Recurrent MIMO CoFAR Probing Waveform Design for Learning-Aided Clutter Estimation

Kunwar Pritiraj Rajput[†,1], M. R. Bhavani Shankar[†,2], Kumar Vijay Mishra[‡,3],
Muralidhar Rangaswamy[+,4] and Björn Ottersten[†,5]
[†]Interdisciplinary Centre for Security, Reliability and Trust, University of Luxembourg
[‡]United States CCDC Army Research Laboratory, Adelphi, Maryland, USA
[+]United States Air Force Research Laboratory, Wright-Patterson Air Force Base, Ohio, USA
Email: {[1]kunwar.rajput, [2]bhavani.shankar, [5]bjorn.ottersten}@uni.lu, [3]kvm@ieee.org, [4]muralidhar.rangaswamy@us.af.mil

***Abstract*—A cognitive fully adaptive radar system (CoFAR) represents an advanced radar architecture predicated on the principles of sensing, learning, and adaptation. However, the efficacy of such an adaptive radar system hinges upon a comprehensive understanding of its operational environment. The presence of unwanted echoes stemming from clutter can significantly impede the performance of a CoFAR. To address this challenge, this study introduces a sparse Bayesian learning framework aimed at estimating the underlying joint sparse clutter channel impulse response. Additionally, a recurrent transmit probing waveform is devised to minimize the resultant mean square error in subsequent iterations. Leveraging the majorization-minimization framework, we derive a closed-form expression for the overall waveform design vector. Waveform optimization brings the radar a step closer to cognitive mode of operation, enabling it to adaptively learn and adjust its parameters in response to changing environmental conditions. Extensive numerical simulations validate our analytical formulations and illustrate the superior performance of the proposed methodology compared to scenarios where optimal waveform design is not implemented.**

***Index Terms*—Bayesian Cramér-Rao bound, cognitive fully adaptive radar, majorization-minimization, RFView, sparse Bayesian learning.**

I. Introduction

A radar system known as cognitive fully adaptive radar (CoFAR) strategically optimizes both its adaptive transmit and receive functions [1]–[5] based on acquired knowledge of the target environment. The conventional radars work on Feed-forward mechanism, on the contrary, the CoFAR system uses the feedback mechanism and employs various optimization criterion to improve its sensing efficiency, optimizing for various outcomes like target detection [6] and tracking [7]. In [8], authors have shown a hardware implementation of CoFAR system capable of tracking a target in a non-linear range measurement model.

In order to understand the target environment properly, a key factor is the estimation of the complex clutter [9], [10], which includes unwanted echoes like ground reflections, atmospheric effects, and electromagnetic interference [11]–[15]. Clutter can severely deteriorate the radar's detection and estimation performance [16]. The conventional clutter model [17], [18] shows a non-linear relationship between signal-dependent covariance and the transmit radar waveform, which results in inaccurate clutter covariance matrix estimation. In order to alleviate this, [19] introduced a multi-input multi-output (MIMO) clutter model with a "stochastic transfer function" approach that mitigates the non-linear dependence. Since, the transfer function approach results in signal-independence, and hence joint optimization of transmit waveform and receiver filters is possible.

The authors in [19] developed a new MIMO CoFAR model for a CoFAR. Furthermore, joint optimization of space-time waveform and receiver function is carried out. Furthermore, a low-complexity frequency-domain approach outlined in [20] was introduced for channel matrix estimation. Integrating this into the optimization aligns with CoFAR's objective of concurrently designing radar transmit-receive functions, offering a straightforward and real-time implementable technique. However, achieving accurate channel impulse response (CIR) estimates at low signal-to-noise Ratio (SNR) levels may require probing signals with substantial pulse-widths, an allocation of CoFAR resources that might be undesirable.

For optimal CoFAR resource utilization, leveraging structural details or relationships among nearby channel transfer functions during estimation proves advantageous. For example, demonstrated in [21], [22], exploiting the strong correlation in CIRs of neighboring pulses allows a constrained maximum likelihood (ML) estimate with a cosine similarity constraint, aiding in CIR estimation. In [23], a Constrained CIR estimation (CCIRE) was introduced, incorporating both cosine similarity constraint and CIR sparsity. However, the resulting optimization posed computational challenges, leading to a computationally expensive semi-definite program (SDP). Further, authors in [24] proposed a sparse Bayesian learning (SBL)-based CCIRE in single-input multiple-output (SIMO) CoFAR which does not require apriori knowledge about the sparsity profile of the underlying sparse channel. Taking this paradigm further, work in [25] deals with CCIRE in a MIMO CoFAR system. It is based on multiple measurement vector scenario and uses covariance free SBL (a low complexity implementation of SBL) to estimate the row sparse CCIR matrix. It also capitalizes the different sparsity scenarios like Group, Joint, and Joint-cum-Group sparsity, and develops the sparse CCIR matrix estimation algorithm for each scenario. However, to the best of our knowledge, none of the works so far have address the problem of joint CCIRE and probing waveform design in a CoFAR system.

This paper takes the sparse CCIRE scenario originally considered in [25] into account. Since, the estimation error is a function of probing waveform matrix therein, one can design the probing waveform to further reduce the estimation error and yield an improved estimate of the underlying sparse channel. We employ majorization minimization (MM) algorithm to solve the resulting optimization problem and propose a closed-form solution for the overall transmit waveform vector. Since, the transmit waveform matrix is a convolution matrix in [25], which is challenging to solve. Our numerical section demonstrate the improved performance of the proposed probing waveform

This work from the University of Luxembourg is partially supported by the grant on "Active Learning for Cognitive Radars" from the European Office of Aerospace Research & Development, part of the US Airforce Office of Scientific Research. This work was supported by the Luxembourg National Research Fund (FNR) through the SENCOM : C20/IS/14799710/SENCOM.

design against the linear frequency modulated (LFM) and random waveforms.

The remainder of the paper is organized as follows. Next section introduces the system model for MIMO CoFAR briefly followed by a brief discussion of SBL-based joint sparse CCIR technique in Section-III, followed by the majorization-minimization based probing waveform design in Section IV. Numerical experiments to validate our models and methods are presented in Section V, and the paper concludes with Section VI.

