

USING RANDOM CIRCULAR MODELS IN AQUIFER FLOW SIMULATION

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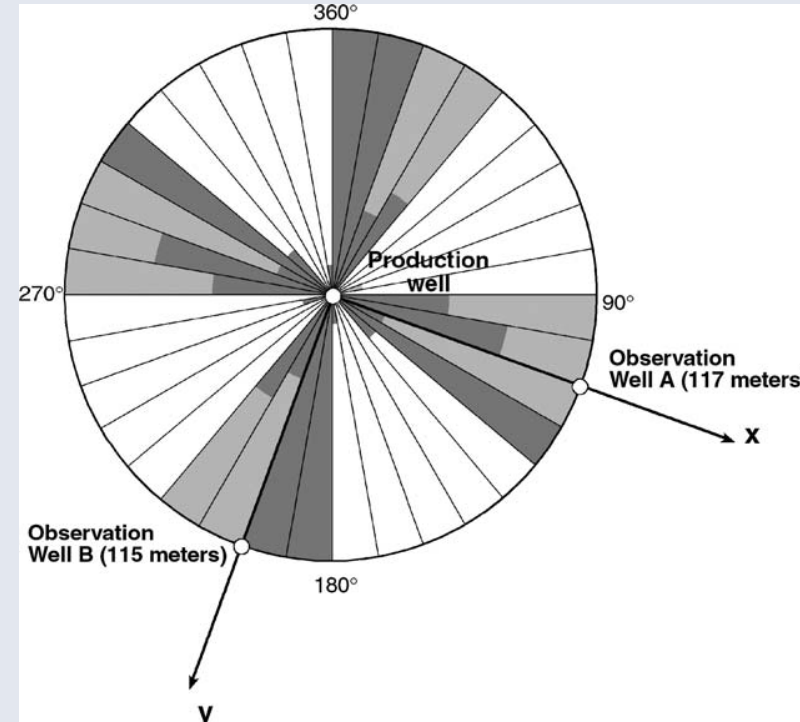
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FRACTURES AND HYDRAULIC CONDUCTIVITY

The water flow in many aquifers is driven by strong anisotropy created by preferential flow features such as fractures and faults. We modeled this anisotropy hydraulic conductivity (AHC) in our poroelastic model.

From Heilweil and Hsieh, 2006



OBJECTIVE

- Overall goal:** assimilate InSAR data into an aquifer model.
- In this work:** develop a flexible stochastic prior model of the AHC tensor that respects its underlying symmetry and positive definiteness.

UNCERTAINTY IN AHC TENSOR

- Because of reported $\approx 20\%$ uncertainty in Heilweil's AHC results,
- Assuming AHC's principal direction aligns with the fracture direction.
- Introducing a stochastic model, accounting for random variance in both the direction and magnitude of AHC using spectral decomposition (Shivanand, Rosić, and Matthies, 2024).

$$k = Q\Lambda Q^T =$$

Diagram illustrating the spectral decomposition of the hydraulic conductivity tensor k . The tensor is represented as a product of three matrices: Q (eigenvectors), Λ (eigenvalues), and Q^T (transpose of eigenvectors). The eigenvectors are shown as a set of $d \times d$ vectors, and the eigenvalues are shown as a set of $d \times d$ scalars. The resulting tensor k is a $d \times d$ matrix.

