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Title: **Thinking about numbers in different tongues: An overview of the influences of multilingualism on numerical and mathematical competencies**

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Abstract

In an increasingly multilingual and multicultural world, understanding the interactions between language and mathematics is critical, especially when individuals must acquire and exercise their mathematical competencies in multiple languages. Indeed, research shows that, overall, L2 language learners are at an academic disadvantage compared to their L1 peers. The current article briefly overviews how multilingualism influences basic and advanced mathematical skills and interacts with mathematical learning difficulties. We first outline the traditional cognitive models of number learning and language processing. We then discuss the particularities of multilingualism and how it impacts numerical skills such as counting and building lexical-semantic associations, transcoding and arithmetic, mathematical word problems and mathematical performance tests, and dyscalculia diagnosis. We end this review by outlining challenges, recommendations, and solutions for multilingual educational settings. The article is intended as a guide for numerical cognition researchers who work with diverse populations and for mathematics educators and educational policy-makers facing the challenges of a multilingual classroom.

1. Introduction

Throughout human history, scholars and scientists alike often viewed mathematical and language skills as independent and even opposing areas of experience. Even nowadays, stereotypes such as if an individual is "good" at math, then they must be "bad" at language skills (or vice versa) persist. However, extensive research has demonstrated that language and mathematical competencies are strongly connected. For instance, research in numerical cognition has shown that the linguistic environment plays a crucial role in numerical competencies across all developmental stages, from the first steps in learning to counting to doing arithmetic operations as an adult. The importance of language becomes especially pertinent in today's increasingly multilingual and multicultural world. In the current article, we provide an overview of the influence of multilingualism on a range of basic and advanced numerical and mathematical competencies in children and adults. We also outline challenges and future directions and list tentative solutions for educators and researchers working with multilingual populations.

1.1. Cognitive models of early number acquisition

In mathematically literate societies, numerical information can be represented in a symbolic (e.g., digits "2" or number words, "two") and in a non-symbolic format (e.g., as a collection of items such as "●●"). So far, research has established that pre-verbal and evolutionarily determined core systems underlie the processing of non-symbolic numbers. These core systems are commonly known as the Object Tracking System (OTS, also referred to as Parallel Individuation) and the Approximate Number System (ANS) (Carey, 2009; Dehaene, 2001; Feigenson et al., 2004). On the one hand, the OTS represents numbers as individual elements in a set (i.e., [●●●] = [i], [j], [k]), which enables very fast and precise processing, but the capacity of this system is limited to 4 or 5 items (Carey et al., 2017; Hyde, 2011; vanMarle et al., 2018). On the other hand, the ANS processes numbers as an approximate sum of sets of objects (i.e., [●●●●●●] ≈ [iiiiii]), which again allows us to extract the numerical information from the environment in a fast but error-prone manner. In contrast to the OTS, however, the representational capacity of the ANS is, in principle, unrestricted. Because of its unlimited representational capacity, the ANS has traditionally been posited as *the* foundational mechanism for acquiring culturally evolved symbolic numbers (Dehaene & Cohen, 1995; Piazza, 2010; vanMarle et al., 2018), although an increasing corpus for evidence challenges this status quo (Carey et al., 2017; Carey & Barner, 2019; Reynvoet & Sasanguie, 2016; Wilkey & Ansari, 2020).

Indeed, according to the most widely used model in numerical cognition – the Triple Code Model (TCM), distinct but overlapping neurocognitive mechanisms underlie the processing of the symbolic visual code (i.e., digits), the symbolic verbal code (i.e., number words) and the non-symbolic code

(Dehaene & Cohen, 1995; Skagenholt et al., 2021). Here, the symbolic numbers are acquired and processed by being mapped onto their corresponding non-symbolic counterparts (e.g., “six” = “6” = [●●●●●●]). Moreover, according to the TCM, the semantic meaning of the symbolic numbers is derived from the non-symbolic code (Dehaene & Cohen, 1995). Mapping between these three numerical notations is a crucial building block for later mathematics competencies, but how exactly children learn to associate the different numerical formats is yet unclear.

In line with the TCM, some developmental models argue that children first learn to map between number words and dots, then between digits and dots, and only after both symbolic formats have been mapped onto the non-symbolic quantities, children can learn the correspondence between the two symbolic formats (i.e., “six” ↔[●●●●●●], “6” ↔ [●●●●●●], only then “six” ↔ “6” (Benoit et al., 2013; Odic et al., 2015; Von Aster & Shalev, 2007). For example, the four-step developmental model by Von Aster and Shalev (2007) posits the ANS as first developmental step in infancy and as the “necessary precondition for children to learn to associate a perceived number of objects or events with spoken or, later, written and Arabic symbols” (Von Aster & Shalev, 2007, p.870). At this developmental step, infants possess basic abilities such as subitizing, approximation, and comparison. In the second developmental step, children acquire the number words, which also enable them to engage in counting procedures, counting strategies, and fact retrieval in preschool. In the third step, in school, children acquire the understanding and mastery of digits, enabling the full development of calculation skills. Finally, in the fourth stage, children acquire the knowledge of ordinality (i.e., the relative position of an item in a set), which supports the development of more complex skills such as arithmetic thinking.

