Who Benefits from Supplier Encroachment in the Presence of Manufacturing Cost Learning?

Manufacturing cost plays a crucial role in suppliers' encroachment decisions. A high manufacturing cost impedes suppliers' capacity to encroach. However, cost learning may reduce this cost sufficiently enough to make encroachment profitable for the supplier at a later point in time. Accordingly, he may have an incentive to boost production so as to promote cost learning. Thus, he may drop the wholesale price to induce the retailer to buy more. On the one hand, cost learning may enable encroachment, which may be detrimental to the retailer. On the other hand, cost learning results in a lower manufacturing cost which may translate into a lower future wholesale price, benefiting the retailer. Therefore, the retailer faces a dilemma: should she increase her order quantity to advance cost learning or not? As the retailer may order fewer units in the initial period to limit future direct channel sales, the supplier faces a challenge: should he, instead of dropping his initial wholesale price, raise it to signal his intention of not encroaching so as to induce the retailer to sell a higher quantity in the first period? We model the supplier-retailer interaction as a two-period Stackelberg game to address the retailer's dilemma and to identify the optimal supplier response. We uncover a new outcome, which arises in the presence of cost learning, where the supplier encroaches but decides not to sell anything through the direct channel. In addition, we find that supplier encroachment may reduce or eliminate the retailer's incentive to advance cost learning. This results in lower sales by the retailer, which impedes cost learning, leading to a higher future manufacturing cost (compared to the no encroachment setting). As a result, encroachment, which is typically viewed as advantageous for the supplier, may become detrimental to him. Surprisingly, the supplier continues to encroach and sell directly unless he can credibly assure the retailer that he will not encroach in the future.

Key words: supplier encroachment, cost learning, game theory

1 Introduction

Encroachment is a common practice whereby an upstream supplier establishes a direct channel in parallel to an existing indirect channel involving retailers. For example, in 2020, Apple launched a direct channel in India to accompany its sales through the indirect channel.[1] In 2021, Hyundai started selling its electric car, Ioniq 5, through a direct channel in New Zealand, in addition to the existing dealerships.[2] Similarly, LG India also sells some of its products (e.g., selected TV models)

[1] In 2020, Apple started selling directly in India through its direct online store (apple.com/in/). This direct channel is in addition to its existing resellers such as Unicorn (shop.unicornstore.in) and Imagine (imagineonline.store/).

[2] In 2021, Hyundai New Zealand started selling its Ioniq 5 model through its direct online channel (hyundai.co.nz/store/build-and-buy-ioniq-5) in addition to the existing dealerships such as Auckland Hyundai (aucklandhyundai.co.nz/) and Manukau Hyundai (manukauhyundai.co.nz).

through direct channel, in addition to selling them through indirect retail channels.[3] Such selling of the goods through the direct channel is widely acknowledged to increase the supplier's profit (Liu et al. 2021). It gives the supplier more control over the market, helps him maintain a higher service level, and serves as a "threat of replacement", which ensures smoother functioning of the indirect channel (Anderson and Coughlan 1987).

Extensive literature has been developed to study the effects of encroachment in supply chains (e.g., Arya et al. 2007, Li et al. 2014, Guan et al. 2019, Hotkar and Gilbert 2021). These studies generally focus on the impact of the direct selling cost, which is the per unit cost the supplier incurs to bypass the retailer and sell directly to consumers. Naturally, the supplier is unable to encroach when this direct selling cost is sufficiently high. However, the direct selling cost is only one element in the interplay between the upstream supplier and the downstream retailer. The role of the manufacturing cost, although critical, has been largely ignored in the encroachment literature. A higher manufacturing cost limits the sales of the good, which reduces the supplier's incentive to open a direct channel. This is evident from the growing electric vehicles (EV) sector, wherein automotive companies that have traditionally relied on a network of dealerships and more recently ventured into the EV sector, have avoided selling their EVs directly during the early years when EV sales were low (Filippo and Taylor 2022). Nevertheless, as EV prices decrease—and sales go up—the suppliers' incentive to sell directly (online) shall increase (Fischer et al. 2021). While the manufacturing cost is common to both the direct and the indirect channels, its impact on the direct channel is instrumental: it severely affects the ability of the supplier to encroach and sell directly. The supplier can sell directly only if the market price covers both the manufacturing cost and the direct selling cost. If the manufacturing cost is sufficiently high, the range of direct selling cost values that allow the supplier to profitably encroach shrinks considerably. In summary, there is a crucial interplay between the manufacturing cost and the direct selling cost, suggesting that encroachment requires a sufficiently low manufacturing cost for it to become viable.

Even though the manufacturing cost may be too high to facilitate encroachment initially, cost learning can promote it by reducing the manufacturing cost sufficiently over time. Cost learning is related to the cumulative production quantity: the manufacturing cost reduces as the firm produces larger quantities (Levitt et al. 2013, Wright 1936, Weiss et al. 2019). It is common in manufacturing industries, whereby opportunities for improvements leading to a reduction in the manufacturing cost are identified and implemented (Gray et al. 2009). For instance, LG India sells TV models based on established technologies like Smart and UHD TVs—whose costs have dropped significantly (at a rate of 17% to 27%) over the years due to cost learning (Desroches and Ganeshalingam

[3] LG India sells some of its models through its direct channel, www.lg.com/in, in addition to selling them through the retail channel.

2015)—through both its direct and retail channels. However, it generally sells TV models with the latest technologies like Signature OLED, OLED Evo, NanoCell, and QNED, which are relatively expensive and yet to realize cost learning (Park et al. 2013), only through the retail channel.[4] Returning to the EV sector, a major cost component of EVs is their lithium-ion batteries whose prices have fallen by more than 64% between 2015 and 2020 due to cost learning (BloombergNEF 2020, Ziegler and Trancik 2021). Recently (in 2020), the battery pack prices have dropped below \$100/KWh for the first time, and it is at around this price that automakers can mass market EVs. Cost learning in the EV sector, which is one of the major contributors to the reduction in the EV's cost (Ewing 2023), is not limited to batteries[5]: the overall EV cost learning rate is estimated at 30-34%, which resulted in an annual price drop of EVs of about 22% (Grieve 2023, Weiss et al. 2019). Eventually, this drop in the manufacturing cost shall increase the EV sales (Filippo and Taylor 2022), encouraging the suppliers to open direct channels (Fischer et al. 2021).

Given the interplay between cost learning and supplier encroachment, the downstream retailer faces a challenge. On the one hand, the retailer has an incentive to increase her orders to expedite the supplier's learning in expectation of a lower wholesale price in the future, induced by the supplier's manufacturing cost reduction (Li et al. 2015). On the other hand, as discussed above, such a reduction in the manufacturing cost can intensify direct selling, which can be detrimental to the retailer. Therefore, it is unclear if the retailer should support the supplier in his cost learning efforts by increasing her order quantity or if she should reduce it to impede cost learning. Anticipating the retailer's response, should the supplier decrease his wholesale price to induce the retailer to order more and enhance his cost learning, or should he increase his wholesale price and maximize his initial period profit at the expense of diminished cost learning? Moreover, given these dynamics, are the supplier and the retailer better off due to encroachment in the presence of cost learning, or can they be worse off?

To address the above questions, we consider a two-period game accounting for the above discussed supplier-retailer interactions. Our study provides some important insights. First, we find that when the manufacturing cost is sufficiently low, the supplier encroaches and profitably sells directly, to which we refer as *active encroachment.* When the manufacturing cost is intermediate, the supplier encroaches but does not sell any units through the direct channel. We refer to this region as *mute encroachment.* However, if the manufacturing cost is sufficiently high, the supplier cannot even use the direct channel as a credible threat and, therefore, he does not encroach.

[4] Refer to Section EC.3 for illustrations from the LG India's website.

[5] Lithium-ion batteries represented around 49% of the total EV cost in 2016, which has now reduced to about 28% (Carlier 2023).

Second, we uncover a new region, which arises due to cost learning when the direct selling cost is moderate. Specifically, we find that the mute encroachment region actually consists of two distinct sub-regions, which we term as *forced mute encroachment* and *voluntarily mute encroachment*. The former coincides with the "threat of encroachment" studied by Guan et al. (2019), whereby the retailer orders sufficiently enough units, reducing the retail price to such a level that selling any positive quantity through the direct channel is not profitable. Therefore, the supplier is forced to only mute encroach.[6] The latter is the new region in which the supplier decides not to sell any units through the direct channel in the second period, even though he can do so profitably. Accordingly, the supplier signals to the retailer that he will abstain from selling directly in the second period by raising his first-period wholesale price. This results in lower first-period sales and a relatively higher second-period manufacturing cost, which leads to mute encroachment in the second period. As the increase in profit due to a higher first-period wholesale price exceeds the profit forgone from direct selling coupled with slower cost learning, the supplier (voluntarily) prefers to mutely encroach rather than actively encroach. Thus, we characterize a richer equilibrium solution compared with the existing literature. Specifically, we find that as the direct selling cost increases, the equilibrium transitions from (i) the supplier encroaching and actively selling some units through the direct channel, to (ii) the supplier encroaching but voluntarily deciding not to sell any units through the direct channel, to (iii) the supplier encroaching but is prevented from selling directly due to the retailer's over-ordering in the second period, to (iv) the supplier not encroaching at all.

The third insight relates to the impact of encroachment on the retailer's initial period order quantity and cost learning. Our study reveals that encroachment leads to lower first-period sales when the direct selling cost is within some intermediate range. This reduction in sales diminishes cost learning, makes the second-period manufacturing costlier than the no encroachment case, and lowers the consumer surplus for a certain range of the direct selling cost values. Otherwise, encroachment results in higher initial period sales, enhancing cost learning and leading to a lower manufacturing cost in the second period. This can be explained as follows. Supplier encroachment may reduce or eliminate the retailer's benefit from cost learning. Hence, the retailer does not support the supplier in his cost learning efforts (through larger order quantities). The supplier, therefore, drops the first-period wholesale price to induce the retailer to order more. However, when the direct selling cost is in the intermediate range, the direct channel sales decrease, dropping the supplier's benefit from faster cost learning. Consequently, the supplier's incentive to cut his

[6] The term "threat of encroachment" used by Guan et al. (2019) highlights the use of the direct channel by the supplier to influence the retailer's sales quantity. However, we use the term mute encroachment to highlight that no units are sold through the direct channel.

wholesale price to accelerate cost learning diminishes, resulting in lower first-period sales, leading to slower cost learning and relatively higher second-period manufacturing cost.

Fourth, we show that, quite surprisingly, encroachment can be detrimental to the supplier himself. This happens when the learning rate is sufficiently high and the direct selling cost is intermediate. What is even more surprising is that the supplier still encroaches and sells directly despite its detrimental effect because he cannot credibly commit to the retailer that he will not sell directly. In the absence of credible commitment, the retailer does not help the supplier move along the learning curve. This results in lower first-period sales, which leads to reduced cost learning and, hence, a higher manufacturing cost in the second period than in the no encroachment case. As a result, encroachment can be detrimental to the supplier while beneficial to the retailer (lose-win), detrimental to both (lose-lose), beneficial to both the players (win-win), or beneficial to only the supplier (win-lose). The last two outcomes are reported by Arya et al. (2007), Hotkar and Gilbert (2021), Ha et al. (2016), while we uncover two new outcomes: lose-win and lose-lose. This result echoes the following remark by Sa Vinhas and Heide (2015): "competition is not just a standard feature of a dual-channel; it has the distinct potential to provoke potentially dysfunctional actions." In this study, this potential dysfunctional action manifests itself in terms of supply chain members' inability to take full advantage and reap the gains of cost learning. For instance, when Compaq (now acquired by HP) opened web stores, it experienced a 12% drop in commercial PC revenues as a result of retaliation from retailers, who objected to the manufacturer's online selling (Tedeschi 2000). Similarly, a survey conducted by Shopatron, a technology company that provides e-commerce solutions to manufacturers and retailers, found that more than 60% of retailers reduced their purchases from suppliers who establish a direct channel (Rueter 2011). This reduction in the retailers' sales eventually results in lower cost learning and, hence, relatively higher manufacturing cost later. The negative reaction from retailers might explain why many firms like Casablanca Fan Co. and Stephen Products Co. proactively decide against entering the direct channel and instead sell only through their retailers (Liang et al. 2023).

In summary, this paper reveals that encroachment may be detrimental to the supplier under certain conditions. More importantly, however, our key message is that manufacturing cost, cost learning, and encroachment are all intertwined. Cost learning, on the one hand, can encourage supplier encroachment and enhance direct selling. On the other hand, encroachment may hinder the supplier's cost learning, adversely impacting his future manufacturing cost. Therefore, practitioners must be wary of this while considering adding a direct channel.

The rest of the paper is organized as follows. Section 2 presents a review of the related literature. Section 3 discusses the model setting, followed by the analysis in Section 4. Section 5 assesses the impact of encroachment on the supplier's and the retailer's profits in the presence of cost learning. Section 6 overviews several extensions to our base model (which are provided in full in the appendix). Section 7 concludes the study and offers directions for future research.

2 Literature Review

This paper is related to two major streams of literature: supplier encroachment and cost learning.

2.1 Supplier Encroachment

The research on channel decisions dates back to the early work by McGuire and Staelin (1983), in which the authors investigate how product substitutability affects the supplier's channel decisions in a duopoly market. Avery et al. (2012) study the impact of adding an indirect sales channel to an existing direct online channel, whereas Deleersnyder et al. (2002) study the impact of adding a direct online channel to an existing indirect channel. The literature often refers to the latter as supplier encroachment.

The early literature on encroachment suggests that opening a direct channel by the supplier can make the retailer worse-off (Frazier and Lassar 1996). However, Arya et al. (2007) find that though encroachment is always beneficial for the supplier, it may also benefit the retailer. Arya et al. (2007) term this phenomenon as the "bright side of encroachment". Tsay and Agrawal (2004) integrate product promotion issues with supplier encroachment decisions. Ha et al. (2016) and Guo et al. (2022) consider product quality decisions along with the supplier's encroachment decision. Hotkar and Gilbert (2021) and Liu et al. (2021) study the impact of supplier encroachment in multi-supplier and multi-retailer settings, respectively. Wang and Li (2021) analyze the impact of supplier encroachment in the presence of a dual purpose retailer (who maximizes the sum of her profit as well as consumer surplus).

A number of papers study encroachment in a two period setting to analyze the dynamics of inventory carry over, where the supplier possibly establishes the direct channel only in the second period. One such example is Guan et al. (2019), who investigate the impact of strategic inventory on supplier encroachment. They find that direct selling is always beneficial for the supplier, while it is beneficial for the retailer when the direct selling cost is high. Li et al. (2021) allow the supplier to operate the direct channel via a subsidiary. They show that when the retailer can carry strategic inventory, letting the subsidiary run the direct channel is better than the supplier-run direct channel for both the supplier and the retailer. Xiong et al. (2012) and Yan et al. (2018) study the role of encroachment in the case of durable goods. Consistent with the existing literature, they find that encroachment can be both beneficial as well as detrimental for the retailer, depending on the direct selling cost.

The above studies assume that both the supplier and the retailer have complete information (e.g., about demand, quality of the product, or direct selling cost). Some recent papers study the impact of information asymmetry on the supplier's encroachment decision. Li et al. (2014) and Sun et al. (2021) study the case when the retailer has private demand information, while Guan et al. (2020) and Gao et al. (2021) analyze the setting where the supplier has private information about the product quality and the direct selling cost, respectively.

The encroachment literature generally normalizes the manufacturing cost to zero, thus abstracting away its role in the encroachment decision (e.g., Arya et al. 2007, Guan et al. 2019, Wang and Li 2021, Hotkar and Gilbert 2021, Ha et al. 2022). Sun et al. (2021) and Yoon (2016) are two exceptions. Yoon (2016) analyses the impact of the supplier's (manufacturing) cost-reducing investment on encroachment and finds that such an investment benefits both the supplier and the retailer. In his work, cost reduction stems from an investment by the supplier in improving production processes or technology improvements, whereas in our study, cost reduction is obtained from accumulated production experience, i.e., learning. Sun et al. (2021) consider information asymmetry and assume that the supplier always encroaches. They find the the retailer never benefits from cost learning. Further, it can be inferred from their model that encroachment enhances cost learning, and as it results with lower manufacturing cost, it is advantageous to the supplier. Thus, as the supplier ends up being worse off, in their model, this can only due to information asymmetries and not because of diminished cost learning. This is in stark contrast to our work, where we eliminate the informational asymmetries and endow the retailer with a lever to determine the magnitude of cost reduction due to learning. As such, we show the the retailer mostly gains from cost learning.

2.2 Cost Learning

Cost learning, whereby the production cost decreases with an increase in production quantity, plays a crucial role in enhancing a firm's competitive advantage (Gray et al. 2009). Wright (1936) is one of the early studies to observe that the labour cost reduces with an increase in the cumulative production quantity. Evidence supporting the notion of cost learning has been documented in a range of manufacturing industries like ships, airplanes, lithium ion batteries, semiconductors, and automotive (Benkard 2000, Levitt et al. 2013, Ziegler and Trancik 2021).

Previous studies suggest that experiences from past production can help build a knowledge base that can give an impetus for process and technology innovation, which increases productivity and decreases production cost (Gray et al. 2009). Many researchers argue that improvements based on previous knowledge are highly discontinuous or 'episodic' processes (e.g., Gray et al. 2009, Tyre and Orlikowski 1993). In particular, Tyre and Orlikowski (1993) make the following remark:

"whenever possible, the managers batch modifications together systematically and implement them in one intensive episode of adaptation. Very often these episodes are timed to coincide with other major changes, such as product-model changeovers, releases of new software versions or yearly factory shutdowns". In summary, although production-based learning accumulates continuously, the exploitation of the knowledge to reduce manufacturing cost generally happens in batches (Gray et al. 2009). Such batch learning is popularly captured in the literature relying on two-period models (e.g., Shum et al. 2017, Li et al. 2015, Gray et al. 2009).

3 Model Framework

We consider a two-period dyadic supply chain setting with one supplier (he) and one retailer (she). Following Guan et al. (2019), the supplier sells through the indirect channel (i.e., through the retailer) in both periods, and he may encroach and also sell through the direct channel in the second period.[7] To that extent, the supplier continuously identifies improvements through leaning-by-doing while implementing them in one intensive episode of adaptation at the end of Period 1, as in Gray et al. (2009) and Tyre and Orlikowski (1993). Although each period can be perceived as a succession of multiple smaller orders from the retailer, we adopt a two-period model as a commonly used abstraction of the real world from the cost learning literature (Gray et al. 2009, Li et al. 2015, Shum et al. 2017). Throughout our analysis, we use the following notation: the superscript j denotes the player, with $j=R$ representing the retailer and $j=S$ representing the supplier, and the subscript t denotes the period, $t\in\{1,2\}$.

We assume identical and independent linear inverse demand functions in both the periods, given by $Q_t = a - bp_t$, where a is the market size, b is the slope, p_t is the retail price, and Q_t is the total quantity sold in period t, respectively. Such a linear demand setting is common in the pertinent literature (Arya et al. 2007, Guan et al. 2019, Wang and Li 2021). Note that p_2 represents the common retail price of the product sold through the indirect and direct channels in the second period since we assume no differentiation (in quality, delivery time, etc.) between the products in the two channels. We relax this assumption in Section EC.8 (Appendix). For simplicity of exposition, following Liu et al. (2021) and Hotkar and Gilbert (2021), we normalize the market size and slope to one, i.e., $a=1$ and $b=1$. Let c_t denote the manufacturing cost in period t. Adopting the learning model of Fudenberg and Tirole (1983) and Cabral and Riordan (1997), we assume a linear batch cost learning with learning rate δ, $0\le\delta\le 1$. Accordingly, the second-period manufacturing cost

[7] The supplier's delayed opening of the direct channel could be due to the time required to accumulate market knowledge, production capabilities, market demand, or in anticipation of higher profits (Benito et al. 2005, Putzhammer et al. 2020, Dong et al. 2018). Nevertheless, we also analyze a model with an option to open a direct channel in both periods in Section EC.5 and note that the main insights remain unchanged. In our main model, we consider direct channel only in the second period to allow a consistent comparison with Arya et al. (2007) and Guan et al. (2019).

decreases linearly in the first-period manufacturing quantity, i.e., $c_2 = c_1 - \delta q_1^R$. Similar to Arya et al. (2007), we normalize the retailer's selling cost to zero, while the supplier incurs a per unit cost d, $0 \le d \le 1$, to sell the product directly. Direct selling cost may arise due to the supplier's lack of experience in direct selling, lack of contact with the consumers, or due to the additional cost of e-commerce (Arya et al. 2007, Guan et al. 2019, Ha et al. 2016). The complete notation is given in Table 1.

