



EXPRESSING GENERAL CONSTITUTIVE MODELS USING ALGORITHMIC AUTOMATIC DIFFERENTIATION IN DOLFINx

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19th Jun, 2024

The Unified Form Language (UFL) (Alnaes et al., 2013) is a widely-used package for writing variational forms of partial differential equations. However, many notable solid mechanics problems cannot be naturally expressed in UFL e.g. complex plasticity and neural network constitutive models. This limitation has slowed the adoption of FEniCS in the solid mechanics community. In this talk we show a framework (Latyshev & Hale, 2024) for DOLFINx (Baratta et al., 2023) that enables the inclusion of arbitrary constitutive models using any third-party package that supports ndarray-like objects (e.g. JAX (Frostig et al., 2018), Numba (Lam et al., 2015) and others). We achieve this by leveraging three recent developments: the ExternalOperator extension of UFL (Bouziani & Ham, 2021), the new data-centric design in DOLFINx, and its automatic code generation for UFL Expressions. A key outcome of this work is the support for algorithmic automatic differentiation techniques in DOLFINx, demonstrated by implementing a non-associative plasticity model of Mohr-Coulomb type using forward-mode automatic differentiation in the JAX library.

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