Alternative developmental models argue that the acquisition of number words and digits (partially) bypasses the ANS in favor of symbolic semantic associations (Carey, 2009; Carey et al., 2017; Carey & Barner, 2019; Hurst et al., 2017; Jiménez Lira et al., 2017; Marinova et al., 2021; Reynvoet & Sasanguie, 2016; Sella et al., 2021; see also Bialystok, 1992, 2000). Within this framework, children acquire the correspondence between non-symbolic quantities and number words as a first step, after which they learn the correspondence between a number word and a digit. Only after learning the correspondence between two symbolic numbers, children then learn to map between digits and dots (i.e., “four” ↔[●●●●], “four” ↔ “4”, only then “4” ↔ [●●●●] (Hutchison et al., 2020; Jiménez Lira et al., 2017; Marinova et al., 2021). Although these models have some subtle differences, their general frameworks share two common developmental stages when it comes to the acquisition of numerical concepts and semantics. As a first developmental step, children may indeed rely on their core competencies and particularly the OTS to acquire the small number words by associating them with small sets of items (e.g., “four” = [i], [j], [k], [l] = [“mama”], [“papa”], [“my sibling”], and [“me”]). The second developmental stage usually involves a bootstrapping procedure through which children become capable of transferring their knowledge of small numbers, to larger ones. Several mechanisms, such as counting, ordinality, cardinality, and directionality (n – 1 and n +1) understanding, seem to be involved in this bootstrapping stage (Reynvoet & Sasanguie, 2016; Sarnecka, 2015; Sarnecka & Carey, 2008; Sella et al., 2021). In addition to these domain-specific mechanisms, domain-general factors such as language and environmental factors (e.g., socioeconomic status) also influence the development of both basic and advanced numerical competencies (for an overview, see Menon & Chang, 2021; Silver & Libertus, 2022).

1.2. The influence of language on numeracy acquisition and mathematics competencies

Extensive research has demonstrated that language has an encompassing influence on mathematical skills (Peng et al., 2020): from basic numerical abilities (e.g., symbol acquisition, enumeration, simple arithmetic; Dehaene, 2001; Dehaene et al., 1999; Marchand et al., 2020; Schneider et al., 2020; Slusser et al., 2019) to more advanced skills (e.g., fractions, geometry; Kleemans & Segers, 2020; Kleemans et al., 2018; Vukovic, & Lesaux; 2013). The effect of language on number competencies is complex and possibly due to various factors such as: the amount of language exposure (e.g., language experience, or receiving language input more frequently), the type of exposure (one vs two or more languages), the transparency of the number system of a particular language (e.g., in Russian “one-ten” for “11” is transparent, while “eleven” in English is not), and the lexico-grammatical structure of the language(s) (Dowker & Nuerk, 2016). For instance, children who receive richer linguistic exposure and input by their caregiver acquire number words faster (Gunderson & Levine, 2011), than children who receive less linguistic exposure (Piantadosi et al. 2014). Furthermore, studies suggest that the grammatical structure of the language influences the understanding and acquisition of the first number words. Concretely, children speaking languages with plural markings (e.g., in English one candy, but two cand*ies*; cf. Carey, 2009; Le Corre & Carey, 2007; Sarnecka et al., 2007) acquire the first number words faster than children who speak languages with no such obligatory plural distinctions (Marušič et al., 2021; Sarnecka et al., 2007). In sum, it is apparent that language may substantially influence the acquisition of numerical competencies already at the very fundamental stages.

The effects of language on mathematical performance and academic achievement are well documented in both native speakers (L1) and second-language learners (L2)[1]. For instance, in an L1 sample using a cross-sectional design, Kleemans et al. (2018) demonstrated that children’s basic language competencies, such as phonological and grammatical skills, are related to arithmetic abilities both early

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[1] We use the terms L2-speaker and L2-learner interchangeably to refer to individuals for whom the language of instruction is not their native tongue.

(1st grade, i.e., 7 years old; see also Kleemans et al., 2014) and later (5th grade, i.e., 11 years-old; Kleemans et al., 2018) in development. Moreover, Kleemans and colleagues demonstrated that advanced language skills, such as academic vocabulary and verbal reasoning, were directly related to advanced mathematical skills, such as geometry and fractions. This relationship pattern is similar for L1 and L2 speakers, however, L2 speakers demonstrate lower scores on geometry and fraction tasks, as well as significantly lower academic vocabulary and verbal reasoning scores compared to their L1 peers. (Kleemans & Segers, 2020; Vukovic & Lesaux, 2013). Such a pattern of results shows that language plays a foundational role in acquiring basic and advanced mathematical concepts (Vukovic & Lesaux, 2013) and that the ability to handle mathematical tasks with increasing complexity in the upper grades of the school system draws upon increasingly complex language skills.

However, acquiring basic and advanced mathematical skills can prove especially challenging for L2-speaking children, who often struggle to learn mathematics in an instructional language different from their home language. These disadvantages are due to many complex factors, including lower language proficiency and socioeconomic status (e.g., Sarnecka et al., 2018). Indeed, research has demonstrated that not mastering the language of mathematics instruction is a significant reason why L2 speakers are generally at a disadvantage compared to their L1 peers (Greisen et al., 2021; Kleemans et al., 2014; Kleemans & Segers, 2020; see also Peng et al., 2020).

2. Multilingualism

Although the world's population is becoming increasingly multilingual (Grosjean, 2013), there is still no consensus regarding the definition of "multilingualism" or "bilingualism"[2]. Precise definitions of these terms depend on multiple factors such as level and format of proficiency (e.g. spoken vs written), acquisition stage (e.g., early in childhood or in adulthood) and number of languages spoken. In general, however, these various definitions converge around the point that bi- and multilingualism refers to the individual's ability to use two or more languages daily for effective communication and learning (Bhatia & Ritchie, 2013; Grosjean, 2010; Hélot & Erfurt, 2016).

2.1. What counts in a multilingual profile

Working with multilingual populations within educational and research settings is often challenging because linguistic profiles can be very heterogeneous (Greisen et al., 2021; Martini et al., 2021). One must consider the following factors to fully account for an individual's profile. First, the

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[2] Because of the versatility of the definitions, in the current article, bilingualism and multilingualism are used interchangeably.

developmental stage at which languages were acquired. Attention should be paid to whether an individual acquired the languages simultaneously since birth (L1 + L2) or sequentially (L1 → L2), with the L2 introduced later than L1 (e.g., after the age of 3 years; Baker, 2011; Couëtoux-Jungmann et al., 2010). Second, the context in which the languages are acquired (e.g., at home) and used (e.g., at school). Third, the language proficiency level in absolute (e.g., for foreign language teachers) or relative (e.g., balanced vs unbalanced) terms. Fourth, the linguistic properties (e.g., morpho-syntax, grammar) of each language the individual uses and the degree of similarity between these languages (e.g., English + German vs French + German). Fifth, the status of the languages within the communities the individual is a member of. Put otherwise, does the multilingual person use the language of the majority (e.g., the official language of the country) or of a minority. Last but not least, when working with multilinguals, one must remember that the whole is more than the sum of its parts: a multilingual individual is a specific case and not simply the sum of two (or more) monolinguals (Grosjean, 2022).

