FACULTY OF LAW, ECONOMICS AND FINANCE

Department of Economics and Management

Discussion Paper

2024-07

Economics

Department of Economics and Management
University of Luxembourg

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September, 2024

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To Deploy or Not to Deploy CCS Abatement, and When: A Differential Game Perspective*

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September 11, 2024

Abstract

Carbon capture and storage (CCS) can be considered as one of the key tools in the fight against climate change, providing a promising method to reduce human-generated CO_2 emissions. Despite its potential, the high cost of CCS deployment leads to an uneven adoption across countries. This paper employs a differential game model with heterogeneous countries facing transboundary pollution to determine the optimal timing to initiate CCS projects, and delivers analytical results for the existence of Markov Perfect Equilibria and the numerical illustration. We show that: (1) The trigger threshold for CCS deployment depends not only on a country's own costs, but also on the costs of other countries and the costs associated with pollution damage. (2) The optimal timing for different countries to initiate their CCS projects occurs when a country's pollution level reaches a critical threshold. (3) Countries are more inclined to free-ride on the pollution abatement efforts of others when the pollution damage costs are symmetric rather than asymmetric. (4) Finally, we provide sufficient conditions under which some countries refrain from engaging in CCS, despite facing the same pollution damage costs as others.

Keywords: Carbon capture and storage, optimal timing, Markovian perfect equilibrium
JEL classification: Q53, Q58, C61, C72

*We appreciate the useful comments and suggestions from Raouf Boucekkine and Herbert Dawid. This research was supported by the Social Science Foundation of Jiangsu Province, China (grant number 22EYD005).

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1 Introduction

Carbon capture and storage (CCS) or utilization (CCU) of the captured CO_2 have a critical role to play in the world's quest for net zero, which requires a gradual phase-out of existing coal-fired power plants and industrial facilities. These technologies offer a pathway to significantly reduce human-generated greenhouse gas emissions while mitigating the risk of stranded assets. According to estimates by Det Norske Veritas (DNV), a complete shift to a fossil-free energy system by 2050 is unrealistic; instead, fossil fuels are projected to still constitute 16% of the global energy mix by then. In this scenario, CCS is expected to play a crucial role, accounting for 20% of the required emissions reductions (IEA, 2022; Salt, 2022)[1]. It thus comes as no surprise that, in the Intergovernmental Panel on Climate Change Working Group III Report (2022), CCS is comprised in nearly all the mitigation pathways to limit the global temperature rise to below 1.5°C above pre-industrial levels.

Governments worldwide have acknowledged the importance of deploying CCS technologies and have integrated them into their climate strategies. The United States has developed a comprehensive policy framework to support all stages of CCS deployment. Canada has introduced an Investment Tax Credit (ITC) for CCUS as part of its 2021 Budget. The adoption of the EU Green Deal, the Climate Law, and subsequent proposals to meet ambitious energy and climate targets for 2030 have made CCS a critical component of the EU's decarbonization strategy. In China, around 10 provincial governments have included CCS development in their decarbonization strategies (Global CCS Institute, 2023b). These are just a few examples that illustrate the global recognition of CCS as a key tool in the fight against climate change.

Despite these efforts, the pace of CC(U)S adoption remains sluggish and current investments in CCS projects seem too low to ensure commercial-scale deployment. On the one hand, CCS is deemed crucial to reach emission reduction targets without prematurely abandoning existing technology and infrastructure. However, on the other hand, investing in CCS may extend the reliance on fossil fuels, while diverting funds from renewable technologies which improve over time and are crucial for a sustainable long-term energy supply and environmental equality. Given these complexities, whether to engage in CCS initiatives or not is still under debate, as clearly stated by Moreaux et al. (2024). In the literature, there are several studies that explore the optimal timing and conditions under which a country (or industry) should deploy CC(U)S (Amigues et al., 2016; Durmaz, 2018; Moreaux et al., 2024, etc.). However, these studies are silent on international competition, transboundary pollution control, and free-riding issues. In reality, there is no supranational authority acting as a sole government to enforce pollution control measures or providing incentives for the implementation of CCS facilities. While international treaties and agreements set long-term climate goals, individual countries' efforts to implement CCS vary widely. CCS installations are capital-intensive, requiring significant investments in R&D and the construction of facili-

[1]The DNV is one of the largest providers of technical consultancy and supervisory services to global renewable energy and oil and gas industries.

ties, as well as supportive regulatory frameworks and policies. This means that countries are at different stages of CCS development: while some countries have taken significant steps to deploy CCS, others are lagging far behind. Therefore, different countries must make their optimal choices based on their specific circumstances and capacities.

In this paper, we propose a differential game model aimed at investigating the optimal timing for countries facing transboundary CO_2 pollution to deploy CCS. Our setting considers the situation where technologically advanced countries with a supportive policy environment take the lead in implementing CCS, while other countries may follow at a later point in time. By focusing on country-specific factors, we capture the heterogeneity among countries. We derive explicit results for the following two scenarios: (1) two countries facing identical CO_2 pollution damage costs but differing in development levels and CCS deployment costs; (2) two countries that are asymmetric in in both pollution damage and CCS deployment costs.

We present both, general results on the trigger thresholds for CCS deployment applicable to all countries without assuming specific functional forms and detailed analytical findings using a linear-quadratic game framework (Boucekkine et al., 2023; Dockner et al., 2000; Dockner and Van Long, 1993). Additionally, we identify the order and optimal timing of the implementation of CCS facilities, which are determined by the reachability of the trigger thresholds and depend on a country's technological advancement, efficiency, and willingness to invest in CCS. Nevertheless, in scenarios where pollution damage costs are symmetric, the incentive to free-ride causes countries to depend on others to implement CCS and curb emissions, which ultimately contributes to higher long-term pollution accumulation.

Our model also establishes sufficient conditions under which certain countries may never trigger CCS deployment. This reluctance stems primarily from two factors: the high costs pose a significant barrier to adoption for some countries (Bertinelli et al., 2014; Ferrari et al., 2019), while the costs of pollution damage to a country also play a significant role (Colmer et al., 2020; Greenstone and Jack, 2015). This finding offers clear and actionable policy recommendations. Our model demonstrates that the threshold for triggering CCS increases with a country's own CCS costs but decreases with both the costs faced by rivals and the efficiency of CCS technology. This suggests that one of the most effective ways to accelerate CCS deployment in still-inactive countries is to advance technology, which can significantly reduce future CCS costs and enhance its efficiency.

The remainder of the paper is organized as follows. Section 2 provides an overview of the current state of CCS deployment and reviews the relevant literature. Section 3 presents the model and characterizes the Markovian perfect strategic Nash equilibrium without specifying the functional forms. To provide further insights, Section 4 characterizes the Markovian subgame perfect Nash equilibrium using linear-quadratic functional forms (following Moreaux et al. (2024)) and employs a numerical simulation illustrating the theoretical findings and extending the scope of the theoretical model. Section 5 concludes the paper.

2 State of the art of CCS and related literature

As stated in Chen et al. (2024), CCS presents some unique features that distinguish it from other abatement technologies. Unlike traditional methods that may simultaneously enhance productivity and reduce CO_2 emissions, CCS primarily focuses on capturing and storing carbon emissions. Additionally, the deployment of CCS often requires a substantial energy input, which can paradoxically increase dependence on CO_2-intensive energy sources, potentially undermining efforts to reduce reliance on fossil fuels. In this section, we will not reiterate the distinctive features or additional costs associated with CCS deployment, as they have been well-documented. Instead, we will focus on global CCS deployment and the economic literature related to CCS in the context of transboundary pollution control.

2.1 Worldwide CCS deployment

As mentioned above, many governments have considered CCS as a key tool to meet their net-zero emission targets. The United States, the United Kingdom, Canada, China, and Norway are the leading countries in CCS deployment. In the US, 73 new facilities were added to the pipeline in 2023, reflecting a significant expansion in this sector. The UK government has also made a substantial commitment, allocating £20 billion in its Spring Budget of 2023 to scale up CCS projects throughout the country. China, for the first time, included carbon capture, utilization, and storage (CCUS) in its 14^{th} Five-Year Plan (2021-2025) and has since introduced around 70 CCUS-related policies at the national level, underscoring its growing emphasis on this technology (Global CCS Institute, 2022, 2023b; Ma et al., 2023). Several European countries also have taken (or are taking) part in CCS projects to achieve net-zero emission targets. Notably, Norway's Sleipner CCS project, which began in 1996, stands out as one of the oldest and most successful CCS initiatives globally, having injected over 20 million tons of CO_2 into a saline aquifer beneath the North Sea. Building on this success, the Northern Lights project in Norway, supported by Germany, aims to establish a comprehensive CCS value chain, capturing, transporting, and storing CO_2 from industrial sources across Europe. Meanwhile, the French company TOTAL is developing the Grandpuits CCS project to capture and store up to 650,000 tons of CO_2 per year from the Seine-et-Marne refinery (Global CCS Institute, 2023a; Reuters, 2023).

Despite CCS's significant role in decarbonizing industries, mitigating carbon emissions, and achieving climate change targets, its widespread deployment has been slow to take off. In 2021, the annual capture capacity of existing facilities was approximately 45Mt CO_2 which accounts for only 0.1% of fossil fuel emissions (IEA, 2022). Durmaz (2018) examines the factors determining the demand for CCS and concludes that CCS will not be implemented if the cost of generating fossil energy with CCS is too high. Similarly, Budinis et al. (2018) identify and analyze the barriers to CCS development, emphasizing that the most significant obstacles are not exclusively technical but rather economic, with high costs posing the greatest challenge in the short to medium term.Golombek et al. (2023)

discuss the cooperation issues among market participants and various market imperfections within a CCS value chain, which can result in suboptimal investment levels in CCS projects. Moreover, several studies have developed models for CCS development roadmaps aimed at meeting emissions targets by 2050. These include Ma et al. (2023) for China, Shackley and Verma (2008) and Garg and Shukla (2009) for India, and Nogueira et al. (2014) for Brazil.

2.2 CCS in the context of transboundary pollution: a literature review

The current paper contributes to the growing body of literature on the optimal timing of CCS deployment (Amigues et al., 2016; Durmaz, 2018; Kalkuhl et al., 2015; Lafforgue et al., 2008, etc.) and optimal pollution control (Augeraud-Véron and Leandri, 2014; Jørgensen et al., 2010). Jørgensen et al. (2010) provide an extensive review of the economics and management of pollution. Ayong Le Kama et al. (2013) derive the optimal rate of capture and sequestration, and show that CCS can constitute a long-term solution to decrease carbon emissions. Moreaux and Withagen (2015) identify the conditions under which partial or no CCS deployment might be optimal. The recent work of Moreaux et al. (2024) extends the model of Amigues et al. (2016) to show that if carbon capture and utilization (CCU) is implemented, it would likely begin at the outset of the planning period and cease before the carbon budget has been fully used up. However, if CCS is implemented at the most socially optimal time point, this only occurs once the carbon budget has been depleted.

The current work also closely relates to the literature on transboundary pollution (Benchekroun and Ray Chaudhuri, 2014; De Frutos et al., 2022; La Torre et al., 2021, etc.). Fanokoa et al. (2011) and Berthod and Benchekroun (2019) focus on firms' cooperation versus non-cooperation strategies. La Torre et al. (2021) analyze the implications of transboundary pollution externalities on environmental policy-making in a spatial setting and show that it is not obvious that transboundary externalities lead to inefficiencies.

However, most of these studies focus on optimal CCS policies or the optimal timing of CCS deployment in a one-country model, and draw upon the assumption of a uniform policy across countries. This assumption simplifies the analysis but overlooks the significant variations in climate policies and economic conditions across different countries. Taking into account country heterogeneity in climate policies, Hoel (2011) focuses on carbon taxes and subsidies as policy tools, without considering CCS technologies. In another related study, Hoel and Jensen (2012) consider CCS and carbon taxation within a two-period model where the length of each stage is predetermined. In contrast, our paper considers a more dynamic approach by making the length of the stage an endogenous choice made by the late-entrant country. This distinction allows for a more nuanced understanding of how heterogeneous countries might strategically time their CCS initiatives in response to global climate challenges.

The work most closely related to ours is Bertinelli et al. (2014), who were the first to introduce a differential game into the economic analysis of CCS. In their study, two competing countries decide on the level of effort to undertake to capture and store CO_2. But in this early framework, they only study symmetric outcomes where both countries engage in CCS activities simultaneously. The study does not account for heterogeneity among countries, and the timing of employing CCS is taken as given. Furthermore, they also take pollutant emissions as given and thus ignore the trade-off between the use of CCS, i.e., the effort to improve the abatement rate, and emissions reductions. Benchekroun and Martín-Herrán (2016) consider asymmetry by investigating the impact of foresight and myopia in countries facing transboundary pollution issues. Generally, CCS costs depend not only on the technological development level of each country but also on other factors, such as regional geological and geographical conditions(Ferrari et al., 2019; Global CCS Institute, 2023b).

3 The game and Markovian perfect equilibrium

3.1 The game

Consider two players denoted by i and j. Denote player k's ($k = i, j$) instantaneous rate of emissions as $E_k(t)$. Both players produce a final good with a polluting resource as the only input. The players face an emission-consumption trade-off since they can obtain utility from the consumption of this final good, but at the same time, the production process generates pollution such as CO_2 which yields disutility.

Pollution is of a transboundary nature in the sense that the two neighboring countries share the same CO_2 pollution state, $y(t)$, whose evolution is given by the following equation:[2]

$$\dot{y}(t) = \underbrace{E_i(t) + E_j(t)}_{\text{emissions}} - \underbrace{\beta(x_i(t) + x_j(t))}_{\text{abatement by CCS}} - \underbrace{\delta y(t)}_{\text{nature's absorption}}, \quad t \in [0, \infty), \tag{1}$$

where the initial condition $y(0) = y_0$ is given and the parameter $\delta \in [0, 1)$ measures the natural absorption rate of CO_2 in the atmosphere. x_k is the quantity of CO_2 emissions captured by country k, i.e., the CCS technology's instantaneous abatement rate in country k, and $\beta(> 0)$ denotes its unit abatement efficiency.

The objective of country $k(= i, j)$ is to choose the emissions rate, $E_k(t)$, and the abatement rate, $x_k(t)(\geq 0)$, that maximize its welfare, W_k:

$$W_k = \max_{x_k, E_k} \int_0^\infty e^{-rt} \left[\overline{U}_k(E_k, x_k) - D_k(y)\right] dt = \max_{x_k, E_k} \int_0^\infty e^{-rt} \left[U_k(E_k) - C_k(x_k) - D_k(y)\right] dt \tag{2}$$

subject to the motion of the pollution stock (1). Here, the parameter $r \in (0, 1)$ is the constant social discount rate which we assume to be the same for both countries. While

[2]Bertinelli et al. (2014); Dockner and Van Long (1993); Schumacher and Zou (2008) and Boucekkine et al. (2023) employ a similar pollution accumulation process.

this assumption simplifies the analysis, as discount rates can indeed vary across countries and influence decision-making, the quantitative results remain. The utility function $\overline{U}_k(x_k, E_k)$ represents the combined utility of pollution emissions (generated from the final goods production) and CCS abatement. We assume separability of the utility function: $\overline{U}_k(x_k, E_k) = U_k(E_k) - C_k(x_k)$. Here $U_k(E_k)$ denotes benefit derived from production (equivalently, emissions, due to the emission–consumption trade-off), which is strictly increasing and concave with respect to emissions: $U'_k = \frac{dU_k}{dE_k} > 0, U''_k < 0$. On the other hand $C_k(x_k)$ represents the cost associated with CCS deployment, which is strictly increasing and convex with respect to the amount of abatement: $C'_k(x_k) \geq 0, C''_k(x_k) \leq 0$ and $C_k(0) = 0$. In other words, no cost are incurred when no abatement is undertaken, while increasing efforts are needed for CCS deployment as the level of captured CO_2 increases (Bertinelli et al., 2014; Moreaux et al., 2024). Pollution damage is denoted by the function $D_k(y)$, which is strictly increasing with $D_k(0) = 0$. The accumulated pollution stock may affect the two players differently.