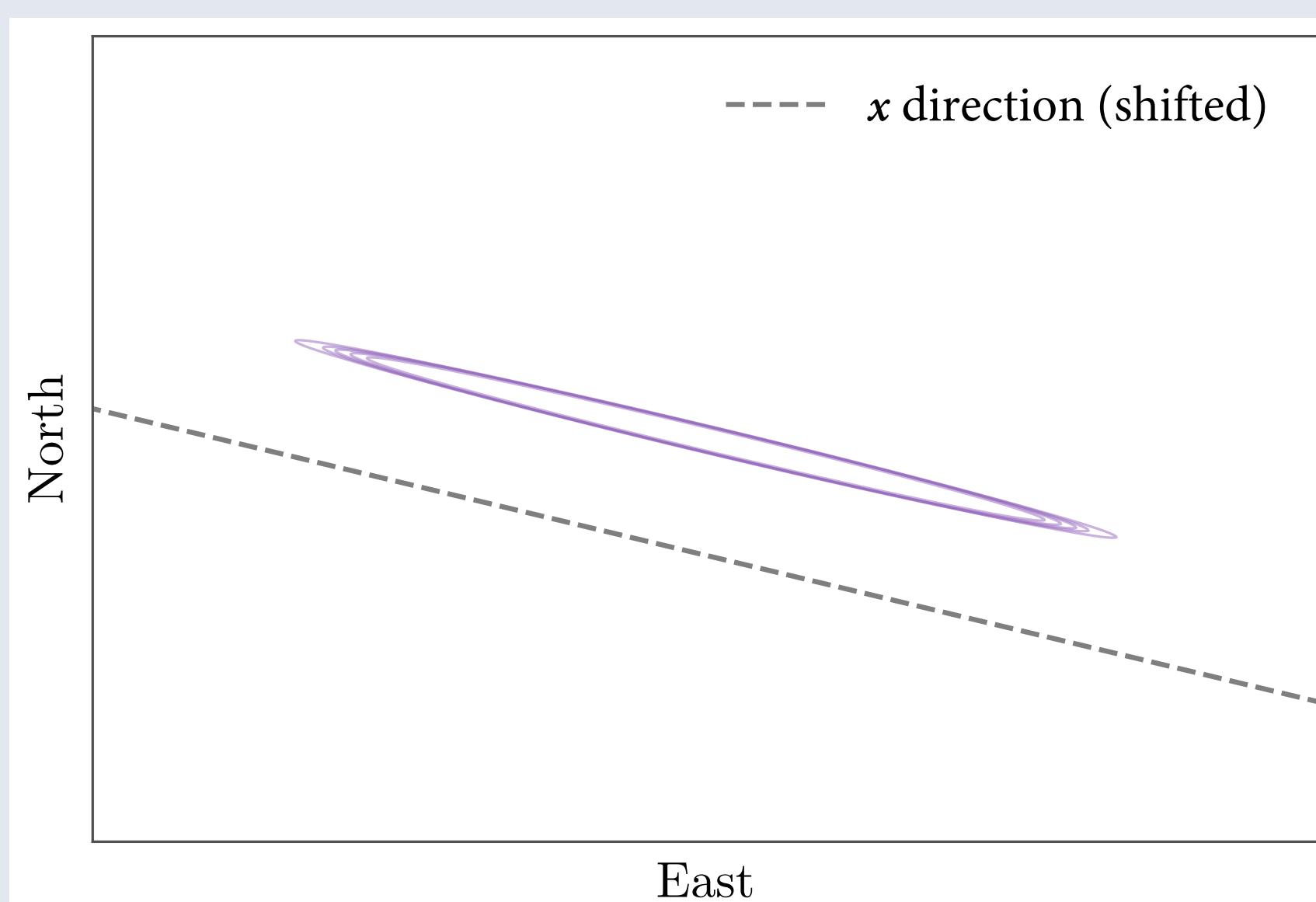
STEP 1: RANDOM SCALING

$$k_s(\omega_s) = \hat{Q}\Lambda_s(\omega)\hat{Q}^T$$