2.2. Cognitive models of multilingual processing

Researchers have introduced various cognitive models to account for reading and speech production in multilinguals. According to these models, language processing in the multilingual brain (e.g., L1 and L2) relies on common or distinct networks, and thus, languages may or may not share one lexicon. Alternatively, languages can also be segregated or integrated depending on the circumstances. When applying these models, however, one should also be sensitive to the heterogeneous profile of multilinguals described above.

Proposed already in 1994, the Revised Hierarchical Model is a model for sequential language acquisition (L1 → L2), which postulates that there are two levels of word representation: lexical (i.e., information about the word form) representation in L1 and in L2, and semantic (or conceptual, i.e., information about the word meaning) representation which is shared across languages (Kroll et al., 2010). Because sequential multilinguals have a larger lexicon in L1, than in L2, the connections between the layers are unbalanced. On the one hand, the connection between the lexical and semantic levels in L1 is more robust than in L2. On the other side, L1 lexical representation is weakly connected to L2 lexical representation, but L2 lexical representation is strongly connected to L1. For instance, consider the digits "5": German (L1), French (L2) bilinguals should translate "cinq" into "fünf" (i.e., L2 → L1) easier than "fünf" into "cinq" (L1 → L2). Although this model has dominated the field of bi- and multilingualism, its plausibility has been seriously scrutinized (Brysbaert & Duyck 2010).

In contrast, the Bilingual Interactive Activation (BIA +) model assumes that the integration of L1 and L2 lexicons and the representations of words are language-independent (Dijkstra & van Heuven, 2002). For example, when a German-French bilingual reads “fünf” or “cinq”, there is a common orthographic, phonological, and semantic process, and only later in the processing will the word be distinguished by languages (see also Dijkstra et al., 2019). The developmental version of this model assumes that increased exposure to L2 leads to weaker lexical associations with L1 and strengthens the L2 lexico-semantic associations (Grainger et al., 2010). Furthermore, the activation levels of L1 and L2 are possibly context-dependent, as Grosjean (2001) suggested. Within this framework, a multilingual individual has different “language modes” according to the situation (e.g., mono- vs multilingual) and the domain (e.g., literature vs mathematics). A German-French bilingual might activate strongly their L2 when in monolingual settings, but may activate both L1 and L2 when in a multilingual environment. Within the same individual, L1 can be the preferred language when discussing literature, while L2 will be reserved for solving mathematical problems.

At the neurocognitive level, imaging studies suggest common networks for lexico-semantic, grammatical, and phonological processing in multilinguals, although some differences remain. For instance, a meta-analysis by Sulpizio et al. (2020) showed that although L1 and L2 recruit largely the same brain structures, lexico-semantic processing in L1 recruits more cortical and subcortical areas compared to L2, especially if the acquisition of these languages is sequential. While L2 also recruits networks related to executive functioning. Differences between L1 and L2 in brain activation pattern concerning grammar and phonology were rare. According to the researchers, these findings suggest that word processing in L1 probably relies on a richer and wider lexico-semantic network than word processing in L2. Additionally, in line with previous data, the executive control network (involving structures such as prefrontal cortices, the inferior parietal lobules, and the left caudate nucleus) plays an important role in the individual’s ability to use the target language correctly (Abutalebi et al., 2008; Sulpizio et al., 2020). The degree of the executive control applied, and particularly inhibition, also depends on a variety of factors such as the age of acquisition and the context, for example, does the individual produce single language, dual language, or mixed language output (Fernández-Coello et al., 2016; Tao et al., 2021)

3. Learning in Diversity: The Good, The Bad, and the Math

There is a wide variety of multilingual school models worldwide (Cummins, 2016). Content and Language Integrated Learning (CLIL), or Content-Based Instruction, in Canada and the USA, is the best-known approach. CLIL combines L2 and educational content learning simultaneously, but the focus nonetheless remains on the content (Ruiz-Cecilia et al., 2023). This is achieved by using the L2 as the medium of instruction for content areas such as mathematics, science, or history (e.g., mathematics classes taught in English in France). Two-way immersion (TWI, also known as bidirectional or bilingual immersion) is another common school model. Here, the speakers of the majority (e.g., English) and minority (e.g., Spanish) languages are taught together in a bilingual environment using both languages, (e.g., math classes taught in both English and Spanish). The main goal of the immersion system is to achieve high-level bilingual mastery. Other multilingual approaches to school settings include the European school, international school and Transitional Bilingual Education (TBE) models.

While the systems mentioned above describe explicitly designed bilingual education environments, a bilingual education situation can also arise incidentally when immigrant children attend a school system in which the language of instruction differs from their language of origin (i.e. spoken at home). These children are also often referred to as “second-language learners” or “non-native learners”. Within these different educational settings, the cognitive processes associated with the control and the use of two (or several) different languages are marking multilinguals’ mathematical thinking and learning.

