59**	**36**	**.960**	**0.951**	**0.058**	**0.035**	**21190.63**	154.89	35	.944	0.928	0.070	0.039	19452.87
	M2	91.28	38	.977	0.966	0.045	0.027	20288.66	103.68	29	.965	0.946	0.061	0.031	21213.07	**73.90**	**29**	**.979**	**0.967**	**0.047**	**0.025**	**19382.14**
	M3	**93.22**	**40**	**.977**	**0.968**	**0.044**	**0.028**	**20283.83**	99.00	31	.968	0.954	0.056	0.031	21203.55	90.80	31	.972	0.959	0.053	0.030	19396.28
Anxiety	M1	344.87	54	.879	0.852	0.088	0.054	20727.80	308.45	44	.863	0.829	0.093	0.059	19992.19	203.40	35	.937	0.919	0.083	0.039	20936.47
	M2	**212.82**	**48**	**.931**	**0.906**	**0.070**	**0.044**	**20572.89**	**175.49**	**38**	**.929**	**0.897**	**0.072**	**0.044**	**19806.99**	**49.07**	**29**	**.992**	**0.988**	**0.031**	**0.018**	**20782.33**
	M3	247.87	50	.918	0.891	0.075	0.050	20622.95	203.14	40	.916	0.884	0.076	0.050	19852.66	64.86	31	.987	0.982	0.040	0.023	20788.29
Hopelessness	M1															95.44	27	.968	0.957	0.060	0.028	16664.80
	M2															65.98	21	.979	0.964	0.055	0.024	16661.98
	M3															**69.20**	**23**	**.978**	**0.966**	**0.054**	**0.025**	**16654.23**
Boredom	M1	251.61	54	.953	0.943	0.072	0.032	22378.91	**94.60**	**35**	**.980**	**0.974**	**0.049**	**0.022**	**20108.36**							
	M2	**215.21**	**48**	**.961**	**0.946**	**0.071**	**0.031**	**22368.67**	76.85	29	.984	0.975	0.049	0.020	20121.73							
	M3	228.10	50	.958	0.945	0.071	0.031	22371.55	82.52	31	.983	0.975	0.049	0.021	20120.89							

Note. $N = 699$. Models selected by the BIC are shown in boldface.

4 Discussion

The Achievement Emotions Questionnaire – Mathematics (AEQ-M) is an important instrument for assessing a broad range of emotions in mathematics. Despite its popularity and frequent use in research on mathematics education, however, the instrument has yet to be published and evidence for several underlying assumptions is either missing or scattered across the literature. Specifically, there is a dearth of evidence for the context-dependence of mathematics-related emotions (i.e., emotions differ between class, learning, and test contexts), their component structure (i.e., emotions reflect a set of interrelated affective, cognitive, motivational, and physiological/expressive processes), and their associations with antecedents (control, value) and outcome variables (motivation, learning strategies, achievement) assumed by the control-value theory of achievement emotions (CVT; Pekrun, 2006). In the present research, we capitalized on both the data originally used to develop the AEQ-M (Study 1) and additional data (Study 2) to scrutinize the validity of these assumptions. Regarding the structural validity of the AEQ-M (i.e., context-dependency, component structure), both studies provided evidence that mathematics-related emotions assessed with the AEQ-M are indeed context-specific. As such, emotions measured in one context might differ from emotions measured in another context (e.g., students might experience more anxiety in tests than in classes). Moreover, Study 2 suggests that mathematics-related emotions are best understood as reflecting a set of interrelated processes. For instance, experiencing anxiety means that students feel anxious (affective), are worried (cognitive), want to leave (motivational), and get queasy (physiological/expressive). These findings corroborate and extend previous research that has investigated some of these assumptions about the internal structure of mathematics-related emotions (e.g., context-dependency in a Portuguese version of the AEQ-M; Moreira et al., 2019). Moreover, they provide a novel set of extended AEQ-M

scales for assessing enjoyment, anger, anxiety, hopelessness, and boredom in mathematics education research.