In this paper, we consider only the subgame perfect Markovian Nash equilibrium in the sense that the optimal choices (E_k and x_k) of player $k = i, j$, depend on the pollution state: $E_k = E_k(y, t)$ and $x_k = x_k(y, t)$ for $\forall (y, t)$. More precisely,

Definition 1 *(Markovian Perfect Nash Equilibrium, MPNE) The 2-tuple* $((E_i, x_i), (E_j, x_j))$ *of functions* $(E_k, x_k) : \mathbb{R}_+ \times [0, \infty) \to \mathbb{R}^2_+$ *for* $k = i, j$, *is called a Markovian Nash equilibrium if, for each* $k \in \{i, j\}$, *an optimal control path* (E^*_k, x^*_k) *of the above optimal control problem (2) subject to (1) exists and is given by the following Markovian strategy:* $E^*_k(t) = E_k(y(t), t), x^*_k(t) = x_k(y(t), t)$, *for all* $t \geq 0$. *If, furthermore, the Markovian Nash equilibrium holds for all* $(y, t) \in \mathbb{R}_+ \times [0, \infty)$, *then the equilibrium is also called a Markovian (subgame) perfect Nash equilibrium.*

3.2 The trigger condition of CCS

Given that the game described above is defined over an infinite time horizon and is autonomous, following Kamien and Schwartz (2012)[3], we can directly study the stationary solutions. Denote the stationary Bellman value function of player $k = i, j$ as $V_k(y)$ for any y with $V'_k(y) < 0$; then V_k must satisfy the following Hamilton-Jacobi-Bellman equation (HJB hereafter):

$$rV_k(y) = \max_{E_k, x_k} \left[U_k(E_k) - C_k(x_k) - D_k(y) + V'_k(y)[E_i + E_j - \beta(x_i + x_j) - \delta y] \right]; \quad x_k \geq 0, \tag{3}$$

with the catching up transversality condition

$$\lim_{t \to \infty} e^{-rt} V_k(y(t)) \leq 0.$$

[3]Generally, the value function V_k depends not only on the state variable y but also on time t. Kamien and Schwartz (2012, p.238) demonstrate that if time enters the objective function only through the discount term, then the value function does not depend on time t explicitly.

For the above maximization problem, we consider only strictly concave value functions, i.e., $V_k''(y) < 0$. The first-order optimal conditions yield that for player $k = i, j$,

$$U_k'(E_k) = -V_k'(y)(> 0), \quad k = i, j, \tag{4}$$

and the Kuhn-Tucker conditions yield, for $k = i, j$,

$$x_k(y) = \begin{cases} 0 \quad \text{if} \quad C_k'(0) \geq -\beta V_k'(y)(> 0), \\ (C_k')^{-1}(-\beta V_k'(y)) > 0 \quad \text{if} \quad C_k'(0) \leq -\beta V_k'(y), \end{cases} \tag{5}$$

where $(C_k')^{-1}$ is the inverse of the marginal deployment cost, and $(V_k')^{-1}$ is the inverse of the marginal Bellman value function.

The trigger condition (5) functions similarly to Tobin's Q, determining whether or not to invest in CCS, depending on the marginal cost and the discounted marginal value. Note that the right hand side of the trigger condition (5), $-\beta V_k'(y)$, is increasing in the accumulated pollution level y since $V_k'(y) < 0$. At low levels of pollution, the marginal costs of deploying CCS outweighs the marginal benefits. However, as pollution accumulates, the threshold for CCS deployment may be triggered. Of course, this threshold is country-specific and depends on a country's economic and technological development as well as the efficiency of the deployed CCS technology.

Rewriting the trigger condition, we find that player k will initiate CCS deployment only when the pollution level reaches the threshold

$$\overline{y}_k \equiv (V_k')^{-1}\left(-\frac{C_k'(0)}{\beta}\right). \tag{6}$$

Condition (6) can also be interpreted as the carbon budget, i.e., the maximum amount of CO_2 that can be emitted within the framework of climate goals, or, as a condition for ceasing CCS. More precisely, when emissions are sufficiently low – such that the natural absorption capacity of the environment is sufficient to absorb the emitted CO_2 – the accumulated pollution will decrease to the point where the first inequality condition in (5) holds. In such cases, CCS deployment can be ceased. This scenario could occur, for example, when renewable energy sources dominate the energy supply and the dependence on fossil fuels is reduced to a minimum (Moreaux et al., 2024).

Combining the optimal conditions (4) and (5), we can derive two key insights: First, the decision to deploy CCS is determined by the carbon budget and is triggered by the level of accumulated pollution, which is influenced by the contribution of both players, due to the transboundary nature of pollution. Second, once CCS is triggered, player k's deployment fulfills the following condition:

$$C_k'(x_k) = \beta U_k'(E_k). \tag{7}$$

Applying the implicit function theorem, (4) implies that $\frac{dE_k}{dy} = -\frac{V_k''(y)}{U_k''(E_k)} < 0$, indicating that the optimal emission level decreases as accumulated pollution increases. Similarly, with CCS deployment in place, equation (5) shows that $\frac{dx_k}{dy} = -\frac{V_k''(y)}{C_k''(x_k)} > 0$, meaning that CCS deployment increases as the pollution stock rises, which is rather straightforward. Combining these two effects, the seemingly counter intuitive result from (7), $\frac{dx_k}{dE_k} = \frac{\beta U_k''(E_k)}{C_k''} < 0$, makes sense. More precisely, emissions E_k decrease only with increasing pollution accumulation, which in turn requires a higher level of CCS deployment.

Nonetheless, it is important to note that there is no guarantee that the players with lower CCS deployment costs will or should start CCS project first, as the pollution damage cost also plays a role. This can be seen from the trigger condition given by equation (6), where the value function has arguments of both pollution damage and deployment costs.

3.3 The Markovian Nash Equilibrium

By applying Theorem 4.4 of Dockner et al. (2000), the above first-order condition and the accompanying analysis – where the second-order optimality conditions also hold – yield the following straightforward results:

Proposition 1 *For the above differential game,*

(a) $((E_i^*(y), x_i^*(y)), (E_j^*(y), x_j^*(y)))$*, for any* $y \geq 0$*, form a Markovian subgame perfect Nash Equilibrium, where* $E_k^*(y)$ *and* $x_k^*(y)$ *are given by (4) and (5), respectively, , with* $k = i, j$*, and* V_i, V_j *are the solutions of the HJB equation system (3) ;*

(b) the equilibrium level of emissions $E_k^*(y)$ *is monotonically decreasing in the level of accumulated pollution,* y*;*

(c) the optimal level of CCS abatement $x_k^*(y)$ *is monotonically increasing in the level of accumulated pollution,* y*, if not zero.*

As mentioned above, the differential game is autonomous and defined over an infinite time horizon. Consequently, the starting time of the game is less important; instead, the starting state is of interest. Hence, we define the start of the game as the point at which one of the two players initiate CCS deployment. If the two countries are identical, they would both deploy CCS simultaneously. In the following, we consider heterogeneity between countries. Without loss of generality, we assume

Assumption 1

$$y_0 \equiv \overline{y}_i < \overline{y}_j,$$

meaning that player i *triggers CCS deployment before player* j*. The game starts at the moment when player* i *deploys CCS.*

3.4 Stop CCS deployment

Given the suggestions of Van der Ploeg and Withagen (2012) and Moreaux et al. (2024), among others, ceasing CCS operations can be an optimal choice under specific conditions. In this subsection, we explore this possibility from two perspectives: first, whether CCS, once initiated, continues indefinitely; and second, whether CCS operations are only temporary, meaning that CCS is phased out in the long term.

The pollution accumulation equation (1) yields that the long-run steady state is determined by the solution of

$$E_i^*(y) + E_j^*(y) = \beta(x_i^*(y) + x_j^*(y)) + \delta y. \tag{8}$$

The left-hand-side is strictly decreasing in y, while the right-hand-side is strictly increasing. Consequently, the long-run steady state is the intersection of the two curves. The following result, whose proof is given in Appendix A.1, is straightforward:

Proposition 2 *Let Assumption 1 hold.*

(1) Suppose the following inequality holds:

$$E_i^*(y_0) + E_j^*(y_0) > \beta x_i^*(y_0) + \delta y_0, \tag{9}$$

then there is one and only one long-run pollution steady state y^{ss}, which checks $y^{ss} > y_0 = \overline{y}_i$. Furthermore,

(1.a) player i will never stop CCS deployment;

(1.b) the steady state of pollution, y^{ss}, varies depending on whether player j deploy CCS or not:

(1.b.1) player j never stops CCS once she triggers the deployment, then $y^{ss} > \overline{y}_j$,

(1.b.2) player j never triggers CCS if and only if $y^{ss} < \overline{y}_j$,

(2) If the opposite direction of the inequality (9) holds, CCS will never be triggered.

When player i initiates CCS, i.e., inequality (9) holds, initially, the joint optimal emission level is higher than the combined abatement efforts, which includes both natural absorption and player i's capture and storage level. Consequently, in the short term, pollution accumulation levels will continue to increase. However, this process will not last forever since joint emissions, $E_i^*(y) + E_j^*(y)$ decrease with accumulated pollution y, while the aggregate abatement efforts, $\beta x_i^*(y) + \delta y$, increase with y, regardless of whether player j implements CCS or not. This trend will not stop until the system reaches the long-run steady state, i.e., $\dot{y} = 0$. In this process, if player j initiates the use of CCS, as depicted in case (1.b.1), she will not have the option to cease CCS deployment. Nonetheless, it may happen that player j never deploys CCS, as in case (1.b.2), a scenario which of course will postpone the time to reach the steady state. This may happen if player j is either less affected by pollution accumulation or finds CCS deployment too costly.

The second part of the proposition is straightforward. When CO_2 emissions are below nature's self-absorption capacity, CCS is not needed. This situation can occur in two distinct scenarios: during the early stages of industrialization, when pollution levels are relatively low, or in later stages of development, when the energy supply is predominantly non-fossil, thus reducing emissions to a level where they no longer pose a significant problem.

The above Proposition 2 provides a different capture regime compared to the recent contribution of Moreaux et al. (2024), who consider partial CCS and the interchangeable use of CCU and CCS, depending on the carbon budget and the initial stock, with CCS potentially being a temporary measure[4]. The reason is that our analysis does not incorporate alternative energy consumption options, such as the transition from fossil fuels to renewable energy sources, or the substitution between CCS and CCU, as in Van der Ploeg and Withagen (2012) and Moreaux et al. (2024). This means that the emission level, which determines the production of the final goods, can not be reduced to a level that is too low. However, when considering the possibility of transitioning to renewable energy, it is plausible that CCS could be utilized only temporarily, that is, before the pollution stock reaches the carbon budget.

Arguably, the above results rely essentially on the trigger condition (6), which is defined by the value function from the HJB equation system (3). Without specifying functional forms for utility, cost and damage, it is impossible to shed more light on the value function. To better illustrate the results from the two propositions, we will employ linear-quadratic functions (Benchekroun and van Long, 1998; Bertinelli et al., 2014; Boucekkine et al., 2023; Dockner and Van Long, 1993; Moreaux et al., 2024) in the subsequent analysis.

With linear-quadratic functional forms, we can derive explicit conditions that determine when both players will deploy CCS and the timing when the second player's CCS initiation. Furthermore, we also provide conditions under which only one player deploys CCS.

4 The linear-quadratic example

In this section we consider the following linear-quadratic functional forms : for $k = i, j$,

$$U_k(E_k) = a_k E_k - \frac{E_k^2}{2}, \qquad C_k(x_k) = b_k\, x_k + \frac{x_k^2}{2} \qquad \text{and} \qquad D_k(y) = \frac{c_k y^2}{2},$$

where the positive constant productivity parameter a_k, measuring the efficiency that converts carbon emissions into the consumption good, is sufficiently large. The parameter b_k denotes the unit investment cost of CCS, including, for example, the development of country-specific technologies, the installation of infrastructure or the maintenance costs. The parameter c_k

[4]Moreaux et al. (2024) show that if CCS and CCU are both used over the planning period, the social planner first uses CCU, then stops with capturing CO_2 and starts using CCS once the carbon budget is exhausted.

represents the unit environmental damage cost. For simplicity we assume that $a_i = a_j = a$, $b_k, c_k > 0$. Thus, the objective function becomes:

$$W_k = \max_{x_k, E_k} \int_0^\infty e^{-rt} \left[\underbrace{aE_k - \frac{E_k^2}{2}}_{\text{individual utility}} - \underbrace{\left(b_k\, x_k + \frac{x_k^2}{2} \right)}_{\text{CCS abatement cost}} - \underbrace{\frac{c_k\, y^2}{2}}_{\text{pollution damage}} \right] dt, \tag{10}$$

and the HJB equation (3) for player k is

$$\begin{aligned} rV_k(y) = \max_{E_k, x_k} \Bigg[& aE_k - \frac{E_k^2}{2} - \frac{c_k\, y^2}{2} - \left(b_k\, x_k + \frac{x_k^2}{2} \right) \\ & + V_k'(y)[E_i + E_j - \beta(x_i + x_j) - \delta y] \Bigg] ; \quad x_k \geq 0, \end{aligned} \tag{11}$$

with the transversality condition

$$\lim_{t \to \infty} e^{-rt} V_k(y(t)) \leq 0.$$

The first-order optimal conditions yield that for player $k = i, j$,

$$E_k = a + V_k'(y), \tag{12}$$

and the Kuhn-Tucker conditions yield

$$x_k = \begin{cases} 0, & \text{if} \quad b_k + \beta V_k'(y) \geq 0, \\ -(b_k + \beta V_k'(y)) > 0 & \text{if} \quad b_k + \beta V_k'(y) \leq 0. \end{cases} \tag{13}$$

Thus, the CCS trigger condition indicates that player k will initiate CCS only if the pollution level reaches

$$y_k = (V_k')^{-1} \left(-\frac{b_k}{\beta} \right), \tag{14}$$

To simplify the analysis and without loss of generality, we denote the player with the lower CCS deployment cost as player i and make the following assumption:

Assumption 2 *For all $x > 0$, $C_i(x) \leq C_j(x)$, i.e., player i's deployment costs are lower than player j's costs, for any level of CCS deployment.*

With the linear-quadratic form, Assumption 2 is simply reduced to $b_i \leq b_j$.

At first sight, the trigger condition in (13) appears to depend solely on the cost-effectiveness ratio, b_k/β. However, the heterogeneity across countries is twofold: it includes both the unit cost of CCS deployment, b_k, and the pollution damage cost, c_k. Arguably, lower CCS costs do not necessarily result in earlier CCS deployment, as the value function also depends on the pollution damage cost c_k.

In the following we need to distinguish among three different cases:

(C.1) $c_i = c_j > 0$;

(C.2) $0 = c_j < c_i$;

(C.3) $c_j > c_i = 0$.

In order to provide explicit solutions, we normalize the smaller damage parameter to 0, which reduces the problem to a polar case. Another polar case is when the damage costs are symmetric for both players. The general case of asymmetric damage is, of course, in between these two polar cases.

In the following sections, we first consider (C.1) and (C.2): Either the pollution damage is the same for both countries (C.1) or higher for the technologically more advanced country (C.2). We then investigate (C.3), where the technologically more advanced country has smaller or zero environmental damage costs. In Subsection 4.3, we will also numerically illustrate the more general cases, in between these two polar cases.

4.1 Period II: both players engage in CCS

We define period II as the stage at which both players engage in CCS. In this section, we focus on the optimal strategies in period II. Naturally, period II only exists if the late-entrant player actually implements CCS in finite time. In this section, we assume that this is true and examine under what conditions period II exists. Subsection 4.2 examines the optimal timing for the late-entrant player to initiate CCS deployment.

4.1.1 Symmetric damage: $c_i = c_j > 0$

We use the subscript s to denote this symmetric case. With the affine-quadratic functional forms and the autonomous system defined over an infinite time horizon, the following results can be directly derived. The detailed proof is provided in the Appendix A.2.