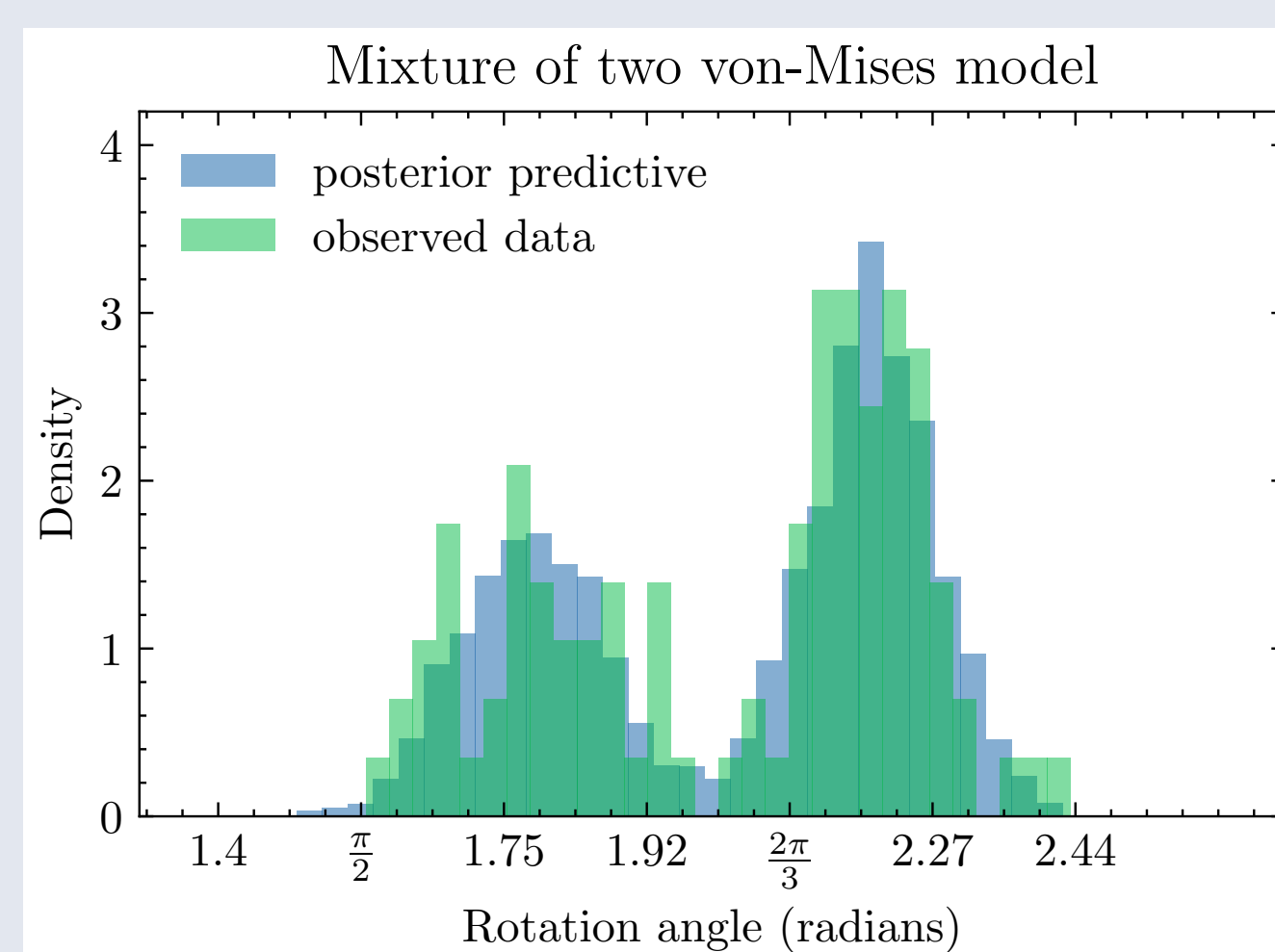
$$\Lambda_s = \begin{bmatrix} \lambda_x & 0 \\ 0 & \lambda_y \end{bmatrix}$$