Regarding the external validity of the AEQ-M, the results of Study 1 showed the theoretically predicted associations between mathematics-related emotions and their core antecedents and outcomes. For instance, students who reported higher levels of control (e.g., higher self-efficacy) and positive value (e.g., higher interest) also reported higher levels of positive emotions and lower levels of negative emotions. In turn, higher levels of positive emotions and lower levels of negative emotions were linked to higher motivation (e.g., effort), different learning styles (e.g., more self-regulation), and higher performance (i.e., better grades). This aligns well with previous research demonstrating the influence of control and value appraisals on mathematics-related emotions (e.g., Frenzel, Pekrun, & Goetz, 2007) and the effect of these emotions on performance (e.g., Pekrun et al., 2017). Besides these main findings, it is noteworthy that the relationship between achievement emotions and performance were substantial (e.g., the latent correlation between enjoyment and grades was $r = .46$). Across both studies, the AEQ-M scales demonstrated good reliability (i.e., it allows to measure emotions precisely; Cronbach's α ranging from .84 to .96) and were correlated with each other in meaningful ways (e.g., higher levels of one negative emotion were associated with higher levels of other negative emotions; $.14 \leq |r| \leq .86$).

Our findings are of great relevance for mathematics education for several reasons. Mathematics is a core subject in school curricula around the world and commonly accompanies students through their entire school life and beyond, especially in STEM-related occupational careers, but also more generally in understanding science and the world (e.g., statistics about diseases and health behavior related to the Covid-19 pandemic). Understanding the emotions students experience in mathematics is therefore of paramount importance (Schukajlow et al., 2017), not only because the

emotional experiences of students in mathematics class should be studied as an outcome variable in itself, but also because emotions are an important predictor of mathematics achievement (Kim et al., 2014). A psychometrically sound, comprehensive, and valid instrument for assessing emotions in mathematics is therefore indispensable, with the need for such an instrument already demonstrated by existing research capitalizing on the AEQ-M. For instance, the AEQ-M has been used to examine the sources of gender differences in mathematics anxiety (Frenzel, Pekrun, & Goetz, 2007) and to investigate the effects of different special education support measures in mathematics on student's emotions (Holm et al., 2020).

The extended AEQ-M scales developed in Study 2 will allow researchers to systematically measure the different components of achievement emotions (i.e., the affective, cognitive, motivational, and physiological/expressive processes of which emotions are composed). In future studies, it could thus be investigated whether these components are differentially affected by control and value appraisals and whether there are differences in the associations with performance. For instance, the cognitive component of mathematics anxiety (e.g., worries) might be more strongly affected by low levels of perceived control and more substantially associated with higher levels of performance than other components of anxiety (Roos, Goetz, Krannich, et al., 2021; see also Barroso et al., 2021). This would have important practical implications for mathematics education, as it might guide the design of interventions (e.g., strengthening self-efficacy beliefs to reduce anxiety).

In addition, the extended AEQ-M scale may inform psychometric research, as the results from studies may allow researchers to compare the components of the AEQ-M with the use of single-item measures to capture achievement emotions (Gogol et al., 2014). Moreover, the systematic coverage of emotion components in the extended AEQ-M scales permits an examination of the interplay of these components across different emotions (e.g., anxiety and boredom might share similar motivational

processes like the urge to leave a situation; Lange & Zickfeld, 2021). This would again greatly benefit mathematics education by identifying possible synergy effects among interventions.