Table 1 Notation

Parameters	
d	Direct selling cost per unit
δ	Learning rate
c_1	Manufacturing cost per unit in period 1
Decision variables	
w_t	Wholesale price in period t, $t \in \{1,2\}$
q_t^R	Indirect retail channel sales in period t, $t \in \{1,2\}$
q_2^S	Supplier's direct channel sales in period 2
Dependent variables	
c_2	Manufacturing cost per unit in period 2
p_t	Price of the product in period t, $t \in \{1,2\}$
π_t^j	Profit of player j in period t, $j \in \{R,S\}$ and $t \in \{1,2\}$
Π^j	Total Profit of player j, i.e., $\Pi^j = \pi_1^j + \pi_2^j$
cs_t	Consumers surplus in period t, $t \in \{1,2\}$
CS	Total consumer surplus, i.e., $CS = cs_1 + cs_2$

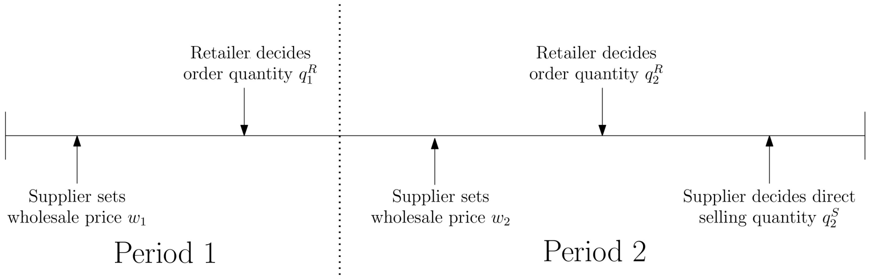
The sequence of decisions, which is illustrated in Figure 1, is as follows. At the beginning of the first period, for a given c_1, the supplier sets the wholesale price, w_1, and then the retailer orders a quantity q_1^R. In the second period, upon observing the new manufacturing cost c_2, first, the supplier decides whether or not to encroach (i.e., open a direct channel). Accordingly, he sets the wholesale price, w_2. Consistent with Guan et al. (2019) and Hotkar and Gilbert (2021), we do not consider the supplier's encroachment decision explicitly in modeling his sequence of decisions since it can be immediately inferred from his second-period wholesale price decision, w_2, depending on whether it is a function of his direct selling cost or not. Upon observing the supplier's wholesale price, the retailer orders q_2^R units, and finally, the supplier determines the direct channel quantity q_2^S (if he encroaches into the retail market). We assume that the supplier determines his direct channel sales quantity after the retailer places her order because the supplier cannot credibly commit that he will not revise his direct channel sales after receiving the order from the retailer (Arya et al. 2007, Li et al. 2014, Guan et al. 2019, Liu et al. 2021).

We impose the following mild assumption on the range of learning rate values.

Assumption 1. $\delta < \min\left\{ \frac{-(1-c_1)+\sqrt{1+2c_1(3-4d)+c_1{}^2}}{1-d}, \frac{-(1-c_1)+\sqrt{1+2c_1(8d-1)+17c_1{}^2}}{2(d+c_1)}, \frac{-4(1-c_1)+\sqrt{1+4c_1+3c_1{}^2}}{3+c_1}, 1 \right\}.$

This assumption is necessary to ensure that the manufacturing cost is strictly positive in the second period. For higher values of the initial manufacturing cost, this assumption is redundant.[8] An analogous assumption is made by Li et al. (2015).

Figure 1 Timeline



4 Model Analysis

In Section 4.1, we consider the no encroachmnet benchmark setting and in Section 4.2, we consider the case where the supplier may decide to open a direct channel in the second period. We solve these two settings by backward induction to arrive at the unique subgame-perfect Nash equilibrium (SPNE) for each setting. All proofs are provided in the Appendix.

4.1 The Benchmark Setting (No Encroachment)

In the no encroachment setting, the profit functions of the supplier and the retailer in period t are given as $\pi_t^S = q_t^R(w_t - c_t)$ and $\pi_t^R = q_t^R(p_t - w_t)$, respectively. After solving the game, we obtain the following unique SPNE: $w_1 = \frac{128(1+c_1)-16(1-c_1)\delta-8(3+c_1)\delta^2-(1-c_1)\delta^3}{32\left(8-\delta^2\right)}$, $w_2 = \frac{1+c_1-\delta q_1^R}{2}$, $q_1^R = \frac{(1-c_1)\delta+8(1-w_1)}{16-\delta^2}$, and $q_2^R = \frac{1-w_1}{2}$.

Lemma 1. *In the benchmark setting (no encroachment model),* $\frac{\partial w_1}{\partial \delta} < 0$, $\frac{\partial w_2}{\partial \delta} < 0$, $\frac{\partial q_1^R}{\partial \delta} > 0$, $\frac{\partial q_2^R}{\partial \delta} > 0$, $\frac{\partial \Pi^S}{\partial \delta} > 0$, $\frac{\partial \Pi^R}{\partial \delta} > 0$, *and* $\frac{\partial CS}{\partial \delta} > 0$.

To take advantage of a higher learning rate, the supplier sets a lower wholesale price in the first period to induce the retailer to order more. Since faster cost learning can benefit the retailer later due to a lower wholesale price, she orders a higher quantity in the first period to accelerate the supplier's cost learning at the expense of her lower profit in the first period. This increase in the cost learning leads to a lower manufacturing cost and, consequently, to a lower wholesale price in

[8] As we will see later, in Figure 4, for a reasonable manufacturing cost (e.g., $c_1 = 0.4$), our model is valid for all direct selling cost values ($0 < d < 1$) even when the learning rate is high (e.g., $\delta = 0.7$).

the second period. This ultimately results in higher second-period sales and higher second period profits for both the supplier and the retailer. Moreover, the increase in first and second-period sales also increases the consumer surplus. In the rest of the paper, we use a superscript B to indicate the equilibrium decisions in the benchmark setting.

4.2 Potential Supplier Encroachment

We now consider the case where the supplier may decide to encroach, i.e., he may open a direct channel in the second period. In that case, the supplier's first and second-period profit functions are given as $\pi_1^S = q_1^R(w_1 - c_1)$ and $\pi_2^S = q_2^R(w_2 - c_2) + q_2^S(p_2 - d - c_2)$, respectively, and the retailer's profit function in period t is given as $\pi_t^R = q_t^R(p_t - w_t)$. We use the superscript E to indicate encroachment. The following lemma characterizes the range of parameter values in which the supplier can encroach and sell directly.

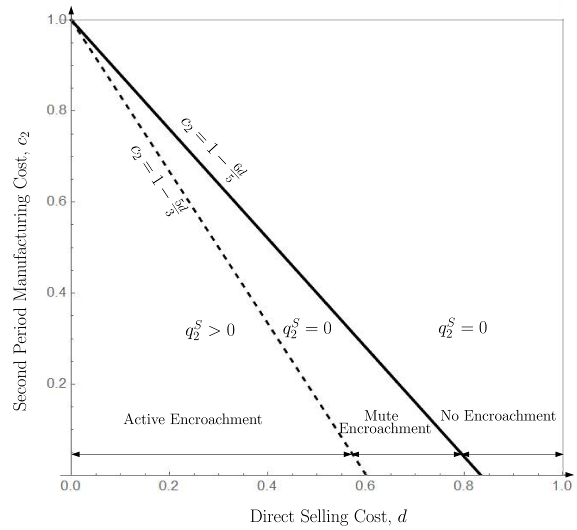
Lemma 2. *In the second period, the supplier can encroach only if $c_2 \leq 1 - \frac{6d}{5}$. Furthermore, he actively sells through the direct channel (i.e., $q_2^S > 0$) if $c_2 < 1 - \frac{5d}{3}$; otherwise, $q_2^S = 0$.*

Lemma 2 highlights the significance of the manufacturing cost to the supplier's encroachment decision. As illustrated in Figure 2, the supplier can encroach (i.e., open a direct channel) and sell directly only if the manufacturing cost is sufficiently low ($c_2 < 1 - \frac{5d}{3}$). We refer to this outcome as active encroachment. When the manufacturing cost is high, the sum of the manufacturing and direct selling costs may exceed the retail price, making direct selling unprofitable. Therefore, the supplier sells directly only when the manufacturing cost is sufficiently low ($c_2 < 1 - \frac{5d}{3}$). Once the manufacturing cost increases beyond this point, the supplier uses the direct channel merely as a threat to induce the retailer to order more. We refer to this outcome as mute encroachment. As the manufacturing cost increases further (hence, the total cost of direct selling increases), the influence of the threat of encroachment diminishes, and the retailer reduces her sales quantity, rendering encroachment unprofitable for the supplier. Hence, the threat of encroachment becomes non-credible, and the retailer simply ignores the direct channel while placing her order.

Our model generalizes that of Arya et al. (2007) by incorporating manufacturing cost and cost learning. Setting $c_2 = 0$ reduces the condition for direct selling to $d < \frac{3}{5}$, which is the same condition as stated by Arya et al. (2007). The condition for encroachment, $c_2 \leq 1 - \frac{6d}{5}$, reduces to $d \leq \frac{5}{6}$, which is the same condition revealed by Guan et al. (2019).

In the presence of cost learning, the later period manufacturing cost decreases in the early period sales. Substituting $c_2 = c_1 - \delta q_1^R$ in the above thresholds reveals that the supplier can encroach only if $q_1^R \geq \frac{6d+5c_1-5}{5\delta}$ and sell directly only if $q_1^R > \frac{5d+3c_1-3}{3\delta}$. To determine the retailer's optimal response and the supplier's consequent reaction, we solve the model and provide the supplier's and the retailer's equilibrium decisions in the following proposition.

Figure 2 The supplier's encroachment decision as a function of the second-period manufacturing cost, c_2, and the direct selling cost, d



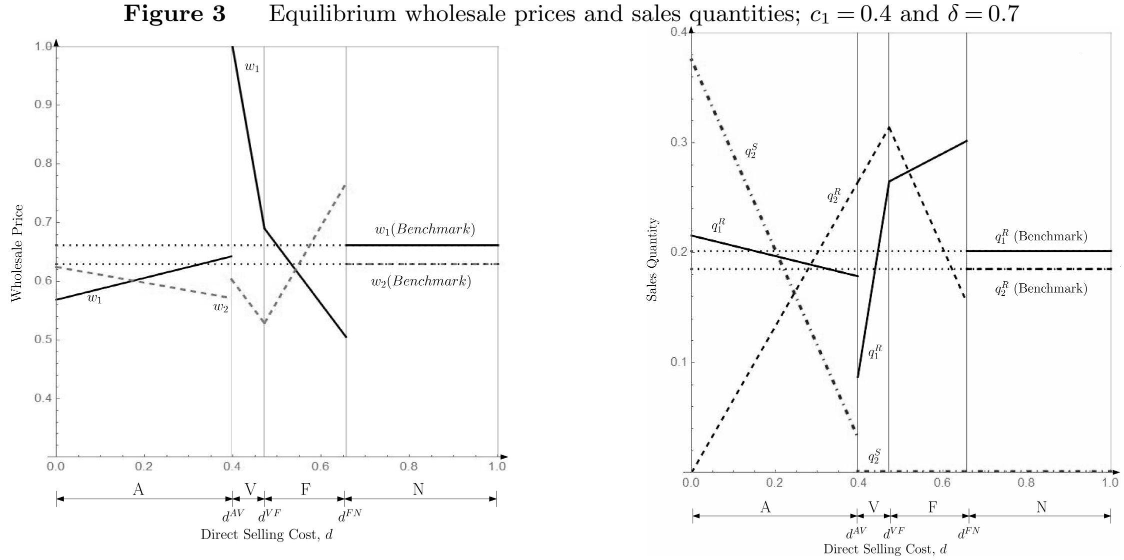
Proposition 1. *In case of potential supplier encroachment, the unique SPNE is given in Table 2, where the thresholds $d^{AV}, d^{VF}, d^{FF'}$, and d^{FN} are as provided in the Appendix.*

Figure 3 illustrates the equilibrium wholesale prices w_1 and w_2 (Panel a) and sales quantities q_1^R, q_2^R, and q_2^S (Panel b) from Table 2 as functions of the direct selling cost. As evident from Table 2 and Figure 3, depending on the direct selling cost, d, the equilibrium decisions can be divided into four regions. When $d \in [0, d^{AV}]$, the supplier encroaches and sells strictly positive quantity through the direct channel, i.e., $q_2^S > 0$. We refer to this region as Active encroachment region, or Region A. In this region, as is evident from Figure 3, the supplier keeps his wholesale prices below the benchmark (no encroachment) in both periods.

When $d \in (d^{AV}, d^{VF}]$, the supplier can sell directly in the second period with a positive margin, but he voluntarily decides not to do so (i.e., he chooses $q_2^S = 0$). Essentially, in this region, the supplier forgoes his second-period profit from selling directly, as he makes a much higher profit by raising his first-period wholesale price substantially above the benchmark wholesale price (which signals his intention to only mutely encroach). We refer to this region as Voluntarily mute encroachment or Region V. This region, which is not explored in the encroachment literature, arises in the presence of cost learning. In the absence of cost learning, i.e., when $\delta = 0$, we have $d^{AV} = d^{VF} = \frac{3(1-c_1)}{5}$ and Region V disappears (refer to the thresholds expressions in the Appendix).

When $d \in [d^{VF}, d^{FN})$, the supplier does not have the option to profitably sell directly in the second period as the retailer orders a quantity in the second period that is sufficient to bring down the retail price, p_2, to the same level as the supplier's total direct selling cost, i.e., $p_2 = 1 - q_2^R = c_2 + d$.

Figure 3 Equilibrium wholesale prices and sales quantities; $c_1 = 0.4$ and $\delta = 0.7$



a. The wholesale prices as a function of the direct selling cost, d

b. Sales quantities as a function of the direct selling cost, d

Table 2 Equilibrium solution in the presence of potential supplier encroachment

	Region A: Active encroachment $d \in [0,\, d^{AV}]$	Region V: Voluntarily mute encroachment $d \in (d^{AV},\, d^{VF}]$
w_1^i	$\frac{4+4c_1-2(1-c_1-d)\delta-\delta^2}{8-\delta^2}$	$\frac{6-6c_1-10d+3\delta+2d\delta^2}{3\delta}$
$q_1^{R,i}$	$\frac{1-w_1^i}{2}$	$\frac{5d+3c_1-3}{3\delta}$
p_1^i	$\frac{6+2c_1-(1-c_1-d)\delta-\delta^2}{8-\delta^2}$	$\frac{3\delta-5d-3c_1+3}{3\delta}$
w_2^i	$\frac{3+3c_2-d}{6}$	$\frac{3+3c_2-d}{6}$
$q_2^{R,i}$	$\frac{2d}{3}$	$\frac{2d}{3}$
$q_2^{S,i}$	$\frac{3-3c_1-5d+3\delta q_1^{R,i}}{6}$	0
p_2^i	$\frac{12(1+c_1)+4d-3(1-c_1)\delta-(1-d)\delta^2}{3\left(8-\delta^2\right)}$	$\frac{3-2d}{3}$

	Region F: Forced mute encroachment $d \in [d^{VF},\, \min\{d^{FF'},\, d^{FN}\})$	Region F: Forced mute encroachment $d \in [\min\{d^{FF'},\, d^{FN}\},\, d^{FN})$	Region N: No encroachment $d \in [d^{FN},\, 1]$
w_1^i	$\frac{2(1+c_1)+(4-6d-4c_1)\delta-c_1\delta^2+(2d+c_1-1)\delta^3}{4-\delta^2}$	$\frac{40-40c_1-48d+20\delta+3d\delta^2}{20\delta}$	$\frac{128(1+c_1)-16(1-c_1)\delta-8(3+c_1)\delta^2-(1-c_1)\delta^3}{32\left(8-\delta^2\right)}$
$q_1^{R,i}$	$\frac{\delta(1-c_1-d)-w_1^i}{2-\delta^2}$	$\frac{\delta(1-c_1-d)-w_1^i}{2-\delta^2}$	$\frac{(1-c_1)\delta+8(1-w_1^i)}{16-\delta^2}$
p_1^i	$\frac{3+c_1-d\delta-\delta^2}{4-\delta^2}$	$\frac{20(1+\delta-c_1)(2-\delta^2)-d\left(48-23\delta^2\right)}{20\delta(2-\delta^2)}$	$\frac{24+8c_1-3(1-c_1)\delta-4\delta^2}{32-4\delta^2}$
w_2^i	$\frac{3(c_2+d)-1}{2}$	$\frac{3(c_2+d)-1}{2}$	$\frac{1+c_2}{2}$
$q_2^{R,i}$	$1-(c_2+d)$	$1-(c_2+d)$	$\frac{1-w_2^i}{2}$
$q_2^{S,i}$	0	0	0
p_2^i	$\frac{4(c_1+d)-(1-c_1)\delta-(c_1+2d)\delta^2}{4-\delta^2}$	$\frac{40-8d-(20-3d)\delta^2}{20(2-\delta^2)}$	$\frac{32(3+c_1)-8(1-c_1)\delta-(15+c_1)\delta^2}{16\left(8-\delta^2\right)}$

Note: $i \in \{A, V, F, N\}$, where $A =$Active encroachment, $V =$ Voluntarily mute encroachment, $F =$ Forced mute encroachment, $N =$No encroachment. To keep the expressions compact, we denote some of the variables as functions of other variables, for example, $q_1^{R,i}$ is expressed as a function of w_1^i.

At this retail price, the supplier's margin from the direct channel sales drops to zero, leaving no incentive for him to sell anything through the direct channel, i.e., $q_2^S = 0$. As the direct selling cost,

d, increases within this region, the retailer reduces her sales quantity just enough to raise the retail price, p_2, and keep the supplier's margin from direct sales at zero. The supplier, therefore, extracts whatever profit margin he can earn from the indirect channel by charging a higher second-period wholesale price. Since in this region of parameter values, the retailer forces the supplier not to sell anything through the direct channel, we refer to this region as Forced mute encroachment or Region F. The existing literature (Guan et al. 2019, Liu et al. 2021, Wang and Li 2021) commonly refer to Region F as the threat of encroachment (see Footnote 6). Towards the higher end of the forced mute encroachment region, i.e., when $d \in [\min\{d^{FF'}, d^{FN}\}, d^{FN})$, the supplier must reduce c_2 sufficiently to ascertain that he can leverage the forced mute encroachment as a chip to charge a higher wholesale price in the second period. Consequently, the supplier drops the first-period wholesale price to ensure that the second-period manufacturing cost decreases sufficiently. We realize that the region $d \in [\min\{d^{FF'}, d^{FN}\}, d^{FN})$ is very narrow.

The crucial difference between Regions V and F is that in the former, the supplier has an option to choose between direct selling (i.e., active encroachment) and mute encroachment, and he chooses mute over active encroachment as it maximizes his overall two-period profit. By contrast, in Region F, the supplier cannot profitably sell directly, hence he does not have the freedom to choose between active and mute encroachment as only the latter is feasible.