Proposition 3 *(Existence of an affine MPE) Suppose that the unit pollution damage cost is identical for the two players, $c_i = c_j \equiv c > 0$, and that there exists a finite time point $T_s \in [0, \infty)$ from which both players start engaging in CCS deployment. Then, for all $t > T_s$ and for all y,*

(C1-1) there exists a unique affine Markovian subgame perfect Nash equilibrium , characterized by

$$\begin{cases} x^*_{s,k}(y) = -b_k - \beta(B_s + C_s y), \\ E^*_{s,k}(y) = a + B_s + C_s y \end{cases} \quad k = i, j, \tag{15}$$

where

$$C_s = \frac{r + 2\delta - \sqrt{(r + 2\delta)^2 + 12c(1 + \beta^2)}}{6(1 + \beta^2)}(< 0), \quad B_s = \frac{[2a + \beta(b_i + b_j)]C_s}{(r + \delta) - 3(1 + \beta^2)C_s}(< 0);$$

(C1-2) the corresponding optimal pollution accumulation is given by

$$y_s(t) = [y(T_s) - y_s^*]\, e^{(2(1+\beta^2)C_s - \delta)t} + y_s^*,$$

which converges asymptotically to its long-run steady state, y_s^*, given by

$$\begin{aligned} y_s^* &= \frac{2a + \beta(b_i + b_j) + 2(1+\beta^2)B_s}{\delta - 2(1+\beta^2)C_s} \\ &= \frac{(2a + \beta(b_i + b_j))\left[\sqrt{(r+2\delta)^2 + 12c(1+\beta^2)} + 5r + 4\delta\right]}{4\delta(r+\delta) + 12c(1+\beta^2) + \delta(\sqrt{(r+2\delta)^2 + 12c(1+\beta^2)} + r)}, \end{aligned}$$

with the convergence speed given by $\delta - 2(1+\beta^2)C_s$;

(C1-3) the corresponding stationary value functions are

$$V_{s,k}(y) = A_{s,k} + B_s y + \frac{C_s y^2}{2},\ k = i, j, \tag{16}$$

where

$$A_{s,k} = \frac{a^2 + b_k^2}{2r} + \frac{[2a + \beta(b_i + b_j)]B_s}{r} + \frac{3(1+\beta^2)}{2r}B_s^2.$$

In the following, we assume $a > -(B_s + C_s y_s^*)$ ensuring that the optimal emissions $E_{s,k}^* > 0$ at any time. It is evident that for any given pollution level y, the player with lower CCS costs ($b_i < b_j$) contributes more to CO_2 abatement ($x_i^* > x_j^*$). A player's optimal emissions level involves two parts: production efficiency a and pollution damage costs c. When $c = 0$ – indicating that pollution damage costs are negligible – $C_s = 0$ and $B_s = 0$, and the optimal emissions level depends only on the production efficiency parameter a. However, when pollution damage costs are not negligible, the optimal emissions level must take into account those costs. Consequently, efforts must be made to reduce pollution emissions $B_s + C_s y < 0$ for all y.

We now investigate the impact of the unit pollution damage cost on $E_{s,k}^*$, $x_{s,k}^*$ and y_s^*. It is straightforward to show that the optimal emissions decrease with an increase in the unit pollution damage cost: $\partial E_{s,k}^*/\partial c < 0$. The optimal level of abatement increases and the long-run level of accumulated pollution decreases with the unit pollution damage cost: $\partial x_{s,k}^*/\partial c > 0$, $\partial y_s^*/\partial c < 0$.

As mentioned in the Introduction, CCS deployment can be very costly. Naturally, the higher the cost of deploying CCS, the lower the optimal abatement rate of both players, $\partial x_{s,k}/\partial b_k < 0$. When CCS costs become excessively high, both players will prefer to reduce emissions directly rather than investing in abatement technologies in order to stabilize pollution accumulation, as reflected by $\partial E_{s,k}^*/\partial b_k < 0$. However, relying solely on emission reductions does not fully offset the lack of abatement leading to an increase in long-term pollution accumulation, as shown by $\partial y_s^*/\partial b_k > 0$. This suggests that if transitioning to renewable energy is costly, CCS may be an alternative approach to achieving carbon neutrality.

To foster CCS deployment, governments can take several strategic actions to reduce the costs of CCS, such as providing subsidies or investing in R&D projects.[5] The dependence of CCS adoption on deployment costs and climate change damages aligns with the existing literature (Amigues et al., 2016; Moreaux and Withagen, 2015).

For simplicity, we define $\overline{y_k} \equiv y(T_k)$, where T_k denotes the time at which player k initiates CCS and $\overline{y_k}$ represents her pollution stock at that time. We define $\overline{y_k}$ as player k's trigger threshold, above which player k starts to initiate CCS. Taking into account the trigger condition (14), it follows that at $\overline{y_k}$, player k's marginal value function must satisfy

$$V_k'(\overline{y}) = B_s + C_s\overline{y_k} = -\frac{b_k}{\beta}. \tag{17}$$

In other words, $\overline{y_k} = -\dfrac{b_k/\beta + B_s}{C_s}, \quad k = i, j$. It is straightforward to show that

$$\frac{\partial y_j}{\partial b_i} = -\frac{\beta}{r + \delta - 3(1 + \beta^2)C_s} < 0; \quad \frac{\partial y_j}{\partial b_j} = -\frac{1}{\beta C_s}\left(\frac{r + \delta - (3 + 2\beta^2)C_s}{r + \delta - 3(1 + \beta^2)C_s}\right) > 0.$$

Furthermore, taking the derivatives on both sides of (17), it follows that

$$\frac{dV_j'(\overline{y_s})}{d\beta} = \frac{V_j''(\overline{y_s})\partial\overline{y_s}}{\partial\beta} = \frac{b_j}{\beta^2} > 0.$$

By construction, the value function is strictly concave, thus $\partial\overline{y_s}/\partial\beta < 0$.

We conclude the above analysis with the following proposition:

Corollary 1 *Under the assumption of Proposition 3, the following holds:*

- *higher CCS costs will postpone a player's deployment of CCS:* $\partial\overline{y_k}/\partial b_k > 0$*, for* $k = i, j$*,*
- *but higher rival-player costs or higher CCS efficiency will expedite one's own CCS deployment.*

Furthermore, by Assumption 2

$$\overline{y_j} - \overline{y_i} = \frac{b_i - b_j}{C_s\beta} \geq 0.$$

[5] The EU has already financed various research projects related to CCS, including the Accelerating CCS Technologies (ACT) project aiming at accelerating the development of CCS technologies. Additionally, the Steelanol project, funded by the EU's Horizon 2020 program, seeks to develop a low-carbon steel production process using CCU technologies. The CEMCAP project focuses on developing post-combustion CO_2-capture technologies for the cement industry. In the United States, the Department of Energy has supported the Carbon Storage Assurance Facility Enterprise (CarbonSAFE) initiative, which aims to develop and deploy commercial-scale CCS facilities that can safely and permanently store large volumes of carbon dioxide emissions. Another example of a government-supported CCS project is the Petra Nova project, which was the world's largest post-combustion carbon-capture facility when it began operating in 2017.

Thus, player i will be the first to trigger CCS projects. In other words, if pollution damage costs are symmetric for both players, the player with the lower CCS deployment costs will intitiate the CCS project first. By definition, the game starts when one of the players triggers CCS deployment; it follows that the initial pollution stock is given by

$$y_0 = -\frac{b_i/\beta + B_s}{C_s}.$$

To show the trigger condition for the late-entrant player to deploy CCS under symmetric costs, let us denote $\overline{y_s} \equiv \overline{y_j}$ and $T_s \equiv T_j$. Thus, the trigger condition for the late-entrant player can be expressed as:

$$\overline{y_s} = -\frac{b_j/\beta + B_s}{C_s}. \tag{18}$$

Two special cases deserve further consideration: (1) In the long-run, $y_s^* < \overline{y_s}$. This means that, due to both emission reductions and abatement efforts, the long-run pollution level is reduced to below the trigger threshold of player j. In that case, at a certain moment player j may suspend CCS deployment. If the pollution stock continues to decrease, player i may also suspend CCS deployment. Conversely, if (2) $y_s^* > \overline{y_s}$, CCS deployment continues until the accumulated pollution stock reaches the long-run steady-state level y_s^*.

In the remainder of this study, we focus on determining the optimal timing for player j to initiate CCS deployment, rather than the optimal timing for player j to suspend CCS operations. Therefore, our analysis is restricted to the case where CCS deployment, once started, is maintained. For this purpose, the following assumption is imposed:

Assumption 3 *Suppose the initial and long-run pollution stocks satisfy*

$$(y_0 \leq)\ \overline{y_s} < y_s^*.$$

It is worth noting that this assumption imposes constraints on the parameters rather than on the (endogenous) state variable, $y(t)$.

4.1.2 The case of extreme asymmetric damage: $c_i > 0$ and $c_j = 0$

We use the subscript e to denote this extreme asymmetric case. Suppose there exists a finite time $T_e \in (0, \infty)$ such that, from T_e onwards, both players engage in CCS deployment. If we impose a non-negativity constraint on CCS, player j's optimal strategy is $x_j = 0$, as shown in the Appendix A.2. In other words, player j will never initiate CCS deployment if her welfare is not harmed by the accumulated pollution stock.[6] The optimal strategies for the players are summarized in proposition 4, the proof of which is given in the Appendix A.2.

[6] If we do not impose the non-negativity constraint on CCS, the optimal strategy of player j, who does not suffer from the pollution stock, would theoretically be $x_j = -b_j < 0$, which is an unrealistic situationas is shown in the Appendix A.2.

Proposition 4 *Suppose the unit pollution damage costs are $c_i > 0$ and $c_j = 0$, and that there exists a finite time $T_e \in [0, \infty)$ such that from T_e onwards both players engage in CCS deployment. Then, for all $t > T_e$ and for all y,*

(C.2-1) there exists a unique affine Markovian subgame perfect Nash equilibrium given by

$$\begin{cases} x^*_{e,i}(y) = -b_i - \beta(B_{e,i} + C_{e,i}y), & x^*_{e,j}(y) = 0, \\ E^*_{e,i}(y) = a + B_{e,i} + C_{e,i}y, & E^*_{e,j}(y) = a, \end{cases} \tag{19}$$

where

$$C_{e,i} = \frac{r + 2\delta - \sqrt{(r+2\delta)^2 + 4(1+\beta^2)c_i}}{2(1+\beta^2)} (< 0), \; B_{e,i} = \frac{[2a + \beta(b_i + b_j)]C_{e,i}}{(r+\delta) - (1+\beta^2)C_{e,i}} (< 0);$$

(C.2-2) the corresponding optimal pollution accumulation is given by

$$y_e(t) = [y(T_e) - y^*_e] \, e^{((1+\beta^2)C_e - \delta)t} + y^*_e,$$

which converges asymptotically to its long-run steady state, y^*_e, given by

$$y^*_e = \frac{2a + \beta b_i + (1+\beta^2)B_{e,j}}{\delta - (1+\beta^2)C_{e,i}} = \frac{(r+\delta)(2a + \beta b_i)}{\delta(r+\delta) + (1+\beta^2)c_i},$$

with the convergence speed $\delta - (1+\beta^2)C_{e,i}$;

(C.2-3) the corresponding stationary value functions are

$$V_{e,i}(y) = A_{e,i} + B_{e,i}y + \frac{C_{e,i}y^2}{2} \tag{20}$$

and

$$V_{e,j}(y) = A_{e,j} = \frac{a^2}{2r}, \tag{21}$$

where

$$A_{e,i} = \frac{a + b_i^2}{2r} + \frac{2a + \beta(b_i + b_j)}{r} B_i + \frac{1+\beta^2}{2r} {B_i}^2.$$

It is not surprising that player j emits more CO_2 emissions than player i, given that the pollution damage cost of player j is smaller than that of player i. Additionally, $x^*_{e,i}(y) > 0$ if $b_i < -\beta(B_{e,i} + C_{e,i}y)$, a condition we assume to hold. Consequently, in this asymmetric case, $T_e = +\infty$, meaning that player j will not initiate CCS deployment within a finite time frame.

In a more general scenario where the asymmetric pollution damage costs satisfy $0 < c_j < c_i$, the results should lie between the two corner cases discussed above. In this scenario, the player who suffers more from pollution damage will make greater efforts to reduce emissions and will deploy CCS earlier. Conversely, the player who suffers less from the pollution damage will contribute less to pollution control efforts.

4.1.3 Alternative extreme asymmetric damage case: $c_i = 0$ and $c_j > 0$

In this scenario, combined with Assumption 2, we consider an alternative possibility where player j faces both higher CCS deployment costs and higher pollution damage costs. This is in line with Greenstone and Hanna (2014), Landrigan et al. (2018), and Greenstone and Jack (2015) which highlight the highly uneven distribution of environmental damage across nations and regions. Lower-income populations are often disproportionately exposed to and affected by pollution externalities, as noted by Hsiang et al. (2019); Rentschler and Leonova (2023) and the World Economics Forum[7].

Theoretically, we can apply the same calculations as outlined in the Appendix A.2 to analyze this extreme asymmetric case. However, in this scenario, player i does not initiate CCS projects. Instead, player j, who faces higher CCS costs and suffers more from pollution accumulation, becomes the only player to deploy CCS. This finding is consistent with Greenstone's conclusion that substantial emission reductions in non-OECD countries are crucial to limiting climate change, regardless of the actions taken by OECD countries. Low-income countries often bear high abatement costs, given their willingness to pay (Greenstone and Jack, 2015).

It's important to note that low-income countries differ from developed countries in various ways, including regulatory frameworks, access to health care, initial pollution stocks, and population health. Consequently, the impacts of pollution in developing countries are also likely to be different (Arceo et al., 2016; Currie et al., 2014; Graff Zivin and Neidell, 2013; Greenstone and Jack, 2015). All these factors contribute to their willingness to invest in improving the environmental quality.

However, as noted in the Introduction and based on current CCS deployment facts (Section 2.2), most CCS projects are concentrated in developed economies such as the US and EU countries, or in rapidly developing countries such as China. Conversely, less developed economies — often characterized by lower per capita CO_2 emissions — tend to lack CCS mitigation initiatives. While low emissions play a role in this disparity, willingness to pay for such technologies is another key factor. For countries that are less directly affected by pollution, their commitment to CCS often depends on financial considerations.

In the context of transboundary pollution control, more developed economies generally show a greater willingness and capacity to reduce CO_2 emissions and to transition to renewable energy sources. In contrast, less developed economies tend to prioritize economic growth and development over environmental concerns.

Interestingly, although player i experiences less direct pollution damage, she may be more concerned about transboundary pollution than player j. At the aggregate level, player i should attach greater importance to pollution damage, taking into account not only the direct damage but also the indirect effects, the transboundary consequences, and the long-term effects. This perspective is consistent with the findings discussed in the previous subsection.

[7] See https://www.weforum.org/agenda/2023/01/climate-crisis-poor-davos2023/.

4.1.4 Free riding scenarios

Before concluding this section, we investigate which of the two scenarios discussed above is more prone to free-riding behavior. To make the comparison feasible, we focus on the case where the total pollution damage in each scenario is the same: $c_{s,i} = c_{s,j} = c$ in the symmetric case and $c_{e,i} = 2c$, $c_{e,j} = 0$ in the extreme asymmetric case. We thus have

$$C_{e,i} = \frac{r + 2\delta - \sqrt{(r+2\delta)^2 + 8(1+\beta^2)c}}{2(1+\beta^2)},\ C_{e,j} = 0.$$

It is straightforward to show that $C_{e,i} < C_s,\ B_{e,i} < B_s$, and in turn the emission levels and abatement efforts of each country in the two cases can be ranked accordingly:

$$\begin{aligned} &E^*_{e,i}(y) < E^*_{s,i}(y) = E^*_{s,j}(y) < E^*_{e,j}(y), \\ &x^*_{e,i}(y) > x^*_{s,i}(y) \geq x^*_{s,j}(y) > x^*_{e,j}(y),\ \ \forall y. \end{aligned}$$

Country i, which faces higher pollution damage costs, emits less compared to each country in the symmetric damage cost scenario. The emissions of both countries in the symmetric case are lower compared to the emissions of country j facing $c_{e,j} = 0$. This is because the country facing higher pollution damage costs will make greater efforts to reduce pollution, while the country facing lower damage costs has less incentive to reduce emissions. Moreover, the country with lower implementation costs (i.e., country i) always abates more than its counterpart, regardless of the damage cost structure. Not surprisingly, country i (resp. country j) emits more and abates less in the symmetric (resp. asymmetric) case.