Throughout this document, vectors and matrices are indicated by lower- and upper-case boldface letters, respectively. The superscripts $*$, T, and H signify the conjugate, transpose, and Hermitian (conjugate transpose) operations, respectively. Statistical expectation and the trace operator are denoted as $\mathbb{E}\{\cdot\}$ and $\mathrm{Tr}\{\cdot\}$, respectively. The notation $\mathbf{x} \sim \mathcal{CN}(\mu, \mathbf{\Sigma})$ represents a complex vector quantity $\mathbf{x}$ with mean μ and covariance matrix $\mathbf{\Sigma}$. The i-th element of a vector quantity $\mathbf{x}$ is represented as $\mathbf{x}(i)$, while the (i,i)-th element of a matrix $\mathbf{X}$ is denoted as $\mathbf{X}(i,i)$. The notation $\otimes$ defines the Kronecker product. $\mathrm{vec}(\mathbf{X})$ represents a vector quantity which is obtained by stacking the columns of matrix $\mathbf{X}$ one below the other. $\Re\{.\}$ denotes the real part of the underlying quantity.

II. System Model

An uniform linear array (ULA) configuration based MIMO radar is considered with N_T transmit and N_R receive antennas, respectively. The received signal vector at the j-th receive antennas for the l-th transmit pulse can be modelled as [19], [26]

$$\mathbf{y}_{j,l} = \sum_{i=1}^{N_T} \mathbf{h}_{j,i,l} * \mathbf{x}_i + \mathbf{v}_{j,l}, \tag{1}$$

where $\mathbf{y}_{j,l} = [y_{j,l}[0], y_{j,l}[1], \ldots, y_{j,l}[L+R-2]]^T \in \mathbb{C}^{(L+R-1)\times 1}$, $\mathbf{x}_i = [x_i[0], x_i[1], \ldots, x_i[L-1]]^T \in \mathbb{C}^{L\times 1}$ is transmit radar signal vector from the i-th antenna and $\mathbf{h}_{j,i,l} = [h_{j,i,l}[0], h_{j,i,l}[1], \ldots, h_{j,i,l}[R-1]]^T \in \mathbb{C}^{R\times 1}$ represents the CCIR vector corresponding to the j-th receive and i-th transmit antenna and the l-th transmitted pulse with R range-bins. The stacked noise vector $\mathbf{v}_{j,l} \in \mathbb{C}^{(L+R-1)\times 1}$. Furthermore, one can also rewrite the above equation as

$$\mathbf{y}_{j,l} = \sum_{i=1}^{N_T} \mathbf{X}_i \mathbf{h}_{j,i,l} + \mathbf{v}_{j,l}, \tag{2}$$

where

$$\mathbf{X}_i = \begin{bmatrix} x_i[1] & 0 & \ldots & 0 \\ x_i[2] & x_i[1] & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ x_i[L] & x_i[L-1] & \ldots & x_i[1] \\ 0 & x_i[L] & \ldots & x_i[2] \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & x_i[L] \end{bmatrix} \in \mathbb{C}^{(L+R-1)\times R}, \tag{3}$$

denotes the transmit signal matrix from i-th antenna, and $x_i[l]$, $1 \le l \le L$ represents the waveform samples transmitted from the i-th antenna. Furthermore, stacking the received vectors $\mathbf{y}_{j,l}$ for all the $1 \le j \le N_R$ receive antennas, one obtains

$$\begin{aligned} \mathbf{y}_l &= \underbrace{(\mathbf{I}_{N_R} \otimes \mathbf{X})}_{\widetilde{\mathbf{X}} \in \mathbb{C}^{(L+R-1)N_R \times N_T N_R R}} \begin{bmatrix} \mathbf{h}_{1,l} \\ \mathbf{h}_{2,l} \\ \vdots \\ \mathbf{h}_{N_R,l} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_{1,l} \\ \mathbf{v}_{2,l} \\ \vdots \\ \mathbf{v}_{N_R,l} \end{bmatrix}, \\ \mathbf{y}_l &= \widetilde{\mathbf{X}} \mathbf{h}_l + \mathbf{v}_l, \end{aligned} \tag{4}$$

where $\mathbf{y}_l = \left[\mathbf{y}_{1,l}^T, \mathbf{y}_{2,l}^T, \ldots, \mathbf{y}_{N_R,l}^T\right]^T \in \mathbb{C}^{(L+R-1)N_R\times 1}$, $\mathbf{X} = \left[\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_{N_T}\right] \in \mathbb{C}^{L+R-1\times N_T R}$, $\mathbf{h}_{i,l} = \left[\mathbf{h}_{j,1,l}^T, \mathbf{h}_{j,2,l}^T, \ldots, \mathbf{h}_{j,N_T,l}^T\right]^T \in \mathbb{C}^{N_T R\times 1}$, $\mathbf{h}_l = \left[\mathbf{h}_{1,l}^T, \mathbf{h}_{2,l}^T, \ldots, \mathbf{h}_{N_R,l}^T\right]^T \in \mathbb{C}^{N_T N_R R\times 1}$, and $\mathbf{v}_l = \left[\mathbf{v}_{1,l}^T, \mathbf{v}_{2,l}^T, \ldots, \mathbf{v}_{N_R,l}^T\right]^T \in \mathbb{C}^{N_R(L+R-1)\times 1}$. Since very few range bins are active or present in the entire range bin domain, each clutter CIR vector $\mathbf{h}_l$ is sparse in nature. If a particular range bin continues to be populated across transmit-receive antenna pairs (co-located). Further, the channel vector for the l-th pulse, $\mathbf{h}_l \in \mathbb{C}^{N_T N_R R\times 1}$, considered in this paper is obtained by stacking across the transmit antennas and followed by the receive antennas. Thus, coupled with the aforementioned fact renders structure to the joint sparsity, which we have attempted to exploit in the next section. Finally, our goal is to estimate the sparse clutter CIR vector $\mathbf{h}_l$ from the received vector $\mathbf{y}_l$. Towards this end, we employ a Bayesian learning-based framework which is described next.