3.1. Switching language between learning and retrieval

Multilingualism can influence how numbers are processed by affecting cognitive processes such as learning and retrieval. Consequently, when multilingual individuals transition from one language to another, they may encounter Language Switching Cost (LSC, Marian & Fausey, 2006) – worse performance when they have to switch between languages (e.g., French vs German) compared to settings where they stay within one language. LSC has been found in short-term (i.e., between trials) and long-term (i.e., training, learning, and testing) settings. Short-term LSC is present, for example, in naming tasks: when the digit “5” has to be first named in French “cinq” and then in German “fünf” (i.e., Jackson et al., 2001, for a review, see Declerck & Philipp, 2015). Long-term LSC occurs when switching languages between training and testing. For instance, in their seminal study, Spelke and Tsivkin (2001) tested Russian-French bilinguals by training their arithmetic in Russian or French. When tested, performance was better in the trained language than in the non-trained language, irrespective of the individual’s language dominance, thus suggesting that exact arithmetic solutions are language-dependent (see also Grabner et al., 2012; Hahn et al., 2019; Saalbach et al., 2013 for replications of this finding in classroom settings). LSC can also occur when testing in a language different from the one used for learning mathematics (i.e., LM). For example, Philippino native speakers who learned mathematics in English retrieve arithmetic facts faster and more accurately in English compared to their mother tongue (Bernardo, 2001). In sum, multilingualism can affect cognitive processes of learning and retrieval, thus incurring a cost when individuals have to switch between different languages.

3.2. A cognitive advantage for multilinguals?

Because multiple language acquisition, processing, and production are tightly linked to executive functioning, some researchers suggest that being multilingual could function as a cognitive training, thus enhancing executive functions and acting as a neuroprotector by building cognitive reserve throughout the lifespan. Indeed, studies suggest that acquiring an additional language increases the grey matter volume and may potentially enhance other cognitive processes such as executive functioning (Hosoda et al., 2013; Mårtensson et al., 2012; Tao et al., 2021), which as a domain-general mechanism also plays an important role in mathematics (Bull & Lee, 2014). In line with this claim, some studies have reported that bilingual individuals show superior mathematics performance compared to their monolingual peers (Hartanto et al., 2018; Kempert et al., 2011; Saalbach et al., 2016; Stocco & Prat, 2014). However, the data showing that multilinguals enjoy a cognitive advantage over monolinguals are inconclusive.

Despite the initial popularity and plausibility (Bialystok et al., 2004) later studies using a variety of tasks and a large population of adults and children did not observe a cognitive advantage (Dick et al., 2019; Paap & Greenberg, 2013; Nichols et al., 2020). Meta-analysis investigations also provide mixed findings, with some researchers reporting evidence in favour of the cognitive advantage hypothesis (Adesope et al., 2010; Dentella et al., 2024; Grundy, 2020), while others did not after controlling for publication bias (de Bruin et al. 2015; Hilchey & Klein, 2011; Lehtonen et al., 2018; Paap et al., 2024). Respectively, differences in executive functioning can be due to other factors such as heterogeneity of language profiles and socioeconomic status. For instance, Gillet et al. (2021) compared different bilingual school constellations in Belgium and reported that only children immersed in a French-Dutch, but not those immersed in a French-English education system showed better performances in arithmetic compared to monolingual control children (Gillet et al., 2021). The specific characteristics (e.g., the languages learned in the multilingual educational settings) of the bilingual situation thus seem to modulate the influence of bilingualism on mathematics performance.

3.3. A learning disadvantage for multilinguals?

Large-scale studies demonstrate that L2 learners (i.e., individuals who at home speak a language different from the language of instruction) underperform in the language of (mathematics) instruction (i.e., L(M)I) comprehension (e.g., listening, reading, writing) and in mathematics competencies compared to their L1 peers (Beal et al., 2010; Heppt et al., 2015; Greisen et al., 2021; Paetsch et al., 2015; Ugen et al., 2013; 2021). Indeed, within incidental bilingual school settings, proficiency in the language of instruction accounts for a critical part of the performance differences between L1 and L2 speakers over and above other contributing factors such as socioeconomic status (Greisen et al., 2021; Paetsch et al., 2015). We elaborate on these examples in section 4.4. For example, Greisen et al. (2021) examined the relationship between the language of instruction and mathematical skills in two large cohorts of 3rd graders (total N = 10428) using the Luxembourgish national school monitoring test scores. Results showed that the children's mathematics performance was predicted by their reading comprehension of German (i.e., the language of mathematics instruction in Luxembourgish primary schools). The effect was especially prominent for L2 children who spoke a language with a very different linguistic structure than German at home (e.g., Portuguese). Put otherwise, L1 German speakers (i.e., children who spoke the language of instruction at home or as their mother tongue) outperformed L2 German speakers because the L2 speakers have lower comprehension of the instructional language.

Furthermore, examples from the French (CLIS, ULIS, SEGPA) and Luxembourgish educational systems indicate that L2 language learners often attain lower school grades than their native peers (Serre-Pradère et al., 2014; see also Martini et al., 2021) and are overrepresented in special education classes which aim to support children with special education needs (Couëtoux-Jungman et al., 2018). Overall, research suggests that L2 language learners are at an academic disadvantage compared to their L1 peers. However, it is worth noting that some research also suggests that L2 speakers are not generally disadvantaged, but only when the tasks are particularly language-demanding (Xu et al., 2022; Xu, Lafay et al., 2022). In the following section, we provide a review of the various ways in which multilingualism impacts both basic and advanced mathematical competencies. In this section, we review these influences from the bottom up regarding the complexity of the underlying cognitive processes.

4. The Effects of Multilingualism on Mathematical Competencies

4.1. Counting and lexico-semantic associations

Contrary to monolinguals, bilingual individuals must deal with more numerical representations: numerosities, digits, and (at least) two formats for number words (in L1 and L2), instead of one. Therefore, it is possible that for bilingual individuals, the numerical representations in one language are not automatically available in another (Saalbach et al., 2013; Spelke & Tsivkin, 2001). Marchand et al. (2020) explicitly tested this claim in 5 to 7-year-old French – English bilingual children attending schools using predominantly French as language of instruction. This study showed that the non-symbolic estimation skills (e.g., naming rapidly presented dots) differed between languages, showing an advantage for English compared to French. These differences persisted even when the researchers controlled for the number word familiarity across languages. These results imply that genuine differences exist in how children represent symbolic numbers in each language, consequently impacting how they are mapped onto the non-symbolic numbers.