It can be verified that $C_{e,i} < 2C_s,\ B_{e,i} < 2B_s$, and in turn the aggregate emissions in the two cases satisfy

$$E_{e,i} + E_{e,j} = 2a + B_{e,i} + C_{e,i}\, y < 2E_{s,k} = 2a + 2B_s + 2C_s y,$$

while the aggregate CCS abatement levels satisfy

$$x_{e,i}(y) + x_{e,j}(y) = -b_i - \beta B_{e,i} - \beta C_{e,i} y > x_{s,i}(y) + x_{s,j}(y) = -b_i - b_j - 2\beta B_s - 2\beta C_s y,\ \ \forall y.$$

For the same pollution stock y, total emissions are higher and total abatement is lower in the symmetric case than in the asymmetric case, indicating that a free-riding mechanism is at work under symmetry. This finding is consistent with the results of Boucekkine et al. (2023) and Dutta and Radner (2009), who demonstrate that symmetry often leads to more intensive free riding. It is also consistent with the literature on unilateral pollution control commitments, which shows that unilateral emission reductions can lead to lower cumulative emissions in the long run (Bertinelli et al., 2018; Hoel, 1991; Zagonari, 1998, etc.).

4.2 Existence of Period I and threshold reachability conditions

We now investigate whether player j actually triggers CCS in finite time, as assumed in Proposition 3. More precisely, we investigate under which conditions a finite trigger time $T < +\infty$ exists, and analyze the players' optimal strategies in period I. Note that if no such finite time T exists, the two-period game is reduced to a single-period optimal control problem for the only player deploying CCS. Arguably, if player k experiences no pollution damage, as stated in Proposition 4, this player will never trigger CCS deployment.

In this section, we focus on the symmetric damage-cost situation: $c_i = c_j > 0$.

4.2.1 Threshold reachability conditions

Looking for optimal choices for the differential game in Period I is equivalent to looking for a solution to HJB equation (11) with the terminal condition at $\overline{y}$. Given that $x_j = 0$ in the first period, player i's HJB equation is

$$\begin{aligned} rV_i(y) = \max_{E_i, x_i} \Bigg[\left(aE_i - \frac{E_i^2}{2} - \frac{c_i\, y^2}{2} \right) - \left(b_i\, x_i + \frac{x_i^2}{2} \right) \\ + V_i'(y)[E_i + E_j - \beta x_i - \delta y] \Bigg], \quad \forall t \in [0, T], \end{aligned} \tag{22}$$

where the terminal condition $V_i(\overline{y})$ is given by (16) with $k = i$ and $c_i = c_j \equiv c$.

Thus, taking E_j as given, the first-order condition with respect to player i's optimal emission level and abatement rate yields, $\forall y, \ \ \forall t \in [0, T]$:

$$E_i(y) = a + V_i'(y) \ \text{ and } \ x_i = -b_i - \beta V_i'(y). \tag{23}$$

Similarly, player j's HJB equation can be simplified as

$$rV_j(y) = \max_{E_j} \left[aE_j - \frac{E_j^2}{2} - \frac{c_j\, y^2}{2} + V_j'(y)\left[E_i + E_j - \beta x_i - \delta y\right] \right], \quad \forall t \in [0, T], \tag{24}$$

where the terminal condition $V_j(\overline{y})$ is given by (16) with $k = j$ and $c_i = c_j \equiv c$.

Thus, player j's optimal emission choice is, $\forall y, \ \ \forall t \in [0, T]$,

$$E_j(y) = a + V_j'(y). \tag{25}$$

Substituting each player's optimal choices above into the right hand side of their HJB equations, we obtain the following equivalent HJB equations:

$$\begin{cases} rV_i = \dfrac{a^2 + b_i^2}{2} + (2a + \beta b_i)V_i' + \dfrac{(1+\beta^2)}{2}\,(V_i')^2 + V_i'V_j' - \delta y V_i' - \dfrac{c_i y^2}{2} \\ rV_j(y) = \dfrac{a^2}{2} + (2a + \beta b_i)V_j' + \dfrac{(V_j')^2}{2} + (1+\beta^2)V_i'V_j' - \delta y V_j' - \dfrac{c_j y^2}{2}. \end{cases} \tag{26}$$

To solve the above system of value functions, we take the derivatives with respect to y on both sides of the two HJB equations and denote the marginal value functions by $P_i(y) = V_i'(y)$, $P_j(y) = V_j'(y)$. The HJB equations above can then be rewritten as

$$\begin{cases} [(1+\beta^2)P_i + P_j - \delta y + (2a + \beta b_i)]P_i'(y) = (r + \delta - P_j')P_i + c_i y, \\ [(1+\beta^2)P_i + P_j - \delta y + (2a + \beta b_i)]P_j'(y) = (r + \delta - (1+\beta^2)P_i')P_j + c_j y, \end{cases} \tag{27}$$

which are nonlinear differential equations of the marginal value functions, $P_i(y)$ and $P_j(y)$.

The terminal conditions of the differential equations system (27) are given by the continuity of the shadow values at the switching times (Boucekkine et al., 2013; Makris, 2001; Tomiyama, 1985):

$$P_i(\overline{y}) = V_i'(\overline{y}) = B_i + C_i\overline{y}; \quad P_j(\overline{y}) = V_j'(\overline{y}) = B_j + C_j\overline{y} \tag{28}$$

and $V_i(\overline{y})$ is given by (16) with $k = i$ and $V_j(\overline{y})$ is given by (16) with $k = j$ and $c_1 = c_2 = c$.

For a more general situation $0 \leq c_i, c_j < +\infty$ and $c_i \neq c_j$, the explicit forms of the MPEs are difficult to obtain. Nonetheless, the HJB equation systems for the value functions are the same as in (26), or equivalently in (27). We will solve the general asymmetric case in the numerical section.

If we can solve the marginal value system (27) embodied in the terminal condition (28) for $P_i(y)$ and $P_j(y)$ with $y_0 \leq y \leq \overline{y}$, and then substitute $P_i(y)$ and $P_j(y)$ into the system (26), the value functions $V_i(y)$ and $V_j(y)$ can be obtained by solving

$$V_k(y) = V_k(\overline{y}) - \int_y^{\overline{y}} P_k(y(s))ds, \quad k = i, j, \quad \forall y \in [y_0, \overline{y}].$$

Having obtained the above results, we can further investigate (a) whether player j triggers CCS deployment in finite time and (b) if j does trigger CCS, the timing T_j, and the optimal choices of both players in this period.[8]

Given the optimal emissions E_i and E_j and player i's abatement rate, x_i, there are two possible scenarios: (1) the pollution accumulation level is not high enough in finite time to prompt player j to start deploying CCS; (2) the pollution stock continues to grow and eventually reaches the trigger threshold $\overline{y}$. In other words, according to the pollution accumulation equation evaluated at $\overline{y}$, if

$$\dot{y}(\overline{y}) = E_i(\overline{y}) + E_j(\overline{y}) - \beta x_i(\overline{y}) - \delta\overline{y} \leq 0, \tag{29}$$

the trigger threshold $\overline{y}$ is never reached. The reason for this is straightforward. Equation (29) indicates that before $\overline{y}$ is reached, the pollution stock has already reached its long-run steady state, given by $\dot{y} = 0$, and is asymptotically stable. Thus, either $\overline{y}$ is the steady state

[8]In the Appendix A.3, we demonstrate the reachability conditions.

that can only be reached asymptotically, or it is unattainable. However, if the sign in (29) is reversed, the pollution accumulation stock has not yet reached its long-run steady state before the threshold level $\overline{y}$ is reached. In this case the trigger threshold will be reached in finite time. The following reachability conditions are detailed in Appendix A.3:

Proposition 5 ***(Reachability conditions of $\overline{y}$)*** *Suppose there exists a solution $(P_i(y), P_j(y))$ to differential system (27) with the terminal condition (28). Then the following holds:*

(5.1) The threshold $\overline{y}$ will never be reached if one of the following conditions holds:

(5.a) If $2a + \beta b_i < \delta\overline{y}$;

(5.b) If $2a + \beta b_i > \delta\overline{y}$ and

$$|(1+\beta^2)\Delta_i - \Delta_j| \geq \frac{3}{2}(2a + \beta b_i - \delta\overline{y})^2; \tag{30}$$

(5.c) If $2a + \beta b_i > \delta\overline{y}$,

$$|(1+\beta^2)\Delta_i - \Delta_j| < \frac{2\sqrt{3}-3}{4}(2a + \beta b_i - \delta\overline{y})^2, \tag{31}$$

and

$$\max\{(1+\beta^2)\Delta_i, \;\; \Delta_j\} > 0; \tag{32}$$

(5.2) If $2a + \beta b_i > \delta\overline{y}$, inequality (31) holds but

$$\max\{(1+\beta^2)\Delta_i, \;\; \Delta_j\} \leq 0. \tag{33}$$

Then for any initial condition $y_0 < \overline{y}$, there exists a finite time $T_s \in (0, \infty)$ such that at T_s, $\overline{y} = y(T_s)$ is reached and

$$T_s = \int_{y_0}^{\overline{y}} \frac{1}{2a + \beta b_i - \delta y + (1+\beta^2)P_i(y) + P_j(y)} \, \mathrm{d}y,$$

where the net-value-functions-at-threshold Δ_i and Δ_j are

$$\Delta_i = 2rV_i(\overline{y}) - b_i^2 - \left(a^2 - c_i\overline{y}^2\right), \quad \Delta_j = 2rV_j(\overline{y}) - \left(a^2 - c_j\overline{y}^2\right).$$

In the above proposition, (5.1) provides sufficient conditions for player j never to initiate a CCS project, while (5.2) provides a sufficient condition for player j to trigger CCS in finite time.

Note that condition (5.a) indicates that if player j's trigger threshold is too high, player j will not deploy CCS. This is because the pollution stock will have asymptotically reached its long-run steady-state before reaching the trigger condition. It is worth noting that the inequality condition depends on $\bar{y}$, the production efficiency parameter a and the unit deployment cost and the abatement efficiency parameter, βb_i, but does not depend on the unit damage costs c_i, c_j.

Nonetheless, condition (5.b) indicates that a relatively low trigger threshold alone does not guarantee that player j will adopt CCS technologies. The decision also depends on the difference in the net value functions, $|(1+\beta^2)\Delta_i - \Delta_j|$. If the difference is large, as shown in (30), it may prevent player j from initiating CCS. This situation may arise if either player j derives significantly less benefit from CCS deployment than player i, i.e., $(1+\beta^2)\Delta_i > \Delta_j + \frac{3}{2}(2a+\beta b_i - \delta\overline{y})^2$, or if player j's net welfare remains sufficiently high even without deploying CCS, i.e., $\Delta_j > (1+\beta^2)\Delta_i + \frac{3}{2}(2a+\beta b_i - \delta\overline{y})^2$.

In contrast, if the difference is not too large, such that condition (33) holds, there is still no guarantee that player j will trigger CCS in finite time, since CCS adoption also depends on the sign of the net value functions at the threshold, as depicted in (5.c). The intuition behind (5.2) can be combined with the reachability conditions in (5.2) by rewriting the inequality condition (33) as follows

$$rV_i - \frac{b_i^2}{2} \leq \frac{a^2}{2} - \frac{c_i\overline{y}^2}{2} \tag{34}$$

for player i and

$$rV_j \leq \frac{a^2}{2} - \frac{c_j\overline{y}^2}{2} \tag{35}$$

for player j. The left-hand side of (34) (resp. (35)) is the discounted social welfare net of CCS costs. The right-hand side reflects the gain from emissions (production) net of pollution accumulation damage, with the squared terms of a and b_i arising from the quadratic functional forms, which are nonessential for our analysis. Given the assumption that both players face the same level of pollution damage, the inequality condition (33) via (34) and (35) indicates that player i's CCS deployment alone is not sufficient to control pollution accumulation. Consequently, player j will need to initiate CCS deployment as well.

In a linear-quadratic setting, the trigger condition is essentially a combination of the conditions outlined in (31) and (33). As a special case, when $r \to 0^+$, the condition (33) is more likely to be met. This implies that a more patient late entrant is more likely to initiate CCS deployment compared to a less patient late entrant.

It is worth pointing out that Proposition 5 provides sufficient conditions while may being neither necessary nor exhaustive.[9]

[9] For example, if $2a+\beta b_i > \delta\overline{y}$ and $\frac{2\sqrt{3}-3}{4}(2a+\beta b_i-\delta\overline{y})^2 < |(1+\beta^2)\Delta_i - \Delta_j| < \frac{3}{2}(2a+\beta b_i - \delta\overline{y})^2$, then Proposition 5 does not provide any result.

4.2.2 Optimal strategies when player j never deploys CCS

In this section, we investigate the optimal strategies of the players and the resulting pollution accumulation outcome when player j never initiates CCS. The analysis is similar to that in Section 4.2 in terms of the game structure, with the only difference being the absence of the reachability conditions and trigger times. The structure of the differential game is similar to the setup in Section 4.1 in the sense that the game is autonomous and defined over an infinite time horizon. Thus, the same Bellman value functions and HJB equations, i.e., (22) and (24), should hold $\forall t \geq 0$. Consequently, similar methods to those used in Section 4.1 can be employed to look for a Markovian perfect Nash equilibrium under both symmetric and asymmetric cost structures.

Suppose $c_i = c_j = c$. Even though both players face the same pollution damage costs, this does not necessarily imply that both players will engage in CCS deployment at the same time or even at all. If the reachability conditions outlined in Proposition 5 fail to hold, the inactive player will never trigger CCS and the MPE can be characterized as follows (the proof is given in the Appendix A.4).

Proposition 6 *Suppose $c_i = c_j = c$ and one of the conditions in (5.1) of Proposition 5 holds. In this case, player i deploys CCS at $t = 0$ while player j never initiates CCS deployment. Furthermore, there exists an affine MPE, which is given by*

$$\widehat{E_i}(y) = a + \widehat{V}_i'(y), \;\; \widehat{x_i} = -b_i - \beta\widehat{V}_i'(y) \;\text{ and }\; \widehat{E_j}(y) = a + \widehat{V}_j'(y), \;\; \widehat{x_j} = 0, \;\; \forall t \geq 0, \forall y,$$

where the affine-quadratic value functions $\widehat{V}_i(y) = \widehat{A_i} + \widehat{B_i}y + \frac{\widehat{C_i}y^2}{2}$ and $\widehat{V}_j(y) = \widehat{A_j} + \widehat{B_j}y + \frac{\widehat{C_j}y^2}{2}$ are given in the Appendix A.4.

It is straightforward to show that the pollution accumulation, following the equilibrium outlined in Proposition 6, yields the long-run steady state

$$\widehat{y} = \frac{2a + \beta b_i + (1+\beta^2)\widehat{B_i} + \widehat{B_j}}{\delta - (1+\beta^2)\widehat{C_i} - \widehat{C_j}}.$$

It is worth mentioning that the two steady states $\widehat{y}$ and y_s^* cannot be directly compared as they are obtained from different conditions. More precisely, $\widehat{y}$ is the direct consequence of (5.1) in Proposition 5. Failing to reach the threshold $\overline{y_s}$ means that for given b_i, b_j, and for a given CCS efficiency parameter, β, $\overline{y_s}$ is too high for player j to initiate CCS projects, while y_s^* comes from Assumption 3 combined with the sufficient condition (5.2) in Proposition 5.

Nevertheless, according to Assumption 3, for given b_i, b_j, and β, $\overline{y_s}$ must be above y_s^*, and so is $\widehat{y}$. Otherwise, the threshold condition $\overline{y_s}$ must be triggered in finite time. Therefore, the long-term cumulative pollution is higher if only one player deploys CCS than if both players do so.