$$\lambda_x = \text{lognorm}(k_{xx}, 0.2)$$

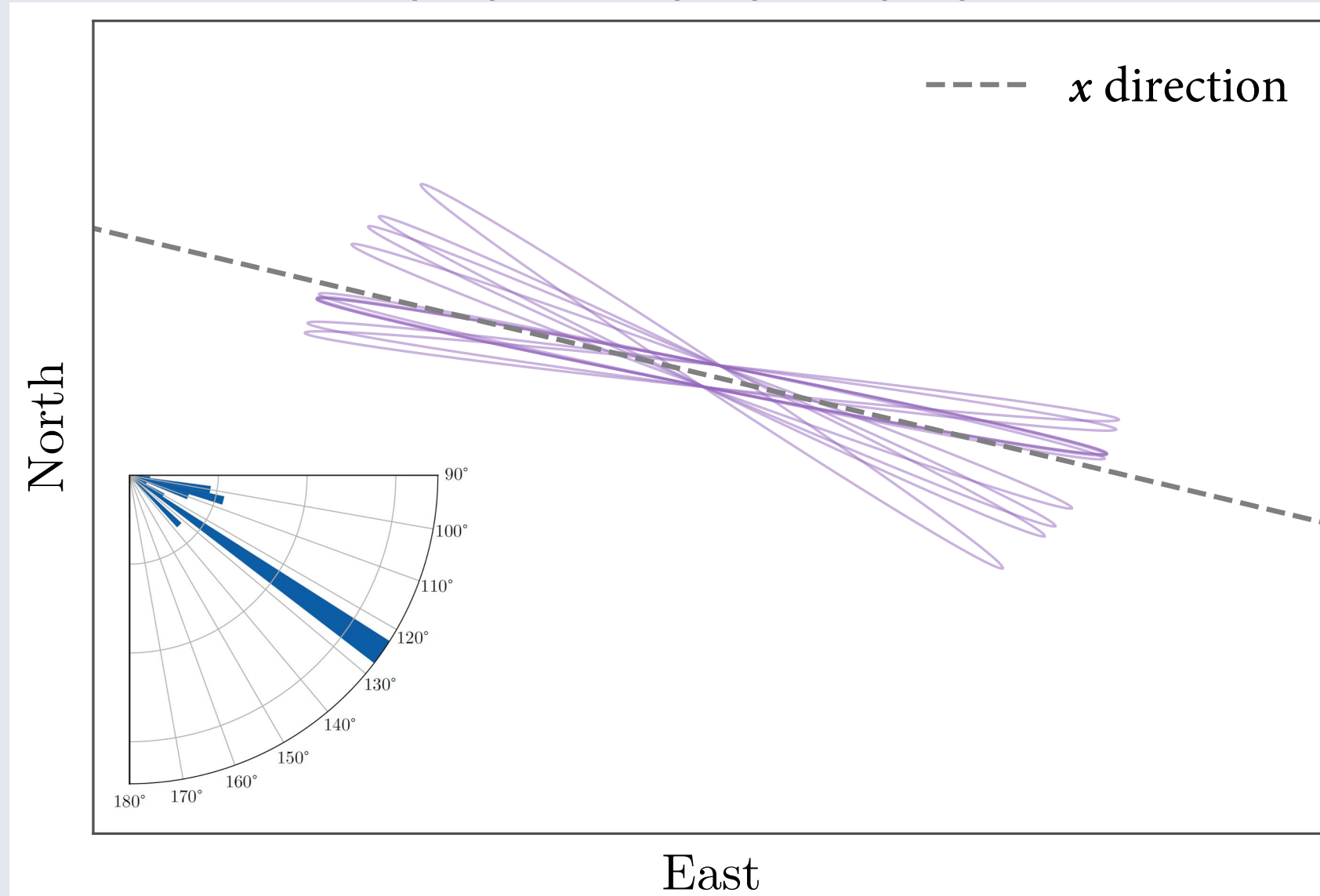
$$\lambda_y = \text{lognorm}(k_{yy}, 0.2)$$