III. SBL-Based Joint Clutter CIR Estimation

In SBL framework [27], the parameterized Gaussian prior probability density function (PDF) of the unknown clutter CIR vector denoted as $\mathbf{h}_{j,i,l}$ is given as

$$p(\mathbf{h}_{j,i,l}, \boldsymbol{\psi}) = \prod_{r=1}^{R} (\pi\psi_r)^{-1} \exp\left(-\frac{|\mathbf{h}_{j,i,l}(r)|^2}{\psi_r}\right), \tag{5}$$

where the hyperparameter ψ_r represents the sparsity of the r-th range bin of each channel vector $\mathbf{h}_{j,i,l}$ for all j, i and l. Furthermore, let $\boldsymbol{\psi} = [\psi_1, \psi_2, \ldots, \psi_R]^T \in \mathbb{R}^{R\times 1}$ and $\widetilde{\mathbf{\Psi}} = \mathrm{diag}[\boldsymbol{\psi}] \in \mathbb{R}^{R\times R}$, the hyperparameter covariance matrix for this scenario is given as

$$\mathbf{\Psi} = \left(\mathbf{I}_{N_R} \otimes \left(\mathbf{I}_{N_T} \otimes \widetilde{\mathbf{\Psi}}\right)\right) \in \mathbb{C}^{N_T N_R R\times N_T N_R R} \tag{6}$$

Hence, the prior PDF of the CCIR vector $\mathbf{h}_l$ is

$$p(\mathbf{h}_l, \mathbf{\Psi}) = \prod_{i=1}^{N_T} \prod_{j=1}^{N_R} \prod_{r=1}^{R} (\pi\psi_r)^{-1} \exp\left(-\frac{|\mathbf{h}_{j,i,l}(r)|^2}{\psi_r}\right). \tag{7}$$

In order to estimate the unknown hyperparameter vector $\boldsymbol{\psi} = [\psi_1, \psi_2, \ldots, \psi_R]^T \in \mathbb{R}^{R\times 1}$, one can employ the sparse Bayesian learning (SBL) approach [27]. This method evaluates the hyperparameter vector $\boldsymbol{\psi}$ by marginalizing it over the CCIR vector $\mathbf{h}_l$. Subsequently, it performs Bayesian evidence maximization using type II maximum-likelihood (ML) optimization. In the prior assignment mentioned earlier, when the hyperparameter ψ_r tends towards 0, the corresponding channel component $\mathbf{h}_l(r)$ approaches 0 as well. This implies that the estimation of the clutter CIR vector $\mathbf{h}_l$ is analogous to the estimation of the associated hyperparameter vector $\boldsymbol{\psi}$.

A. EM Procedure

Represent the estimation of the hyperparameter matrix $\mathbf{\Psi}$ in the k-th iteration as $\widehat{\mathbf{\Psi}}^{(k)}$.

E-Step: In the k-th iteration, compute the average log-likelihood $\mathcal{L}\left(\mathbf{\Psi}|\mathbf{\Psi}^{(k)}\right)$ for the complete data set $\mathbf{y}_l, \mathbf{h}_l$ using:

$$\begin{aligned}\mathcal{L}\left(\mathbf{\Psi}|\mathbf{\Psi}^{(k)}\right) &= \mathbb{E}_{\mathbf{h}_l|\mathbf{y}_l;\widehat{\mathbf{\Psi}}^{(k)}}\left\{\log p\left(\mathbf{y}_l, \mathbf{h}_l; \mathbf{\Psi}\right)\right\} \\ &= \mathbb{E}_{\mathbf{h}_l|\mathbf{y}_l;\widehat{\mathbf{\Psi}}^{(k)}}\left\{\log p\left(\mathbf{y}_l|\mathbf{h}_l\right) + \log p\left(\mathbf{h}_l; \mathbf{\Psi}\right)\right\}. \quad (8)\end{aligned}$$

To evaluate the conditional expectation in (8), one requires the posterior PDF of $\mathbf{h}_l$, given as $p\left(\mathbf{h}_l|\mathbf{y}_l; \widehat{\mathbf{\Psi}}^{(k)}\right) \sim \mathcal{CN}\left(\mu_h^{(k)}, \mathbf{\Sigma}_h^{(k)}\right)$, where the [28]:

$$\boldsymbol{\mu}_h^{(k)} = \mathbf{\Sigma}_h^{(k)} \widetilde{\mathbf{X}}^H \mathbf{R}_v^{-1} \mathbf{y}_l, \quad (9)$$

$$\mathbf{\Sigma}_h^{(k)} = \left(\widetilde{\mathbf{X}} \mathbf{R}_v^{-1} \widetilde{\mathbf{X}}^H + \left(\mathbf{\Psi}^{(k)}\right)^{-1}\right)^{-1}. \quad (10)$$

M-Step: The estimates $\widehat{\boldsymbol{\psi}}^{(k+1)}$ in the k-th EM iteration are obtained as:

$$\begin{aligned}\widehat{\psi}_r^{(k+1)} &= \mathbb{E}_{\mathbf{h}_l|\mathbf{y}_l;\widehat{\mathbf{\Psi}}^{(k)}}\left\{|\mathbf{h}_l(r)|^2\right\} \\ &= \mathbf{\Sigma}_h^{(k)}(r,r) + \left|\boldsymbol{\mu}_h^{(k)}(r)\right|^2, \forall; r. \quad (11)\end{aligned}$$

Iterate the EM algorithm until convergence or reaching the maximum iterations $k_{\max}$. Upon convergence, the SBL-based estimate of the sparse clutter CIR vector $\mathbf{h}_l$ is given by $\widehat{\mathbf{h}}_l = \boldsymbol{\mu}^{(k)}$. The resultant means square error is given

$$\mathrm{MSE} = \mathrm{Tr}\left\{\left(\widetilde{\mathbf{X}} \mathbf{R}_v^{-1} \widetilde{\mathbf{X}}^H + \left(\mathbf{\Psi}^{(k)}\right)^{-1}\right)^{-1}\right\}, \quad (12)$$

where $\mathbf{R}_v = \sigma_v^2 \mathbf{I}$. Once can readily observed that the MSE is a function of the waveform matrix $\widetilde{\mathbf{X}}$. Hence, one can improve the CCIR estimation performance by further designing the transmit waveform. Towards this end, we develop a MM-based iterative procedure to design optimal transmit waveform.