4.3 Numerical illustration

To illustrate the findings related to CCS efficiency, β, and time preference, r, particularly in the context of the asymmetric pollution damage situation, we rely on numerical simulations.

4.3.1 Parameter setting and calibration

The model features the following set of parameters: $\{r, \delta, \beta, a, c_i, c_j, b_i, b_j\}$. The first three are standard parameters, while the last five require a more careful selection and calibration. Following the literature (Hoel and Karp, 2002; Nordhaus, 2007; Stern, 2006), we set the rate of time preference r to 0.022 as a benchmark, and allow r to take different values while holding the other parameters constant at the benchmark level in order to investigate the impact of the policymakers' patience. The capacity of nature to absorb CO_2 as noted by Benchekroun and Ray Chaudhuri (2014); Mason et al. (2017) among others, is set to $\delta = 0.015$.

Regarding β, unit CCS efficiency, Budinis et al. (2018) estimate that the capture rate of CCS technology should range between 85% to 90%. However, a recent review by the Institute for Energy Economics and Financial Analysis (IEEFA) highlights that no existing project has consistently achieved such a high carbon capture rate (Schlissel and Juhn, 2023). Furthermore, as noted in Section 2, CCS actually increases the energy input requirement, suggesting the need to consider an even lower efficiency parameter. Therefore, in the following analysis, we take $\beta = 0.56$ as a benchmark and study the impact of varying β.

In a linear-quadratic setting considering international cooperation to mitigate pollution damage, Mason et al. (2017), following the calibration by Karp and Zhang (2006, 2012), estimates that the pollution damage parameter c lies within the range of $0.00001 \leq c \leq 0.005$. For our numerical analysis, we set $c_i, c_j \in (0.0002, 0.0004)$ and consider both symmetric and asymmetric damage scenarios, thus extending the scope of the above theoretical study.

Different from Moreaux et al. (2024), where the initial pollution condition can be chosen for a specific year, we rearrange the timeline and set the initial condition as the trigger condition when player i starts to deploy CCS. In other words, the initial condition, y_0, and the cost parameter, b_i, are determined by the trigger condition (13), i.e., $y_0 = (V_i')^{-1}\left(-\frac{b_i}{\beta}\right)$, where $(V_i')^{-1}$ is the inverse function to be solved in the numerical analysis below. It is important to note that y_0 will vary depending on the various parameter settings.

Moreaux et al. (2024) calibrate separately the capture, transport, and storage costs of CCS. Combining their examples with the willingness to pay, we set $b_i = 13\text{US\$/tCO}_2$. The CCS cost for player j, b_j, should be higher than b_i. Therefore, different values can be considered, including scenarios where player j never triggers CCS deployment. Considering the reachability condition in Proposition 5, we set $b_j = 20\text{US\$/tCO}_2$.

As mentioned in Mason et al. (2017), the range of the productivity parameter a, which converts the CO_2 emissions into the final consumption goods, can be quite large. In our analysis we set $a = 116$, which satisfies all the inequality conditions imposed in the theoretical framework above.

The benchmark parameters are presented in the following table:

Table 1: Benchmark parameter setting

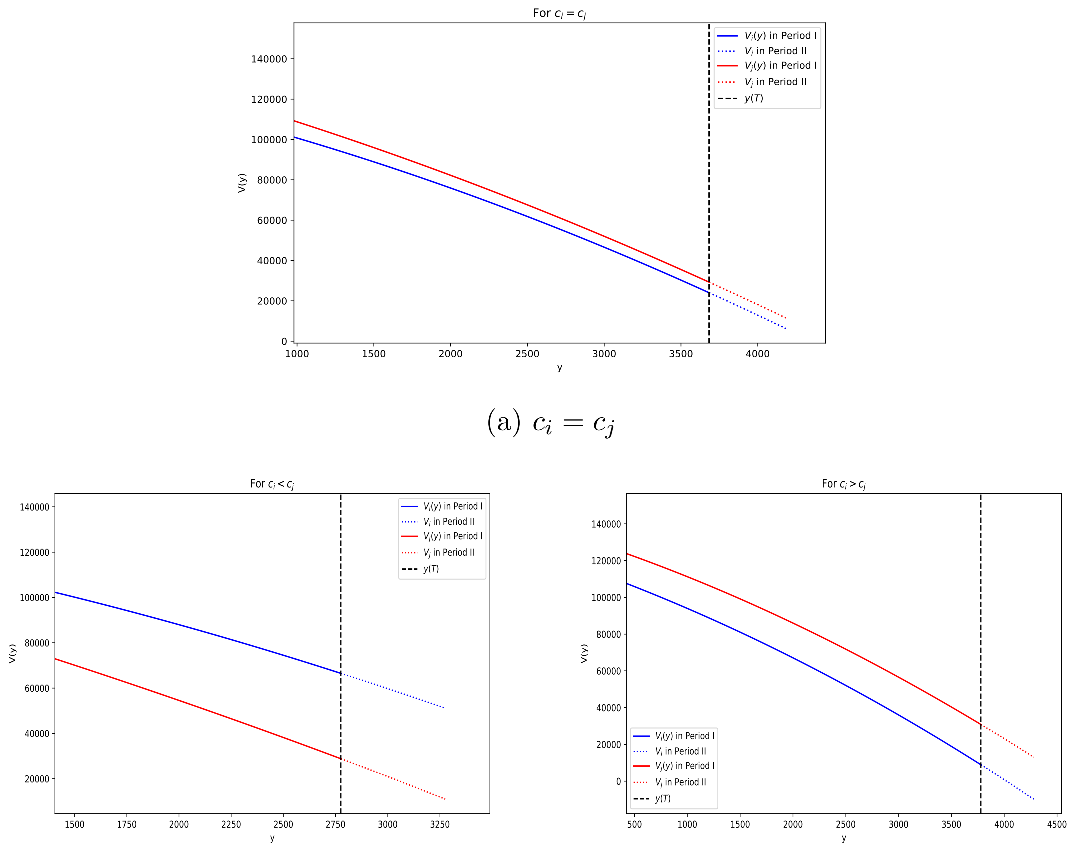
Parameters	Interpretation	Benchmark Value
r	Discount rate	0.022
δ	Nature's CO_2 absorption rate	0.015
β	CCS efficiency	0.58
c_i	player i's unit environmental damage cost	0.000282
c_j	player j's unit environmental damage cost	0.000282
b_i	Deployment cost of player i	13
b_j	Deployment cost of player j	20
a	Productivity	116

4.3.2 Numerical analysis

The contributions of this numerical study are threefold: (1) to illustrate the theoretical findings derived above; (2) to perform a numerical analysis of more general cases where $c_i > c_j$ and $c_i < c_j$; and (3) to further investigate the impact of policymakers' time preference r and efficiency parameter β on social welfare, emissions and CCS abatement.

Value functions, emissions and CCS deployment across two periods

Under different environmental damage scenarios, i.e., various combinations of c_i and c_j, Figure 1 illustrates the strictly concave and decreasing parts of the value functions (social welfare) for the two players, V_i and V_j, in terms of pollution accumulation. The horizontal axis represents pollution accumulation (y), while the vertical axis represents social welfare. In this and all subsequent figures, the red curve represents player j and the blue curve represents player i. Figure (1a) depicts the symmetric case where $c_i = c_j$, while Figures (1b) and (1c) show the asymmetric cases where $c_i < c_j$ and $c_i > c_j$, respectively. The vertical black dotted line marks the trigger pollution level ($\overline{y}$) for player j, indicating the start of the second period.



(a) $c_i = c_j$

(b) $c_i < c_j$

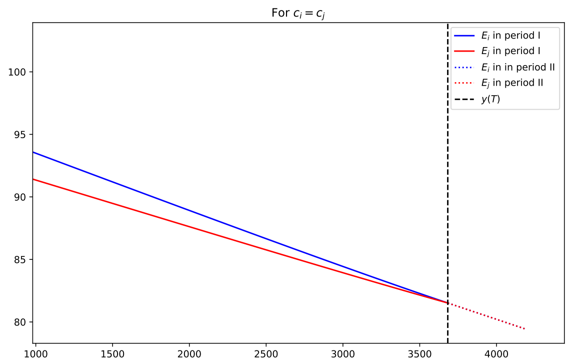
(c) $c_i > c_j$

Figure 1: Effect of different unit environmental damage costs on social welfare.

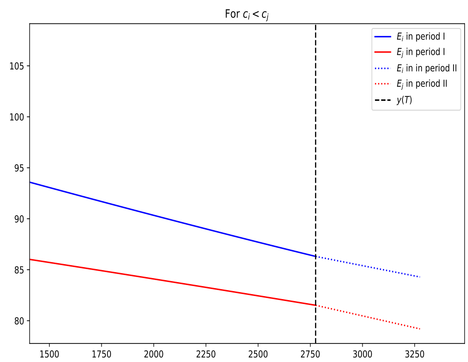
Under different environmental damage scenarios, i.e., various combinations of c_i and c_j, Figure 1 illustrates the strictly concave and decreasing parts of the value functions (social welfare) for the two players, V_i and V_j, in terms of pollution accumulation. The horizontal axis represents pollution accumulation (y), while the vertical axis represents social welfare. In this and all subsequent figures, the red curve represents player j and the blue curve represents player i. Figure (1a) depicts the symmetric case where $c_i = c_j$, while Figures (1b) and (1c) show the asymmetric cases where $c_i < c_j$ and $c_i > c_j$, respectively. The vertical black dotted line marks the trigger pollution level ($\overline{y}$) for player j, indicating the start of the second period.

According to the theoretical framework, player i initiates CCS deployment earlier than player j due to a lower deployment costs. Figure (1a) and (1c) demonstrate that when $c_i \geq c_j$, the delayed CCS deployment by player j results in a social welfare advantage for her, regardless of whether she ultimately deploys CCS. This advantage is evident as the red curve consistently dominates the blue curve. Player j benefits from player i's abatement efforts, achieving a higher welfare level by delaying or even avoiding CCS deployment, which indicates a free-riding behavior. In Figure (1c), the welfare gain for player j over player i is more pronounced when there is a disparity in pollution damage, i.e. when $c_i > c_j$.

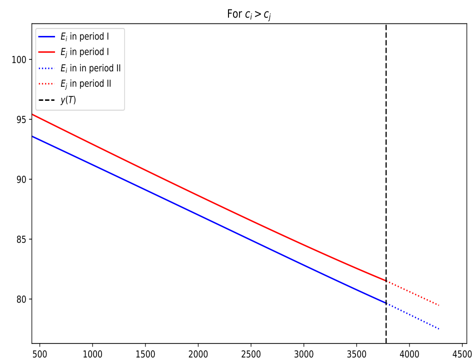
However, when $c_i < c_j$, with all other factors being equal, player j's value function (red curve) is dominated by player i's value function (blue curve). In this scenario, the impact of pollution damage outweighs the benefits of free-riding. Although player i starts CCS deployment earlier and faces relatively less pollution damage, the overall reduction in accumulated pollution is insufficient. As a result, the damage to player j increases, eventually forcing her to deploy CCS as well. This can be observed in Figures 2 and 3, which show emissions and CCS abatement, respectively. As before, the vertical black dotted line marks the trigger pollution level at which player j begins CCS deployment, and the solid (dotted) curves represent the first (second) period.



(a) Emissions with $c_i = c_j$



(b) Emissions with $c_i < c_j$

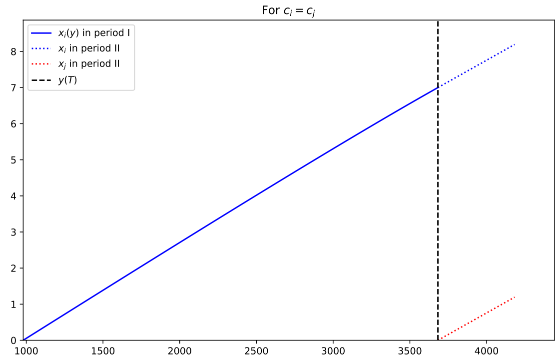


(c) Emissions with $c_i > c_j$

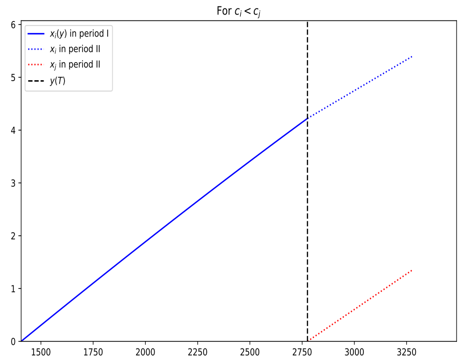
Figure 2: Emissions over two periods for various damage scenarios.

Figure 2 illustrates emissions across two periods under both symmetric and asymmetric pollution damage scenarios, based on our set of benchmark parameters. It is evident that the player who experiences less pollution damage tends to emit more. Specifically, when $c_i < c_j$, player i' emission (blue curve) exceed those of player j (red curve), as shown in Figure (2b). Conversely, when player i faces greater damage ($c_i > c_j$), her emissions are lower in both periods, as depicted in Figure (2c). In the symmetric case shown in Figure (2a) where $c_i = c_j$, player i emits more in the first period while being the only one deploying CCS. However, in the second period, both players emit at the same level, consistent with the predictions of Proposition 3.

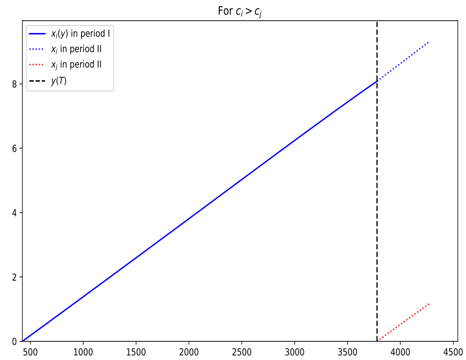
Figure 3 displays CCS deployment over two periods under both symmetric and asymmetric pollution damage scenarios, using the same set of benchmark parameters.



(a) CCS with $c_i = c_j$



(b) CCS with $c_i < c_j$



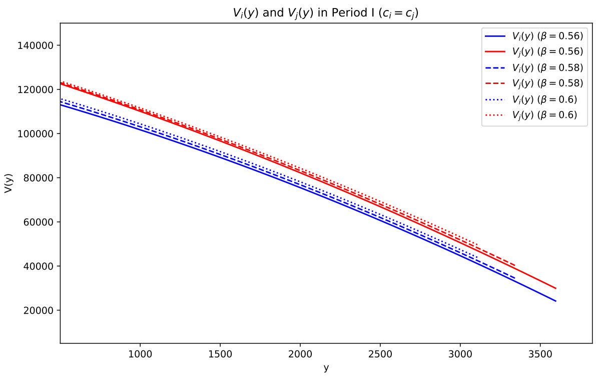
(c) CCS with $c_i > c_j$

Figure 3: CCS deployment across the two periods

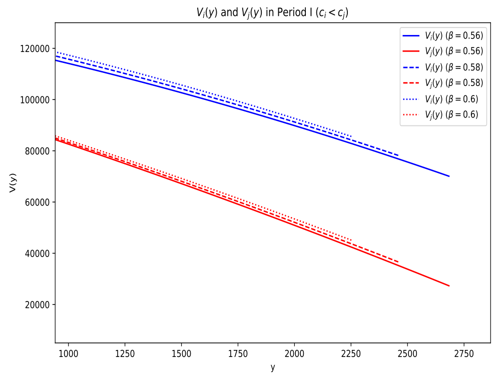
As expected, in Figure (3b), where $c_i < c_j$, player j initiates her CCS deployment much earlier compared to the other two scenarios shown in Figures (3a) and (3c).