In the present research, we developed extended AEQ-M scales to systematically cover all theoretically assumed components of emotions. However, the expanding of a scale may be a double-edged sword, as adding items to scales can be beneficial in terms of increasing reliability and ensuring that all relevant aspects of a construct may be captured, but it may also render the scales less convenient to administer (e.g., increasing time, decreasing compliance). This is particularly relevant for repeated assessments, for instance, in the context of experience sampling studies (Goetz et al., 2016). Experience sampling assesses emotions at the moment of their experience, which is increasingly used to study students' and teachers' emotions in the domain of mathematics (e.g., Bieg et al., 2017). A complementary approach would be the development of scales with fewer items that still cover each emotion component, which would address the balance between brevity and comprehensiveness. In terms of the domain-general AEQ, such a short-form has already been developed (the AEQ-S uses four items to measure each component of an emotion; Bieleke et al., 2021). It would therefore be possible to develop similar short-form versions of the AEQ-M, as adopting an already domain-specific questionnaire to a different domain may be less complex than adopting versions of a domain-specific questionnaire like the AEQ.

There are several substantive research questions that could be addressed with the AEQ-M. For instance, it would be important to examine differences in students' emotions regarding various domains of mathematics (e.g., analysis versus geometry). In addition, the association between emotions assessed with the AEQ-M and alternative indicators of emotions (e.g., physiological measures; automated facial expression analysis) could be assessed. Finally, the AEQ-M could be

systematically used to conduct cross-cultural research (see Frenzel, Thrash, et al., 2007) and to evaluate the effectiveness of interventions in mathematics.

There are also some limitations of the present research that should be considered when interpreting our findings. Firstly, both studies are based on samples from German secondary schools. And while there already is evidence on the invariance of the AEQ-M across cultures (Frenzel, Thrash, et al., 2007), it would be desirable to investigate whether our results generalize to other age groups and educational settings (e.g., university students). This seems particularly important for the newly developed extended AEQ-M scales. Relatedly, we did not focus on gender differences as CVT assumes structural equivalence across gender – for instance, the association between control and value, and achievement emotions should be similar across female and male students (e.g., Pekrun et al., 2007; for empirical evidence, see Frenzel, Pekrun, & Goetz, 2007). Moreover, measurement equivalence of the AEQ-M across genders has already been demonstrated (e.g., Moreira et al., 2019).

Secondly, our validation of the AEQ-M in terms of the core antecedents and outcomes of achievement emotions relied on self-reports. This mirrors the approach commonly taken in related research on achievement emotions (Bieleke et al., 2021; Pekrun et al., 2011), however it should still be complemented by more objective measures in future research. For example, physiological measures provide information beyond self-report and could be used to further validate the AEQ-M (Roos, Goetz, Voracek, et al., 2021).

Thirdly, the fit of models representing different component structures of emotions did not always meet the thresholds recommended in the literature (e.g., for the learning- and test-related enjoyment scales). This may indicate a need for further refinement of these scales in future research, especially when the focus is on differentiating between the different components of emotions. However, it should be noted that these recommended cut-off criteria were derived from simulated

datasets and are often not met with data sets derived from more complex studies, suggesting that they should be used with caution (Heene et al., 2011; Marsh et al., 2004). Relatedly, future research might use different analytic approaches to investigate the assumptions behind the AEQ-M. For instance, the context-dependency of mathematics-related emotions and their component structure could be jointly examined in one comprehensive model rather than in two separate steps.

Fourthly, our data in Study 1 is correlational and does not allow us to draw causal inferences about the relations between achievement emotions and their antecedents and outcomes, or to capture mediated relations between these constructs. While the observed correlations are in line with CVT propositions, experimental or longitudinal data are necessary to examine the causal effects generating these correlations (see, e.g., Forsblom et al., 2022; Pekrun et al., 2017).

5 Conclusion

Across two independent studies, we examined the internal structure and external relations of mathematics-related emotions measured with the AEQ-M, a widely used instrument that has not been published yet and that lacked dedicated evaluations of the validity of its assumptions. Our results indicate that the structural properties of the AEQ-M correspond closely to predictions that can be derived from the control-value theory of achievement emotions and are similar to those observed for achievement emotions in other school domains. Specifically, mathematics-related emotions depend on the academic context in which they occur (i.e., class, learning, and test), represent a set of interrelated psychological processes (i.e., affective, cognitive, motivational, and physiological/expressive components), and are linked to their assumed antecedents (control, value) and outcomes (motivation, learning strategy, achievement). We introduced a set of extended AEQ-M scales that researchers in mathematics education can use to conduct a valid, reliable, and systematic examination of the component structure of several mathematics-related emotions in future studies.