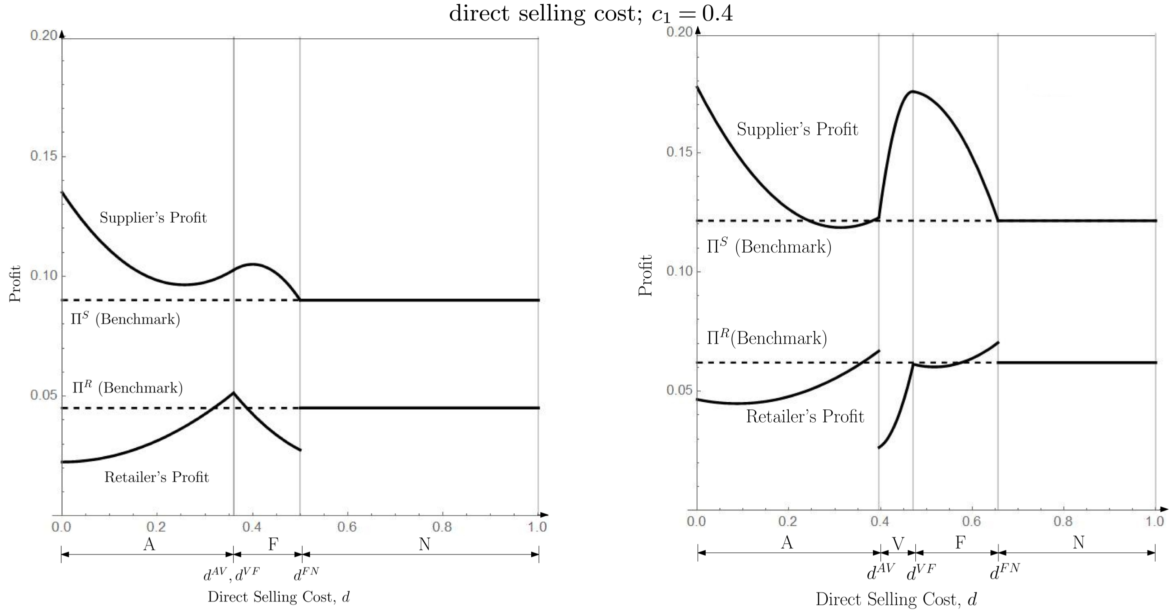
When $d \in [d^{FN}, 1]$, the supplier has no incentive to encroach and the retailer behaves as if the direct channel does not exist. In this case, the equilibrium solution is exactly the same as in the benchmark setting of no encroachment (Section 4.1).

Based on the above discussion, the notation E for encroachment introduced at the beginning of this subsection can denote A, V, or F, depending on the direct selling cost value. In Figure 4, we illustrate the supplier's and the retailer's total profits in the two periods as a function of the direct selling cost when the supplier may open a direct channel, and contrast them with their respective profits in the benchmark setting (no encroachment). Observe that in the absence of cost learning (Panel a), as d increases, the equilibrium transitions from active to forced mute and, ultimately, to no encroachment. However, in the presence of positive cost learning (Panel b), the new region emerges as the equilibrium transitions, as d increases, from active to voluntarily mute, then to forced mute and, finally, to no encroachment.

It is evident from Proposition 1 that the supplier encroaches only when the direct selling cost is less than the threshold d^{FN}, and sells directly only when the direct selling cost is less than the threshold d^{AV}. The following corollary highlights the impact of cost learning on these thresholds.

COROLLARY 1. $\frac{\partial d^{AV}}{\partial \delta} > 0$ *and* $\frac{\partial d^{FN}}{\partial \delta} > 0$.

Figure 4 The players' profits with (solid lines) and without (dashed lines) encroachment as a function of the direct selling cost; $c_1 = 0.4$



a. $\delta = 0$ **b.** $\delta = 0.7$

This corollary highlights that a higher learning rate can enhance the supplier's capability to encroach and sell directly. Specifically, a higher learning rate increases the range of direct selling cost values for which the supplier can encroach and sell directly, as is evident from a comparison between Figure 4(a) and Figure 4(b). Recall from Lemma 2 that the supplier can encroach only if $d \leq \frac{5(1-c_2)}{6}$ and can sell directly only if $d < \frac{3(1-c_2)}{5}$. The presence of cost learning can help reduce the manufacturing cost c_2 so that the supplier can encroach and sell directly even for higher direct selling costs.

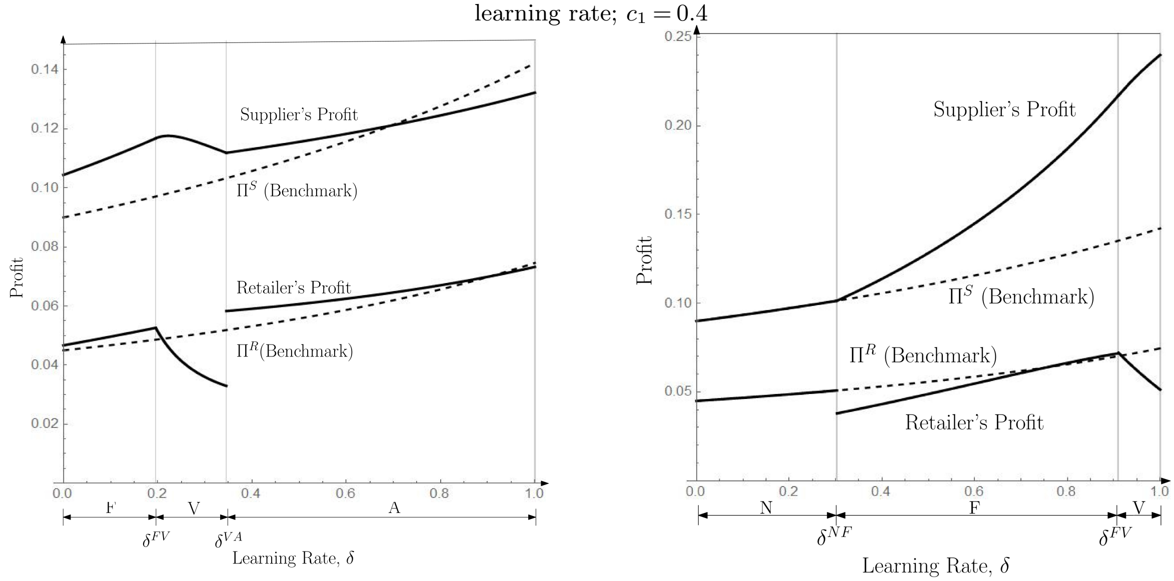
In Figure 5, we present the supplier's and the retailer's total profits as a function of the learning rate for $d = 0.38$ and $d = 0.55$.[9] When $d = 0.38$, with an increase in the learning rate, the equilibrium solution shifts from forced mute encroachment to voluntary mute encroachment to active encroachment. However, when $d = 0.55$, the equilibrium solution shifts from no encroachment to forced mute encroachment to voluntarily mute encroachment region.

4.2.1 Active Encroachment

When $d \in [0, d^{AV}]$, the supplier opens the direct channel in the second period and sells a strictly positive quantity through this channel. In the next lemma, we discuss the impact of cost learning on the players' profits and consumer surplus.

[9] The specific values of $d = 0.38$ and $d = 0.55$ are selected to showcase certain transitions in the equilibrium solution. Figure 6 later demonstrates the full range of transitions between the regions, and the two values illustrated here can be perceived as vertical slices of Figure 6. The thresholds δ^{FV}, δ^{VA}, and δ^{NF} can be found by solving $d^{VF} = d$, $d^{AV} = d$, and $d^{FN} = d$, respectively.

Figure 5 The players' profits with (solid lines) and without (dashed lines) encroachment as a function of the learning rate; $c_1 = 0.4$



a. $d = 0.38$ **b.** $d = 0.55$

LEMMA 3. *In the active encroachment region, i.e., when $d \in [0, d^{AV}]$,*
1. $\frac{\partial \pi_1^{R,A}}{\partial \delta} > 0$, $\frac{\partial \pi_2^{R,A}}{\partial \delta} = 0$, *and* $\frac{\partial \Pi^{R,A}}{\partial \delta} > 0$.
2. $\frac{\partial \pi_1^{S,A}}{\partial \delta} < 0$, $\frac{\partial \pi_2^{S,A}}{\partial \delta} > 0$, *and* $\frac{\partial \Pi^{S,A}}{\partial \delta} > 0$.
3. $\frac{\partial cs_1^{A}}{\partial \delta} > 0$, $\frac{\partial cs_2^{A}}{\partial \delta} > 0$, *and* $\frac{\partial CS^{A}}{\partial \delta} > 0$.

A key observation from this lemma is that in the active encroachment region, the retailer does not benefit from the supplier's cost learning in the second period ($\frac{\partial \pi_2^{R,A}}{\partial \delta} = 0$), even though some cost savings are passed on to her in terms of a lower second-period wholesale price ($\frac{\partial w_1}{\partial c_2} > 0$). The intuition is as follows. A lower second-period manufacturing cost (due to cost learning) stimulates second-period sales. However, our analysis reveals that all the additional sales due to a lower c_2 come from the direct channel (as $\frac{\partial q_2^{S,A}}{\partial c_2} < 0$ and $\frac{\partial q_2^{R,A}}{\partial c_2} = 0$), while the sales through the indirect channel remain constant ($q_2^{R,A} = \frac{2d}{3}$). As larger quantities are brought to the market, the second-period retail price decreases, which in turn, negates the retailer's benefit of a lower wholesale price, resulting in a profit margin that is identical to the case of no cost learning ($p_2^A - w_2^A = \frac{d}{3}$). Thus, the retailer's profit in the second period, $\pi_2^{R,A}(q_1^{R,A}) = \frac{2d^2}{9}$, is the same as in the case of no cost learning and is actually independent of the first-period order quantity. This result echoes the observation made by Gray et al. (2009), although in a different context, where an OEM does not benefit from the cost savings achieved by a contract manufacturer in the second period.

To capitalize on a higher learning rate, the supplier reduces the first-period wholesale price in δ to induce the retailer to order more, which increases his cost learning. Thus, a higher learning rate has two effects on the supplier: (i) the first-period wholesale price drops, which decreases the supplier's first-period profit ($\frac{\partial \pi_1^{S,A}}{\partial \delta} < 0$), while increasing the retailer's profit and the consumer surplus in this period ($\frac{\partial \pi_1^{R,A}}{\partial \delta} > 0$ and $\frac{\partial cs_1^{A}}{\partial \delta} > 0$), and (ii) the second-period direct channel sales increases on account of a faster cost learning, raising the supplier's second-period profit and the consumer surplus ($\frac{\partial \pi_2^{S,A}}{\partial \delta} > 0$ and $\frac{\partial cs_2^{A}}{\partial \delta} > 0$). Overall, in the active encroachment region, the supplier, the retailer, and the consumers, all benefit from an increase in the learning rate, i.e., $\frac{\partial \Pi^{S,A}}{\partial \delta} > 0$, $\frac{\partial \Pi^{R,A}}{\partial \delta} > 0$, and $\frac{\partial CS^{A}}{\partial \delta} > 0$.

Proposition 2. *In the active encroachment region, i.e., when $d \in [0, d^{AV}]$,*
a. if $d \le \frac{1-c_1}{4}$, then $q_1^{R,A} \ge q_1^{R,B}$ (hence, $c_2^A \le c_2^B$); otherwise $q_1^{R,A} < q_1^{R,B}$ (hence, $c_2^A > c_2^B$).
b. $\frac{\partial |c_2^A - c_2^B|}{\partial \delta} > 0$.

Proposition 2 reveals that in the active encroachment region, when the direct selling cost is low ($d \le \frac{1-c_1}{4}$), the retailer orders a higher quantity in the first period, enhancing cost learning. However, when the direct selling cost is high, the retailer reduces her first-period sales, which impedes the supplier's cost learning and makes manufacturing costlier in the second period.

In the benchmark setting, the supplier's and the retailer's incentives are aligned with respect to cost learning, i.e., both benefit from an increase in cost learning (Lemma 1). As explained before in the benchmark setting, the retailer benefits from the supplier's cost learning in the second period. Hence, the retailer orders a higher quantity in the first period, helping the supplier progress along the learning curve, which reduces her first-period profit. This drop in the retailer's first-period profit is more than compensated by the increase in her second-period profit due to cost learning. However, in the active encroachment region, a higher cost learning bears no impact on the retailer's profit, i.e., $\frac{\partial \pi_2^{R,A}}{\partial \delta} = 0$ (Lemma 3). Accordingly, she has no incentive to increase her order quantity in the first period to support the supplier's cost learning efforts. As a result, she orders as if cost learning does not exist, leading to a lower first-period order quantity for the same wholesale price, i.e., $q_1^{R,A}(w_1) \le q_1^{R,B}(w_1)$.

Anticipating a drop in the retailer's sales, the supplier counteracts by setting a lower wholesale price than the benchmark setting, $w_1^A < w_1^B$, to induce the retailer to order more and foster his cost learning. When the direct selling cost is sufficiently low, $d \le \frac{1-c_1}{4}$, the latter effect dominates, hence the retailer orders a higher quantity in the first period, i.e., $q_1^{R,A} \ge q_1^{R,B}$, and making the second-period manufacturing more economical, $c_2^A \le c_2^B$. Otherwise, the former effect dominates and the retailer sells a lower quantity in the first period, $q_1^{R,A} < q_1^{R,B}$, rendering second-period manufacturing costlier than the benchmark setting, $c_2^A > c_2^B$. Further, we find that depending on the first period manufacturing cost, the direct selling cost, and the learning rate, active encroachment can result in a drop of as much as 15.79% or an increase of up to 9.09% in the first period sales quantity, i.e., $\frac{q_1^{R,A}}{q_1^{R,B}} \in [0.8421, 1.0909]$.[10] This relative increase in the second-period manufacturing cost eventually affects the supplier's second-period profit negatively.

Note that $|c_2^A - c_2^B|$ is increasing in the learning rate, i.e., $\frac{\partial |c_2^A - c_2^B|}{\partial \delta} > 0$. Consequently, the higher the learning rate, the higher the deviation of the second-period manufacturing cost from the benchmark setting (no encroachment model). If the direct selling cost is low, then a higher learning rate amplifies the negative deviation of the manufacturing cost from the benchmark setting. Whereas, if the direct selling cost is high, a higher learning rate amplifies the positive deviation of the manufacturing cost from the benchmark setting. We further discuss the consequences of this result in Section 5.

4.2.2 Mute Encroachment

Lemma 2 established that the supplier sells directly only if the product's manufacturing cost is sufficiently low; otherwise, if the manufacturing cost is in an intermediate range, the supplier mutely encroaches. Recall (from the Introduction) that for its more novel and costlier models, LG India redirects buyers to a physical retailer while offering the cheaper models both directly and indirectly. This is how LG India mutely encroaches.

According to Proposition 1, the supplier mutely encroaches into the retail market when the direct selling cost is such that $d \in (d^{AV}, d^{FN})$. This range of d values contains two regions: in the first region, the supplier voluntarily mutely encroaches, whereas, in the second region, he is forced to mutely encroach. Below, we discuss these two regions.

As seen in the discussion following Lemma 2, in the active encroachment region, i.e., when $d \in [0, d^{AV}]$, the supplier lowers his first-period wholesale price (as compared to the benchmark setting) to enhance his cost learning and promote direct selling. This drop in the wholesale price decreases his first-period profit, but the gain from direct selling in the second period exceeds this loss. However, when the direct selling cost increases beyond d^{AV}, i.e., when $d \in (d^{AV}, d^{VF}]$, the supplier cannot completely recoup the profit forgone in the first period. Hence, he voluntarily decides to mutely encroach in the second period. He signals his decision to mutely encroach by setting a sufficiently high first-period wholesale price (as evident from Figure 3). This increase in the first-period wholesale price increases his first-period profit, which more than compensates for the profit forgone from direct selling. Anticipating mute encroachment in the second period, the retailer reacts by increasing her first-period sales quantity $(q_1^{R,V}(w_1) > q_1^{R,A}(w_1))$ to stimulate cost learning and induce the supplier to lower the second-period wholesale price. However, the retailer

[10] Refer to Section EC.2 for analytical expressions for the impact of potential supplier encroachment on the retailer's order quantity and the supplier's profit.

limits her sales to a level $q_1^R = \frac{5d+3c_1-3}{3\delta}$, leading to $c_2 = 1 - \frac{5d}{3}$. Any further increase in the first-period sales quantity does not increase the retailer's second-period profit, as it may result in active encroachment, in which case she will not benefit any more from the supplier's cost learning in the second period (see Lemmas 2 and 3), i.e., $\frac{\partial \pi_2^{R,V}}{\partial q_1^{R,V}} \geq 0$ if $q_1^R \leq \frac{5d+3c_1-3}{3\delta}$. In the next proposition, we shift our attention to the forced mute encroachment region and analyze the impact of the increase in the direct selling cost on the retailer's profit.

PROPOSITION 3. *In the forced mute encroachment region, i.e., when* $d \in [d^{VF}, \min\{d^{FF'}, d^{FN}\})$, *if* $d \leq d^{RF} = \frac{(1-c_1)\left(16-2\delta-8\delta^2+\delta^3+\delta^4\right)}{16-6\delta^2}$, *then* $\frac{\partial \Pi^{R,F}}{\partial d} \leq 0$; *otherwise* $\frac{\partial \Pi^{R,F}}{\partial d} > 0$. *Further,* $\frac{\partial d^{RF}}{\partial \delta} < 0$.

In the forced mute encroachment region, the retailer's profit is convex in the direct selling cost, reaching a minimum when $d = d^{RF}$ (see Figure 4). Furthermore, d^{RF} decreases in the learning rate. This implies that if the learning rate is high, an increase in the direct selling cost is advantageous to the retailer. This result complements the previous literature, which says that in the absence of cost learning, $\delta = 0$, a higher direct selling cost always harms the retailer in the forced mute encroachment region (Guan et al. 2019, Wang and Li 2021, Liu et al. 2021). We show that in the presence of cost learning, however, the retailer might benefit from a higher direct selling cost, i.e., $\frac{\partial \Pi^{R,F}}{\partial d} > 0$ when $d \in [d^{VF}, d^{RF}]$.

As evident from Figure 3(a), in the forced mute encroachment region, the supplier raises his second-period wholesale price in the direct selling cost ($\frac{\partial w_2^F}{\partial d} > 0$), and it may even be greater than the benchmark wholesale price, w_2^B. That is, in the forced mute encroachment region, the supplier may exploit the direct channel by charging a higher wholesale price in the second period. At the same time, the retailer orders a quantity $q_2^{R,F} = 1 - (c_2^F + d)$, leading to a retail price equal to the total cost of direct selling, i.e., $p_2^F = c_2^F + d$. This means that as the total direct selling cost $(c_2 + d)$ increases, it becomes possible for the retailer to prevent the supplier from direct selling even at a lower ordering quantity, $q_2^{R,F}$. In the presence of cost learning, the supplier can limit this reduction in sales by dropping the first-period wholesale price ($\frac{\partial w_1^F}{\partial d} < 0$) aimed at decreasing the second-period manufacturing cost, c_2^F. Further, a higher learning rate amplifies the supplier's incentive to reduce w_1^F. Thus, an increase in the direct selling cost has two effects on the retailer's profit. On the one hand, it hurts the retailer due to an increase in w_2^F. On the other hand, it benefits the retailer due to a decrease in w_1^F. When the direct selling cost is high, the latter effect dominates, and the retailer's profit increases in the direct selling cost. Otherwise, the former effect dominates, and the retailer's profit decreases in the direct selling cost. The next lemma reveals the impact of cost learning on the players' profits and consumer surplus in the voluntarily and forced mute encroachment regions.