The impact of the efficiency parameter β

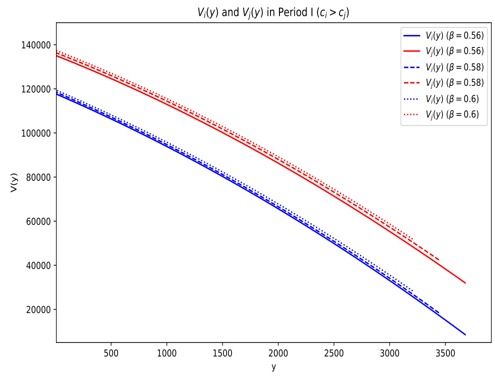
The impact of the CCS efficiency parameter is presented in Figure 4. The solid curves represent the benchmark case ($\beta = 0.56$), while the dashed and dotted curves correpsond to $\beta = 0.58$ and $\beta = 0.6$, respectively. In this figure, we only plot the first period. The second period begins where the current curves stop.



(a) V_i, V_j in terms of β when $c_i = c_j$



(b) V_i, V_j in terms of β when $c_i < c_j$



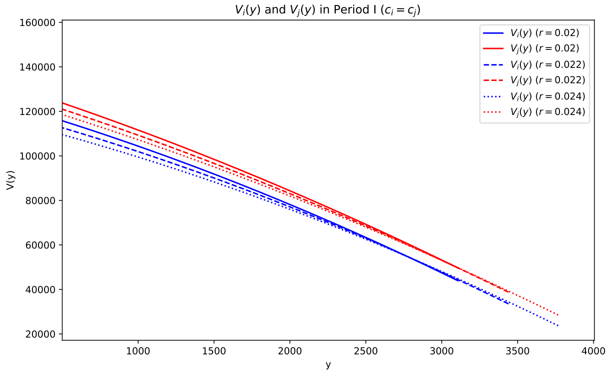
(c) V_i, V_j in terms of β when $c_i > c_j$

Figure 4: Effect of β on the value functions.

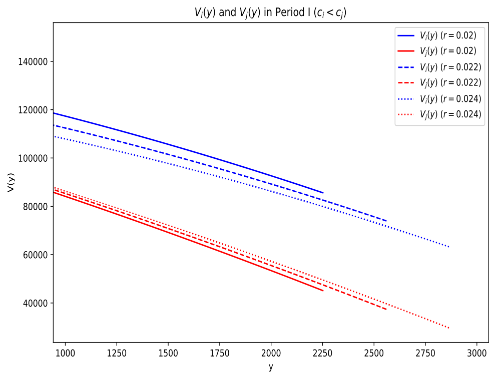
On the one hand, Figure 4 confirms the statement from Figure 1 that, regardless of the efficiency of the CCS facilities β, (1) the player who suffers more from accumulated pollution experiences relatively lower social welfare, and (2) player j free rides on player i's CCS efforts when they face the same pollution damage. On the other hand, Figure 4 shows that higher CCS efficiency leads to a monotonic increase in both players' social welfare. Most importantly, higher efficiency (larger β values) prompts player j to initiate CCS deployment earlier, irrespective of the damage scenario. As expected, Figure (4b) further illustrates that when player j suffers more from pollution damages than player i, she starts CO_2 abatement earlier than in the other two cases, depicted in Figures (4a) and (4c).

The impact of the policymakers' patience r

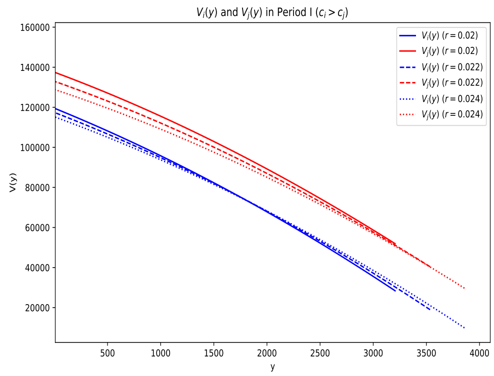
Unlike the efficiency parameter (β), which has a symmetric and monotonic impact on the players' welfare (with social welfare increasing as CCS efficiency improves), the effect of the time preference reflecting the decision makers' patience, r, is different for the two players depending on the unit environmental damage cost (c_i, c_j) and the level of accumulated pollution. This finding is illustrated in Figure 5, where the solid, dashed and dotted curves represent the benchmark case and the alternative cases with $r = 0.020$ and 0.024, respectively.



(a) V_i, V_j in terms of r when $c_i = c_j$



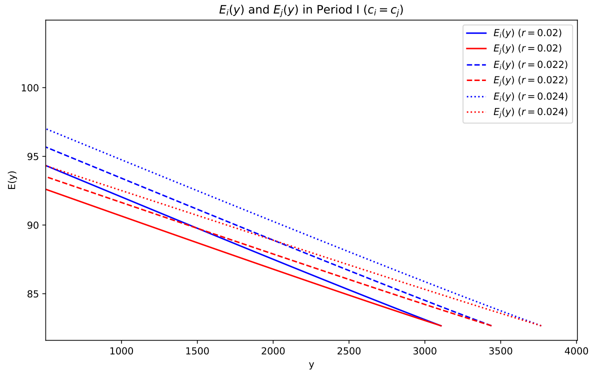
(b) V_i, V_j in terms of r when $c_i < c_j$



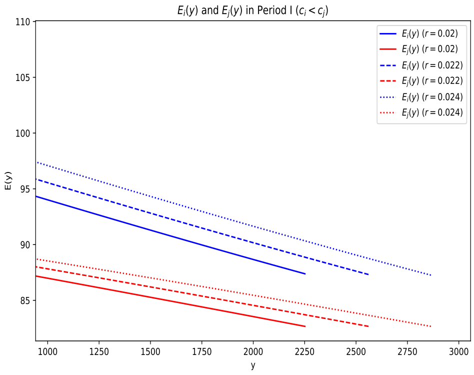
(c) V_i, V_j in terms of r when $c_i > c_j$

Figure 5: Effect of r on the value functions

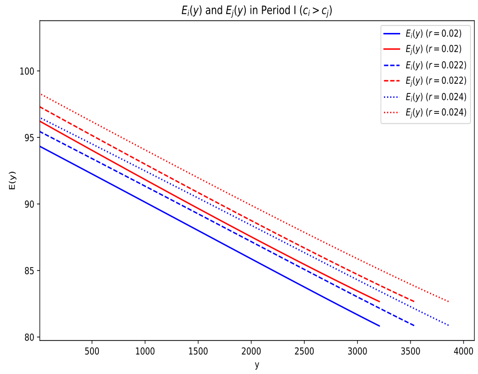
Figures (5a)-(5c) illustrate three different damage scenarios with $c_i = c_j$, $c_i < c_j$, and $c_i > c_j$, respectively. In the symmetric cost scenario (Figure 5a), social welfare decreases symmetrically with the discount rate r for both players. An impatient policymaker often prioritizes immediate gains over long-term sustainability. This short-term focus can lead to decisions that increase pollution by raising current emission levels and reducing current CCS efforts, ultimately diminishing the overall welfare. This trend is illustrated in Figures (6a) and (7a), which show the levels of emissions and CCS abatement for different values of r, respectively. However, as the accumulated pollution level y increases, the advantage in social welfare for a patient one diminishes. Just before transitioning into the second period, an opposite trend appears to emerge.



(a) E_i, E_j in terms of r when $c_i = c_j$



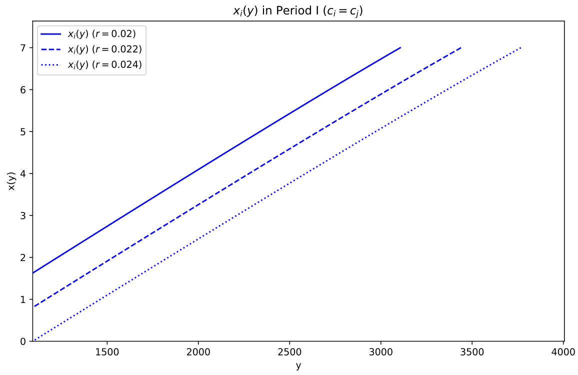
(b) E_i, E_j in terms of r when $c_i < c_j$



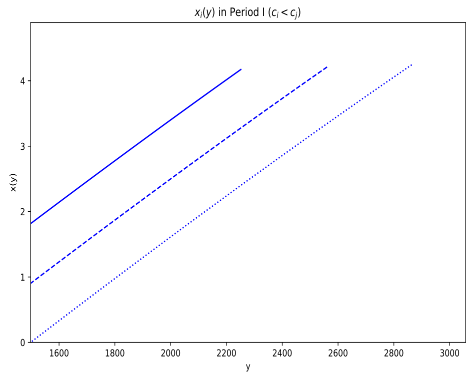
(c) E_i, E_j in terms of r when $c_i > c_j$

Figure 6: Effect of r on emissions

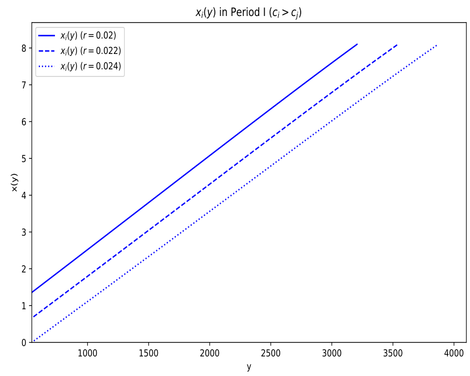
Figure (5c), where $c_i > c_j$, further illustrates this pattern: social welfare is higher when r is smaller at low levels of pollution. However, as pollution levels rise, the benefits of a lower r diminish, especially for player i who faces higher climate change damage costs and initiates CCS earlier. On the one hand, as pollution accumulates and reaches higher levels, the marginal damage from additional pollution increases sharply. Consequently, even for a policymaker with a higher discount rate, her efforts toward pollution control become crucial (Figures (6c) and Figures (7c)). On the other hand, at low levels of accumulated pollution, a more patient policymaker (with a lower r) prioritizes long-term outcomes and is thus willing to invest in emission reductions and CCS abatement to mitigate future climate change damage. Since pollution is still manageable, the costs of these efforts remain relatively low, and the benefits of preventing future damage are significant. As a result, a more patient one experiences higher net social welfare because her forward-looking action is effective and not overly burdensome at this stage. In contrast, as pollution levels increase, the dynamics shift. A patient policymaker who has consistently invested in emission reductions and CCS abatement now faces substantial cumulative costs from these ongoing efforts due to the convex nature of these expenses. As shown in Figures (6c) and Figures (7c), the trajectories for emissions (for both player i and player j) and CCS abatement (for player i) are steeper at lower values of r compared to higher values. While the benefits of preventing future damage remain, the cost burden has escalated significantly due to the sustained pollution control efforts over time. As pollution continues to accumulate, these rising costs can start to outweigh the benefits.



(a) x_i in terms of r when $c_i = c_j$



(b) x_i in term of r where $c_i < c_j$

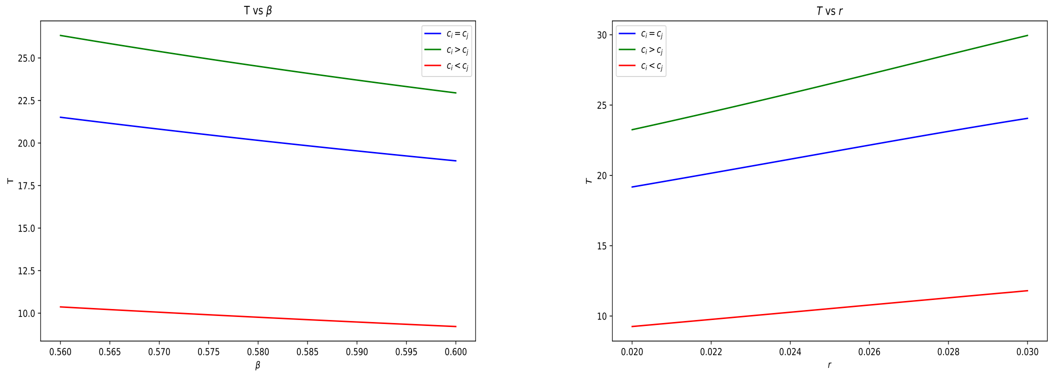


(c) x_i in term of r where $c_i > c_j$

Figure 7: Effect of r on the CCS deployment of player i in period I.

A notable exception is observed in Figure (5b), where $c_i < c_j$. In this case, the social welfare trends for the two player (player i and j) move in opposite directions with respect to the parameter r. Player j with higher climate damage costs, paradoxically benefits from a higher discount rate. Furthermore, player i's welfare remains consistently higher than that of player j. Although both players' emissions increase with r, as shown in Figure (6b), player i emits more than player j, enjoys higher current consumption, and thus achieves higher social welfare. This pattern differs from those in (6a) and (6c). Additionally, Figure (7b) shows that player i's contribution to CCS deployment is also lower than in the other two cases depicted in Figure 7, which prompts player j to deploy CCS earlier when r is smaller.

Optimal trigger time



(a) T in terms of β

(b) T in terms of r

Figure 8: Effect of β and r on the optimal trigger time T

Figure 8 illustrates the moment when the second period begins, expressed in terms of β or r, assuming that the second period exists. The information provided is twofold: on one hand, the benchmark parameters were selected to ensure that Condition (5.2) in Proposition 5 holds, meaning that within a finite time T, player j initiates her CCS deployment. On the other hand, it demonstrates how T varies across different damage scenarios (c_i, c_j), CCS efficiency (β), and time preference (r).

In Figure 8, the green, blue, and red curves correspond to the three different scenarios: $c_i > c_j$, $c_i = c_j$, and $c_i < c_j$. The vertical axis indicates the trigger moment T, while the horizontal axis represents β (Figure 8a), respectively r (Figure 8b).

It is expected that when $c_i < c_j$, player j who faces a higher accumulated pollution damage, will trigger her CCS deployment earlier, regardless of the pollution level y, CCS efficiency β,or time preference r. It is straightforward that player j prefers to delay the deployment of CCS technologies when she is less affected by the pollution damage (as shown by the green curves consistently being above the blue and red ones).

All curves in Figures 8a and 8b are monotonic. An increase in the CCS efficiency parameter (β) prompts player j to deploy CCS earlier, assuming all other parameters remain constant. Conversely, an increase in the time preference (r) delays player j's CCS deployment, which is consistent with the pattern shown in Figure 6.

5 Conclusion

In this paper, we employed a differential game model to investigate the optimal timing for different countries to engage in CCS deployment. We characterize the structure of the equilibria, the dynamics of pollution accumulation that maximize social welfare and the respective long-run pollution accumulation. We also determine the conditions under which a country should start CCS deployment, providing valuable guidance for policymakers in designing strategies for effective pollution control and CCS implementation..

The main findings of this paper can be summarized as follows. (i) Even when countries face the same level of pollution damage costs, differences in development levels, financial resources, and institutional capacities can lead to varied timings for implementing CCS technologies. In some cases, countries may never initiate CCS. (ii) Regardless of the costs and the development level, countries that suffer less from pollution accumulation have fewer incentives to invest in CCS, despite their CO_2 emissions negatively affecting others. This disparity underscores the need for international cooperation to address the global impact of pollution effectively. (iii) When countries face symmetric pollution damage costs, they are more likely to free-ride on a neighboring country's CCS investment efforts compared to scenarios with asymmetric damage costs. As a result, symmetric pollution damage costs tends to result in higher long-run pollution accumulation compared to the asymmetric damage cost case.

Despite the ongoing advancements in the development and deployment of CCS technologies, significant challenges remain, particularly in reducing CCS costs and removing regulatory barrier in order to ensure commercial-scale deployment. As a result, continued investment in research, development, and the deployment of CCS technologies is essential for scaling up these technologies and achieving significant emissions reductions. This paper offers the first steps toward establishing a comprehensive framework for the theoretical analysis of CCS, as well as for the even more promising CCUS technologies.

Ultimately, the deployment of CCS technologies must be a global effort, requiring collaboration among all countries to develop and deploy these technologies effectively. Only through such collective action can we achieve significant emissions reductions and make meaningful progress in combating climate change.