IV. Probing Waveform Design

Using the SBL algorithm discussed in the previous section, one can estimate the hyperparameter vector $\boldsymbol{\psi}$ which serves as a partial prior knowledge for the subsequent clutter channel impulse response (CCIR) estimation. The resulting MSE is a function of the matrix $\widetilde{\mathbf{X}}$ which, in turn, depends on $\mathbf{x}$. Therefore, we design the optimal transmit vector $\mathbf{x}$ to reduce the MSE further and yield a further improved CCIR estimate. The optimal waveform design is obtained by solving the following optimization problem

$$\begin{aligned}&\underset{\mathbf{x}}{\text{minimize}} \quad \mathrm{Tr}\left\{\left(\widetilde{\mathbf{X}}^H \mathbf{R}_v^{-1} \widetilde{\mathbf{X}} + \mathbf{\Psi}^{-1}\right)^{-1}\right\} \\ &\text{subject to} \quad |x_i[l]| = 1, \forall\, i, l,\end{aligned} \quad (11)$$

where the unit modulus constraint arises from the power amplifier efficiency perspective [29]. Since, we are considering the optimization of (11) for each k and hence would be omitting the superscript k for ease of notations. Next, after employing the matrix inversion lemma [30], one obtains

$$\mathrm{Tr}\left\{\left(\widetilde{\mathbf{X}}^H \mathbf{R}_v^{-1} \widetilde{\mathbf{X}} + \mathbf{\Psi}^{-1}\right)^{-1}\right\} = \mathrm{Tr}\left\{\mathbf{\Psi} - \mathbf{\Psi} \widetilde{\mathbf{X}}^H \mathbf{C}^{-1} \widetilde{\mathbf{X}} \mathbf{\Psi}\right\}, \quad (14)$$

where $\mathbf{C} = \widetilde{\mathbf{X}} \mathbf{\Psi} \widetilde{\mathbf{X}}^H + \mathbf{R}_v \succ 0 \in \mathbb{C}^{(L+R-1)N_R \times (L+R-1)N_R}$. Employing the MM framework, one needs to introduce a majorizer for our modified objective function $u(\widetilde{\mathbf{X}}, \mathbf{C}) = \mathrm{Tr}\{\mathbf{\Psi} - \mathbf{\Psi}\widetilde{\mathbf{X}}^H \mathbf{C}^{-1} \widetilde{\mathbf{X}} \mathbf{\Psi}\}$. This is stated in the following lemma.

Lemma 1. *The function $u(\widetilde{\mathbf{X}}, \mathbf{C})$ is jointly concave on $\widetilde{\mathbf{X}}$ and $\mathbf{C}$ and is majorized by*

$$q\left(\widetilde{\mathbf{X}}, \boldsymbol{C}; \widetilde{\mathbf{X}}_l, \boldsymbol{C}_l\right) = \mathrm{Tr}\left\{\mathbf{E}_l^H \boldsymbol{C}\right] - 2\Re\left\{\mathrm{Tr}\left\{\mathbf{F}_l^H \widetilde{\mathbf{X}}\right\}\right\} + \mathrm{Tr}\{\mathbf{\Psi}\} \quad (15)$$

where $\mathbf{E}_l = \mathbf{C}_l^{-1} \widetilde{\mathbf{X}}_l \mathbf{\Psi}^2 \widetilde{\mathbf{X}}_l^H \mathbf{C}_l^{-1} \in \mathbb{C}^{(K+R-1)M \times (K+R-1)M}$ *and* $\mathbf{F}_l = \mathbf{C}_l^{-1} \widetilde{\mathbf{X}}_l \mathbf{\Psi}^2 \in \mathbb{C}^{(K+R-1)M \times NMR}$.

Proof: The function $g(\widetilde{\mathbf{X}}, \mathbf{C}) = \mathbf{\Psi} - \mathbf{\Psi}\widetilde{\mathbf{X}}^H \mathbf{C}^{-1} \widetilde{\mathbf{X}} \mathbf{\Psi}$ is jointly concave with respect to $\widetilde{\mathbf{X}}$ and $\mathbf{C}$ provided $\mathbf{C} \succ 0$ [12, pp. 108-111]. Since the function $\mathrm{Tr}\{.\}$ is linear and non-decreasing in nature, hence the function $u(\widetilde{\mathbf{X}}, \mathbf{C}) = \mathrm{Tr}\{g(\widetilde{\mathbf{X}}, \mathbf{C})\}$ is also jointly concave with respect to $\widetilde{\mathbf{X}}$ and $\mathbf{C}$. Furthermore, it is upper bounded as [31]

$$\begin{aligned}&u(\widetilde{\mathbf{X}}, \mathbf{C}) \leq q\left(\widetilde{\mathbf{X}}, \boldsymbol{C}; \widetilde{\mathbf{X}}_l, \boldsymbol{C}_l\right) \\ &= \mathrm{Tr}\left\{\boldsymbol{C}_l^{-1} \widetilde{\mathbf{X}}_l \mathbf{\Psi}^2 \widetilde{\mathbf{X}}_l^H \boldsymbol{C}_l^{-1}\left(\boldsymbol{C} - \boldsymbol{C}_l\right)\right\} \\ &-2\,\mathrm{Re}\left\{\mathrm{Tr}\left\{\mathbf{\Psi}^2 \widetilde{\mathbf{X}}_l^H \boldsymbol{C}_l^{-1}\left(\widetilde{\mathbf{X}} - \widetilde{\mathbf{X}}_l\right)\right\}\right\} + u\left(\widetilde{\mathbf{X}}_l, \boldsymbol{C}_l\right),\end{aligned} \quad (16)$$

and simplifies to (15) through algebraic manipulations, omitted here for brevity. This concluding the proof. ■