STEP 2: RANDOM ROTATION



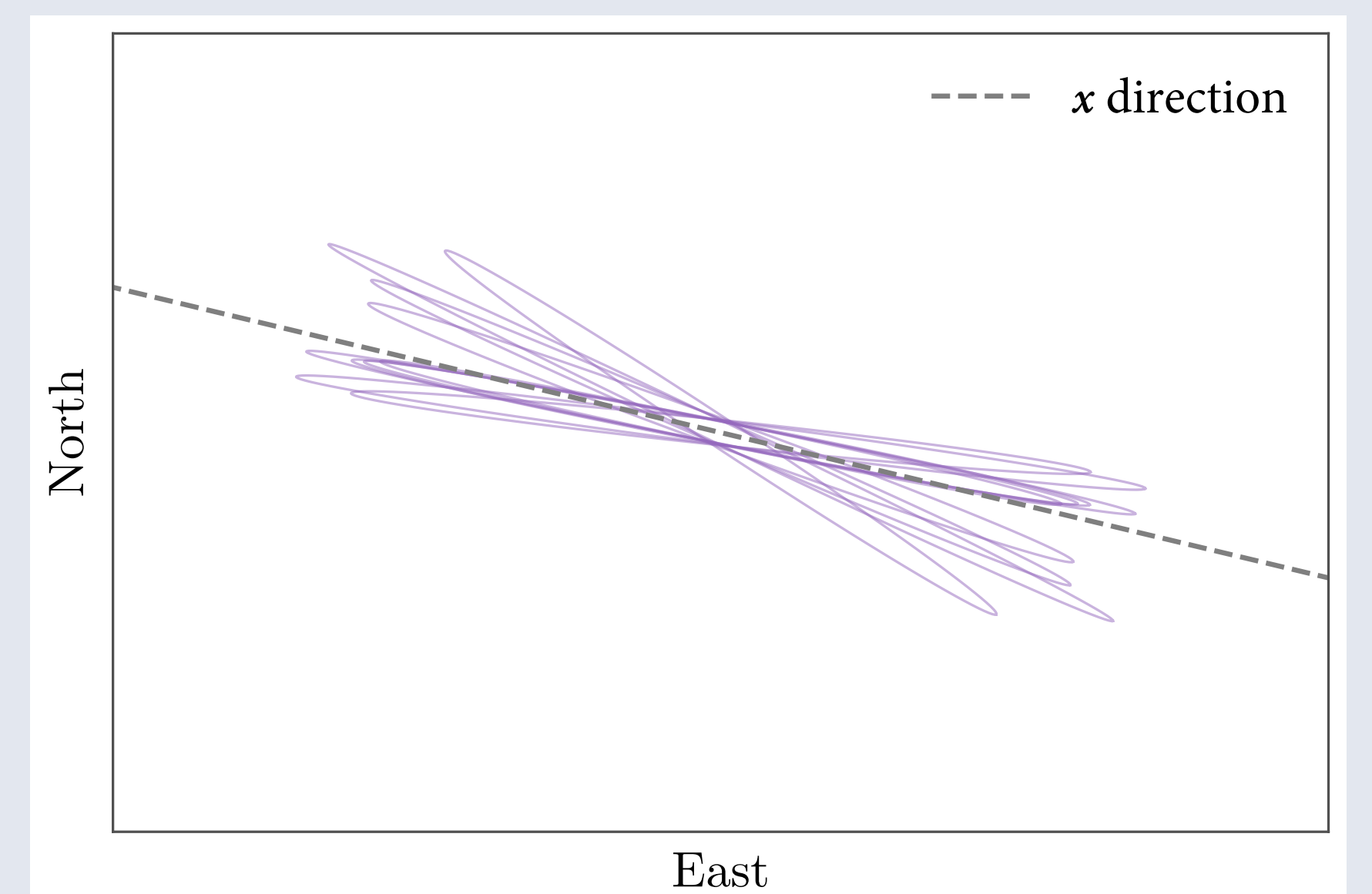
$$k_r(\omega_r) = R(\omega_r)\hat{k}R(\omega_r)^T$$



STEP 3: RANDOM BOTH

We consider these two approaches (fixed orientation and fixed scaling) independent, so we can combine them.

$$k_{rs}(\omega_{rs}) = R(\omega_r)k_s(\omega_s)R(\omega_r)^T$$



Random Hydraulic
Conductivity Tensor
($k_{rs}(\omega_{rs})$)