Similarly, Wagner and colleagues (2015) examined the acquisition of small number words in English – Spanish and English – French bilingual children aged between 2 and 5 years. The results of this study showed that the early stages of counting proficiency (i.e., being one- two- or three-knower) in L1 did not predict counting proficiency in L2. Put otherwise, children who already knew the meaning of “three” in English did not acquire “tres” in Spanish faster – acquiring these first number words was instead predicted by age and experiences. In contrast, children’s mastery of the cardinality principle (i.e., CP = that the last number (*n*) in a set denotes how many items there are in a set) in L2 was strongly associated by being CP – knower in L1 and their counting ability in L2. Overall, the results of Wagner et al. (2015) suggest that in bilingual children, the acquisition of the first number words (“one”, “two”, “three”) is language-dependent, while the acquisition and mastery of the cardinality of larger numbers is not restricted to a particular language. Rather, it is related to learning the logic of counting in general.

Going one step further, in a cross-cultural study Schneider et al. (2020) demonstrated that the productive rules of language influence the acquisition of the counting sequences. Under this assumption, the acquisition of numerical concepts follows the same rules as the acquisition of natural language and thus relies on recursive rules. In their study Schneider and colleagues (2020) tested 3.5 – 6.5 years old children and examined whether differences in the morpho-syntactic structure of the counting list in transparent (Cantonese, Slovenian, and English) and opaque (Hindi and Gujarati) languages affected children’s understanding of the successor function (n+1). The results showed that more transparent language systems were associated with higher counting proficiency and understanding of successor functions than more opaque languages. Notwithstanding these cross-language differences, there was also a robust relation between counting proficiency and the successor knowledge for every language. These results suggest that the mechanisms involved in mastering the counting list are similar across languages. Still, languages with more transparent structures may provide a natural learning boost because they enable more accessible rule-based learning.

Notably, the acquisition of early number skills is affected not only by the language profile of the individual and the particularities of each language but also by the language of mathematics instruction (LM). In a large-scale study involving over 500 children aged between 3 and 6 years from diverse backgrounds, (Sarnecka et al., 2023) examined the effects of language context on a wide range of early number skills (e.g., counting, CP, number line estimation). The overall results showed no notable performance differences between monolingual and bilingual (English – Spanish) children. Moreover, for those who spoke both English and Spanish, the performance was similar across languages and even slightly better in the language of mathematics instruction (LM = English) despite the more extensive vocabulary in their home language (LH = Spanish), thus emphasizing the critical role the instruction language plays in the learning process. Overall, although studies in multilingual populations are still rare, these findings suggest that the language profile (bilingual vs monolingual), the particularities of each language, and the language of mathematics instructions may influence early mathematical competencies such as counting possibly because the lexico-semantic associations also differ between languages.

To examine the strength of lexico-semantic associations in multilinguals, Lachelin et al. (2023) tested proficient German (LM1) – French (LM2) adults, who had sequentially learned mathematics first in German (LM1), then in French (LM2). Here, individuals were shortly primed with either German or French number words followed by an Arabic digit target, which had to be named. The result showed a prime–target distance effect only with German number words, but not with French (Lachelin et al., 2023). Similarly, in a study with Dutch (L1) – French (L2) bilinguals, Arabic digit primes were followed by L1 or L2 number word targets, which had to be named. Targets had to be read or translated (i.e. switching). A distance effect was found when reading in or translating to L1 (i.e. prime = 2, target = “vijf” or “cinq” , response = /vijf/ or /vijf/). However, the effect was absent when naming in or switching to L2 (i.e. prime = 2, target = “cinq” or “vijf”, response = /cinq/ or /cinq/; Duyck et al., 2008), suggesting a privileged (number) semantic access for L1 compared to L2. These results indicate that the lexico-semantic associations of number words may vary in strength between a multilinguals’ different languages.

4.2. Transcoding and Arithmetic

Another fundamental cognitive process influenced by language is transcoding – converting from one numerical code to another. For instance, naming a digit involves converting the visual numerical code (i.e., “5”) into a verbal one (i.e., “/five/”). In multilingual individuals, transcoding has an additional layer of complexity because multiple verbal codes exist (e.g., “5” → “cinq” and 5 → “fünf”). Consequently, the strength of the relationships across the numerical formats (i.e., digits, number words, non-symbolic magnitudes) may vary depending on the language status and experience (for a meta analysis, see Garcia et al., 2021). For instance, when the language spoken at home (LH) differs from the language of mathematics instruction at school (LM), the latter is a strong predictor of digit naming performance. In a transversal developmental study including primary school children, adolescents, and adults, Lachelin et al. (2022) observed that irrespective of age, individuals are slower naming double-digit numbers in LM2 (i.e., French), compared to their LM1 (i.e., German). Similar results were found when adult bilinguals had to name single-digit numbers (Lachelin et al., 2023).