LEMMA 4. *Consider the mute encroachment region.*

1. *When* $d \in (d^{AV}, d^{VF}]$*, i.e., in the voluntarily mute encroachment region, if* $\delta < \delta^M = \frac{4(5d+3c_1-3)}{3(1-c_1)}$*, then* $\frac{\partial \Pi^{S,V}}{\partial \delta} > 0$*; otherwise,* $\frac{\partial \Pi^{S,V}}{\partial \delta} \leq 0$*. Further,* $\frac{\partial \Pi^{R,V}}{\partial \delta} < 0$ *and* $\frac{\partial CS^V}{\partial \delta} < 0$*.*
2. *When* $d \in [d^{VF}, d^{FN})$*, i.e., in the forced mute encroachment region,* $\frac{\partial \Pi^{S,F}}{\partial \delta} > 0$*,* $\frac{\partial \Pi^{R,F}}{\partial \delta} > 0$*, and* $\frac{\partial CS^F}{\partial \delta} > 0$*.*

According to Lemma 4, in the voluntarily mute encroachment region, an increase in the learning rate benefits the supplier until it reaches a certain threshold, δ^M. Quite surprisingly, above this threshold, an increase in the learning rate hurts the supplier. Further, in the entire voluntary mute encroachment region, an increase in the learning rate is always detrimental to the retailer and the consumers.

The intuition behind is simple. In the voluntary mute region, the supplier increases his first-period wholesale price, w_1, as his learning rate increases, i.e., $\frac{\partial w_1^V}{\partial \delta} > 0$, to credibly signal his intention of mute encroachment in the second period. This increase in w_1 benefits the supplier but hurts the retailer. At the same time, with an increase in the learning rate, δ, the retailer drops her first period sales, i.e., $\frac{\partial q_1^{R,V}}{\partial \delta} = \frac{\partial \frac{5d+3c_1-3}{3\delta}}{\partial \delta} < 0$, which results in a higher first-period retail price, hurting the supplier and the consumers. We find that when learning is sufficiently low, $\delta < \delta^M$, the former effect dominates, and an increase in the learning rate benefits the supplier. Otherwise, the latter effect dominates, and the supplier becomes worse off due to an increase in the learning rate. However, the consumers and the retailer always become worse off due to an increase in the learning rate due to lower first-period sales and a higher wholesale price.

As the retailer reduces her first-period sales in δ, the impact of an increase in the learning rate gets nullified and the total cost learning of the supplier and the second-period manufacturing cost remains unchanged (i.e., independent of learning rate), $c_2^V = c_1 - \delta q_1^{R,V} = c_1 - \delta\left(\frac{5d+3c_1-3}{3\delta}\right) = 1 - \frac{5d}{3}$. Hence, an increase in the learning rate has no impact on the second-period wholesale price, w_2^V, the ordering quantity, $q_2^{R,V}$, consumer surplus, the supplier's second-period profit, and the retailer's second-period profit.

The second part of the Lemma 4 states that in the forced mute encroachment region, an increase in the learning rate always benefits the consumers, the supplier, and the retailer. In this region, the retailer benefits from the supplier's cost learning in the second period. Hence, she increases her first-period sales quantity, compared to the no-learning case, to accelerate cost learning. This increase in cost learning results in a decrease in the second-period manufacturing cost and, consequently, a decrease in second-period wholesale prices, increasing the second-period sales, the supplier's profit, the retailer's profits, and consumer surplus.

In Proposition 2, we discussed the impact of supplier encroachment on the manufacturing cost vis-a-vis the benchmark setting. The next proposition extends that result to the mute encroachment region.

PROPOSITION 4. *In the mute encroachment region, i.e., when* $d \in [d^{AV}, d^{FN}]$,

a. if $d < d^M = \frac{(1-c_1)(96+24\delta-3\delta^2)}{160-20\delta^2}$, *then* $q_1^{R,E} < q_1^{R,B}$ *(hence,* $c_2^E > c_2^B$*); otherwise* $q_1^{R,E} \geq q_1^{R,B}$ *(hence,* $c_2^E \leq c_2^B$*). Further,* $\frac{\partial d^M}{\partial \delta} > 0$.

b. $\frac{\partial |c_2^E - c_2^B|}{\partial \delta} > 0$.

Proposition 4 reveals that in the mute encroachment region, cost learning is subdued when the direct selling cost is sufficiently low, which makes second-period manufacturing costlier than the benchmark setting. By contrast, if the direct selling cost is sufficiently high, mute encroachment makes manufacturing more economical in the second period. In the voluntarily mute encroachment region, the retailer sells quantity, $q_1^{R,V} = \frac{5d+3c_1-3}{3\delta}$, resulting in $c_2^V = 1 - \frac{5d}{3}$. Observe that c_2^V decreases in the direct selling cost. Hence, when the direct selling cost is sufficiently high ($d \geq d^M$), the retailer orders a higher quantity in the first period, enhancing cost learning. By contrast, when the direct selling cost is low, the retailer orders a lower quantity, resulting in lower cost reduction than the benchmark setting. Depending on the first period manufacturing cost, the direct selling cost, and the learning rate, we find that voluntary mute encroachment may boost the first-period sales quantity by as much as 69.69% or drop it by 64.39%, i.e., $\frac{q_1^{R,V}}{q_1^{R,B}} \in [0.3561, 1.6969]$.[11]

In the voluntarily mute encroachment region, as the learning rate increases, the retailer drops her first-period sales quantity, nullifying the benefit of an increase in learning rate on the second period manufacturing cost, i.e., $c_2 = 1 - \frac{5d}{3}$. On the contrary, in the benchmark setting, the retailer raises her first-period sales quantity in the learning rate (Lemma 1), accelerating the impact of cost learning and resulting in a lower second-period manufacturing cost, i.e., $\frac{\partial c_2^B}{\partial \delta} < 0$. As a result, for a higher learning rate, the range of direct selling cost values for which the voluntarily mute encroachment results in higher second-period manufacturing cost (compared to the benchmark setting), increases, i.e., $\frac{\partial d^M}{\partial \delta} > 0$.

In the forced mute encroachment, the supplier passes on to the retailer a greater portion of the savings from a reduction in second-period manufacturing costs than he does in the benchmark setting, i.e., $\frac{\partial w_2^F}{\partial c_2} > \frac{\partial w_2^B}{\partial c_2} > 0$. Therefore, the retailer has a stronger incentive to help the supplier move further along the learning curve, and to that end, she orders a higher quantity in the first period, $q_1^{R,F}(w_1) \geq q_1^{R,B}(w_1)$, aimed at reducing the second-period manufacturing cost, leading to an even lower cost than the benchmark setting, i.e., $c_2^F \leq c_2^B$. We find that forced mute encroachment can result in an increase of up to 98.80% in the first-period sales quantity depending on the manufacturing cost, the direct selling cost, and the learning rate, i.e., $\frac{q_1^{R,F}}{q_1^{R,B}} \in [1, 1.988]$.

Using the results from Proposition 2 (for active encroachment) and Proposition 4 (for mute encroachment), we state the following general result.

[11] Refer to Section EC.2 for the analytical expressions.

Corollary 2. *In the presence of potential supplier encroachment, if* $\frac{1-c_1}{4} < d < \frac{(1-c_1)(96+24\delta-3\delta^2)}{160-20\delta^2}$*, then* $q_1^{R,E} < q_1^{R,B}$ *(hence,* $c_2^E > c_2^B$*); otherwise, if either* $d \leq \frac{1-c_1}{4}$ *or* $d \geq \frac{(1-c_1)(96+24\delta-3\delta^2)}{160-20\delta^2}$*, then* $q_1^{R,E} \geq q_1^{R,B}$ *(hence,* $c_2^E \leq c_2^B$*).*

The next section discusses the impact of this diminished cost learning on the supplier's and the retailer's profit.

5 Who Benefits from Encroachment?

In this section, we analyze the impact of encroachment on the supplier's and the retailer's profit and consumer surplus. We show that depending on the direct selling cost, d, encroachment can be beneficial or detrimental to either the supplier, the retailer, or both. We define the thresholds d^R, d^{S1}, d^{S2}, d^{F1} and d^{F2} (given in the Appendix) to characterize the domains where the supplier and the retailer are better off, or worse off, due to encroachment. Further, let S and R represent the sets (d^{S1}, d^{S2}) and $[d^R, d^{AV}]$, respectively. Sets $\overline{S}$ and $\overline{R}$ are the complement sets of S and R, respectively. That is, $\overline{S} = [0, d^{AV}] \setminus (d^{S1}, d^{S2})$ and $\overline{R} = [0, d^{AV}] \setminus [d^R, d^{AV}]$. In Figure 6, we demonstrate the impact of encroachment on the supplier's, and the retailer's profitability as a function of the direct selling cost (x-axis) and the learning rate (y-axis).

We start our discussion with the impact of active encroachment on profits. So far, we have seen that when direct selling is expensive, active encroachment hampers cost learning; otherwise, it enhances cost learning. The next proposition explores how this increase or decrease in cost learning impacts profits compared to the benchmark setting.

Proposition 5. *In the active encroachment region, i.e., when* $d \in [0, d^{AV}]$*, encroachment leads to:*

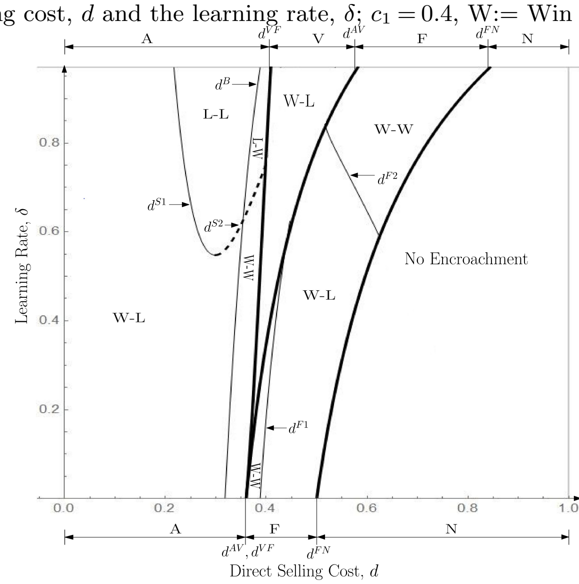
1. *Lose-Win:* $\Pi^{S,A} < \Pi^{S,B}$ *and* $\Pi^{R,A} \geq \Pi^{R,B}$*, when* $d \in S \cap R$*,*
2. *Lose-Lose:* $\Pi^{S,A} < \Pi^{S,B}$ *and* $\Pi^{R,A} < \Pi^{R,B}$*, when* $d \in S \cap \overline{R}$*,*
3. *Win-Win:* $\Pi^{S,A} \geq \Pi^{S,B}$ *and* $\Pi^{R,A} \geq \Pi^{R,B}$*, when* $d \in \overline{S} \cap R$*,*
4. *Win-Lose:* $\Pi^{S,A} \geq \Pi^{S,B}$ *and* $\Pi^{R,A} < \Pi^{R,B}$*, when* $d \in \overline{S} \cap \overline{R}$*.*

Further, $\frac{\partial (d^{S2} - d^{S1})}{\partial \delta} > 0$.

Proposition 5 reveals that contrary to the wisdom from the existing literature, encroachment might become detrimental to the supplier and this happens when $d \in S$. Further, depending on the value of the direct selling cost and the learning rate, encroachment may lead to either lose-win, lose-lose, win-win, or win-lose outcomes.

Similar to Arya et al. (2007), when the direct selling cost is within an intermediate range, $d \in R$, encroachment is beneficial to the retailer. On the one hand, the retailer benefits from encroachment due to a decrease in the second-period wholesale price as the direct selling cost increases, i.e., $\frac{\partial w_2^A}{\partial d} < 0$ (recall the bright side of encroachment by Arya et al. 2007). On the other hand, direct

Figure 6 Impact of supplier encroachment on the supplier's and the retailer's profitability as a function of the direct selling cost, d and the learning rate, δ; $c_1 = 0.4$, W:= Win and L:=Lose



selling by the supplier hurts the retailer as it cannibalizes the retailer's demand. When the direct selling cost is sufficiently low, the latter factor dominates, rendering the retailer worse off due to encroachment. Otherwise, the former factor dominates, and the retailer benefits from supplier encroachment. In the absence of cost learning, when $d \in [d^R|_{\delta=0}, d^{AV}|_{\delta=0}]$, both the supplier and the retailer become better off due to encroachment. Therefore, the existing literature often refers to this region as the "bright side of encroachment" (Arya et al. 2007, Wang and Li 2021, Guan et al. 2019). However, in the presence of cost learning, when $d \in [d^R|_{\delta>0}, d^{AV}|_{\delta>0}]$, encroachment is not necessarily "bright", as it can hurt the supplier.

When $d \in S$, the supplier becomes worse off due to encroachment. This happens when δ is high (see Figure 6).[12] Recall from Proposition 2 that a higher direct selling cost makes manufacturing costlier in the second period compared to the benchmark setting ($c_2^A > c_2^B$), and a higher learning rate amplifies this deviation ($\frac{\partial |c_2^A - c_2^B|}{\partial \delta} > 0$), which negatively affects the supplier's second-period profit. As a result, the Set S increases in δ. That is, the range of the direct selling cost values for which the supplier becomes worse off due to encroachment increases in the learning rate. On top of that, the supplier cannot recover this loss from selling directly in the second period, as it is less profitable when d is high. Interestingly, even though direct selling is detrimental to the supplier in this region, he still sells directly since he is not able to credibly commit to the retailer that he will abstain from direct selling. To credibly commit, the second-period manufacturing cost should be sufficiently high, which requires a high wholesale price in the first period, i.e., $w_1 \geq$

[12] Relaxing the assumption that the demand slope, b, equals 1 reveals that $d \in S$ only if $\delta \geq \frac{0.548}{b}$. This implies that for higher values of b, direct selling can be detrimental even for significantly lower learning rate values.

$w^A = \frac{6+3\delta-10d-6c_1}{3\delta}$. This hurts the supplier even more than selling directly when $d \le d^{AV}$. Hence, the supplier decides to sell directly, despite its detrimental effects. We find that depending on the manufacturing cost, the direct selling cost, and the learning rate, active encroachment can result in a drop of up to 7.52% or an increase of up to 50% in the supplier's profit, i.e., $\frac{\Pi^{S,A}}{\Pi^{S,B}} \in [0.9248, 1.5]$.

Building on the above discussion we further explore the four sub-regions: lose-win, lose-lose, win-win and win-lose. When $d \in S \cap R$, the supplier becomes worse off, whereas the retailer becomes better off due to encroachment, i.e., lose-win. As seen above, when direct selling is expensive and the learning rate is high, selling directly makes the supplier worse off. However, the retailer becomes better off due to the lower wholesale price offered in the second period. Essentially, even though the second-period manufacturing becomes costlier with an increase in the direct selling cost, the supplier still reduces w_2^A, which results in the retailer becoming better off at the supplier's expense. To the best of our knowledge, this outcome has not been observed previously in the case of encroachment with complete information.

The lose-lose outcome is characterized by an intermediate direct selling cost and a high learning rate, $d \in S \cap \overline{R}$. In this region, both the supplier and the retailer become worse off due to encroachment. When $d < d^R$, the second-period wholesale price is not sufficiently low to offset the retailer's loss due to the supplier's encroachment. From the supplier's perspective, the drawback of opening the direct channel (i.e., reduced first-period profit and increased second-period manufacturing cost) exceeds the benefit.

The win-win and win-lose outcomes have been extensively studied in the literature (Arya et al. 2007, Ha et al. 2016, Guan et al. 2019, Hotkar and Gilbert 2021). The intuition is simple. When the learning rate is low, there is a minor deviation in the manufacturing cost from the benchmark setting (Proposition 2). Hence cost learning has a smaller impact on the players' profits. Consequently, the situation faced by the players becomes similar to what has been analyzed in Arya et al. (2007). When the direct selling is expensive, i.e., $d \in R$, the supplier drops the second-period wholesale price. This reduction in the wholesale price increases the retailer's profit in the second period, which results in a win-win outcome. Otherwise, the retailer's loss due to competition from the direct channel outweighs the benefit of the reduced wholesale price, rendering the retailer worse off due to encroachment.

In summary, direct selling misaligns the retailer's and the supplier's incentives to accelerate cost learning. As a result, in the first period, the retailer orders as if cost learning does not exist, which may be detrimental to the supplier and the retailer. Therefore, when the learning rate is high and the direct selling cost is intermediate, the supplier should credibly commit to no encroachment. This may be in terms of a non-compete agreement with the retailer. This will assure the retailer that no encroachment will take place so that she can help the supplier move further along the learning curve by placing large orders. Such an approach is consistent with Fein and Anderson (1997), who suggest that if a player makes specific investments in a relationship, she demands a reciprocal pledge from the other player.

Our insight that encroachment can make the supplier worse off due to a reduction in the retailer's order quantity necessities a comparison with the model by Li et al. (2014), who study encroachment under information asymmetry. They find that the retailer reduces the order quantity to credibly signal a low market size and the impact of the reduction in her order quantity becomes intensified when the probability of a large market size is low.[13] In our model, she reduces her initial period order quantity $\left(q_1^{R,A}(w_1) \leq q_1^{R,B}(w_1)\right)$, because she does not benefit from the supplier's cost learning when the latter sells directly. The impact of this reduction is enhanced when the learning rate is high. Hence, our analysis establishes another plausible explanation for the supplier becoming worse off due to encroachment without considering information asymmetry.

Now, we shift our attention to the impact of mute encroachment on the players' profits. The existing literature on encroachment suggests that the retailer benefits from mute encroachment only if the direct selling cost is sufficiently low (Guan et al. 2019). However, our study reveals that in the presence of cost learning, the retailer benefits even when the direct selling cost is sufficiently high.

PROPOSITION 6. *In the forced mute encroachment region, i.e., when $d \in [d^{VF}, d^{FN})$, encroachment may lead to:*

1. *Win-Lose: $\Pi^{S,F} \geq \Pi^{S,B}$ and $\Pi^{R,F} < \Pi^{R,B}$, when $d \in (d^{F1}, d^{F2})$.*
2. *Win-Win: $\Pi^{S,F} \geq \Pi^{S,B}$ and $\Pi^{R,F} \geq \Pi^{R,B}$, when $d \in [d^{VF}, d^{FN}) \setminus (d^{F1}, d^{F2})$.*

This proposition reveals that forced mute encroachment is always beneficial to the supplier, but detrimental to the retailer when $d \in (d^{F1}, d^{F2})$, and beneficial otherwise. The intuition is as follows. Recall from our discussion following Proposition 3 that an increase in the direct selling cost induces a higher w_2^F but a lower w_1^F, and that the rate of decrease of w_1^F is amplified in the learning rate. When the direct selling cost is sufficiently low, the retailer benefits from encroachment due to a lower second-period wholesale price, while when the direct selling cost is sufficiently high, she benefits due to a lower first-period wholesale price, leading to a win-win outcome. By contrast, when the direct selling cost takes an intermediate value, the negative impact of higher w_2^F outweighs the increase in the retailer's profit from lower w_1^F, resulting in a win-lose outcome.

[13] Li et al. (2014) consider encroachment under information asymmetry, where the supplier does not know the actual market size, while the retailer can observe it. Market size can either be large (with probability λ) or small (with probability $1-\lambda$). The retailer signals the market size to the supplier through her order quantity. Accordingly, the supplier decides his direct selling quantity.

Voluntarily mute encroachment predominantly leads to a win-lose outcome. However, we note that it can also lead to lose-lose or win-win outcomes when d is very close to d^{AV} or d^{VF}, respectively. However, these regions are extremely small.[14] Overall, depending on the manufacturing cost, the direct selling cost, and the learning rate, voluntary mute encroachment can result in a drop of up to 5.73% or an increase of up to 70.4% in the supplier's profit, i.e., $\frac{\Pi^{S,V}}{\Pi^{S,B}} \in [0.9427, 1.704]$. Furthermore, the forced mute encroachment can help raise the supplier's profit by 68.73% while never hurting him, i.e., $\frac{\Pi^{S,F}}{\Pi^{S,B}} \in [1, 1.6873]$.

In the next proposition, we discuss the impact of supplier encroachment on consumer surplus. We define thresholds d^{VC} and d^{FC} (given in the Appendix), which belong to voluntary and forced mute encroachment regions, respectively, to characterize the domain where the consumers become better off or worse off due to encroachment.