A Appendix

A.1 Proof of Proposition 2

Following the optimal choice of emissions and CCS abatement, recall that the pollution accumulation equation is given by

$$\dot{y} = [E_i^*(y) + E_j^*(y)] - [\beta(X_i^*(y) + x_j^*(y)) + \delta y],$$

with a given initial condition $y_0 = \overline{y_i}$. From Proposition 1, the first term on the right hand side of the equation is decreasing, while the second term is increasing with respect to y. In other words, there is one and only one potential long-run steady state given by the solution of equation (8), or

$$E_i^*(y) + E_j^*(y) = \beta(X_i^*(y) + x_j^*(y)) + \delta y. \tag{36}$$

Thus, depending on at the initial condition $y_0 = \overline{y_i}$, there are two possibilities: Either $E_i^*(y_0) + E_j^*(y_0) > \beta X_i^*(y_0) + \delta y_0$ or the inequality holds in the opposite direction. Figure 9 illustrates the first case while a similar figure can depict the latter one.

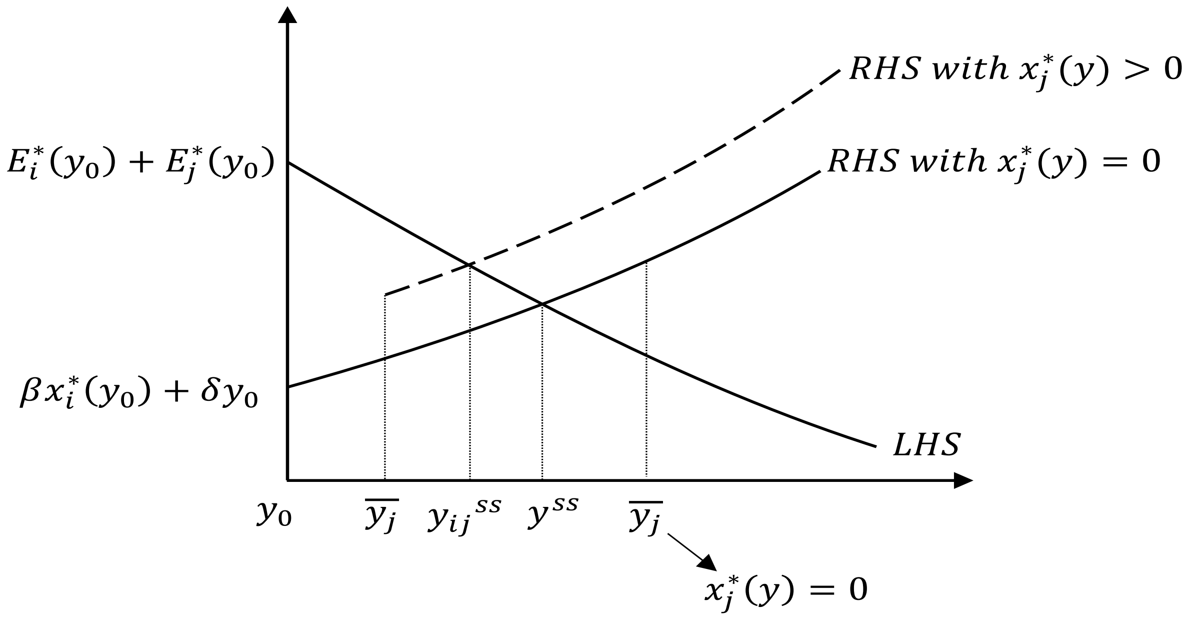


Figure 9: Illustration of the state equation

In Figures 9, LHS and RHS represent the left-hand-side, respectively the right-hand-side of the steady state equation (36), in terms of y.

In Figure 9, depending on trigger condition of player j, two cases emerge: if the trigger condition is such that $\overline{y_j} > y^{ss}$, player j will never start deploying CCS. In this scenario, the unique long-run steady state is y^{ss} which satisfies $y^{ss} > y_0 = \overline{y_i}$. However, if $\overline{y_j} < y^{ss}$, player j will, sooner or later, trigger CCS deployment. In this case, the LHS curve shifts from the solid curve to the dashed one, and the long-run steady state will be y_{ij}^{ss}, which corresponds to the long-run steady state when both players deploy CCS. As a result, $y_{ij}^{ss} < y^{ss}$. This proves the first part of Proposition 2.

Alternatively, consider the scenario where player i starts CO_2 abatement and it satisfies $E_i^*(y_0) + E_j^*(y_0) < \beta X_i^*(y_0) + \delta y_0$, where the pollution level immediately drops below the trigger condition of player i. Under these circumstances, there is not even the need for player i to start CCS deployment, not to mention player j. This scenario can occur when the emissions from both countries are relatively low compared to the natural absorption capacity.

That completes the proof.

A.2 Proof of Proposition 3 and 4

We complete the proof in 3 steps: Step 1 is about the common parts of Proposition 3 and 4; Step 2 sets out the proof for Proposition 3, and Step 3 finishes the proof of Proposition 4.

Step 1: The common part.

Recall that the HJB equation of player k is given by

$$rV_k(y) = \max_{E_k, x_k} \left[\left(a_k E_k - \frac{E_k^2}{2} - \frac{c_k\, y^2}{2} - \left(b_k\, x_k + \frac{x_k^2}{2} \right) \right) \right. \\ \left. + V_k'(y)[E_i + E_j - \beta(x_i + x_j) - \delta y] \right].$$

The optimality conditions yield, for $k = i, j$ and $t \geq T$,

$$E_k = a_k + V_k'(y)$$

and

$$x_k = -(b_k + \beta V_k'(y)).$$

Given the linear-quadratic structure of the optimal control problem, a linear-quadratic guess of the Bellman value function is made

$$V_k(y) = A_k + B_k y + \frac{C_k}{2} y^2$$

where A_k, B_k and C_k are constants to be determined. It is easy to see that $V_k'(y) = B_k + C_k y$. Thus

$$E_k = a_k + B_k + C_k y$$

and

$$x_k = -[b_k + \beta(B_k + C_k y)].$$

Substituting x_k into the HJB equations and comparing the coefficients of powers of y, it follows that, for i and j, respectively

$$\begin{cases} rA_i = \frac{a^2+b_i^2}{2} + (2a + \beta(b_i + b_j))B_i + \frac{(1+\beta^2)(B_i^2+2B_iB_j)}{2}, \\ (r+\delta)B_i = (1+\beta^2)[B_iC_i + B_iC_j + B_jC_i] + (2a + \beta(b_i + b_j))C_i, \\ (r+2\delta)C_i = (1+\beta^2)C_i^2 + 2(1+\beta^2)C_iC_j - c_i \end{cases}$$
$$\begin{cases} rA_j = \frac{a^2+b_j^2}{2} + (2a + \beta(b_i + b_j))B_j + \frac{(1+\beta^2)(B_j^2+2B_iB_j)}{2}, \\ (r+\delta)B_j = (1+\beta^2)[B_jC_j + B_iC_j + B_jC_i] + (2a + \beta(b_i + b_j))C_j, \\ (r+2\delta)C_j = (1+\beta^2)C_j^2 + 2(1+\beta^2)C_iC_j - c_j \end{cases} \tag{37}$$

For general damage cost structure $0 < c_i, c_j < +\infty$, it is difficult to obtain explicit solution from the last two equations. In the following, we will thus consider two special cases.

Step 2: Symmetric costs: $c_i = c_j = c$

With the assumption that $c_i = c_j = c$, it can be shown that V_i is a valid value function if and only if $C_i = C_j \equiv C_s$. As a result, the last equation in (37) is a quadratic equation of the variable C_s. Usually, there are two real roots: one positive and one negative,

$$C_s = \frac{r + 2\delta \pm \sqrt{(r+2\delta)^2 + 12c(1+\beta^2)}}{6(1+\beta^2)}.$$

Picking only the root with the negative sign yields a valid concave value function. We thus have

$$C_s = \frac{(r + 2\delta) - \sqrt{(r+2\delta)^2 + 12c(1+\beta^2)}}{6(1+\beta^2)}(< 0).$$

Substituting C_s into the second equation in (37), it follows that $B_i = B_j \equiv B_s$, where the expression of B_s is provided in Proposition 1.

Substituting $x^*_{s,k}(y), E^*_{s,k}(y)$ into the dynamic equation of pollution accumulation, it follows that

$$\dot{y} = E_1 + E_2 - \beta(x_1 + x_2) - \delta y = [2a + \beta(b_i + b_j) + 2(1+\beta^2)B_s] + [2(1+\beta^2)C_s - \delta]y.$$

With this, the steady state and trajectory path are straightforward to derive and given in the Proposition.

The uniqueness of the affine strategy follows the same argument as in Boucekkine et al. (2021).

Step 3: Asymmetric costs: $c_i > 0$ **and** $c_j = 0$

With the assumption that $c_i > 0$ and $c_j = 0$, it is easy to see that the last equation in (37) yields a concave value function for players i and j, if and only if

$$C_{e,j} = 0 \text{ and } C_{e,i} = \frac{r + 2\delta - \sqrt{(r+2\delta)^2 + 4c_i(1+\beta^2)}}{2(1+\beta^2)}.$$

In other words, if $C_{e,j} \neq 0$, there is no valid value function for player i.

Accordingly, we have

$$B_{e,j} = 0 \text{ and } B_{e,i} = \frac{(2a + \beta(b_i + b_j))C_{e,i}}{r + \delta - (1+\beta^2)C_{e,i}}(< 0),$$

and

$$A_{e,j} = \frac{a^2 + b_j^2}{2r} \text{ and } A_{e,i} = \frac{a^2 + b_i^2}{2r} + \frac{2a + \beta(b_i + b_j)}{r}B_{e,i} + \frac{1+\beta^2}{2r}B_{e,i}^2.$$

Thus, it is straightforward to show that,

$$V_{e,i}(y) = A_{e,i} + B_{e,i}y + \frac{C_{e,i}y^2}{2} \text{ and } V_{e,j}(y) = A_{e,j},$$

That completes the proof.

A.3 Proof of Proposition 5

Whether the trigger condition $\overline{y}$ is reachable or not essentially depends on the terminal condition (28). We complete the analysis in three steps. Step 1 presents the explicit forms of the terminal conditions $P_i(\overline{y})$ and $P_j(\overline{y})$; in Step 2 we rewrite the dynamic equation of pollution accumulation using the explicit forms obtained from the first step; and in Step 3 we derive the results of Proposition 5 from the newly formed dynamic equation.

Step 1: The explicit forms of $P_i(\overline{y})$ and $P_j(\overline{y})$

We rewrite the affine-quadratic form of (28) in the following way:

$$\begin{cases} (1+\beta^2)P_i^2(\overline{y}) + 2[(2a+\beta b_i) - \delta\overline{y}]P_i(\overline{y}) + 2P_j(\overline{y})P_i(\overline{y}) - [2rV_i(\overline{y}) + c_i\overline{y}^2 - (a^2+b_i^2)] = 0, \\ P_j^2(\overline{y}) + 2[(2a+\beta b_i) - \delta\overline{y}]P_j(\overline{y}) + 2(1+\beta^2)P_i(\overline{y})P_j(\overline{y}) - [2rV_j(\overline{y}) + c_j\overline{y}^2 - a^2] = 0. \end{cases}$$

To shorten the notation, we set $\mu \equiv 2P_i(\overline{y})P_j(\overline{y})$, and

$$\Delta_i \equiv 2rV_i(\overline{y}) + c_i\overline{y}^2 - (a^2+b_i^2), \quad \Delta_j \equiv 2rV_j(\overline{y}) + c_j\overline{y}^2 - a^2.$$

It follows that the roots of $P_i(\overline{y})$ and $P_j(\overline{y})$ are given by

$$P_i(\overline{y}) = \frac{-(2a+\beta b_i - \delta\overline{y}) \pm \sqrt{(2a+\beta b_i-\delta\overline{y})^2 + [(1+\beta^2)(\Delta_i - \mu)]}}{1+\beta^2} \tag{38}$$

and

$$P_j(\overline{y}) = -(2a+\beta b_i - \delta\overline{y}) \pm \sqrt{(2a+\beta b_i-\delta\overline{y})^2 + [\Delta_j - (1+\beta^2)\mu]}. \tag{39}$$

Given that at $\overline{y}$, the marginal value function $P_k(\overline{y}) = B_{II} + C_{II}\overline{y} \leq 0$ for both $k = i, j$, only the negative roots can be taken. Furthermore, if both roots are negative, we take the most concave one when $y < \overline{y}$ and $y \to \overline{y}^-$. Therefore

(For P_i)

- In (38), the positive sign is taken, if

$$2a + \beta b_i - \delta\overline{y} > 0 \text{ and } \Delta_i < \mu.$$

- In (38), the negative sign is taken, if

$$2a + \beta b_i - \delta\overline{y} > 0;$$

or

$$2a + \beta b_i - \delta \overline{y} < 0 \text{ and } \Delta_i > \mu.$$

(For P_j)

- In (39), the positive sign is taken, if

$$2a + \beta b_i - \delta \overline{y} > 0 \text{ and } \Delta_j < (1+\beta^2)\mu.$$

- In (39), the negative sign is taken, if

$$2a + \beta b_i - \delta \overline{y} > 0;$$

or

$$2a + \beta b_i - \delta \overline{y} < 0 \text{ and } \Delta_j > (1+\beta^2)\mu.$$

To clear the notation, let's set $\sigma_k = 1$ if the positive sign is taken and $\sigma_k = -1$ if the negative sign is taken in (38) or (39), respectively.

Thus, μ must be a solution of

$$\begin{aligned}(1+\beta^2)\mu = 2&\left[-(2a+\beta b_i - \delta\overline{y}) \pm \sqrt{(2a+\beta b_i-\delta\overline{y})^2 + [(1+\beta^2)(\Delta_i - \mu)]}\right] \\ &\left[-(2a+\beta b_i - \delta\overline{y}) \pm \sqrt{(2a+\beta b_i-\delta\overline{y})^2 + [\Delta_j - (1+\beta^2)\mu]}\right].\end{aligned} \tag{40}$$

Step 2: Forming the new pollution accumulation equation

Recall that the dynamic equation of pollution accumulation is given by

$$\begin{aligned}\dot{y} &= E_i + E_j - \beta x_i - \delta y \\ &= a + V_i'(y) + a + V_j'(y) - \beta(-b_i - \beta V_i'(y)) - \delta y \\ &= 2a + \beta b_i - \delta y + (1+\beta^2)P_i(y) + P_j(y),\end{aligned}$$

determined by the optimal choices (23) and (25).

Let's define

$$f_i(y) \equiv (1+\beta^2)P_i(y) + \frac{2a+\beta b_i - \delta y}{2} \quad \text{and} \quad f_j(y) \equiv P_j(y) + \frac{2a+\beta b_i - \delta y}{2}.$$

Then

$$\dot{y} = f_i(y) + f_j(y), \quad \forall y \leq \overline{y}. \tag{41}$$

Furthermore, from (38) and (39), it is easy to see that at $y = \overline{y}$

$$\begin{aligned}f_i(\overline{y}) + f_j(\overline{y}) = &-(2a+\beta b_i-\delta\overline{y}) + \sigma_i\sqrt{(2a+\beta b_i-\delta\overline{y})^2 + [(1+\beta^2)(\Delta_i-\mu)]} \\ &+\sigma_j\sqrt{(2a+\beta b_i-\delta\overline{y})^2 + [\Delta_j - (1+\beta^2)\mu]}.\end{aligned} \tag{42}$$

Apparently,

- If $f_i(\overline{y}) + f_j(\overline{y}) > 0$: At $\overline{y}$, from equation (41), $\dot{y} = f_i(\overline{y}) + f_j(\overline{y}) > 0$. At $\overline{y}$, pollution accumulation is thus still increasing, meaning that the trigger condition will be reached in finite time T.
- If $f_i(\overline{y}) + f_j(\overline{y}) \leq 0$: At $\overline{y}$, $\dot{y} \leq 0$, indicating that $\overline{y}$ can never be reached because the dynamic system has already surpassed its asymptotic stable long-run steady state (where $\dot{y} = 0$).

Therefore, the reachability conditions are essentially determined by $f_i(\overline{y}) + f_j(\overline{y}) \lesseqgtr 0$. In the following section of this proof, we will examine the specific conditions under which each of theses possibilities occurs.