In addition, the transparency of the particular language’s counting system can also influence the number transcoding in multilinguals. In languages with inversion, such as Dutch and German, the decade comes before the unit (e.g., “twenty-four” in English vs “four-and-twenty” in Dutch (“vier-en-twintig”) or German (“vier-und-zwanzig”), resulting in a unit-decade discrepancy between the digits and the number words. Similarly, in French, the large number words are computed according to a base-20 system and are thus opaque (e.g., 99 = 4* 20 + 10 + 9). For example, Xenidou-Dervou et al. (2023) examined the effects of number words inversion in a sample of adult Dutch – English bilinguals. Participants performed an audio-visual numerical matching task (i.e., are the number they hear and the number they see same or different in terms of quantity), where the unit-decade congruency and the quantity match were orthogonally manipulated, resulting in the following conditions: congruent match (“twenty and four” vs “24”), congruent non-match (“two and forty” vs “24”), incongruent match (“four and twenty” vs “24”), incongruent non-match (“forty and two” vs “24”). Results showed that both in their L1 and L2 individuals were equally fast judging the real number words, although L1 number words involved inversion and the L2 did not, but they were more accurate in L1 trials involving artificially transparent number words. More importantly, this effect of transparency was stronger for individuals with higher L2 (i.e., English) proficiency. In contrast, the language profile did not impact morpho-syntactic processing in other studies. When Lafay et al. (2023) examined the transcoding performance in 8 – 10 years old children using a number writing task, the researchers found no difference between the error rate of L1 and L2 French speakers. Instead, children’s performance was related to their overall vocabulary skills. Overall, in multilingual individuals, transcoding seems to be better in the language of mathematics instruction. Language profile and the transparency of the counting system may impact transcoding, although these factors and their interaction effects may be subject of developmental changes and proficiency.

Similarly to transcoding, multilingual’s performance in simple arithmetic is typically better in the language they learned mathematics in (LM(1)), compared to other languages – a finding traditionally explained by the idea that arithmetic facts are stored in and retrieved from long-term memory in a language-dependent manner (Bernardo, 2001; Spelke & Tsivkin, 2001; Van Rinsveld et al., 2015; 2016). Accordingly, neurocognitive studies suggest that solving arithmetic problems in LM2 requires more effort and recruits additional brain areas compared to LM1 (Salillas & Wicha, 2012; Van Rinsveld et al., 2017; Wang et al., 2007).

However, it has also been suggested that arithmetic performance in multilinguals is not solely restrained to language-dependent memory but is also significantly influenced by language experience (for a review, see Cerda & Wicha, 2023). Indeed, in simultaneous English-Spanish bilinguals, Cerda et al. (2019) used an arithmetic verification task and observed no differences between languages in the N400 component - an event-related potential component indicating semantic processing. Similarly, the N400 observed during an arithmetic verification task peaked earlier in Spanish-English bilingual teachers when they were tested in the language they teach, compared to the language they learned mathematics in, suggesting that intense language exposure can also influence the automaticity of the lexico-semantic associations (Martinez-Lincoln et al., 2015). Moreover, factors such as language context also play a role when solving arithmetic. For example, bilingual participants performed better in solving arithmetic problems in LM2 when they first read a sentence compatible with the language in which the arithmetic problem was presented (Van Rinsveld et al., 2016), suggesting that the performance efficiency in multilingual is also related to activating the corresponding "language mode" (Grosjean, 2001). Overall, multilingual individuals are more efficient in transcoding and solving simple arithmetic problems in the language in which they have learned mathematics. However, early language acquisition and language context might also influence arithmetic fact retrieval in different languages.

4.3. Word problems and mathematical language

Mathematical word problems present concrete situations that need to be conveyed into a mathematical equation to come to a result (Greer et al., 2002). Word problems are considered a challenging part of the mathematical school curriculum (Daroczy et al., 2015) and require different steps to be resolved: from understanding the given context through retrieving relevant information to creating a mathematical model and solution (Verschaffel et al., 2000). Because word problems are in the form of written text, an individual's reading skills play a critical role, which has an important implication for multilingual settings (Fuchs et al., 2015). For instance, in third-graders, Powell et al. (2020) demonstrated that L2 English speakers had a lower level of English mathematics vocabulary (i.e., terms related to mathematics) compared to their monolingual peers. Particularly because L2 speakers of the instruction language typically demonstrate lower language proficiency and a lower domain-specific vocabulary than their L1 peers, this may put the former at a disadvantage when solving word problems.

Kempert et al. (2011) compared Turkish (L1) - German (L2) bilingual children to German monolinguals to examine this question more in detail. Results showed that the level of proficiency in the instructional language (i.e., German) predicted the bilinguals' performance on word problems such that children with low proficiency in German scored lower on simple word problems than their monolingual peers. In contrast, for more complicated word problems requiring attentional control, there were no significant differences between the monolinguals and the L2 speakers with a high command of German. Finally, in bilinguals with a high command of German, the performance in word problems was superior to that in Turkish (i.e., their L1). Such results again emphasize that mastering the language of instruction plays a critical role in math performance, even above the individual's native tongue, although some studies report that children perform better on word problems in their L1 compared to their L2 (e.g., Bernardo & Calleja, 2005).

Word problem performance in multilinguals is also related to language experience, the maturation of critical cognitive functions such as working memory, and their interaction. Concretely, Ester et al. (2021) demonstrated that first-grade bilingual children were less effective in finding solutions to word problems in their second language, whereas this was not the case in second-grade students. Studies in Spanish (L1)-English (L2) bilingual primary school children found that the central executive component of WM predicts the accuracy of mathematical word problem solutions in both L1 and L2 (Swanson et al., 2019; Swanson et al., 2022). Overall, these findings demonstrate that L2 speakers, particularly those with a low proficiency in the language of mathematical instruction, can be at a disadvantage when solving word problems. However, the performance in this group depends on additional factors such as language experience and executive functioning, which tend to improve over time.

4.4. Advanced Mathematics achievement

Deploying advanced mathematical skills can prove challenging for L2-speaking children. Experimental data from older children (i.e., 10-12 years old), revealed L2 Dutch speakers demonstrated lower scores on geometry and fraction tasks, as well as significantly lower academic vocabulary and verbal reasoning scores compared to their L1 peers (Kleemans & Segers, 2020; see also Kleemans et al., 2014). Since such tasks presuppose a good understanding of the domain-specific terminology, they also indirectly assess student reading comprehension in addition to mathematical knowledge (Boonen et al., 2013). Given that L2 speakers often have poorer language skills (e.g., Kleemans & Segers, 2020; Mancilla-Martinez & Lesaux, 2010) compared to their L1 peers, it is highly likely that the performance gap will only become more prominent for advanced mathematical skills. Language-related difficulties are probably greatest for students who learned mathematics solely in their native language before transferring to a mathematics course taught in another language (Kempert et al., 2011; Spelke & Tsivkin, 2001), as they must (re)learn specific terminology (Dehaene et al., 1999) and may engage in time-consuming translations. This is particularly true when considering domains where students must rely on a well-developed academic vocabulary (geometry or word problems; Kempert et al., 2011).