Proposition 7. *If $d \in [0, d^{AV}] \cup [d^{VC}, d^{FC}]$, then supplier encroachment benefits the consumers; otherwise, it hurts them.*

The above proposition suggests that in the entire active encroachment region, i.e., when $d \in [0, d^{AV}]$, and in the mute encroachment region if $d \in [d^{VC}, d^{FC}]$, supplier encroachment benefits the consumers. Active encroachment increases retail competition in the second period, lowering the second-period price ($p_2^A < p_2^B$) and increasing consumer surplus. However, when the direct selling cost is high, $d > \frac{1-c_1}{4}$ (see Proposition 2), active encroachment results in lower first-period sales, higher retail price, and lower consumer surplus. Our analysis reveals that the increase in the second-period consumer surplus is always higher than the decrease in the first-period consumer surplus. Hence, overall active encroachment benefits consumers.

In the mute encroachment region, consumers are worse off due to encroachment when $d < d^{VC}$ or $d > d^{FC}$'; otherwise, consumers are better off (d^{VC} and d^{FC} are given in the Appendix). Recall from Proposition 4a that in the voluntarily mute encroachment region, the retailer reduces her first-period sales when $d < \frac{(1-c_1)(96+24\delta-3\delta^2)}{160-20\delta^2}$, resulting in a higher retail price. In the second period, consumers benefit from an increase in indirect channel sales, which lowers the retail price (as compared to the non-encroachment setting). When the direct selling cost is less ($d < d^{VC}$), the first effect dominates, and consumers are worse off; otherwise, encroachment benefits consumers. In the forced mute encroachment region, the retailer always sells more in the first period (see Proposition 4a), benefiting first-period consumers. In the second period, the retailer reduces sales

[14] For example, for $\delta = 0.9$ and $c_1 = 0.4$, the thresholds d^{AV} and d^{VF} are 0.405 and 0.544, respectively. In this case, the voluntarily mute encroachment leads to a lose-lose outcome when $d \in (0.405, 0.408)$ and to a win-win outcome when $d \in (0.542, 0.544)$.

as the direct selling cost rise, reducing consumer surplus. When $d > d^{FC}$, the drop in second-period consumer surplus is greater than the increase in first-period consumer surplus, making encroachment harmful to consumers.

To conclude, we have studied the effect of cost learning on pricing and the ordering decisions in supply chains when the supplier may have the option to encroach. Our analysis revealed three key insights. (i) In the presence of cost learning, a new region—voluntary mute encroachment—emerges such that as the direct selling cost increases, the equilibrium transitions from active encroachment to voluntarily mute encroachment, then to forced mute encroachment and, finally, to no encroachment. (ii) In the active encroachment region, the retailer does not benefit from the supplier's cost learning in the second period. As a result, the retailer does not support the supplier in moving along the learning curve and orders as if cost learning does not exist. (iii) Due to the reduction in the retailer's first-period sales quantity, the second-period manufacturing cost becomes relatively higher than the benchmark setting, rendering encroachment detrimental for the supplier.

6 Extensions

Insights (ii) and (iii) summarized in the last section raise the following interesting question: can the supplier compensate for the drop in the retailer's order quantity by over-producing in the first period and carrying inventory to the next period so as to progress faster along the learning curve? As the supplier's inventory will lower his second-period production quantity, diminishing the benefit reaped from cost learning, it is unclear if the supplier should carry inventory or not. We address this question in Section EC.4. In essence, we find that in the benchmark setting, the supplier will carry some inventory, but only negligibly so, as he would rather delegate the role of increasing the scope of production to the retailer. When accounting for encroachment, which induces higher levels of sales in the second period, the supplier may indeed carry significant level of inventory for some range of direct selling cost. In another extension (see Section EC.5), we ask if our main insights will change if the supplier can also encroach in the first period, in addition to the possibility of encroaching in the second period. Interestingly, we find that in the presence of cost learning (i.e., when $\delta > 0$), in the first period, as the direct selling cost increases, the equilibrium transitions from active encroachment to no encroachment, then surprisingly to forced mute encroachment and, finally, to no encroachment. In the second period, as in Proposition 1, the equilibrium transitions from active encroachment to voluntarily mute encroachment to forced mute encroachment, and finally to no encroachment. We find that our main results hold true in both extensions.

Additional extensions are presented in the appendix such as stochastic learning rate (Section EC.6), where we assume that the learning rate follows a two-point distribution and may take either low or high value with some probability; the presence of multiple retailers

(Section EC.7) revealing that our results follow through; and imperfect substitution between the two channels (Section EC.8), which gives rise to additional dynamics associated with the wholesale price set by the retailer as a function of the substitution level.

7 Conclusion

A vast amount of literature on supplier encroachment discusses the importance of direct selling cost in making encroachment decisions. However, most contributions are silent on the role of manufacturing cost, which in fact, plays an equally important role in determining the supplier's encroachment strategy. This study reveals that a higher manufacturing cost restricts the supplier from opening a direct channel and selling directly. Nevertheless, the manufacturing cost does not remain constant as it reduces with the experience gained from past productions due to cost learning.

Extensive studies on supplier encroachment show that encroachment is always beneficial to the supplier and can also be beneficial to the retailer depending on the supplier's direct selling cost, leading to a win-win situation for the supplier and the retailer. However, our study shows that the supplier's encroachment decision may diminish cost learning, hurting the supplier while benefiting the retailer. Hence, firms should be wary of this impact of encroachment on the manufacturing cost while considering adding a direct channel.

Admittedly, our study has some limitations which provide opportunities for future explorations. First, we assumed a linear demand function. However, the demand function might be unknown, and the supplier and the retailer might try to learn the actual demand to maximize their revenues (Besbes and Zeevi 2015). Second, we assumed that only the manufacturing cost decreases due to learning. However, as the supplier gains market knowledge, the direct selling cost can also reduce. It would be interesting to analyze the impact of this drop in the direct selling cost on the players' profits.

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E-Companion

Appendix. Thresholds

$$d^{AV} = \frac{3(1-c_1)\left(\left(\delta^4-\delta^3-12\delta^2+20\delta+80\right)-\sqrt{\delta^3\left(\delta^5-2\delta^4-34\delta^3-24\delta^2+208\delta+320\right)}\right)}{11\delta^4-120\delta^2+400}$$

$$d^{VF} = \frac{3(1-c_1)\left(4+\delta-\delta^2\right)}{20-8\delta^2}$$

$$d^{FF'} = \frac{20(1-c_1)\left(\delta^4-\delta^3-6\delta^2+2\delta+8\right)}{43\delta^4-180\delta^2+192}$$

$$d^{FNa} = \frac{(1-c_1)\left(4\left(2\delta^4-\delta^3-24\delta^2+8\delta+64\right)+\sqrt{2\delta^8+32\delta^7+114\delta^6-160\delta^5-1464\delta^4-1664\delta^3+2880\delta^2+7168\delta+4096}\right)}{16\left(\delta^4-11\delta^2+24\right)}$$

$$d^{FNb} = \frac{5(1-c_1)(2-\delta^2)\left(8\left(-3\delta^6+23\delta^5+124\delta^4-232\delta^3-992\delta^2+384\delta+1536\right)+z\right)}{2\left(249\delta^8-5276\delta^6+36448\delta^4-90624\delta^2+73728\right)}, \text{ where,}$$

$$z = \sqrt{2\delta^3\left(39\delta^9-432\delta^8-1604\delta^7+15168\delta^6+49568\delta^5-90624\delta^4-386560\delta^3-73728\delta^2+581632\delta+393216\right)}$$

$d^{FN} = d^{FNa}$ if $d^{FNa} < d^{FF'}$; otherwise $d^{FN} = d^{FNb}$

$$d^{RF} = \frac{(1-c_1)\left(16-2\delta-8\delta^2+\delta^3+\delta^4\right)}{16-6\delta^2}$$

$$d^{W} = \frac{(1-c_1)\left(31\delta^4-24\delta^3-396\delta^2+160\delta+1088\right)}{64(8-\delta^2)(3-\delta^2)} \text{ if } \delta \le 0.899; \text{ otherwise } d^{W} = \frac{3(1-c_1)\left(\delta^4-8\delta^3-48\delta^2+128\delta+512\right)}{64\left(8-\delta^2\right)\left(5-\delta^2\right)}$$

$$d^{R} = \frac{3(1-c_1)\left(48\delta^2+96\delta+\sqrt{14\delta^8-96\delta^7-1025\delta^6+592\delta^5+15184\delta^4+8960\delta^3-69632\delta^2-32768\delta+131072}\right)}{16\left(2\delta^4-23\delta^2+128\right)}$$

$$d^{S1} = \frac{(1-c_1)\left(12(\delta+4)-\sqrt{-3\delta^4+48\delta^3+378\delta^2+480\delta-384}\right)}{8\left(14-\delta^2\right)}$$

$$d^{S2} = \frac{(1-c_1)\left(12(\delta+4)+\sqrt{-3\delta^4+48\delta^3+378\delta^2+480\delta-384}\right)}{8\left(14-\delta^2\right)}$$

$$d^{F1} = \frac{(1-c_1)\left(8\left(\delta^4+\delta^3-8\delta^2-2\delta+16\right)-\frac{4-\delta^2}{8-\delta^2}\sqrt{64\delta^8+128\delta^7-1109\delta^6-2160\delta^5+5576\delta^4+9600\delta^3-9600\delta^2-10240\delta+8192}\right)}{16\left(8-3\delta^2\right)}$$

$$d^{F2} = \frac{(1-c_1)\left(8\left(\delta^4+\delta^3-8\delta^2-2\delta+16\right)+\frac{4-\delta^2}{8-\delta^2}\sqrt{64\delta^8+128\delta^7-1109\delta^6-2160\delta^5+5576\delta^4+9600\delta^3-9600\delta^2-10240\delta+8192}\right)}{16\left(8-3\delta^2\right)}$$

$$d^{VC} = \frac{3(1-c_1)\left(80(8-\delta^2)+\sqrt{\delta^2\left(4\delta^6-64\delta^5-423\delta^4+4720\delta^3+28176\delta^2+32000\delta-14336\right)}\right)}{(64\delta^2+400)(8-\delta^2)}$$

$$d^{FC} = \frac{(1-c_1)\left(16\left(2\delta^4-2\delta^3-12\delta^2+3\delta+16\right)\left(8-\delta^2\right)-\sqrt{\left(4-\delta^2\right)^2\left(4\delta^8-64\delta^7+305\delta^6+4848\delta^5+9888\delta^4-11264\delta^3-40704\delta^2-12288\delta+16384\right)}\right)}{16\left(4\delta^4-15\delta^2+16\right)\left(8-\delta^2\right)}$$

$$c_1^{AN} = 1 - \frac{d\left(40-10\delta-3\delta^2+2\delta^3\right)+\sqrt{d^2\delta^2\left(4\delta^4-24\delta^3-7\delta^2+92\delta-28\right)}}{24}$$

$$d^{I} = \frac{(2-h)(256-64\delta)-512c_1+128\delta c_1-\delta^2(8-8c_1-12h)-\delta^3 h}{2\left(512-128\delta-16\delta^2-\delta^3\right)}$$

$$d^{I1} = \frac{512(2-h)-4c_1\left(\delta^3-12\delta^2-64\delta+256\right)-\delta^4 h+\delta^3(4-16h)-24\delta^2(2-3h)-128\delta(2-h)}{(4-\delta)\left(512-128\delta-16\delta^2-\delta^3\right)}$$

$$d^{I2} = \frac{192\left(16(2-h)-c_1\left(32-8\delta-\delta^2\right)-\delta^2(1-2h)-8\delta\right)}{\left(20-5\delta-2\delta^2\right)\left(512-128\delta-16\delta^2-\delta^3\right)}$$

EC.1 Proof of Results in Sections 4 and 5

Proof of Lemma 1: We solve the game using backward induction to arrive at the SPNE. The supplier's profit function is given by $\Pi^S = (w_1 - c_1)q_1^R + (w_2 - c_2)q_2^R$ and the retailer's profit function is given by $\Pi^R = (p_1 - w_1)q_1^R + (p_2 - w_2)q_2^R$, where $p_t = 1 - q_t^R$, $t \in \{1, 2\}$ and $c_2 = c_1 - \delta q_1^R$. We note that Π^R is concave in q_2^R, and from the first order condition, we find that the optimal first-period order quantity is $q_2^R = \frac{1-w_2}{2}$. Next, we substitute the optimal q_2^R in the supplier's profit function to obtain $\Pi^S = (w_1 - c_1)q_1^R + (w_2 - c_2)\frac{1-w_2}{2}$, which is concave in w_2. From the first order condition, we obtain the optimal second-period wholesale price $w_2 = \frac{1+c_1-\delta q_1^R}{2}$. We substitute the optimal q_2^R and w_2 in Π^R. We note that Π^R is concave in q_1^R and from the first order condition, the optimal $q_1^R = \frac{(1-c_1)\delta+8(1-w_1)}{16-\delta^2}$. Finally, we substitute the optimal q_2^R, w_2 and q_1^R in Π^S. As Π^S is concave in w_1, solving the first order condition yields the optimal $w_1 = \frac{128(1+c_1)-16(1-c_1)\delta-8(3+c_1)\delta^2-(1-c_1)\delta^3}{32\left(8-\delta^2\right)}$. Back substituting the supplier's optimal first-period wholesale price, the optimal second-period wholesale price and the retailer's optimal first-period and second-period order quantities are given by: $w_2 = \frac{32(1+c_1)-8\delta(1-c_1)-\delta^2(7+c_1)}{8\left(8-\delta^2\right)}$, $q_1^R = \frac{(1-c_1)(3\delta+8)}{4(8-\delta^2)}$, and $q_2^R = \frac{(1-c_1)(32+8\delta-\delta^2)}{16\left(8-\delta^2\right)}$.

Differentiating the above equilibrium solution w.r.t. δ gives $\frac{\partial q_2^R}{\partial \delta} = \frac{(1-c_1)(\delta+2)(\delta+4)}{2\left(8-\delta^2\right)^2} > 0$, $\frac{\partial w_2}{\partial \delta} = -\frac{(1-c_1)(2+\delta)(4+\delta)}{(8-\delta^2)^2} < 0$, $\frac{\partial q_1^R}{\partial \delta} = \frac{(1-c_1)\left(3\delta^2+16\delta+24\right)}{4\left(8-\delta^2\right)^2} > 0$, $\frac{\partial w_1}{\partial \delta} = -\frac{(1-c_1)(128+128\delta+40\delta^2-\delta^4)}{32(8-\delta^2)^2} < 0$, $\frac{\partial \Pi^R}{\partial \delta} = \frac{(1-c_1)^2\left(384-3\delta^4-8\delta^3+72\delta^2+336\delta\right)}{16\left(8-\delta^2\right)^3} > 0$, $\frac{\partial \Pi^S}{\partial \delta} = \frac{(1-c_1)^2\left(3\delta^2+17\delta+24\right)}{4\left(8-\delta^2\right)^2} > 0$ for all values of δ satisfying $0 \le \delta \le 1$. This completes the proof of Lemma 1. □

Proof of Lemma 2: The supplier's first-period and the second-period profit functions are given by $\pi_1^S = (w_1 - c_1)q_1^R$ and $\pi_2^S = (w_2 - c_2)q_2^R + (p_2 - (c_2 + d))\, q_2^S$, respectively. The retailer's profit in period t, $t \in \{1, 2\}$ is given by $\pi_t^R = (p_t - w_t)q_t^R$, where, $p_1 = 1 - q_1^R$, $p_2 = 1 - (q_2^R + q_2^S)$ and $c_2 = c_1 - \delta q_1^R$. We solve the game using backward induction. First, we determine the supplier's direct channel sales, q_2^S, by solving $\max_{q_2^S} \Pi^S$. Where $\Pi^S = (p_2 - (c_2 + d))q_2^S + (w_2 - c_2)q_2^R + (w_1 - c_1)q_1^R$.

$$q_2^S = \begin{cases} (1 - (c_2 + d) - q_2^R)/2, & if\, 0 \le q_2^R \le 1 - (c_2 + d), \\ 0, & if\, 1 - (c_2 + d) \le q_2^R. \end{cases} \tag{EC.1}$$

Next, we find the retailer's second-period order quantity, q_2^R, by solving $\max_{q_2^R} \Pi^R$, subject to constraints given in EC.1. Where, $\Pi^R = (p_1 - w_1)q_1^R + (p_2 - w_2)q_2^R$. We have two cases:

1. If $0 \le q_2^R \le 1 - (c_2 + d)$, then $q_2^S = (1 - (c_2 + d) - q_2^R)/2$. Substituting q_2^S in the retailer's profit function and performing maximization yields $q_2^R = \frac{(c_2+d)-2w_2+1}{2}$. This solution requires $\frac{(c_2+d)+1}{2} \ge w_2 \ge \frac{3(c_2+d)-1}{2}$; otherwise, if $w_2 \le \frac{3(c_2+d)-1}{2}$, then $q_2^R = 1 - (c_2 + d)$ and if $w_2 \ge \frac{1+(c_2+d)}{2}$, then $q_2^R = 0$.
2. If $q_2^R \ge 1 - (c_2 + d)$, then the supplier does not sell any thing through the direct channel, i.e., $q_2^S = 0$. Substituting $q_2^S = 0$ in the retailer's profit function and performing maximization yields $q_2^R = (1 - w_2)/2$. This solution requires $0 \le w_2 \le 2(c_2 + d) - 1$; otherwise, if $w_2 \ge 2(c_2 + d) - 1$, then $q_2^R = 1 - (c_2 + d)$.

Since $c_2+d \le 1$, we note that $2(c_2+d)-1 \le \frac{3(c_2+d)-1}{2} \le \frac{(c_2+d)+1}{2}$. Combining the four ranges of q_2^R, the retailer's optimal second-period quantity is:

$$q_2^R = \begin{cases} 0, & if\ \frac{(c_2+d)+1}{2} \le w_2, \\ \frac{(c_2+d)-2w_2+1}{2}, & if\ \frac{3(c_2+d)-1}{2} \le w_2 \le \frac{(c_2+d)+1}{2}, \\ 1-(c_2+d), & if\ 2(c_2+d)-1 \le w_2 \le \frac{3(c_2+d)-1}{2}, \\ \frac{1-w_2}{2}, & if\ 0 \le w_2 \le 2(c_2+d)-1. \end{cases} \tag{EC.2}$$

We determine the optimal second-period wholesale price, w_2 by solving $\max\limits_{w_2} \Pi^S$, subject to the constraints given in EC.2. We have following four cases.

1. If the supplier chooses the second-period wholesale price from the range $\frac{(c_2+d)+1}{2} \le w_2$, then $q_2^R = 0$ and the direct selling quantity $q_2^S = \frac{1-(c_2+d)}{2}$. In this case, the supplier's profit is independent of w_2 and his corresponding second-period profit is $\pi_{2a}^S = \frac{(1-(c_2+d))^2}{4}$
2. If the supplier chooses the second-period wholesale price from the range $\frac{3(c_2+d)-1}{2} \le w_2 \le \frac{(c_2+d)+1}{2}$, then $q_2^R = \frac{(c_2+d)-2w_2+1}{2}$ and $q_2^S = (1-(c_2+d)-q_2^R)/2$. Maximising the supplier's profit function yields the optimal second-period wholesale price $w_2 = \frac{3-d+3c_2}{6}$, and it requires $c_2 \le \frac{3-5d}{3}$. The supplier's corresponding second-period profit is $\pi_{2b}^S = \frac{7d^2-6d(1-c_2)+3(1-c_2)^2}{12}$; otherwise, if $c_2 \ge \frac{3-5d}{3}$, then $w_2 = \frac{3(c_2+d)-1}{2}$. In this case, the supplier's corresponding second-period profit is $\pi_{2b}^S = \frac{(1-d-c_2)(3d+c_2-1)}{2}$.
3. If the supplier chooses the second-period wholesale price from the range $2(c_2+d)-1 \le w_2 \le \frac{3(c_2+d)-1}{2}$, then $q_2^R = 1-(c_2+d)$ and $q_2^S = (1-(c_2+d)-q_2^R)/2 = 0$. In this case the supplier's profit is increasing in w_2. Hence, the optimal $w_2 = \frac{3(c_2+d)-1}{2}$ and the supplier's corresponding second-period profit is $\pi_{2c}^S = \frac{(1-d-c_2)(3d+c_2-1)}{2}$.
4. If the supplier chooses the wholesale price from the range $0 \le w_2 \le 2(c_2+d)-1$, then $q_2^R = \frac{1-w_2}{2}$ and $q_2^S = 0$. Maximising the supplier profit function yields the optimal second-period wholesale price $w_2 = \frac{1+c_2}{2}$. This solution requires $c_2 \ge \frac{3-4d}{3}$. The supplier's corresponding profit in this case is $\pi_{2d}^S = \frac{(1-c_2)^2}{8}$; otherwise, if $c_2 < \frac{3-4d}{3}$, then $w_2 = 2(c_2+d)-1$ and the supplier's corresponding profit is $\pi_{2d}^S = (1-c_2-d)(c_2+2d-1)$.