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7 Appendix 1: AEQ-M Instructions

Class-Related Experiences

"Attending classes in mathematics can induce different feelings. This part of the questionnaire refers to emotions you may experience when being in mathematics classes. Before answering the following questions, please recall some typical situations of being in a mathematics class which you have experienced."
(1) Before Class : "The following questions pertain to feelings you may experience BEFORE attending mathematics classes. Please indicate how you feel, typically, before you go to a mathematics class."
(2) During Class: "The following questions pertain to feelings you may experience DURING mathematics classes. Please indicate how you feel, typically, during mathematics classes."
(3) After Class: "The following questions pertain to feelings you may experience AFTER having attended a mathematics class. Please indicate how you feel, typically, after mathematics classes."

Learning-Related Experiences

"Studying and doing homework assignments in mathematics can induce different feelings. This part of the questionnaire refers to emotions you may experience when studying and doing homework in math. Before answering the following questions, please recall some typical situations of studying and doing assignments in mathematics which you have experienced."
(1) Before Studying: "The following questions pertain to feelings you may experience BEFORE studying and doing homework in mathematics. Please indicate how you feel, typically, before you begin to study or do an assignment in math."
(2) During Studying: "The following questions pertain to feelings you may experience DURING studying and doing homework in mathematics. Please indicate how you feel, typically, during studying and doing assignments in math."
(3) After Studying: "The following questions pertain to feelings you may experience AFTER having studied or done homework in mathematics. Please indicate how you feel, typically, after having studied or done assignments in math."

Test-Related Experiences

"Test and exams in mathematics can induce different feelings. This part of the questionnaire refers to emotions you may experience when taking tests or exams in math. Before answering the following questions, please recall some typical situations of taking tests or exams in mathematics which you have experienced."
(1) Before taking the test/exam: "The following questions pertain to feelings you may experience BEFORE taking a test or an exam in mathematics. Please indicate how you feel, typically, before taking a test or an exam in math."
(2) During the test/exam: "The following questions pertain to feelings you may experience DURING taking a test or an exam in mathematics. Please indicate how you feel, typically, during taking a test or an exam in math."
(3) After taking the test/exam: "The following questions pertain to feelings you may experience AFTER taking a test or an exam in mathematics. Please indicate how you feel, typically, after taking a test or an exam in math."

8 Appendix 2: AEQ-M Scales

Enjoyment

I look forward to my mathematics class.
I enjoy my mathematics class.
The material we deal with in mathematics is so exciting that I really enjoy my class.
I enjoy my class so much that I am strongly motivated to participate.
When doing my mathematics homework, I am in a good mood.
I am happy that I understand the material.
I enjoy doing my mathematics homework so much that I am motivated to do extra assignments.
I enjoy taking tests in mathematics.
Because I look forward to getting a good grade, I study hard for the test.
I think that things are going great.

Pride

I think I can be proud of my knowledge in mathematics.
I am proud of my contributions to the mathematics class.
After having done my mathematics homework, I am proud of myself.
I am very motivated because I want to be proud of my achievements in mathematics.
After a mathematics test, I am proud of myself.
I am proud of how well I have done on the math test.

Anger

I am annoyed during my mathematics class.
I am so angry during my mathematics class that I would like to leave.
I get angry because the material in mathematics is so difficult.
I get irritated by my mathematics class.
My mathematics homework makes me angry.
I get angry because my mathematics homework occupies so much of my time.
I am so angry that I would like to throw my homework into the thrash.
I am so angry that I would like to tear the exam paper into pieces.
I am annoyed that the teacher asks such difficult questions.