Employing the above lemma, the majorized problem becomes

$$\begin{aligned}&\underset{\mathbf{X}}{\text{minimize}} \quad \mathrm{Tr}\left\{\mathbf{E}_l^H \mathbf{C}\right\} - 2\Re\left\{\mathrm{Tr}\left(\mathbf{F}_l^H \widetilde{\mathbf{X}}\right)\right\} \\ &\text{subject to} \quad |x_i[l]| = 1, \forall\, i, l.\end{aligned}$$

Next, using $\widetilde{\mathbf{X}} = \mathbf{I}_{N_R} \otimes \mathbf{X}$, and $\mathbf{\Psi} = \mathbf{I}_{N_R} \otimes \widetilde{\mathbf{\Psi}}$, one can simplify the first term of the above objective function as follows:

$$\mathrm{Tr}\left\{\mathbf{E}_l^H \boldsymbol{C}\right\} = \mathrm{Tr}\left\{\mathbf{E}_l^H\left((\mathbf{I}_{N_R} \otimes \mathbf{X})(\mathbf{I}_{N_R} \otimes \widetilde{\mathbf{\Psi}})(\mathbf{I}_{N_R} \otimes \mathbf{X}^H)\right)\right\}. \quad (13)$$

Furthermore, one can also rewrite the matrix $\mathbf{E}_l$ as

$$\begin{aligned}\mathbf{E}_l = \mathbf{C}_l^{-1} \widetilde{\mathbf{X}}_l \mathbf{\Psi}^2 \widetilde{\mathbf{X}}_l^H \mathbf{C}_l^{-1} &= \left[\mathbf{I}_{N_R} \otimes \left(\mathbf{X}_l \widetilde{\mathbf{\Psi}} \mathbf{X}_l^H + \sigma_v^{-2} \mathbf{I}_{\tilde{k}}\right)^{-1}\right] \\ &\left(\mathbf{I}_{N_R} \otimes \mathbf{X}_l \widetilde{\mathbf{\Psi}}^2 \mathbf{X}_l^H\right)\left[\mathbf{I}_{N_R} \otimes \left(\mathbf{X}_l \widetilde{\mathbf{\Psi}} \mathbf{X}_l^H + \sigma_v^{-2} \mathbf{I}_{\tilde{k}}\right)^{-1}\right],\end{aligned}$$

where $\mathbf{C}_l = \widetilde{\mathbf{X}}_l \mathbf{\Psi} \widetilde{\mathbf{X}}_l^H + \mathbf{R}_v = \mathbf{I}_{N_R} \otimes \mathbf{A}_l$, and $\mathbf{A}_l = \left(\mathbf{X}_l \widetilde{\mathbf{\Psi}} \mathbf{X}_l^H + \sigma_v^{-2} \mathbf{I}_{\tilde{k}}\right) \in \mathbb{C}^{(L+R-1)\times(L+R-1)}$. Using the above results, and defining $\widetilde{\mathbf{A}}_l = \mathbf{A}_l^H \mathbf{X}_l \mathbf{\Lambda}^2 \mathbf{X}_l^H \mathbf{A}_l \in \mathbb{C}^{(L+R-1)\times(L+R-1)}$, one obtains

$$\begin{aligned}&\mathrm{Tr}\left[\mathbf{E}_l^H \mathbf{C}\right] = \mathrm{Tr}\left[\mathbf{I}_{N_R} \otimes \widetilde{\mathbf{A}}_l^H\left((\mathbf{I}_{N_R} \otimes \mathbf{X})(\mathbf{I}_{N_R} \otimes \widetilde{\mathbf{\Psi}})(\mathbf{I}_{N_R} \otimes \mathbf{X}^H)\right)\right] \\ &= \mathrm{Tr}\left[\mathbf{I}_{N_R} \otimes \widetilde{\mathbf{A}}_l^H \mathbf{X} \widetilde{\mathbf{\Psi}} \mathbf{X}^H\right] = M\,\mathrm{Tr}\left[\widetilde{\mathbf{A}}_l^H \mathbf{X} \widetilde{\mathbf{\Psi}} \mathbf{X}^H\right] \\ &= \mathrm{vec}\left(\mathbf{X}\right)^H\left(\widetilde{\mathbf{\Psi}}^T \otimes \widetilde{\mathbf{A}}_l^H\right)\mathrm{vec}\left(\mathbf{X}\right),\end{aligned} \quad (14)$$

where the first equality follows from the property $\left(\mathbf{A} \otimes \mathbf{B}\right)\left(\mathbf{C} \otimes \mathbf{D}\right) = \left(\mathbf{AC} \otimes \mathbf{BD}\right)$; the second equality follows from $\mathrm{Tr}\left[\mathbf{A} \otimes \mathbf{B}\right] = \mathrm{Tr}\left[\mathbf{A}\right]\mathrm{Tr}\left[\mathbf{B}\right]$; and the last equality follows from $\mathrm{Tr}\left[\mathbf{A}^H \mathbf{BCD}\right] = \mathrm{vec}\left(\mathbf{A}\right)^H\left(\mathbf{D}^T \otimes \mathbf{B}\right)\mathrm{vec}\left(\mathbf{C}\right)$. Using the MM technique, one obtains

$$\begin{aligned}&\mathrm{vec}(\mathbf{X})^H\left(\widetilde{\mathbf{\Psi}}^T \otimes \widetilde{\mathbf{A}}_l^H - \gamma\mathbf{I}\right)\mathrm{vec}(\mathbf{X}) \\ &\leq 2\Re\{\mathrm{vec}(\mathbf{X}_l)^H\left(\widetilde{\mathbf{\Psi}}^T \otimes \widetilde{\mathbf{A}}_l^H - \gamma\mathbf{I}\right)\mathrm{vec}(\mathbf{X})\} + \text{const.} \\ &= 2\Re\left\{\mathrm{Tr}\left\{\left(\widetilde{\mathbf{A}}_l^H \mathbf{X}_l \widetilde{\mathbf{\Psi}} - \gamma\mathbf{X}_l\right)^H \mathbf{X}\right\}\right\}\end{aligned} \quad (15)$$

where $\gamma \triangleq \gamma_{\max}\left(\widetilde{\mathbf{\Psi}}^T \otimes \left(\widetilde{\mathbf{A}}_l^H\right)\right) = \gamma_{\max}(\widetilde{\mathbf{\Psi}}^T)\gamma_{\max}\left(\widetilde{\mathbf{A}}_l^H\right)$ and $\gamma_{\max}(.)$ denotes the largest eigenvalue of its matrix argument and const. represents an unrelated constant term.