We compare the supplier's second-period payoff. We have $\pi_{2a}^S - \pi_{2b}^S < 0$. $\pi_{2b}^S \ge \pi_{2c}^S$ if $c_2 \le \frac{3-5d}{3}$, else if $c_2 \ge \frac{3-5d}{3}$ then $\pi_{2b}^S = \pi_{2c}^S = \frac{(1-d-c_2)(3d+c_2-1)}{2}$. Finally, $\pi_{2c}^S \ge \pi_{2d}^S$ if $c_2 \le \frac{5-6d}{5}$; otherwise, $\pi_{2c}^S \le \pi_{2d}^S$ if $c_2 \ge \frac{5-6d}{5}$. Note that $\frac{3-5d}{3} \le \frac{3-4d}{3} \le \frac{5-6d}{5}$. Combining the above ranges of w_2, the supplier's optimal response function for the second-period wholesale price is:

$$w_2 = \begin{cases} \frac{3-d+3c_2}{6}, & if\ c_2 \le 1-\frac{5d}{3}, \\ \frac{3(c_2+d)-1}{2}, & if\ 1-\frac{5d}{3} \le c_2 \le 1-\frac{6d}{5}, \\ \frac{1+c_2}{2}, & if\ 1-\frac{6d}{5} \le c_2. \end{cases} \tag{EC.3}$$

Back substituting w_2 in Equations EC.1 and EC.2, we note that $q_2^S > 0$, when $c_2 < \frac{3-5d}{3}$. Further, $q_2^S = 0$ and $q_2^R = 1-(c_2+d)$ when $\frac{3-5d}{3} \le c_2 \le \frac{5-6d}{5}$. This completes the proof of Lemma 2. □

Proof of Proposition 1: In Lemma 2, we found the optimal q_2^S, q_2^R and w_2. Now we find the optimal q_1^R by substituting $c_2 = c_1 - \delta q_1^R$ in Equation EC.3, and solving $\max_{q_1^R}(p_2 - w_2)q_2^R + (p_1 - w_1)q_1^R$, subject to the constraints given in EC.3. We have the following cases:

1. If $c_2 \leq \frac{3-5d}{3}$, that is, $q_1^R \geq \frac{5d+3c_1-3}{3\delta}$, then the optimal first-period sales quantity is $q_1^R = (1-w_1)/2$, and it requires $w_1 \leq \frac{6+3\delta-10d-6c_1}{3\delta} = w^A$. The corresponding profit of the retailer is $\Pi_a^R = \frac{8c^2+9(1-w_1)^2}{36}$; otherwise, if $w_1 \geq w^A$, then $q_1^R = \frac{5d+3c_1-3}{3\delta}$ and the retailer's corresponding profit is $\Pi_a^R = \frac{15d(2-2c_1+\delta(1-w_1))-9(1-c_1)(1-c_1+\delta(1-w_1))-d^2\left(25-2\delta^2\right)}{9\delta^2}$.
2. If $\frac{3-5d}{3} \leq c_2 \leq \frac{5-6d}{5}$, that is, $\frac{6d+5c_1-5}{5\delta} \leq q_1^R \leq \frac{5d+3c_1-3}{3\delta}$, then the optimal solution is $q_1^R = \frac{\delta(1-d-c_1)+1-w_1}{2-\delta^2}$ and it requires $w^S \leq w_1 \leq w^{SS}$. The retailer's corresponding profit in this case is $\Pi_b^R = \frac{2d^2+2d(2c_1+\delta(w_1-1)-2)+2{c_1}^2+2c_1(\delta(w_1-1)-2)-2\delta w_1+2\delta+{w_1}^2-2w_1+3}{4-2\delta^2}$; otherwise, if $w^{SS} \leq w_1$, then $q_1^R = \frac{6d+5c_1-5}{5\delta}$. The retailer's corresponding profit in this case is $\Pi_b^R = \frac{d^2\left(\delta^2-72\right)-60d(2c_1+\delta(w_1-1)-2)-50(c_1-1)(c_1+\delta(w_1-1)-1)}{50\delta^2}$; otherwise, if $w_1 \leq w^S$, then $q_1^R = \frac{5d+3c_1-3}{3\delta}$ and the retailer's corresponding profit is $\Pi_b^R = \frac{15d(2-2c_1+\delta(1-w_1))-9(1-c_1)(1-c_1+\delta(1-w_1))-d^2\left(25-2\delta^2\right)}{9\delta^2}$. Where, $w^S = \frac{6-6c_1-10d+3\delta+2d\delta^2}{3\delta}$ and $w^{SS} = \frac{d\left(\delta^2-12\right)+5(-2c_1+\delta+2)}{5\delta}$.
3. If $\frac{5-6d}{5} \leq c_2$, that is, $q_1^R \leq \frac{6d+5c_1-5}{5\delta}$, then optimal q_1^R is $\frac{(1-c_1)\delta+8(1-w_1)}{16-\delta^2}$. This solution requires $w_1 \geq \frac{40-40c_1-48d+20\delta+3d\delta^2}{20\delta} = w^N$. In this case, the retailer's corresponding profit is $\Pi_c^R = \frac{{c_1}^2\left(-\delta^3+16\delta+32\right)+c_1\left(\delta^3(w_1+1)-8\delta^2(w_1-1)+16\delta(w_1-3)+64(2w_1-3)\right)+\delta^3(-w_1)+8\delta^2\left(2{w_1}^2-3w_1+1\right)-16\delta(w_1-2)+32\left(-4{w_1}^2+4w_1+1\right)}{\left(\delta^2-16\right)^2}$; otherwise, $q_1^R = \frac{6d+5c_1-5}{5\delta}$ and the corresponding profit of the retailer is $\Pi_c^R = \frac{d^2\left(\delta^2-72\right)-60d(2c_1+\delta(w_1-1)-2)-50(c_1-1)(c_1+\delta(w_1-1)-1)}{50\delta^2}$.

Note that $w^S < w^N < w^{SS}$. Further, when $w^N \leq w_1$, then $\Pi_c^R(w_1) - \Pi_b^R(w_1) \geq 0$. Therefore, if $w^N \leq w_1$, then $q_1^R = \frac{(1-c_1)\delta+8(1-w_1)}{16-\delta^2}$. Combining the above ranges, the optimal first-period order quantity is:

$$q_1^R = \begin{cases} \frac{1-w_1}{2}, & if\, 0 \leq w_1 \leq w^A, \\ \frac{5d+3c_1-3}{3\delta}, & if\, w^A \leq w_1 \leq w^S, \\ \frac{\delta(1-d-c_1)+1-w_1}{2-\delta^2}, & if\, w^S \leq w_1 \leq w^N, \\ \frac{(1-c_1)\delta+8(1-w_1)}{16-\delta^2}, & if\, w^N \leq w_1 \leq 1. \end{cases} \tag{EC.4}$$

Rewriting the above constraint in Equation EC.4 in terms of d, we obtain:

$$q_1^R = \begin{cases} \frac{1-w_1}{2}, & if\, 0 \leq d \leq d^A, \\ \frac{5d+3c_1-3}{3\delta}, & if\, d^A \leq d \leq d^S, \\ \frac{\delta(1-d-c_1)+1-w_1}{2-\delta^2}, & if\, d^S \leq d \leq d^N, \\ \frac{(1-c_1)\delta+8(1-w_1)}{16-\delta^2}, & if\, d^N \leq d \leq 1. \end{cases} \tag{EC.5}$$

Where, $d^A = \frac{(6(1-c_1)+3\delta(1-w_1))}{10}$, $d^S = \frac{6(1-c_1)+3\delta(1-w_1)}{10-2\delta^2}$, $d^N = \frac{40(1-c_1)+20\delta(1-w_1)}{48-3\delta^2}$.

Case 1. When $0 \leq d \leq d^A$. We note that $q_1^R(w_1) - q_1^{R,B}(w_1) = \frac{1-w_1}{2} - \frac{(1-c_1)\delta+8(1-w_1)}{16-\delta^2} < 0$. We consider the cases sequentially.

Case 2. When $d^A \le d \le d^S$. Solving $q_1^R(w_1) - q_1^B(w_1) = \frac{5d+3c_1-3}{3\delta} - \frac{(1-c_1)\delta+8(1-w_1)}{16-\delta^2} = 0$ for d, reveals that when $d < d_1 = \frac{48-48c_1+24\delta-24\delta w_1}{80-5\delta^2}$, then $q_1^R(w_1) < q_1^{R,B}(w_1)$; otherwise $q_1^R(w_1) \ge q_1^{R,B}(w_1)$. Further, $d_1 - d^A = \frac{3\delta^2(2(1-c_1)+\delta(1-w_1))}{10\left(16-\delta^2\right)} > 0$ and $d_1 - d^S = -\frac{33\delta^2(2(1-c_1)+\delta(1-w_1))}{10\left(80-21\delta^2+\delta^4\right)} < 0$. Hence, $d^A \le d_1 \le d^S$.

Case 3. When $d^S \le d \le d^N$. Solving $q_1^R(w_1) - q_1^{R,B}(w_1) = \frac{\delta(1-d-c_1)+1-w_1}{2-\delta^2} - \frac{(1-c_1)\delta+8(1-w_1)}{16-\delta^2} = 0$ for d reveals that $q_1^R(w_1) < q_1^{R,B}(w_1)$, when $d > \frac{14c_1+7\delta w_1-7\delta-14}{\delta^2-16} = d_2$; otherwise $q_1^R(w_1) \ge q_1^{R,B}(w_1)$. But, $d_2 - d^N = \frac{2(1-c_1)+\delta(1-w_1)}{48-3\delta^2} > 0$. Hence, when $d^S \le d \le d^N$, then $q_1^R \ge q_1^{R,B}$.

Case 4. when $d^N \le d \le 1$. In this case $q_1^R(w_1) = q_1^{R,B}(w_1)$.

We have already found the optimal q_2^S, q_2^R, w_2, and q_1^R. Now, we find the optimal w_1 by solving $\max_{w_1}(p_2-(c_2+d))q_2^S + (w_2-c_2)q_2^R + (w_1-c_1)q_1^R$, subject to constraints given in EC.4. We have the following cases:

1. If $0 \le w_1 \le w^A$, then the optimal w_1 is $\frac{4+4c_1-2(1-c_1-d)\delta-\delta^2}{8-\delta^2} = w_1^A$, and it requires $d \le \frac{3(1-c_1)(\delta+4)}{20-\delta^2} = d^T$. The supplier's and the retailer's corresponding profits are $\Pi_a^S = \frac{d^2\left(14-\delta^2\right)-3d(1-c_1)(\delta+4)+3(1-c_1)^2(\delta+3)}{3\left(8-\delta^2\right)}$ and $\Pi_a^R = \frac{d^2\left(2\delta^4-23\delta^2+128\right)-18d(1-c_1)\delta(\delta+2)+9(1-c_1)^2(\delta+2)^2}{9\left(8-\delta^2\right)^2}$, respectively. When $d \ge d^T$, then $w_1 = w^A$ and the supplier's and the retailer's corresponding profits are $\Pi_a^S = \frac{d^2\left(4\delta^2-50\right)-15d(c_1-1)(\delta+4)-9(c_1-1)^2(\delta+2)}{9\delta^2}$ and $\Pi_a^R = \frac{d^2\left(2\delta^2+25\right)-30d(1-c_1)+9(1-c_1)^2}{9\delta^2}$, respectively.
2. If $w^A \le w_1 \le w^S$, then the supplier's profit is increasing in w_1, hence the optimal $w_1 = w^S$. The corresponding profits of the supplier and the retailer are $\Pi_b^S = \frac{3d(1-c_1)\left(20+5\delta-2\delta^2\right)-9(1-c_1)^2(\delta+2)-2d^2\left(25-7\delta^2\right)}{9\delta^2}$ and $\Pi_b^R = \frac{d^2\left(25-8\delta^2\right)-6d(c_1-1)\left(\delta^2-5\right)+9(c_1-1)^2}{9\delta^2}$, respectively.
3. If $w^S \le w_1 \le w^N$, then the optimal first-period wholesale price is $w_1 = \frac{2(1+c_1)+(4-6d-4c_1)\delta-c_1\delta^2+(2d+c_1-1)\delta^3}{4-\delta^2} = w_1^F$, and it requires $d^{VF} \le d \le d^{FF'}$, and the corresponding profits of the supplier and the retailer are $\Pi_c^S = \frac{2d(1-c_1)\left(8+\delta-2\delta^2\right)-4d^2\left(3-\delta^2\right)-(1-c_1)^2\left(3-\delta^2\right)}{2\left(4-\delta^2\right)}$ and $\Pi_c^R = \frac{d^2\left(16-6\delta^2\right)-2d(1-c_1)\left(\delta^4+\delta^3-8\delta^2-2\delta+16\right)}{2\left(4-\delta^2\right)^2} + \frac{(1-c_1)^2\left(\delta^4-9\delta^2+18\right)}{2\left(4-\delta^2\right)^2}$, respectively. Else, if $d \ge d^{FF'}$, then $w_1 = w^N$. The corresponding profits of the players are $\Pi_c^S = \frac{40d(1-c_1)\left(-3\delta^6+23\delta^5+106\delta^4-94\delta^3-392\delta^2+96\delta+384\right)}{800\delta^2\left(2-\delta^2\right)^2} + \frac{d^2\left(249\delta^6-3284\delta^4+10176\delta^2-9216\right)-800(1-c_1)^2(\delta+2)\left(2-\delta^2\right)^2}{800\delta^2\left(2-\delta^2\right)^2}$ and $\Pi_c^R = \frac{d^2\left(129\delta^4-1408\delta^2+2304\right)}{800\delta^2\left(2-\delta^2\right)} - \frac{120d(1-c_1)\left(\delta^4-18\delta^2+32\right)+800(1-c_1)^2\left(2-\delta^2\right)}{800\delta^2\left(2-\delta^2\right)}$, respectively. Else, if $d \le d^{VF}$, then $w_1 = w^S$ and the corresponding profits of the supplier and the retailer are $\Pi_c^S = \frac{3d(1-c_1)\left(20+5\delta-2\delta^2\right)-9(1-c_1)^2(\delta+2)-2d^2\left(25-7\delta^2\right)}{9\delta^2}$ and $\Pi_c^R = \frac{d^2\left(25-8\delta^2\right)-6d(1-c_1)\left(5-\delta^2\right)+9(1-c_1)^2}{9\delta^2}$, respectively. Where $d^{VF} = \frac{3(1-c_1)\left(4+\delta-\delta^2\right)}{20-8\delta^2}$ and $d^{FF'} = \frac{20(1-c_1)\left(\delta^4-\delta^3-6\delta^2+2\delta+8\right)}{43\delta^4-180\delta^2+192}$.

4. If $w^N \le w_1 \le 1$, then the optimal solution is $w_1 = \frac{128(1+c_1)-16(1-c_1)\delta-8(3+c_1)\delta^2-(1-c_1)\delta^3}{32\left(8-\delta^2\right)} = w_1^N$, and it requires $d \ge d^N$. The corresponding profits of the players are $\Pi_d^S = \frac{(1-c_1)^2\left(\delta^2+48\delta+128\right)}{64\left(8-\delta^2\right)}$ and $\Pi_d^R = \frac{(1-c_1)^2\left(7\delta^4-48\delta^3-176\delta^2+768\delta+2048\right)}{256\left(8-\delta^2\right)^2}$. Else, $w_1 = w^N$. In this case the profits of the players are $\Pi_d^S = \frac{15d(1-c_1)\left(32+8\delta-\delta^2\right)-36d^2\left(8-\delta^2\right)-100(1-c_1)^2(\delta+2)}{100\delta^2}$ and $\Pi_d^R = \frac{9d^2\left(16-\delta^2\right)-15d(1-c_1)\left(16-\delta^2\right)+100(1-c_1)^2}{100\delta^2}$. Where, $d^N = \frac{5(1-c_1)\left(32-\delta^2+8\delta\right)}{24\left(8-\delta^2\right)}$.

Note that $d^T < d^{VF} < d^N < d^{FF'}$. When $d < d^T$, we compare the supplier's profits in the above four cases. We find that the optimal first-period wholesale price is $w_1 = w_1^A$ when $d \le d^{AV}$; otherwise $w_1 = w^S$. Considering the case when $d^T \le d \le d^{VF}$ and comparing the supplier's profit. We find that the optimal first-period wholesale price is $w_1 = w^S$. When $d^{VF} < d < d^N$, comparing the supplier's profit reveals that the optimal first-period wholesale price is $w_1 = w_1^F$. When $d^N \le d < d^{FF'}$, we find that the optimal first-period wholesale price is $w_1 = w_1^F$, if $d < d^{FNa}$; otherwise $w_1 = w_1^N$. In case $d^{FNa} \ge d^{FF'}$, the optimal wholesale price in the range $d^N \le d < d^{FF'}$ is $w_1 = w_1^F$. Finally, When $d \ge d^{FF'}$, we compare the supplier's profits to find the optimal first-period wholesale price. In case $d^{FNa} \ge d^{FF'}$, then $w_1 = w^N$ when $d < d^{FNb}$; otherwise $w_1 = w_1^N$. For the case $d^{FNa} < d^{FF'}$, the optimal first-period wholesale price for the range $d \ge d^{FF'}$ is $w_1 = w_1^N$. Where, $d^{FNa} = \frac{(1-c_1)\left(4\left(2\delta^4-\delta^3-24\delta^2+8\delta+64\right)+\sqrt{2\delta^8+32\delta^7+114\delta^6-160\delta^5-1464\delta^4-1664\delta^3+2880\delta^2+7168\delta+4096}\right)}{16\left(\delta^4-11\delta^2+24\right)}$, and

$d^{FNb} = \frac{5(1-c_1)(2-\delta^2)\left(8\left(-3\delta^6+23\delta^5+124\delta^4-232\delta^3-992\delta^2+384\delta+1536\right)+z\right)}{2\left(249\delta^8-5276\delta^6+36448\delta^4-90624\delta^2+73728\right)}$, where,

$z = \sqrt{2\delta^3\left(39\delta^9-432\delta^8-1604\delta^7+15168\delta^6+49568\delta^5-90624\delta^4-386560\delta^3-73728\delta^2+581632\delta+393216\right)}$.