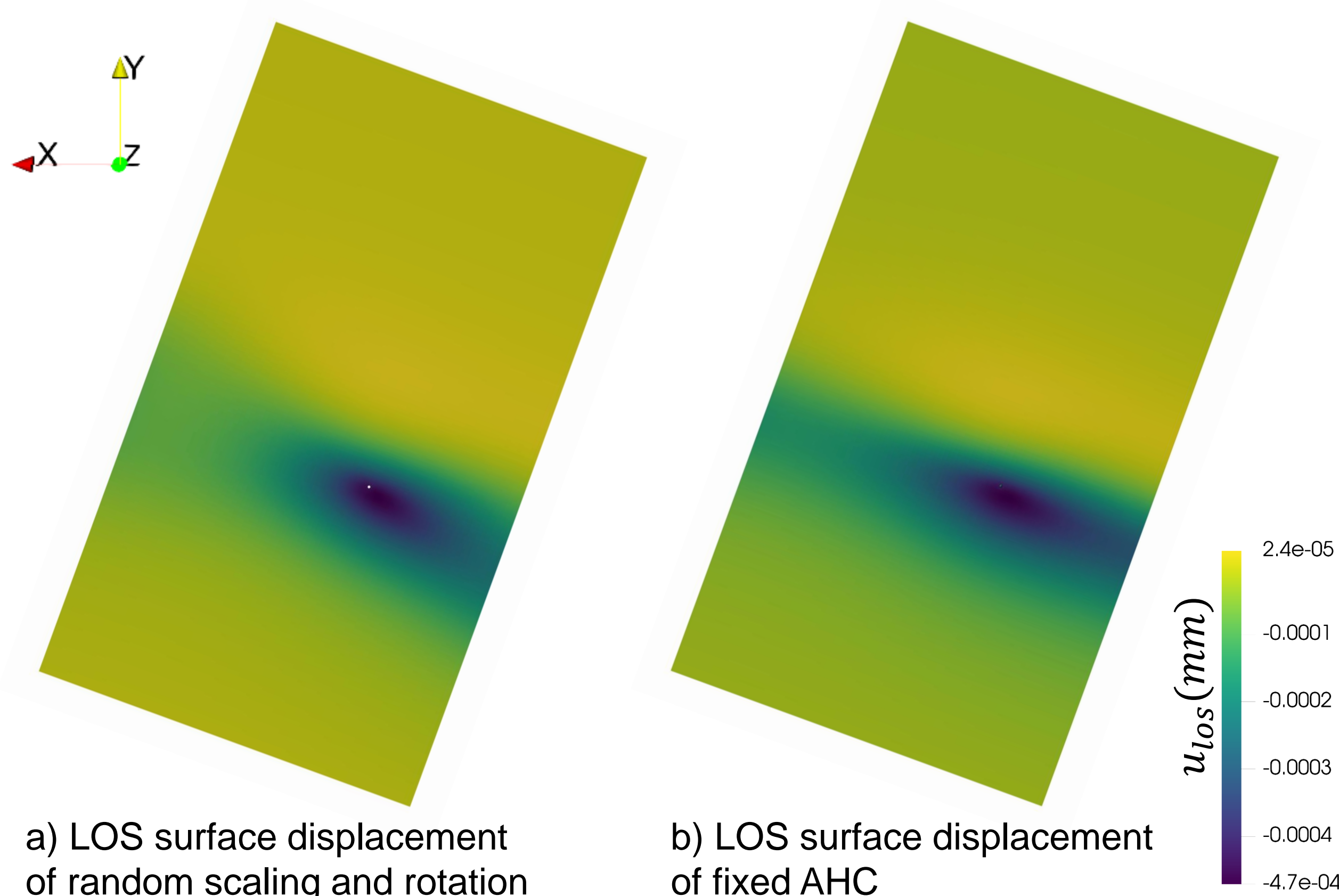
Running Parallel
Poroelastic Finite Element
Model in HPC

Outputs (line of sight
deformation)

Statistical Analysis (mean,
standard deviation, ...)

DISPLACEMENT OF AQUIFER MODEL

The results show that the randomness in hydraulic conductivity is reflected in the surface displacement.



a) LOS surface displacement
of random scaling and rotation

b) LOS surface displacement
of fixed AHC

TAKE-HOME MESSAGE

Regenerating rotation angles using a mixture of two von Mises models and employing them to model random symmetric and positive definite (SPD) tensors were addressed in this study. The focus is on modelling uncertainties in such a way as to have a fine control independently over the scaling and directional attributes of the tensor—given that the material symmetry is fixed.

ACKNOWLEDGEMENTS

This work was funded in whole, or in part, by the Luxembourg National Research Fund (FNR), grant reference PRIDE/17/12252781.

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- Shivanand, Sharana Kumar, Bojana Rosić, and Hermann G. Matthies (2024). *Stochastic Modelling of Symmetric Positive Definite Material Tensors*. arXiv: 2109.07962 [math.NA].