On similar lines, the second term $\text{Tr}\left\{\mathbf{F}_l^H\widetilde{\mathbf{X}}\right\}$ is simplified as

$$\begin{aligned}
&\text{Tr}\left\{\mathbf{F}_l^H\left(\mathbf{I}_{N_R}\otimes\mathbf{X}\right)\right\}\\
&=\text{Tr}\left\{(\mathbf{I}_{N_R}\otimes\widetilde{\mathbf{\Psi}}^2)(\mathbf{I}_{N_R}\otimes\mathbf{X}_l^H)(\mathbf{I}_{N_R}\otimes\mathbf{A}_l^{-H})(\mathbf{I}_{N_R}\otimes\mathbf{X})\right\}\\
&=N_R\text{Tr}\left\{\widetilde{\mathbf{\Psi}}^2\mathbf{X}_l^H\mathbf{A}_l^{-H}\mathbf{X}\right\}. \qquad (16)
\end{aligned}$$

Furthermore, defining the matrix $\mathbf{B}_l=\widetilde{\mathbf{A}}_l^H\mathbf{X}_l\widetilde{\mathbf{\Psi}}-\gamma\mathbf{X}_l-\mathbf{F}_l$, and ignoring the constant terms, the MSE minimization problem reduces to

$$\begin{aligned}
&\underset{\mathbf{X}}{\text{minimize}} && \Re\left[\text{Tr}\left\{\boldsymbol{B}_l^H\mathbf{X}\right\}\right]\\
&\text{subject to} && |x_i[l]|=1,\forall\, i,l.
\end{aligned} \qquad (17)$$

The matrix $\mathbf{X}=[\mathbf{X}_1,\mathbf{X}_2,\ldots,\mathbf{X}_N]\in\mathbb{C}^{(L+R-1)\times N_TR}$, where each $\mathbf{X}_n$ for $i=1,2,\ldots,N_T$ is a convolution matrix. This structure makes the problem difficult to solve. We now simplify the above objective function

$$\text{Tr}\left\{\mathbf{X}\mathbf{B}_l^H\right\}=\text{Tr}\left\{\mathbf{X}\left[\mathbf{b}_1,\mathbf{b}_2,\ldots,\mathbf{b}_{L+R-1}\right]\right\}, \qquad (18)$$

where $\mathbf{b}_n\in\mathbb{C}^{N_TR\times 1}$ is the n-th column of $\mathbf{B}_l^H$, where $n=1,2,\ldots,(L+R-1)$. Further, let $\mathbf{b}_n=\left[\mathbf{b}_{n,1}^T,\mathbf{b}_{n,2}^T,\ldots,\mathbf{b}_{n,N_T}^T\right]^T$, where $\mathbf{b}_{n,i}\in\mathbb{C}^{R\times 1}$ for $i=1,2,\ldots,N_T$. We write $\mathbf{X}\mathbf{b}_n$ as

$$\mathbf{X}\mathbf{b}_n=\sum_{i=1}^{N_T}\mathbf{X}_i\mathbf{b}_{n,i}.$$

Since $\mathbf{X}_i$ is a convolution matrix, we have

$$\mathbf{X}_i\mathbf{b}_{n,i}=\mathbf{B}_{n,i}\mathbf{x}_i,$$

where $\mathbf{x}_i$ is the transmitted sample vector from the i-th antenna and $\mathbf{B}_{i,i}\in\mathbb{C}^{L+R-1\times K}$ is the convolution matrix of the vector $\mathbf{b}_{n,i}$. Using this, one obtains the relation

$$\mathbf{X}\mathbf{b}_n=\sum_{i=1}^{N_T}\mathbf{X}_i\mathbf{b}_{n,i}=\sum_{i=1}^{N_T}\mathbf{B}_{i,i}\mathbf{x}_i$$

Hence, Eq. (18) is equivalently written as

$$\text{Tr}\left\{\mathbf{X}\mathbf{B}_l^H\right\}=\text{Tr}\Bigg\{\left[\mathbf{B}_1,\mathbf{B}_2,\ldots,\mathbf{B}_{L+R-1}\right]\left(\mathbf{I}_{L+R-1}\otimes\mathbf{x}\right)\Bigg\}, \qquad (19)$$

where each $\mathbf{B}_i=[\mathbf{B}_{i,1},\mathbf{B}_{i,2},\ldots,\mathbf{B}_{i,N_T}]\in\mathbb{C}^{(L+R-1)\times N_T}$. Hence, the optimization problem becomes

$$\underset{\mathbf{x}}{\text{mininize}}\ \Re\left\{\left(\sum_{j=1}^{L+R-1}\boldsymbol{b}_{j.}\right)^H\mathbf{x}\right\} \qquad (20)$$

$$\text{subject to}\ \ |x_i[l]|=1,\forall\, i,l. \qquad (23)$$

where $\boldsymbol{b}_{j.}\in\mathbb{C}^{NL\times 1}$ represents the j-th row of the matrix $\mathbf{B}_j$. Note that the optimization objective in (20) is the real part of dot product between two vector quantities, where the elements of vector $\mathbf{x}$ are unit magnitude. Hence, the closed-form solution for the overall transmitted vector $\mathbf{x}$ is

$$\mathbf{x}=-e^{j\arg\left(\sum_{j=1}^{L+R-1}b_{j.}\right)}, \qquad (21)$$

where $\arg(\mathbf{z})$ denotes the phase of complex vector quantity $\mathbf{z}$.