Combining the above ranges of w_1, we have:

- When $d^{FNa} < d^{FF'}$

$$w_1 = \begin{cases} \frac{4+4c_1-2(1-c_1-d)\delta-\delta^2}{8-\delta^2}, & if\, 0 \le d \le d^{AV}, \\ w^S, & if\, d^{AV} \le d \le d^{VF}, \\ \frac{2(1+c_1)+(4-6d-4c_1)\delta-c_1\delta^2+(2d+c_1-1)\delta^3}{4-\delta^2} = w_1^F, & if\, d^{VF} \le d < d^{FNa}, \\ \frac{128(1+c_1)-16(1-c_1)\delta-8(3+c_1)\delta^2-(1-c_1)\delta^3}{32\left(8-\delta^2\right)}, & if\, d^{FNa} \le d \le 1. \end{cases} \tag{EC.6}$$

- When $d^{FNa} \ge d^{FF'}$

$$w_1 = \begin{cases} \frac{4+4c_1-2(1-c_1-d)\delta-\delta^2}{8-\delta^2}, & if\, 0 \le d \le d^{AV}, \\ w^S, & if\, d^{AV} < d \le d^{VF}, \\ \frac{2(1+c_1)+(4-6d-4c_1)\delta-c_1\delta^2+(2d+c_1-1)\delta^3}{4-\delta^2}, & if\, d^{AV} \le d < d^{FF'}, \\ w^N, & if\, d^{FF'} \le d < d^{FNb}, \\ \frac{128(1+c_1)-16(1-c_1)\delta-8(3+c_1)\delta^2-(1-c_1)\delta^3}{32\left(8-\delta^2\right)}, & if\, d^{FNb} \le d \le 1. \end{cases} \tag{EC.7}$$

For brevity, we define d^{FN} as : $d^{FN} = d^{FNa}$ if $d^{FNa} < d^{FF'}$, else if $d^{FNa} \ge d^{FF'}$, then $d^{FN} = d^{FNb}$. □

Proof of Corollary 1: After differentiating d^{AV} w.r.t. δ we have

$$\frac{\partial d^{AV}}{\partial \delta}=\frac{3\left(1-c_1\right)4\delta\left(60-11\delta^2\right)\left(80+20\delta+\delta^4-\delta^3-12\delta^2-\sqrt{\delta^3\left(\delta^5-2\delta^4-34\delta^3-24\delta^2+208\delta+320\right)}\right)}{\left(11\delta^4-120\delta^2+400\right)^2}$$
$$+\frac{3\left(1-c_1\right)\left(11\delta^4-120\delta^2+400\right)\left(4\delta^3-3\delta^2+\frac{\left(7\delta^4-4\delta^5+102\delta^3+60\delta^2-416\delta-480\right)\sqrt{\delta}}{\sqrt{\left(\delta^5-2\delta^4-34\delta^3-24\delta^2+208\delta+320\right)}}-24\delta+20\right)}{\left(11\delta^4-120\delta^2+400\right)^2}.$$

We can rearrange the above equation as $\frac{\partial d^{AV}}{\partial \delta}\equiv\frac{a+b}{\left(11\delta^4-120\delta^2+400\right)^2}$. Where, $a=3\left(1-c_1\right)4\delta\left(60-11\delta^2\right)\left(80+20\delta+\delta^4-\delta^3-12\delta^2-\sqrt{\delta^3\left(\delta^5-2\delta^4-34\delta^3-24\delta^2+208\delta+320\right)}\right)$ and $b=3\left(1-c_1\right)\left(11\delta^4-120\delta^2+400\right)\left(4\delta^3-3\delta^2+\frac{\left(7\delta^4-4\delta^5+102\delta^3+60\delta^2-416\delta-480\right)\sqrt{\delta}}{\sqrt{\left(\delta^5-2\delta^4-34\delta^3-24\delta^2+208\delta+320\right)}}-24\delta+20\right)$. Observe that the denominator of $\frac{\partial d^{AV}}{\partial \delta}$ is always positive, hence we just need to prove that $a+b$ is positive. We start with the term a. Clearly, $(1-c_1)>0$ for $0<c_1<1$. Next, we find the minimum and the maximum values of $(\delta^5-2\delta^4-34\delta^3-24\delta^2+208\delta+320)$. Since $\frac{\partial\left(\delta^5-2\delta^4-34\delta^3-24\delta^2+208\delta+320\right)}{\partial\delta}<0$, we can show that minimum and the maximum values of the function is at $\delta=0$ and $\delta=1$, respectively. Substituting $\delta=0$ and $\delta=1$, we obtain $469\geq(\delta^5-2\delta^4-34\delta^3-24\delta^2+208\delta+320)\geq320$. Further, $(80+20\delta+\delta^4-\delta^3-12\delta^2-\sqrt{\delta^3\left(\delta^5-2\delta^4-34\delta^3-24\delta^2+208\delta+320\right)})\geq(80+7\delta-\sqrt{\delta^3\left(\delta^5-2\delta^4-34\delta^3-24\delta^2+208\delta+320\right)})\geq(80+7\delta-\sqrt{469\delta^3})\geq(80+7\delta-\sqrt{469})>7\delta+58$. Since $7\delta^4-4\delta^5>0$ (as $0\leq\delta\leq1$), we can show that $(102\delta^3+60\delta^2-416\delta-480)\sqrt{\delta}\leq(7\delta^4-4\delta^5+102\delta^3+60\delta^2-416\delta-480)\sqrt{\delta}<0$. Hence, $\frac{\left(7\delta^4-4\delta^5+102\delta^3+60\delta^2-416\delta-480\right)\sqrt{\delta}}{\sqrt{\left(\delta^5-2\delta^4-34\delta^3-24\delta^2+208\delta+320\right)}}>\frac{(102\delta^3+60\delta^2-416\delta-480)\sqrt{\delta}}{\sqrt{320}}$. Now, we focus on the term b. Since $0\leq\delta\leq1$, we can show that $(11\delta^4-120\delta^2+400)>280$. Combining the above inequalities, we can write:

$$4\delta\left(60-11\delta^2\right)\left(80+20\delta\delta^4-\delta^3-12\delta^2-\sqrt{\delta^3\left(\delta^5-2\delta^4-34\delta^3-24\delta^2+208\delta+320\right)}\right)$$
$$+\left(11\delta^4-120\delta^2+400\right)\left(4\delta^3-3\delta^2+\frac{\left(7\delta^4-4\delta^5+102\delta^3+60\delta^2-416\delta-480\right)\sqrt{\delta}}{\sqrt{\left(\delta^5-2\delta^4-34\delta^3-24\delta^2+208\delta+320\right)}}-24\delta+20\right)>$$
$$4\delta\left(60-11\delta^2\right)\left(7\delta+58\right)+280\left(4\delta^3-3\delta^2+\frac{\left(102\delta^3+60\delta^2-416\delta-480\right)\delta}{\sqrt{320\delta}}-24\delta+20\right).$$

Expanding the above expression and analyzing analytically, we obtain $5600-3360\sqrt{5}\sqrt{\delta}+(7200\delta-2912\sqrt{5}\delta^{3/2})+840\delta^2+(420\sqrt{5}\delta^{5/2}-1432\delta^3)+(714\sqrt{5}\delta^{7/2}-308\delta^4)\geq5600-3360\sqrt{5}\sqrt{\delta}+688\delta+840\delta^2-493\delta^3+1288\delta^{7/2}\geq5600-3360\sqrt{5}\sqrt{\delta}+688\delta+347\delta^2+1288\delta^{7/2}=f(\delta)$. We find that $f(\delta)$ is convex in δ and its minimum value is always positive. Hence, $\frac{\partial d^{AV}}{\partial\delta}>0$.

Next, we differentiate d^{FN} w.r.t. δ.

$$\frac{\partial d^{FNa}}{\partial\delta}=\frac{\left(1-c_1\right)\left(24-11\delta^2+\delta^4\right)\left(4\left(8\delta^3-3\delta^2-48\delta+8\right)+\frac{\sqrt{2}\left(4\delta^7+56\delta^6+171\delta^5-200\delta^4-1464\delta^3-1248\delta^2+1440\delta+1792\right)}{\sqrt{\delta^8+16\delta^7+57\delta^6-80\delta^5-732\delta^4-832\delta^3+1440\delta^2+3584\delta+2048}}\right)}{16\left(\delta^4-11\delta^2+24\right)^2}$$
$$+\frac{\left(1-c_1\right)2\delta\left(11-2\delta^2\right)\left(8\delta^4-4\delta^3-96\delta^2+32\delta+256+\sqrt{2}\sqrt{\delta^8+16\delta^7+57\delta^6-80\delta^5-732\delta^4-832\delta^3+1440\delta^2+3584\delta+2048}\right)}{16\left(\delta^4-11\delta^2+24\right)^2}$$

We can rearrange the above equation as $\frac{\partial d^{FNa}}{\partial\delta}\equiv\frac{a+b}{16\left(\delta^4-11\delta^2+24\right)^2}$, where

$$a=\left(1-c_1\right)\left(\left(24-11\delta^2+\delta^4\right)\left(4\left(8\delta^3-3\delta^2-48\delta+8\right)\right)+2\delta\left(11-2\delta^2\right)\left(8\delta^4-4\delta^3-96\delta^2+32\delta+256\right)\right),$$

$$b = (1-c_1)\left(24-11\delta^2+\delta^4\right)\frac{\sqrt{2}\left(4\delta^7+56\delta^6+171\delta^5+(1792-200\delta^4-1464\delta^3)+1248\delta(1-\delta)+192\delta\right)}{\sqrt{\delta^8+16\delta^7+57\delta^6-80\delta^5-732\delta^4-832\delta^3+1440\delta^2+3584\delta+2048}}$$
$$+(1-c_1)\,2\delta\left(11-2\delta^2\right)\left(\sqrt{2}\sqrt{\delta^8+16\delta^7+57\delta^6-80\delta^5-732\delta^4-832\delta^3+1440\delta^2+3584\delta+2048}\right).$$

Clearly, the term 'b' and the denominator of $\frac{\partial d^{FNa}}{\partial\delta}$ are always positive for $0\le\delta\le 1$ and $0<c_1<1$, hence, if 'a' is positive, then $\frac{\partial d^{FNa}}{\partial\delta}$ is positive. Term 'a' can be rewritten as $a=(1-c_1)(4\delta^6+16\delta^5-52\delta^4-256\delta^3+64\delta^2+1024\delta+768)=(1-c_1)(4\left(8-\delta^2\right)^2\left(\delta^2+4\delta+3\right))$, which is always positive for $0\le\delta\le 1$ and $0\le c_1<1$. Hence, $\frac{\partial d^{FNa}}{\partial\delta}>0$. On the similar lines, we can prove that $\frac{\partial d^{FNb}}{\partial\delta}>0$. $\square$

Note: In the active encroachment region, $d\le d^T=\frac{3(1-c_1)(\delta+4)}{20-\delta^2}$ (refer proof of Proposition 1). The inequality $d\le d^T$ can be rewritten as $c_1+\frac{d(20-\delta^2)}{3(\delta+4)}=c_1+kd\le 1$, where $k=\frac{20-\delta^2}{3(\delta+4)}$. Since $k>1$, we note that in the active encroachment region $c_1+d<1$.

Proof of Lemma 3: The first derivative of $\pi_1^{R,A}$, $\pi_2^{R,A}$, $\Pi^{R,A}$, $\pi_1^{S,A}$, $\pi_2^{S,A}$, and $\Pi^{S,A}$ w.r.t. δ are as follows: $\frac{\partial\pi_1^{R,A}}{\partial\delta}=\frac{2\left((2(1-c_1)+\delta(1-c_1-d))\left((1-c_1-d)(8+\delta^2)+4\delta(1-c_1)\right)\right)}{(8-\delta^2)^3}>0$, $\frac{\partial\pi_2^{R,A}}{\partial\delta}=\frac{\partial(\frac{2d^2}{9})}{\partial\delta}=0$, $\frac{\partial\Pi^{R,A}}{\partial\delta}=\frac{2\left((2(1-c_1)+\delta(1-c_1-d))\left((1-c_1-d)(8+\delta^2)+4\delta(1-c_1)\right)\right)}{(8-\delta^2)^3}>0$, $\frac{\partial\pi_1^{S,A}}{\partial\delta}=-\frac{\delta(4(1-c_1-d)+\delta(1-c_1))\left((1-c_1-d)\left(\delta^2+8\right)+4\delta(1-c_1)\right)}{\left(8-\delta^2\right)^3}<0$, $\frac{\partial\pi_2^{S,A}}{\partial\delta}=\frac{2(4(1-c_1-d)+\delta(1-c_1))\left(8\delta(1-c_1-d)+(1-c_1)(\delta^2+8)\right)}{\left(8-\delta^2\right)^3}>0$, and $\frac{\partial\Pi^{S,A}}{\partial\delta}=\frac{4d^2\delta+\left(\delta^2+8\delta+8\right)(1-c_1)(1-c_1-d)}{\left(8-\delta^2\right)^2}>0$.

The first derivative of $q_1^{R,A}$ and $(q_2^{R,A}+q_2^{S,A})$ w.r.t to δ are $\frac{\partial q_1^{R,A}}{\partial\delta}=\frac{(8+\delta^2)(1-c_1-d)+4\delta(1-c_1)}{\left(8-\delta^2\right)^2}>0$, and $\frac{\partial q_2^{R,A}+q_2^{S,A}}{\partial\delta}=\frac{8(1-c_1)+8\delta(1-d-c_1)+\delta^2(1-c_1)}{\left(8-\delta^2\right)^2}>0$. Since $q_1^{R,A}$ and $(q_2^{R,A}+q_2^{S,A})$ are increasing in δ, we conclude that $\frac{\partial cs_1^A}{\partial\delta}>0$, $\frac{\partial cs_2^A}{\partial\delta}>0$, $\frac{\partial CS^A}{\partial\delta}>0$. $\square$

Proof of Proposition 2: We compare the equilibrium solutions obtained in Lemma 1 and Proposition 1. First, $w_1^A-w_1^B=\frac{\delta\left(64d-(1-c_1)\left(48+8\delta-\delta^2\right)\right)}{32\left(8-\delta^2\right)}$. Solving $w_1^A-w_1^B=0$ for d reveals threshold $d_1=\frac{(1-c_1)\left(48+8\delta-\delta^2\right)}{64}$, such that $w_1^A<w_1^B$ when $d<d_1$. But, $d_1-d^T>0$; hence, in Region A, $w_1^A<w_1^B$. Next, solving $q_1^A-q_1^B=\frac{4\delta(4d+c_1-1)}{\left(8-\delta^2\right)^2}=0$ for d reveals threshold $\frac{1-c_1}{4}$, such that if $d>\frac{1-c_1}{4}$, then $q_1^B>q_1^A$; otherwise $q_1^B\le q_1^A$. Since $c_2=c_1-\delta q_1^R$, we have $c_2^A>c_2^B$ if $d>\frac{1-c_1}{4}$; otherwise $c_2^A\le c_2^B$. Finally, we note that $\frac{\partial|c_2^A-c_2^B|}{\partial\delta}=\frac{4\delta|1-4d-c_1|}{\left(8-\delta^2\right)^2}>0$. $\square$

Proof of Proposition 3: When $d\in[d^{VF},min\{d^{FF'},d^{FN}\}]$, i.e, SPNE is forced mute encroachment, then $\Pi^{R,F}=\frac{d^2\left(16-6\delta^2\right)+2d(c_1-1)\left(\delta^4+\delta^3-8\delta^2-2\delta+16\right)+(c_1-1)^2\left(\delta^4-9\delta^2+18\right)}{2\left(4-\delta^2\right)^2}$ and $\frac{\partial\Pi^{R,F}}{\partial d}=\frac{d\left(16-6\delta^2\right)-(1-c_1)\left(\delta^4+\delta^3-8\delta^2-2\delta+16\right)}{\left(4-\delta^2\right)^2}$. Solving $\frac{\partial\Pi^{R,F}}{\partial d}=0$ for d reveals threshold $d^{RF}=\frac{(1-c_1)(16-2\delta-8\delta^2+\delta^3+\delta^4)}{16-6\delta^2}$, such that, $\frac{\partial\Pi^{R,F}}{\partial d}>0$, if $d>d^{RF}$; otherwise $\frac{\partial\Pi^{R,F}}{\partial\delta}\le 0$. The first derivative of d^{RF} w.r.t. δ is $\frac{\partial d^{RF}}{\partial\delta}=-\frac{(1-c_1)(16-18\delta^2+3\delta^4+6\delta^5+32\delta(1-\delta^2))}{2\left(8-3\delta^2\right)^2}$. Now, we prove that $\frac{\partial d^{RF}}{\partial\delta}<0$.

To prove: $\frac{\partial d^{RF}}{\partial\delta}<0$. Let $f(\delta)=16-18\delta^2+3\delta^4$, $\frac{\partial f(\delta)}{\partial\delta}=-12\delta(3-\delta^2)$. For $0\le\delta\le 1$, $\frac{\partial f(\delta)}{\partial\delta}<0$. Hence, $min\,(f(\delta))=f(\delta=1)=1>0$. Hence, $16-18\delta^2+3\delta^4>0$, for $0\le\delta\le 1$. Thus, $(16-18\delta^2+3\delta^4+6\delta^5+32\delta(1-\delta^2))>0$. This implies that $\frac{\partial d^{RF}}{\partial\delta}=-\frac{(1-c_1)(16-18\delta^2+3\delta^4+6\delta^5+32\delta(1-\delta^2))}{2\left(8-3\delta^2\right)^2}<0$. $\square$

Proof of Lemma 4 The first derivative of $\Pi^{S,V}$ w.r.t δ is $\frac{\partial \Pi^{S,V}}{\partial \delta} = \frac{(5d+3c_1-3)(20d-3(1-c_1)(\delta+4))}{9\delta^3}$. Solving $\frac{\partial \Pi^{S,V}}{\partial \delta} = 0$ for δ, reveals that $\frac{\partial \Pi^{S,V}}{\partial \delta} > 0$, when $\delta < \frac{4(5d+3c_1-3)}{3(1-c_1)}$; otherwise, $\frac{\partial \Pi^{S,V}}{\partial \delta} \leq 0$. Next, we find that $\frac{\partial \Pi^{R,V}}{\partial \delta} = -\frac{2(5d+3c_1-3)^2}{9\delta^3} < 0$ and $\frac{\partial CS^V}{\partial \delta} = -\frac{(5d+3c_1-3)^2}{9\delta^3} < 0$.

Next, we find that $\frac{\partial \Pi^{S,F}}{\partial \delta} = \frac{4d^2\delta d(1-c_1)\left(\delta^2+4\right)+(1-c_1)^2\delta}{\left(4-\delta^2\right)^2} > 0$. Further, $\frac{\partial \Pi^{R,F}}{\partial \delta} = \frac{(1+d\delta-c_1)\left((1-c_1)\delta^3-d\left(8-6\delta^2\right)\right)}{\left(4-\delta^2\right)^3}$. Solving $\frac{\partial \Pi^{R,F}}{\partial \delta} = 0$ for d, reveals that $\frac{\partial \Pi^{R,F}}{\partial \delta} \leq 0$ when $d \leq \frac{3(c_1-1)\left(\delta^2-\delta-4\right)}{20-8\delta^2}$; otherwise $\frac{\partial \Pi^{R,F}}{\partial \delta} > 0$. However, $\frac{3(c_1-1)\left(\delta^2-\delta-4\right)}{20-8\delta^2} < d^{VF}$. Hence, $\frac{\partial \Pi^{R,F}}{\partial \delta} > 0$. Next, we find that $\frac{\partial q_1^{R,F}}{\partial \delta} = \frac{d\left(\delta^2+4\right)+2(1-c_1)\delta}{\left(4-\delta^2\right)^2} > 0$ and $\frac{\partial q_2^{R,F}+q_2^{S,F}}{\partial \delta} = \frac{8d\delta+(4+\delta^2)(1-c_1)}{\left(4-\delta^2\right)^2} > 0$. Since $\frac{\partial q_1^{R,F}}{\partial \delta} > 0$ and $\frac{\partial q_2^{R,F}+q_2^{S,F}}{\partial \delta} > 0$, we conclude that $\frac{\partial CS^F}{\partial \delta} > 0$. □

Proof of Proposition 4: Comparing the first-period wholesale price of the mute encroachment region and the benchmark setting, we have the following case:

Case 1. Region V: solving $w_1^V - w_1^B = 0$ for d reveals that $w_1^V > w_1^B$ if $d < d^{W1}$; otherwise $w_1^V \leq w_1^B$. Where, $d^{W1} = \frac{3(1-c_1)\left(\delta^4-8\delta^3-48\delta^2+128\delta+512\right)}{64\left(8-\delta^2\right)\left(5-\delta^2\right)}$. Further, $d^{W1} - d^{VF} = \frac{3(1-c_1)\delta^2\left(14\delta^4-171\delta^2-88\delta+208\right)}{64\left(8-\delta^2\right)\left(5-\delta^2\right)\left(5-2\delta^2\right)}$. Hence, $d^{W1} - d^{VF} > 0$ if $(14\delta^4 - 171\delta^2 - 88\delta + 208) > 0$. The equation $(14\delta^4 - 171\delta^2 - 88\delta + 208) = 0$ has only one root in the range $0 \leq \delta \leq 1$, which is $\delta = 0.899$. When $\delta > 0.899$, then $(14\delta^4 - 171\delta^2 - 88\delta + 208) < 0$, hence, $d^{W1} < d^{VF}$; otherwise $d^{W1} \geq d^{VF}$.