Anxiety

When thinking about my mathematics class, I get nervous.
I worry if the material is much too difficult for me.
When thinking of my mathematics class, I get queasy.
Mathematics scares me so much that I would rather not attend school.
I worry whether I will ever be able to completely understand the material.
I start sweating because I am worried I cannot complete my assignments in time.
I am tense and nervous.
I'm so scared of my mathematics assignments that I would rather not start them.
When taking the mathematics test, I am tense and nervous.
When taking the mathematics test, I worry I will get a bad grade.
I am very nervous.
Even before I take the mathematics test I worry I could fail.
I am so anxious that I would rather not take the mathematics test.
When I have an upcoming mathematics test, I get sick to my stomach.
I am so anxious that I can't fully concentrate.

Shame

When I say something in my mathematics class, I can tell that my face gets red.
I am ashamed that I cannot answer my mathematics teacher's questions well.
When I say something in my mathematics class, I feel like embarrassing myself.
I am embarrassed about my lack of knowledge in mathematics.
When I don't understand something in my mathematics homework, I don't want to tell anybody.
When I discuss the homework assignments with my classmates, I avoid eye contact.
After taking a test in mathematics, I feel ashamed.
I start sweating because my performance on the mathematics exam embarrasses me.

Hopelessness

I feel down.
During the mathematics test, I feel hopeless.
I keep thinking that I don't understand the material.
I keep thinking that I will never get good grades in mathematics.
I would prefer to give up.
I have no energy.

Boredom

I think the mathematics class is boring.
I can't concentrate because I am so bored.
I am so bored that I can't stay awake.
Just thinking of my mathematics homework assignments makes me feel bored.
My mathematics homework bores me to death.
I'm so bored that I don't feel like studying any more.

9 Appendix 3: Extended AEQ-M Scales

Items highlighted in italics are also part of the AEQ-M. Note that German items were used in Study 2 and the following table provides their translation to English.

Enjoyment

I look forward to my mathematics class.
I enjoy my mathematics class.
The material we deal with in mathematics is so exciting that I really enjoy my class.
I enjoy my class so much that I am strongly motivated to participate.
I am cheerful in my mathematics class.
I look forward to learning a lot in my mathematics class.
I am happy that I understood the material.
I could listen enthusiastically for hours.
In my mathematics class, I feel my heart pounding with joy.
I smile at my teacher with joy about my mathematics class.

When doing my mathematics homework, I am in a good mood.
I am happy that I understand the material.
I enjoy doing my mathematics homework so much that I am motivated to do extra assignments.
In math, I look forward to doing the homework.
I enjoy the homework in math.
I am happy that the assignments are so exciting.
I am happy that the mathematics homework did not cause me any problems.
Because I enjoy math, I engage in it more than I need to.
When it goes well, I feel my heart pounding with joy.
When mathematics homework went well, I beam with joy.

I enjoy taking tests in mathematics.
Because I look forward to getting a good grade, I study hard for the test.
I think that things are going great.
Mathematics tests are tests that I enjoy.
I am happy that I can show what I have learned right away.
I think I did pleasingly well on the mathematics test.
When I notice that I am doing well, I try harder.
When the mathematics test goes well, I smile with joy.
I am so happy that I feel warm.
I feel my heart beat faster with joy.

Anger

I am annoyed during my mathematics class.
I am so angry during my mathematics class that I would like to leave.
I get angry because the material in mathematics is so difficult.
I get irritated by my mathematics class.
I am upset.

My mathematics class really makes me angry.
I am irritated with the mathematics teacher.
I would like to take my anger out on my classmates.
I am getting hot with anger.
I am boiling with rage inside.
In my mathematics class, I look irritated.