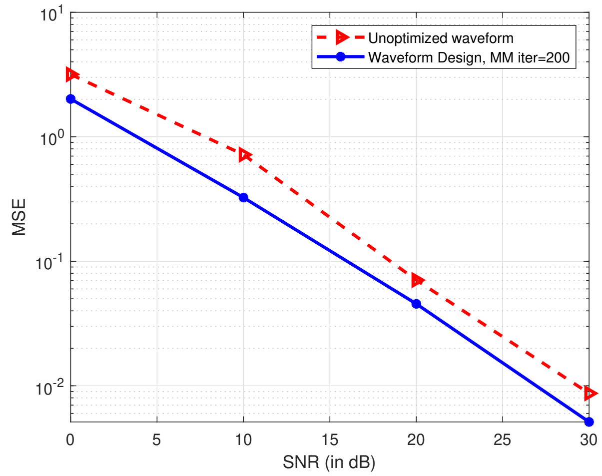

Figure 1. MSE versus SNR for the proposed SBL-based CCIR estimation and MM-based probing waveform design.

Algorithm 1 Joint sparse CCIRE and probing waveform design

Input: $\mathbf{y},\widetilde{\mathbf{X}}$, ϵ (Threshold), maximum iterations $k_{\max}$
Output: $\widehat{\boldsymbol{\mu}}_h$, $\mathbf{x}$
1: **Initialize:** $\mathbf{\Psi}=\mathbf{I}$, and $\mathbf{x}$ using N_T LFM waveform samples
2: **while** $||\boldsymbol{\psi}(k)-\boldsymbol{\psi}(k-1)||\leq\epsilon$ or $k\leq k_{\max}$ **do**
3: **E-step:** Compute $\boldsymbol{\mu}_h^{(k)}$ and $\mathbf{\Sigma}_h^{(k)}$ using (9) and (10), respectively
4: **M-Step:** Compute $\widehat{\mathbf{\Psi}}^{(k+1)}$ using (11)
5: Calculate $\mathbf{A}_l=\left(\mathbf{X}_l\widetilde{\mathbf{\Psi}}\mathbf{X}_l^H+\sigma_v^{-2}\mathbf{I}_{\tilde{k}}\right)$
6: Calculate $\widetilde{\mathbf{A}}_l=\mathbf{A}_l^H\mathbf{X}_l\mathbf{\Lambda}^2\mathbf{X}_l^H\mathbf{A}_l$
7: Calculate $\widetilde{\mathbf{\Psi}}$ using (6)
8: Calculate $\mathbf{B}_l=\widetilde{\mathbf{A}}_l^H\mathbf{X}_l\widetilde{\mathbf{\Psi}}-\psi\mathbf{X}_l-\mathbf{F}_l$
9: calculate $\mathbf{x}$ using (21)
10: **end while**
11: **return** $\widehat{\boldsymbol{\mu}}_h=\boldsymbol{\mu}_h^{(k)}$ and $\mathbf{x}$

V. Numerical Experiments

We present the numerical experiments to demonstrate the performance of the proposed technique. We test our proposed algorithms against a representative instantiation of the radio frequency (RF) environment provided by the high-fidelity, physics-based, site-specific modeling and simulation software RFView, developed by Information System Laboratories (ISL) . This tool has been vetted against different measured data sets from VHF to Ku band with proven success [26]. An example is given in the Fig. (29) of [26], where it is shown explicitly that the simulated data provided by the RFview matches the measured dataset collected as part of the ARPA/NAVY Mountaintop Program. The RFView generated channel is used, which divides the clutter region into distinct clutter patches using publicly accessible topographical data and land cover classifications. A MIMO scenario is considered with $N_T=2$ transmitter and $N_R=2$ receivers. From a comparison point of view an LFM waveform is use which is having 20 MHz bandwidth, 1.2 GHz center frequency and 1 KHz pulse repetition frequency.

Fig. 1 depicts the MSE performance of the proposed SBL-based joint sparse clutter channel estimation and MM-based transmit waveform design technique as a function of the signal-to-noise (SNR)

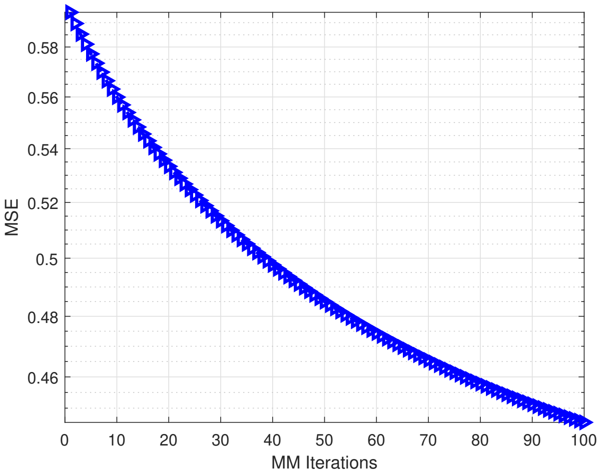

Figure 2. MSE as a function of number of iterations for the proposed technique.

for 200 iterations [1]. As expected, with the increment in SNR the effect of noise get reduced and the MSE performance improves. For performance comparison, we have also plotted the MSE of the channel estimation when transmit waveform is not optimized and a fixed LFM waveform is used. It can be readily observed from the figure that with the waveform optimization we are able to further reduce the MSE which results in an improved estimate of the CCIR.

Fig. 2 presents the MSE performance for the proposed design as a function of number of iterations. It can be observed from the figure that as the iterations are increasing the MSE decreases since the proposed technique is able to obtain a better transmitted waveform which yields an improved MSE performance. This further enhances the quality of the estimated CCIR.

VI. Summary

This work developed a SBL-based joint sparse CCIRE and MM-based probing design waveform algorithm in MIMO CoFAR. The SBL-based algorithm is able to estimate the underlying joint sparse channel, and MM-based probing design waveform is able to design MIMO probing waveforms optimally such that MSE is reduced further and an improved CCIR estimation is obtained. This integrated approach not only enhances CCIR matrix estimation accuracy but also highlights the adaptability and effectiveness of our proposed method.

References

[1] K. V. Mishra, M. R. B. Shankar, and M. Rangaswamy, *Next-Generation Cognitive Radar Systems*. IET Press, 2023.