4.5. Dyscalculia

Developmental dyscalculia, or specific learning disorder in mathematics, is a neurodevelopmental disorder characterised by persistent difficulties in basic numerical and mathematical skills, including number sense, number facts, calculation, and mathematical reasoning (American Psychiatric Association, 2013; World Health Organization, 2019). Without intervention, dyscalculia can negatively impact an individual's functioning in school (Schulte-Körne, 2016), and, consequently – their personal life and professional achievements (Kohn et al., 2013; Parsons & Bynner, 2005; Ritchie & Bates, 2013).

To this date, sufficient direct evidence concerning mathematical difficulties and dyscalculia in multilingual populations is lacking. Swanson and colleagues (2018) conducted a longitudinal study to investigate how increasing bilingual proficiency influences mathematics performance in Spanish (L1)-English (L2) bilingual children with severe mathematics difficulties. Children's bilingual proficiency level was determined based on their combined English and Spanish vocabulary scores. The findings indicated that in children with mathematics difficulties, higher bilingual proficiency came along with higher mathematics and working memory-related performances, both concurrently and longitudinally (Swanson et al., 2018). Similarly, Powell and colleagues (2020) observed that differences in mathematical vocabulary between English-monolingual and English-L2-speaking students experiencing mathematics difficulties depended on the type of deficits they presented. Concretely, when experiencing difficulties in arithmetic, L2-speakers underperformed compared to their L1 monolingual peers. In contrast, there were no differences in mathematical vocabulary between mono- and bilingual individuals encountering difficulties in mathematical word problem solving (Powell et al., 2020; 2022). These findings show that bilingualism can lead to differential results in children with difficulties or a disorder in mathematics. Specific intervention and teaching methods may support bilingual children with dyscalculia and enhance their performance in mathematical problems (King & Powell, 2023; Lariviere et al., 2022; Lei et al., 2020 for a meta-analysis). Yet, to accurately identify multilingual children at risk, it is crucial to consider individual's language profiles before an official diagnosis. Nonetheless, it is difficult to draw generalisations based on these data because the criteria for dyscalculia or mathematics difficulties differed between studies (e.g., below the 13th or 25th percentile), and mixed populations were included (i.e., children who had either a disorder or difficulties). Different study designs (e.g., comparing within bilinguals versus bilinguals and monolinguals) also make interpretations and potential conclusions for interventions more difficult. Thus, more research is needed to better understand the processes involved in bilingual individuals with dyscalculia or mathematics learning difficulties. We elaborate more on dyscalculia in multilinguals in the following section.

5. Challenges, recommendations, and future directions

Individuals in multilingual educational settings, such as Luxembourg, Lebanon, Singapore, and South Africa, become balanced and highly proficient multilinguals, which is generally viewed as an asset (e.g., Bialystock, 2018). However, diverse linguistic backgrounds can also create barriers and inequalities between students. Multilingualism also poses a high demand on students, given that the instructional languages are mostly not formally taught as foreign languages, and individuals are expected to master them independently. For instance, as we saw in the current review, mathematical attainment can vary vastly, depending on whether an individual speaks the language of instruction and whether they speak it fluently. Below, we outline some tentative proposals we believe are worth considering by educational practitioners and researchers in numerical cognition. However, we want to point out that these suggestions are based on the current, rather, sparse literature and our experience working with multilingual populations. Since research on multilingual classrooms is still in its infancy, more data are needed, particularly one involving longitudinal designs and interventions.

5.1. What does it take to succeed in multilingual educational settings: the case of Luxembourg

The Grand Duchy of Luxembourg has three official languages (Luxembourgish, German, and French), and a multilingual education system: in kindergarten, the instructional language is Luxembourgish; in primary schools (6 years), the instructional language for all school subjects is German, while from the 7th to 13th grade, the instructional language gradually becomes French (Ministère de l'Éducation Nationale, 2024). Despite its small size, the country of Luxembourg has a super diverse society encompassing individuals from various nationalities, ethnicities, language groups, socioeconomic backgrounds, and identities (STATEC, 2021). Consequently, this poses a challenge for the school system and the teachers, as only about 32% of the primary school students speak Luxembourgish (i.e., the first language of instruction) at home (MENJE & SCRIPT, 2022). Nonetheless, the country fares relatively well in international educational assessments such as PISA (Weis et al., 2020). Consequently, some factors can be identified as pivotal in successful academic performance in multilingual and diverse settings, which could be potentially generalized to other superdiverse environments.

One such factor, expectedly, is the figure of the teacher and the relationship they build with the students. Indeed, in a systematic review of meta-analyses plus a second-order meta-analysis, Emslander et al. (2023) found that teacher-student relationships (TSR) are essential for creating a positive school environment. TSRs were strongly associated with academic achievement, academic emotions, appropriate student behaviour, behaviour problems, executive functions and self-control, motivation, school belonging and engagement, and student well-being. Consequently, focusing on building TSR is one way to improve the learning experience in multilingual settings. Additionally, a critical step to building solid TSRs in multilingual classroom settings would be the teachers' awareness that language can strongly interact with the pupils' learning outcomes, and this interaction can be potentially negative for non-native speakers. Ironically, this L2 learner disadvantage strongly contrasts with teachers' belief that mathematics contains minimal language and fewer linguistic challenges than other topics (Fernandes, 2023). Such an underestimation of mathematical language load is problematic since it prevents the establishment of dedicated learning and instruction measures necessary to support language learners appropriately. Consequently, specific training programs can be implemented and scientifically monitored to inform and prepare educators working in multilingual settings about the influence of language on mathematics.