My mathematics homework makes me angry.
I get angry because my mathematics homework occupies so much of my time.
I am so angry that I would like to throw my homework into the thrash.
I get angry when I do mathematics homework.
I get angry that I have to do so much homework in math.
I would prefer not to start at all out of anger about the mathematics homework.
The mathematics homework makes me so angry that I would like to stop doing it.
Even before I do the mathematics homework, I get upset with anger.
When I have to sit and do mathematics homework for a long time, I get all restless with anger.
When I have done mathematics homework for a long time, I get all tense with anger.

I am so angry that I would like to tear the exam paper into pieces.
I am annoyed that the teacher asks such difficult questions.
I am angry.
After the mathematics test, I feel a real anger in my stomach.
After a test, I'm angry.
I feel angry about how the mathematics test went.
I wish my mathematics teacher would get lost.
I notice how my fists clench in anger during the mathematics test.
I feel my blood rush to my head with anger.
I'm so angry that I feel hot all over.

Anxiety

When thinking about my mathematics class, I get nervous.
I worry if the material is much too difficult for me.
When thinking of my mathematics class, I get queasy.
Mathematics scares me so much that I would rather not attend school.
I am afraid of my math class.
I am nervous in my mathematics class.
Even before my math class, I worry if I will understand enough.
I worry if I will understand less than the others.
Because I'm afraid I won't be able to follow the mathematics class, I try extra hard.
Because I am afraid of saying something wrong, I would prefer not to speak at all.
I am very anxious because of my fear.
In my mathematics class, I start to sweat from fear.

I worry whether I will ever be able to completely understand the material.
I start sweating because I am worried I cannot complete my assignments in time.
I am tense and nervous.

I'm so scared of my mathematics assignments that I would rather not start them.
I am afraid of mathematics assignments.
When I get stuck on my mathematics assignments, I get scared.
I worry that my mathematics assignments will be too difficult for me.
I worry if I will ever get it right.
Because I am afraid that I won't be able to do my mathematics assignments, I try even harder.
When I do my mathematics assignments, I have to get a funny feeling in my stomach from nervousness.
When I can't complete the mathematics assignments, my heart pounds from anxiety.

When taking the mathematics test, I am tense and nervous.
When taking the mathematics test, I worry I will get a bad grade.
I am very nervous.
Even before I take the mathematics test I worry I could fail.
I am so anxious that I would rather not take the mathematics test.
When I have an upcoming mathematics test, I get sick to my stomach.
I am so anxious that I can't fully concentrate.
I worry that the tasks are too hard for me.
I am so scared that I wish I were far away.
During a mathematics test, my hands get clammy.
When I am quizzed, I have wobbly knees.
I am so nervous that I can't remember what I've learned.

Hopelessness
I feel down.
During the mathematics test, I feel hopeless.
I keep thinking that I don't understand the material.
I keep thinking that I will never get good grades in mathematics.
I would prefer to give up.
I have no energy.
Because I feel so hopeless, I don't make any more effort.
I feel paralyzed.
I almost have to cry because I don't know what to do.

Boredom
I think the mathematics class is boring.
I can't concentrate because I am so bored.
I am so bored that I can't stay awake.
I am bored.
In my mathematics class, my mind is often somewhere else.
Being bored, I think that I don't really care about the material.
When I am bored, I think to myself: If only the mathematics class were already over!
In my mathematics class I daydream.
I constantly look at the clock, because the time does not pass.
I try to distract myself out of boredom.
I get restless because I am just waiting for the mathematics class to finally be over.

I notice how I sink down in my chair from boredom.

Just thinking of my mathematics homework assignments makes me feel bored.
My mathematics homework bores me to death.
I'm so bored that I don't feel like studying any more.
I'm thinking about how boring the mathematics homework is again today.
My mind is somewhere else entirely.
I am bored thinking to myself that there is not much point in doing these assignments.
The assignments are so boring that I don't feel like starting.
Out of boredom, I would rather do something I enjoy.
When doing mathematics homework, I get tired quickly from boredom.
I have to yawn because of boredom.