Step 3: The reachability conditions

The reachability conditions can be derived in four parts:

Part 3.a: Suppose $2a + \beta b_i < \delta\overline{y}$.

By (42), we must take $\sigma_i = -1$, $\sigma_j = -1$ and $\mu \leq \min\left\{\Delta_i, \frac{\Delta_j}{1+\beta^2}\right\}$. Hence, the right hand side of (42) is increasing in μ and is no more than its value at $\mu = \min\left\{\Delta_i, \frac{\Delta_j}{1+\beta^2}\right\}$. Then it is straightforward to show that,

$$-\sqrt{(2a + \beta b_i - \delta\overline{y})^2 + [(1+\beta^2)(\Delta_i - \mu)]} < -|2a + \beta b_i - \delta\overline{y}|$$

and

$$-\sqrt{(2a + \beta b_i - \delta\overline{y})^2 + [\Delta_j - (1+\beta^2)\mu]} < -|2a + \beta b_i - \delta\overline{y}|.$$

Thus,

$$f_i(\overline{y}) + f_j(\overline{y}) < -(2a + \beta b_i - \delta\overline{y}) - 2|2a + \beta b_i - \delta\overline{y}| = -|2a + \beta b_i - \delta\overline{y}| < 0.$$

As a consequence, $\overline{y}$ can never be reached.

Part 3.b: Suppose $2a + \beta b_i > \delta\overline{y}$.

We shall show that for either $\sigma_i = -1$ or $\sigma_j = -1$, there is $f_i(\overline{y}) + f_j(\overline{y}) < 0$.

Let us start with $\sigma_i = -1$ and $\sigma_j = 1$. In that case

$$\begin{aligned} f_i(\overline{y}) + f_j(\overline{y}) &= -(2a + \beta b_i - \delta\overline{y}) - \sqrt{(2a + \beta b_i - \delta\overline{y})^2 + (1+\beta^2)(\Delta_i - \mu)} \\ &\quad + \sqrt{(2a + \beta b_i - \delta\overline{y})^2 + (\Delta_j - (1+\beta^2)\mu)} \\ &= P_j(\overline{y}) - \sqrt{(2a + \beta b_i - \delta\overline{y})^2 + (1+\beta^2)(\Delta_i - \mu)} \\ &< 0 \end{aligned}$$

given the marginal value function $P_j(\overline{y}) \leq 0$ from the last section. The same calculations hold if we take $\sigma_i = 1$ and $\sigma_j = -1$.

As a byproduct of $P_j(\overline{y}) \leq 0$ and $P_i(\overline{y}) \leq 0$, together with the assumption that $2a+\beta b_i > \delta\overline{y}$, we must have

$$\Delta_i - \mu < 0 \quad \text{and} \quad \Delta_j - (1+\beta^2)\mu < 0.$$

That is,

$$(1+\beta^2)\mu > \max\{(1+\beta^2)\Delta_i, \Delta_j\}. \tag{43}$$

We can thus conclude that, as long as one of the σ_i or σ_j take -1, we always have $f_i(\overline{y}) + f_j(\overline{y}) < 0$. Therefore, $f_i(\overline{y}) + f_j(\overline{y}) > 0$ is only possible if $\sigma_i = \sigma_j = 1$. However, we shall prove that under condition (30), this is not true.

We argue by contradiction that $f_i(\overline{y}) + f_j(\overline{y}) > 0$, meaning

$$\begin{aligned} f_i(\overline{y}) + f_j(\overline{y}) = -(2a+\beta b_i - \delta\overline{y}) + \sqrt{(2a+\beta b_i-\delta\overline{y})^2 + (1+\beta^2)(\Delta_i - \mu)} \\ + \sqrt{(2a+\beta b_i-\delta\overline{y})^2 + (\Delta_j - (1+\beta^2)\mu)} > 0, \end{aligned}$$

which is equivalent to

$$\begin{aligned} \sqrt{(2a+\beta b_i-\delta\overline{y})^2 + (1+\beta^2)(\Delta_i - \mu)} + \sqrt{(2a+\beta b_i-\delta\overline{y})^2 + (\Delta_j - (1+\beta^2)\mu)} \\ > 2a+\beta b_i - \delta\overline{y}. \end{aligned}$$

By the concavity of the square-root function,

$$\begin{aligned} \sqrt{(2a+\beta b_i-\delta\overline{y})^2 + (1+\beta^2)(\Delta_i - \mu)} + \sqrt{(2a+\beta b_i-\delta\overline{y})^2 + (\Delta_j - (1+\beta^2)\mu)} \\ < 2\sqrt{(2a+\beta b_i-\delta\overline{y})^2 + \frac{(1+\beta^2)\Delta_i + \Delta_j}{2} - (1+\beta^2)\mu}. \end{aligned}$$

Linking the last two inequality together, it follows that

$$(2a+\beta b_i-\delta\overline{y})^2 < 4\left[(2a+\beta b_i-\delta\overline{y})^2 + \frac{(1+\beta^2)\Delta_i + \Delta_j}{2} - (1+\beta^2)\mu\right],$$

which yields

$$(1+\beta^2)\mu < \frac{3}{4}\left(2a+\beta b_i-\delta\overline{y}\right)^2 + \frac{(1+\beta^2)\Delta_i + \Delta_j}{2}. \tag{44}$$

Furthermore, given $\mu = 2P_i(\overline{y})P_j(\overline{y}) > 0$, it is necessary that the right hand side of the above inequality is non-negative:

$$(1+\beta^2)\Delta_i + \Delta_j \geq -\frac{3}{2}\left(2a+\beta b_i-\delta\overline{y}\right)^2.$$

Since

$$\frac{(1+\beta^2)\Delta_i+\Delta_j}{2} = \frac{1}{2}\left[\max\{(1+\beta^2)\Delta_i,\ \Delta_j\} + \min\{(1+\beta^2)\Delta_i,\ \Delta_j\}\right],$$

by condition (30) we have

$$\begin{aligned}\max\{(1+\beta^2)\Delta_i,\ \Delta_j\} &= |(1+\beta^2)\Delta_i - \Delta_j| + \min\{(1+\beta^2)\Delta_i,\ \Delta_j\}\\ &\geq \frac{3}{2}\left(2a+\beta b_i-\delta\overline{y}\right)^2 + \min\{(1+\beta^2)\Delta_i,\ \Delta_j\}\\ &= \frac{3}{2}\left(2a+\beta b_i-\delta\overline{y}\right)^2 + (1+\beta^2)\Delta_i+\Delta_j - \max\{(1+\beta^2)\Delta_i,\ \Delta_j\}.\end{aligned}$$

As a consequence,

$$\max\{(1+\beta^2)\Delta_i,\ \Delta_j\} \geq \frac{3}{4}\left(2a+\beta b_i-\delta\overline{y}\right)^2 + \frac{(1+\beta^2)\Delta_i+\Delta_j}{2}.$$

Combining this with (43), it follows that

$$(1+\beta^2)\mu > \max\{(1+\beta^2)\Delta_i,\ \Delta_j\} \geq \frac{3}{4}\left(2a+\beta b_i-\delta\overline{y}\right)^2 + \frac{(1+\beta^2)\Delta_i+\Delta_j}{2}. \tag{45}$$

Obviously, the inequalities (44) and (45) constitute a contradiction. This proves that under condition (3.b), $f_i(\overline{y}) + f_j(\overline{y}) < 0$. Thus, $\overline{y}$ can never be reached.

Part 3.c: Suppose $2a+\beta b_i-\delta\overline{y} > 0$, $\max\{(1+\beta^2)\Delta_i, \Delta_j\} > 0$ and

$$|(1+\beta^2)\Delta_i - \Delta_j| < \frac{2\sqrt{3}-3}{4}(2a+\beta b_i-\delta\overline{y})^2,$$

then $f_i(\overline{y}) + f_j(\overline{y}) \leq 0$.

We argue by contradiction that $f_i(\overline{y}) + f_j(\overline{y}) > 0$. As already proven in Part 3.b, it is necessary that

$$\max\{(1+\beta^2)\Delta_i, \Delta_j\} < (1+\beta^2)\mu \leq \frac{3}{4}\left(2a+\beta b_i-\delta\overline{y}\right)^2 + \frac{(1+\beta^2)\Delta_i+\Delta_j}{2}.$$

Now consider both sides of equation (40), especially at the two end points of the above interval of $(1+\beta^2)\mu$.

At $(1+\beta^2)\mu = \max\{(1+\beta^2)\Delta_i, \Delta_j\}$, it is easy to see that the right hand side of equation (40) is zero, and by the assumption that $\max\{(1+\beta^2)\Delta_i, \Delta_j\} > 0$, the left hand side of equation (40) is $(1+\beta^2)\mu > 0$.

At $(1+\beta^2)\mu = \frac{3}{4}\left(2a+\beta b_i-\delta\overline{y}\right)^2 + \frac{(1+\beta^2)\Delta_i+\Delta_j}{2}$, the right hand side (RHS) of (40) is

$$\begin{aligned}
RHS &= 2\left[(2a+\beta b_i-\delta\overline{y}) - \sqrt{\frac{(2a+\beta b_i-\delta\overline{y})^2}{4} + \frac{(1+\beta^2)\Delta_i-\Delta_j}{2}}\right] \\
&\quad \left[(2a+\beta b_i-\delta\overline{y}) - \sqrt{\frac{(2a+\beta b_i-\delta\overline{y})^2}{4} + \frac{\Delta_j-(1+\beta^2)\Delta_i}{2}}\right] \\
&< 2\left[(2a+\beta b_i-\delta\overline{y}) - \frac{(2a+\beta b_i-\delta\overline{y})}{2}\right] \\
&\quad \left[(2a+\beta b_i-\delta\overline{y}) - \sqrt{\frac{(2a+\beta b_i-\delta\overline{y})^2}{4} - \frac{|\Delta_j-(1+\beta^2)\Delta_i|}{2}}\right] \\
&< (2a+\beta b_i-\delta\overline{y})^2\left[1-\frac{\sqrt{4-2\sqrt{3}}}{2}\right] \\
&= \frac{3-\sqrt{3}}{2}(2a+\beta b_i-\delta\overline{y})^2,
\end{aligned} \tag{46}$$

in which we used the inequality condition (31) in the second last inequality.

On the other hand, the left hand side (LHS) of (40) is

$$\begin{aligned}
LHS = (1+\beta^2)\mu &= \frac{3}{4}\left(2a+\beta b_i-\delta\overline{y}\right)^2 + \frac{(1+\beta^2)\Delta_i+\Delta_j}{2} \\
&= \frac{3}{4}\left(2a+\beta b_i-\delta\overline{y}\right)^2 + \max\{(1+\beta^2)\Delta_i,\ \Delta_j\} - \frac{|(1+\beta^2)\Delta_i-\Delta_j|}{2} \\
&> \frac{3}{4}\left(2a+\beta b_i-\delta\overline{y}\right)^2 - \frac{2\sqrt{3}-3}{4}\left(2a+\beta b_i-\delta\overline{y}\right)^2 \\
&= \frac{3-\sqrt{3}}{2}(2a+\beta b_i-\delta\overline{y})^2,
\end{aligned} \tag{47}$$

in which we used the condition $\max\{(1+\beta^2)\Delta_i,\ \Delta_j\} > 0$.

Furthermore, it is easy to check that the right hand side of (40) is increasing with μ. The above analysis thus demonstrates that the left hand side of equation (40) is always larger than the right hand side. As a result there is no solution for μ. In other words, $\overline{y}$ is not reachable under the conditions specified in (3.c).

Part 3.d: The conditions in (3.d) are very much the same as in (3.c) with the only difference being that $\max\{(1+\beta^2)\Delta_i,\ \Delta_j\} < 0$.

$\overline{y}$ is reachable if and only if $f_i(\overline{y}) + f_j(\overline{y}) > 0$, which is equivalent to the inequality (44):

$$(1+\beta^2)\mu \leq \frac{3}{4}\left(2a+\beta b_i-\delta\overline{y}\right)^2 + \frac{(1+\beta^2)\Delta_i+\Delta_j}{2}.$$

It is easy to check that with the condition that $\max\{(1+\beta^2)\Delta_i,\ \Delta_j\} < 0$, at $\mu = 0$, the left hand side of (40) is 0, while the right hand side is

$$\left[(2a+\beta b_i-\delta\overline{y})-\sqrt{(2a+\beta b_i-\delta\overline{y})^2+(1+\beta^2)\Delta_i}\right]$$
$$\left[(2a+\beta b_i-\delta\overline{y})-\sqrt{(2a+\beta b_i-\delta\overline{y})^2+\Delta_j}\right]\geq 0.$$

Similar to the above analysis of (46) and (47), we can show that for equation (40) there exists a unique positive solution for μ. At this point, $f_i(\overline{y})+f_j(\overline{y})>0$. Hence, $\overline{y}$ is reached in finite time defined as T.

That completes the proof.

A.4 Proof of Proposition 6

Given the HJB equations are the same as in (22) for player i and the same as in (24) for player j, the same first order conditions can be obtained. However, in this section, the autonomous game is defined over an infinite time horizon, and we can thus guess the same linear-quadratic strategy as in the proof of Proposition 1. We thus try a linear-quadrativ form of the value functions:

$$V_i(y)=A_i+B_iy+\frac{C_iy^2}{2}\quad\text{and}\quad V_j(y)=A_j+B_jy+\frac{C_jy^2}{2}$$

with the coefficients being undetermined. Substituting the value functions and marginal value functions into the HJB equations, rearranging terms, and comparing coefficients of powers for y, it follows that

$$\begin{cases} rA_i=\frac{a^2+b_i^2}{2}+(2a+\beta b_i)B_i+\frac{1+\beta^2}{2}B_i^2+B_iB_j,\\ (r+\delta-(1+\beta^2)C_i)B_i=(2a+\beta b_i)C_i+B_iC_j+B_jC_i,\\ (r+2\delta)C_i=(1+\beta^2)C_i^2+2C_iC_j-c_i,\end{cases}$$
$$\begin{cases} rA_j=\frac{a^2}{2}+(2a+\beta b_i)B_j+\frac{1}{2}B_j^2+(1+\beta^2)B_iB_j,\\ (r+\delta-C_j)B_j=(2a+\beta b_i)C_j+(1+\beta^2)(B_iC_j+B_jC_i),\\ (r+2\delta)C_j=C_j^2+2(1+\beta^2)C_iC_j-c_j,\end{cases}\tag{48}$$

It is easy to see that even with $c_i=c_j$, the system in (48) differs from the system in (37) where both players engage in CCS deployment.

Obviously, the 3^{rd} and the 6^{th} equations are only related to thevariables C_i, C_j and are independent of the others. They can thus be solved separately first. Once C_i and C_j are solved, they can be substituted into the 2^{nd} and the 5^{th} equations, and the variables B_i and B_j can be solved. Variables A_i and A_j follow easily from the 1^{st} and 4^{th} equations. More precisely, we pick up the 3^{rd} and the 6^{th} equations, rearrange terms and obtain:

$$\begin{cases} (1+\beta^2)C_i^2 - (r+2\delta)C_i + 2C_iC_j = c_i, \\ C_j^2 - (r+2\delta)C_j + (1+\beta^2)\,2C_iC_j = c_j. \end{cases} \tag{49}$$

With $c_i = c_j = c$, the above system (49) can be re-written as follows

$$\begin{cases} (1+\beta^2)C_i^2 - (r+2\delta)C_i + 2C_iC_j = c, \\ C_j^2 - (r+2\delta)C_j + (1+\beta^2)\,2C_iC_j = c. \end{cases}$$

By applying Descartes's rule of signs we show that the above 2^{nd} degree polynomial system has one and only one pair of negative roots, denoted as $(\widehat{C_i}, \widehat{C_j})$. Substituting these values into (48), the linear algebra system can be solved explicitly in terms of B_i and B_j. We denote the solution by $\widehat{B_i}$ and $\widehat{B_j}$. The same can be done for $\widehat{A_i}$ and $\widehat{A_j}$.

This pair is the only pair that yields affine-quadratic concave value functions for players i and j.

That completes the proof.

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