[2] S. Haykin, "Cognitive radar: A way of the future," *IEEE Signal Processing Magazine*, vol. 23, no. 1, pp. 30–40, 2006.

[3] J. S. Bergin, J. R. Guerci, R. M. Guerci, and M. Rangaswamy, "MIMO clutter discrete probing for cognitive radar," in *IEEE Radar Conference*, 2015, pp. 1666–1670.

[4] J. R. Guerci, "Cognitive radar: A knowledge-aided fully adaptive approach," in *IEEE Radar Conference*, 2010, pp. 1365–1370.

[5] J. R. Guerci, R. M. Guerci, M. Ranagaswamy, J. S. Bergin, and M. C. Wicks, "CoFAR: Cognitive fully adaptive radar," in *IEEE Radar Conference*, 2014, pp. 0984–0989.

[6] K. L. Bell, C. J. Baker, G. E. Smith, J. T. Johnson, and M. Rangaswamy, "Fully adaptive radar for target tracking part I: Single target tracking," in *IEEE Radar Conference*, 2014, pp. 0303–0308.

[1]Please note the EM and MM iterations both are same and are represented as iterations in the paper.

[7] ——, "Fully adaptive radar for target tracking part II: Target detection and track initiation," in *IEEE Radar Conference*, 2014, pp. 0309–0314.

[8] L. Úbeda Medina and J. Grajal, "Implementation of the fully adaptive radar framework: Practical limitations," in *IEEE Radar Conference*, 2017, pp. 0761–0766.

[9] S. Haykin, B. Currie, and S. Kesler, "Maximum-entropy spectral analysis of radar clutter," *Proceedings of the IEEE*, vol. 70, no. 9, pp. 953–962, 1982.

[10] L. Spafford, "Optimum radar signal processing in clutter," *IEEE Transactions on Information Theory*, vol. 14, no. 5, pp. 734–743, 1968.

[11] W. B. Lewis, "Sea clutter," in *Radar Handbook*, 3rd ed., M. I. Skolnik, Ed. McGraw-Hill, 2008.

[12] J. B. Billingsley, *Low-angle radar land clutter: Measurements and empirical models*. IET, 2002.

[13] K. D. Ward, S. Watts, and R. J. Tough, *Sea clutter: Scattering, the K distribution and radar performance*. IET, 2006, vol. 20.

[14] G. P. Kulemin, *Millimeter-wave radar targets and clutter*. Artech House, 2003.

[15] T. Musha and M. Sekine, "Models of clutter," in *Advanced Radar Techniques and Systems*, G. Galati, Ed. Peter Peregrinus, 1993.

[16] S. Kay, "Optimal signal design for detection of Gaussian point targets in stationary gaussian clutter/reverberation," *IEEE Journal of Selected Topics in Signal Processing*, vol. 1, no. 1, pp. 31–41, 2007.

[17] W. L. Melvin and J. A. Scheer, *Principles of modern radar: Advanced techniques*. IET, 2012, vol. 2.

[18] S. Theodoridis and R. Chellappa, *Communications and radar signal processing*, ser. Academic Press Library in Signal Processing. Academic Press, 2013.

[19] J. R. Guerci, J. S. Bergin, R. J. Guerci, M. Khanin, and M. Rangaswamy, "A new MIMO clutter model for cognitive radar," in *IEEE Radar Conference*, 2016, pp. 1–6.

[20] S. Gogineni, M. Rangaswamy, J. R. Guerci, J. S. Bergin, and D. R. Kirk, "Estimation of radar channel state information," in *IEEE Radar Conference*, 2019, pp. 1–4.

[21] B. Kang, S. Gogineni, M. Rangaswamy, and J. R. Guerci, "Constrained maximum likelihood channel estimation for CoFAR," in *Asilomar Conference on Signals, Systems, and Computers*, 2020, pp. 1162–1166.

[22] B. Kang, S. Gogineni, M. Rangaswamy, J. R. Guerci, and E. Blasch, "Adaptive channel estimation for cognitive fully adaptive radar," *IET Radar, Sonar & Navigation*, 2021.

[23] S. Sedighi, M. R. B. Shankar, K. V. Mishra, and M. Rangaswamy, "Physics-based cognitive radar modeling and parameter estimation," in *IEEE Radar Conference*, 2022, pp. 1–6.

[24] K. P. Rajput, M. R. Bhavani Shankar, K. V. Mishra, M. Rangaswamy, and B. Ottersten, "CoFAR clutter channel estimation via sparse Bayesian learning," in *2023 IEEE Radar Conference (RadarConf23)*, 2023, pp. 1–5.

[25] K. P. Rajput, B. S. M. R., K. V. Mishra, M. Rangaswamy, and B. Ottersten, "CoFAR clutter estimation using covariance-free Bayesian learning," *IEEE Transactions on Aerospace and Electronic Systems*, pp. 1–17, 2024.

[26] S. Gogineni, J. R. Guerci, H. K. Nguyen, J. S. Bergin, D. R. Kirk, B. C. Watson, and M. Rangaswamy, "High fidelity RF clutter modeling and simulation," *IEEE Aerospace and Electronic Systems Magazine*, vol. 37, no. 11, pp. 24–43, 2022.

[27] D. Wipf and B. Rao, "Sparse Bayesian learning for basis selection," *IEEE Transactions on Signal Processing*, vol. 52, no. 8, pp. 2153–2164, 2004.

[28] S. M. Kay, *Fundamentals of statistical signal processing: Estimation theory*. Prentice-Hall, 1993.

[29] Z.-J. Wu, T.-L. Xu, Z.-Q. Zhou, and C.-X. Wang, "Fast algorithms for designing complementary sets of sequences under multiple constraints," *IEEE Access*, vol. 7, pp. 50 041–50 051, 2019.

[30] S. M. Kay, *Fundamentals of statistical signal processing, Volume I: Estimation theory*. Englewood Cliffs, NJ: Prentice Hall, 1993.

[31] Y. Sun, P. Babu, and D. P. Palomar, "Majorization-minimization algorithms in signal processing, communications, and machine learning," *IEEE Transactions on Signal Processing*, vol. 65, no. 3, pp. 794–816, 2017.