Furthermore, as Emslander, Rosa et al. (2023) suggest, the acknowledgement of the students' home language also has the potential to improve educational experiences in multilingual settings by promoting adaptability, inclusion, and acceptance. Support for non-native speakers can also focus on increasing the L2 speakers' proficiency in the language of instruction (e.g., by teaching it as a foreign language), providing educational material in their native tongue, or acknowledging their language preferences. For instance, when high-schoolers take the Luxembourgish national school monitoring test they are offered the choice to solve the item either in German (LM2) or in French (LM2). Interestingly, although the LM is French in high school, many students opt to solve the items in German, probably because this is their preferred language for counting, mental arithmetic, and the first language of mathematics instruction (see Martini, 2021). Nevertheless, there are large groups of students (e.g., French speakers, Portuguese, or Slavic speakers) who learned to count and prefer to do mental arithmetic in a different tongue and for whom the provided language options might not be the best fit (Martini, 2021). Consequently, offering educational material in different languages already in the early grade and/or reducing the language load where appropriate is worth considering. In sum, although multilingual educational environments are often challenging and generate educational inequality, focusing on creating positive TSRs, raising the teacher's awareness about the role of language in learning, and acknowledging language and cultural diversity can create a positive learning environment.

5.2. Learning difficulties in diversity

As we saw in section 4.5, multilingual educational settings may create inequality for typically developed children and children with learning difficulties. A crucial criterion concerning multilingualism and general linguistic diversity listed in the classification manual DSM-5 (American Psychiatric Association, 2013) and ICD-11 (World Health Organization, 2019) is that a lack of understanding of the language of mathematics instruction cannot explain the learning difficulties. This process becomes increasingly challenging also because dyscalculic children, in general, show lower language skills than their peers, regardless of language status (Chow et al., 2021; Forsyth & Powell, 2017; Powell et al., 2020). Indeed, results also suggest that bilingual children with a specific learning disorder experience difficulties in language-related skills compared to their monolingual peers and children without a specific learning disorder (Riva et al., 2021). Additionaly, given that psychometric tests assessing children's mathematical abilities for diagnostic purposes are generally based on language, language proficiency may influence performance in these tests, ultimately increasing the risk of misdiagnosis (Ugen et al., 2021). Consequently, to prevent over- or underdiagnosis, bilingual and multilingual children whose native tongue is not the same as the language of instruction in schools require special attention during the diagnostic process.

A study conducted on data from the national school monitoring program "Épreuves Standardisées" in Luxembourg has revealed how performance differences in mathematics based on language background affect cut-off scores at the low-achievement range (i.e., 25th and 10th percentiles; Martini et al., 2021). Due to significant differences in performance, cut-offs based on the whole sample would lead to different identification rates in different language background groups. In fact, for L2 speakers (i.e., children who do not speak the language of mathematics instruction at home) more than 10% and > 25% would be identified as children at risk of mathematics difficulties, resulting in overdiagnosis. Conversely, since the cut-offs are based on the whole sample, the identification rate in children who speak the instructional language at home would be lower than 10% and < 25%, thus resulting in underdiagnosis. To compensate for the L2-speaker disadvantage and avoid an over-diagnosis, cut-offs for children who do not speak the instructional language at home must be lower than those for the L1 peers, and vice versa (Martini et al., 2021). In sum, in multilingual environments, diagnostic tests should have separate reference norms depending on the children's language background and proficiency (Ugen et al., 2021).

Overall, research shows that multilingualism is essential to consider in dyscalculia. First, language proficiency should be considered during the diagnostic process to allow a fair and adequate diagnosis. Second, affected children should be supported, and their language proficiency levels should be given special attention. Unfortunately, as evidenced by an inventory study by Fischer and Pit-ten Cate (2021), most diagnostic tests are adapted to native or proficient speakers, and only a few consider linguistic heterogeneity. Consequently, the current diagnostic tests are poorly suited for multilingual contexts where proficiency in the testing language varies widely between individuals. Using such tools poses a severe challenge to clinical and educational professionals who must disentangle difficulties originating in poor language skills from a genuine learning disorder. On the positive side, diagnostic test batteries in mathematics tailored to multilingual educational settings for primary school children are being developed.

Finally, today's psychological science has put much effort into increasing and acknowledging diversity. Yet, research articles in numerical cognition rarely report language profiles, even when working with diverse populations. Therefore, we encourage numerical cognition researchers working in multilingual environments to collect and report information regarding their participants' language status whenever appropriate. This can be done in many ways, the simple one of which can be a questionnaire collecting information about the following (see also section 2.1): the number of languages an individual speaks, the order and age of acquisition, the frequency of daily usage, their personal language preferences—in what language they speak with their family members, in what language they prefer to count, etc. Additionally, depending on the research question, individuals' preferences should be considered during experimental testing and interventions to prevent confounding genuine group and individual differences with language proficiency.

6. Conclusions

In the current article, we reviewed the effect of multilingualism on basic and more advanced mathematical competencies and mathematical learning difficulties. Overall, when it comes to basic skills such as counting, building lexico-semantic associations, transcoding, and arithmetic, research suggests that performance is better in the individual native tongue (L1) and/or in the (first) language of mathematics learning (LM(1)). For more complicated skills, such as word problems, which rely on vocabulary and language skills, the importance of L1 increases at least until the mastery of LM is sufficiently high. Nonetheless, L2 learners of the language of instructions tend to underperform compared to their L1 peers in mathematics achievement tests. To overcome this issue, educational practices focusing on building a positive learning environment, increasing LM proficiency, and/or adapting the individual's language needs are needed. Additionally, when considering the influence of multilingualism on mathematics, one must remember that multilingualism is a spectrum, and the language learners' profiles are incredibly diverse. This is especially important when screening for learning difficulties, often done with diagnostic tests unsuitable for multilinguals and doing research in diverse populations.

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