Case 2. Region F: When $d < d^{FF'}$, we solve the equation $w_1^F - w_1^B = 0$ for d, which reveals the root $d^{W2} = \frac{(1-c_1)\left(31\delta^4-24\delta^3-396\delta^2+160\delta+1088\right)}{64(8-\delta^2)(3-\delta^2)}$. When $d < d^{W2}$, then $w_1^F > w_1^B$; otherwise $w_1^F \leq w_1^B$. Further, $d^{W2} - d^{VF} = \frac{(1-c_1)(4-\delta^2)\left(14\delta^4-171\delta^2-88\delta+208\right)}{64\left(5-2\delta^2\right)\left(\delta^4-11\delta^2+24\right)}$, which is positive only if $(14\delta^4 - 171\delta^2 - 88\delta + 208) > 0$. As seen above, the equation $(14\delta^4 - 171\delta^2 - 88\delta + 208) = 0$ has only one root in the range $0 \leq \delta \leq 1$, which is $\delta = 0.899$. Hence, when $\delta > 0.899$, $d^{W2} < d^{VF}$; otherwise $d^{W2} \geq d^{VF}$. When $d \geq d^{FF'}$, from proof of Proposition 1, we have $w_1^F|_{d<d^{FF'}} \geq w^N = w_1^F|_{d \geq d^{FF'}}$. Further $d^{W2} - d^{FF'} = -\frac{(1-c_1)(4-\delta^2)(2-\delta^2)(576+480\delta-53\delta^4-248\delta^3-212\delta^2)}{64\left(8-\delta^2\right)\left(3-\delta^2\right)\left(43\delta^4-180\delta^2+192\right)} < 0$. Hence, $w_1^F < w_1^N$ when $d \geq d^{FF'}$.

From Case 1 and 2, we conclude that, in mute encroachment region, $w_1^E > w_1^B$, when $d < d^W$; otherwise $w_1^E \leq w_1^N$, where, $d^W = d^{W1}$ if $\delta > 0.899$; otherwise $d^W = d^{W2}$ if $\delta \leq 0.899$.

We now analyse the impact of mute encroachment on the second-period manufacturing cost as compared to the benchmark setting.

Case 1. Region V: We solve the equation $c_2^V - c_2^B = -\frac{5d}{3} + \frac{(1-c_1)\left(32-\delta^2+8\delta\right)}{4\left(8-\delta^2\right)} = 0$ for d, which reveals that $c_2^V > c_2^B$ if $d < \frac{(1-c_1)(96+24\delta-3\delta^2)}{160-20\delta^2}$; otherwise $c_2^V \leq c_2^B$. Since $c_2 = c_1 - \delta q_1^R$, we have $q_1^V < q_1^B$ when $d < \frac{(1-c_1)(96+24\delta-3\delta^2)}{160-20\delta^2}$; otherwise $q_1^V \geq q_1^B$.

Case 2a. Region F: when $d^{FNa} < d^{FF'}$. We solve the equation $c_2^F - c_2^B = \frac{\delta^2\left(4d\left(\delta^2-8\right)+(1-c_1)\left(12-3\delta^2-4\delta\right)\right)}{4\left(8-\delta^2\right)\left(4-\delta^2\right)} = 0$ for d, which reveals that $c_2^F \geq c_2^B$, when $d \leq \frac{(1-c_1)\left(12-3\delta^2-4\delta\right)}{4\left(8-\delta^2\right)}$. But, $\frac{(1-c_1)\left(12-3\delta^2-4\delta\right)}{4\left(8-\delta^2\right)}$ is always less than d^{VF}.

Case 2b. Region F: when $d^{FF'} \leq d^{FNa}$ and $d^{FF'} \leq d \leq d^{FNb}$. Solving the equation

$c_2^F - c_2^B = \frac{c\left(23\delta^4-232\delta^2+384\right)-5(1-c_1)\left(\delta^4-8\delta^3-34\delta^2+16\delta+64\right)}{20\left(8-\delta^2\right)\left(2-\delta^2\right)} = 0$ for d, reveals that $c_2^F \geq c_2^B$, when $d \leq \frac{5(1-c_1)\left(2-\delta^2\right)\left(32+8\delta-\delta^2\right)}{\left(8-\delta^2\right)\left(48-23\delta^2\right)}$. But, $\frac{5(1-c_1)\left(2-\delta^2\right)\left(32+8\delta-\delta^2\right)}{\left(8-\delta^2\right)\left(48-23\delta^2\right)}$ is always less than $d^{FF'}$. Hence, from case 2a and case 2b, we conclude that in Region F, i.e., when $d \in [d^{VF}, d^{FN})$, $c_2^F \leq c_2^B$. □

Proof of Proposition 5: We start by proving that when $\Pi^{R,A} < \Pi^{R,B}$ when $d \in [0, d^R)$; otherwise, $\Pi^{R,A} \geq \Pi^{R,B}$ when if $d \in [d^R, d^{AV}]$.

To prove: When $d \in [0, d^{AV})$ then $\Pi^{R,A} < \Pi^{R,B}$. Otherwise $\Pi^{R,A} \geq \Pi^{R,B}$.
We compare the retailer's profit in the benchmark setting and in the active encroachment region. When $0 \leq d \leq d^{AV}$, then $\Pi^{R,B} = \frac{(1-c_1)^2\left(7\delta^4-48\delta^3-176\delta^2+768\delta+2048\right)}{256\left(8-\delta^2\right)^2}$ and $\Pi^{R,A} = \frac{d^2\left(2\delta^4-23\delta^2+128\right)-18d(1-c_1)\delta(\delta+2)+9(1-c_1)^2(\delta+2)^2}{9\left(8-\delta^2\right)^2}$. We note that $\Pi^{R,B} - \Pi^{R,A}$ is concave in d. Solving $\Pi^{R,B} - \Pi^{R,A} = 0$ for d reveals two roots d^R and $d^{R'}$:

$$d^R = \frac{3(1-c_1)\left(48\delta^2+96\delta+\sqrt{14\delta^8-96\delta^7-1025\delta^6+592\delta^5+15184\delta^4+8960\delta^3-69632\delta^2-32768\delta+131072}\right)}{16\left(2\delta^4-23\delta^2+128\right)},$$

$$d^{R'} = \frac{3(1-c_1)\left(48\delta^2+96\delta-\sqrt{14\delta^8-96\delta^7-1025\delta^6+592\delta^5+15184\delta^4+8960\delta^3-69632\delta^2-32768\delta+131072}\right)}{16\left(2\delta^4-23\delta^2+128\right)}.$$

Since $d^{R'} < 0 < d^R$, we conclude that when $d < d^R$ then $\Pi^{R,A} < \Pi^{R,B}$; otherwise $\Pi^{R,A} \geq \Pi^{R,B}$.

Next, we prove that when $d \in (d^{S1}, d^{S2})$ then $\Pi^{S,A} < \Pi^{S,B}$; otherwise, if $d \in [0, d^{AV}] \setminus (d^{s1}, d^{s2})$ then $\Pi^{S,A} \geq d^{S,B}$.

To Prove: When $d \in (d^{S1}, d^{S2})$ then $\Pi^{S,A} < \Pi^{S,B}$. Otherwise, when $d \in [0, d^{AV}] \setminus (d^{s1}, d^{s2})$ then $\Pi^{S,A} \geq \Pi^{S,B}$. In active encroachment region, i.e., $0 \leq d \leq d^{AV}$,the supplier's profit is $\Pi^{S,A} = \frac{d^2\left(14-\delta^2\right)-3d(1-c_1)(\delta+4)+3(1-c_1)^2(\delta+3)}{3\left(8-\delta^2\right)}$. The supplier's profit in the benchmark setting is given by $\Pi^{S,B} = \frac{(1-c_1)^2\left(\delta^2+48\delta+128\right)}{64\left(8-\delta^2\right)}$. We note that $\Pi^{S,B} - \Pi^{S,A}$ is concave in d. Solving $\Pi^{S,B} - \Pi^{S,A} = 0$ for d, reveals two threshold $d^{S1} = \frac{(1-c_1)\left(12(\delta+4)-\sqrt{-3\delta^4+48\delta^3+378\delta^2+480\delta-384}\right)}{8\left(14-\delta^2\right)}$ and $d^{S2} = \frac{(1-c_1)\left(12(\delta+4)+\sqrt{-3\delta^4+48\delta^3+378\delta^2+480\delta-384}\right)}{8\left(14-\delta^2\right)}$, such that $\Pi^{S,B} > \Pi^{S,A}$ if $d^{S1} < d < d^{S2}$, otherwise, $\Pi^{S,B} \leq \Pi^{S,A}$ if $d \leq d^{S1}$ or $d \geq d^{S2}$. Note that d^{S1} and d^{S2} takes a real value only if discriminant, $D = -3\delta^4 + 48\delta^3 + 378\delta^2 + 480\delta - 384$, is non negative, which happens when $\delta \geq 0.548$.

Combining the above analysis we have the following four cases:

1. When $d \in (d^{S1}, d^{S2}) \cap [d^R, d^{AV}]$ then $\Pi^{S,A} < \Pi^{S,B}$ and $\Pi^{R,A} \geq \Pi^{R,B}$,
2. When $d \in (d^{S1}, d^{S2}) \cap [0, d^R)$ then $\Pi^{S,A} < \Pi^{S,B}$ and $\Pi^{R,A} < \Pi^{R,B}$,
3. When $d \in ([0, d^{AV}] \setminus (d^{S1}, d^{S2})) \cap (d^R, d^{AV})$ then $\Pi^{S,A} \geq \Pi^{S,B}$ and $\Pi^{R,A} \geq \Pi^{R,B}$,
4. When $d \in ([0, d^{AV}] \setminus (d^{S1}, d^{S2})) \cap [0, d^R)$ then $\Pi^{S,A} \geq \Pi^{S,B}$ and $\Pi^{R,A} < \Pi^{R,B}$. □

Proof of Proposition 6: First, we prove that in the forced mute encroachment region $\Pi^{S,F} \geq \Pi^{S,B}$.

To prove: $\Pi^{S,F} \geq \Pi^{S,B}$.

Case 1: $d^{FF'} > d^{FNa}$. When $d = d^{VF}$, then $\Pi^{S,F} - \Pi^{S,B} = \frac{(1-c_1)^2\left(4\delta^6+\delta(1104-48\delta^4-348\delta^3-480\delta^2)+415\delta^2+448\right)}{64\left(5-2\delta^2\right)^2\left(8-\delta^2\right)} > 0$ and when $d = d^{FN}$, then $\Pi^{S,F} - \Pi^{S,B} = 0$. Furthermore $\frac{\partial^2(\Pi^{S,F}-\Pi^{S,B})}{(\partial d)^2} = -\frac{4\left(3-\delta^2\right)}{4-\delta^2} < 0$. Hence, when $d \in [d^{VF}, d^{FNa}]$ then $\Pi^{S,F} - \Pi^{S,B} \geq 0$. Observe that when $d^{FF'} = 0$ then $\Pi^{S,F} - \Pi^{S,B} \geq 0$, since $d^{FF'} < d^{FNa}$.

Case 2: When $d^{FF'} \leq d^{FNa}$. From Case 1, we observe that $\Pi^{S,F} - \Pi^{S,B} \geq 0$, when $d = d^{FF'}$. When $d = d^{FNb}$, then $\Pi^{S,F} - \Pi^{S,B} = 0$. Furthermore, $\frac{\partial^2(\Pi^{S,F}-\Pi^{S,B})}{(\partial d)^2} = \frac{249\delta^6-3284\delta^4+10176\delta^2-9216}{400\delta^2\left(2-\delta^2\right)^2}$. Observe that $400\delta^2\left(2-\delta^2\right)^2$ is always positive, therefore $\frac{\partial^2(\Pi^{S,F}-\Pi^{S,B})}{(\partial d)^2} < 0$ if $(249\delta^6 - 3284\delta^4 + 10176\delta^2 - 9216) < 0$. $\frac{\partial(249\delta^6-3284\delta^4+10176\delta^2-9216)}{\partial\delta} = 2\delta(10176 - 6568\delta^2 + 747\delta^4) > 0$. Hence, $\max\{(249\delta^6 - 3284\delta^4 + 10176\delta^2 - 9216)\} = -2075 < 0$. Therefore, $\frac{\partial^2(\Pi^{S,F}-\Pi^{S,B})}{(\partial d)^2} < 0$. From case 1 and case 2, we conclude that when $d \in [d^{VF}, d^{FN}]$, then $\Pi^{S,F} - \Pi^{S,B} \geq 0$.

Now, we prove that when $d^{F1} \leq d \leq d^{F2}$ then $\Pi^{R,F} \leq \Pi^{R,B}$; otherwise $\Pi^{R,F} > \Pi^{R,B}$.

To prove: If $d^{F1} < d < d^{F2}$ then $\Pi^{R,F} < \Pi^{R,B}$; otherwise $\Pi^{R,F} \geq \Pi^{R,B}$.

$\Pi^{R,F} - \Pi^{R,B} =$

$$\frac{256d^2\left(8-3\delta^2\right)-256d(1-c_1)\left(\delta^4+\delta^3-8\delta^2-2\delta+16\right)+\frac{(1-c_1)^2\left(121\delta^8+48\delta^7-2968\delta^6-1152\delta^5+25360\delta^4+6912\delta^3-91392\delta^2-12288\delta+114688\right)}{\left(8-\delta^2\right)^2}}{256\left(4-\delta^2\right)^2}.$$

Solving $\Pi^{R,F} - \Pi^{R,B} = 0$ for d, reveals two roots d^{F1} and d^{F2}, such that, when $d^{F1} < d < d^{F2}$ then $\Pi^{R,F} < \Pi^{R,B}$; otherwise $\Pi^{R,F} \geq \Pi^{R,B}$. Where,

$$d^{F1} = \frac{(1-c_1)\left(8\left(\delta^4+\delta^3-8\delta^2-2\delta+16\right) - \frac{4-\delta^2}{8-\delta^2}\sqrt{64\delta^8+128\delta^7-1109\delta^6-2160\delta^5+5576\delta^4+9600\delta^3-9600\delta^2-10240\delta+8192}\right)}{16\left(8-3\delta^2\right)},$$

$$d^{F2} = \frac{(1-c_1)\left(8\left(\delta^4+\delta^3-8\delta^2-2\delta+16\right) + \frac{4-\delta^2}{8-\delta^2}\sqrt{64\delta^8+128\delta^7-1109\delta^6-2160\delta^5+5576\delta^4+9600\delta^3-9600\delta^2-10240\delta+8192}\right)}{16\left(8-3\delta^2\right)}.$$

Combining the above analysis we conclude that in the forced mute encroachment region, i.e., when $d \in [d^{VF}, d^{FN})$, encroachment leads to :

1. $\Pi^{S,F} \geq \Pi^{S,B}$ and $\Pi^{R,F} < \Pi^{R,B}$, when $d \in (d^{F1}, d^{F2})$: Win-Lose,
2. $\Pi^{S,F} \geq \Pi^{S,B}$ and $\Pi^{R,F} \geq \Pi^{R,B}$, when $d \in [d^{VF}, d^{FN}) \setminus (d^{F1}, d^{F2})$: Win-Win. □

Proof of Proposition 7 Comparing consumer surplus in active encroachment region and benchmark setting, we find that $CS^A - CS^B = \frac{256d^2\left(\delta^4+17\delta^2+16\right)-1536d(1-c_1)\left(\delta^3+7\delta^2+10\delta+16\right)-9(1-c_1)^2\left(\delta^4-16\delta^3-368\delta^2-1792\delta-3072\right)}{4608\left(8-\delta^2\right)^2} \leq 0$ if $d_{c2} \leq d \leq d_{c2}$; otherwise, $CS^A - CS^B > 0$. Where, $d_{c1} = \frac{3\left(16\delta^3+112\delta^2+160\delta+256-16c_1\left(\delta^3+7\delta^2+10\delta+16\right)-\sqrt{(c_1-1)^2\left(\delta^8-16\delta^7-95\delta^6+1520\delta^5+8352\delta^4+13312\delta^3+24832\delta^2+53248\delta+16384\right)}\right)}{16\left(\delta^4+17\delta^2+16\right)}$ and $d_{c2} = \frac{3\left(16\delta^3+112\delta^2+160\delta+256-16c_1\left(\delta^3+7\delta^2+10\delta+16\right)+\sqrt{(c_1-1)^2\left(\delta^8-16\delta^7-95\delta^6+1520\delta^5+8352\delta^4+13312\delta^3+24832\delta^2+53248\delta+16384\right)}\right)}{16\left(\delta^4+17\delta^2+16\right)}$.

However, $d_{c2} > d_{c1} > d^{AV}$. Hence, we conclude that in active encroachment, $CS^A > CS^B$.

Next, we compare consumer surplus in voluntarily mute encroachment region and benchmark setting. we find that $CS^V - CS^B = \frac{\left(256d^2\left(4\delta^2+25\right)-7680d(1-c_1)\right)\left(8-\delta^2\right)^2-9(1-c_1)^2((8-\delta)\delta+32)(\delta(\delta+8)((16-\delta)\delta+16)+512)}{4608\delta^2\left(8-\delta^2\right)^2} \geq 0$ if $d \geq d^{VC} = \frac{3(1-c_1)\left(80(8-\delta^2)+\sqrt{\delta^2\left(4\delta^6-64\delta^5-423\delta^4+4720\delta^3+28176\delta^2+32000\delta-14336\right)}\right)}{(64\delta^2+400)(8-\delta^2)}$; otherwise, $CS^V - CS^B < 0$. Similar, we can prove that $CS^F - CS^B \geq 0$ if $d \leq d^{FC} = \frac{(1-c_1)\left(16\left(2\delta^4-2\delta^3-12\delta^2+3\delta+16\right)\left(8-\delta^2\right)-\sqrt{\left(4-\delta^2\right)^2\left(4\delta^8-64\delta^7+305\delta^6+4848\delta^5+9888\delta^4-11264\delta^3-40704\delta^2-12288\delta+16384\right)}\right)}{16\left(4\delta^4-15\delta^2+16\right)\left(8-\delta^2\right)}$; otherwise, $CS^F - CS^B < 0$. □

EC.2 The impact of potential supplier encroachment on the order quantity and the supplier's profit

Comparing the retailer's first period sales quantity and the supplier's profit in the active encroachment and the no encroachment region gives $\frac{q_1^{R,A}}{q_1^{R,B}} = \frac{4((1-d)\delta-c_1(\delta+2)+2)}{(1-c_1)(3\delta+8)}$ and $\frac{\Pi^{S,A}}{\Pi^{S,B}} = \frac{64\left(d^2\left(14-\delta^2\right)-3d(1-c_1)(\delta+4)+3(1-c_1)^2(\delta+3)\right)}{3(1-c_1)^2\left(\delta^2+48\delta+128\right)}$. Clearly, $\frac{\partial\left(\frac{q_1^{R,A}}{q_1^{R,B}}\right)}{\partial d} = -\frac{4\delta}{(1-c_1)(3\delta+8)} < 0$. Further, $\frac{\partial\left(\frac{q_1^{R,A}}{q_1^{R,B}}\right)}{\partial \delta} = \frac{8(1-4d-c_1)}{(1-c_1)(3\delta+8)^2}$. Clearly, when $d \leq \frac{1-c_1}{4}$, then $\frac{\partial\left(\frac{q_1^{R,A}}{q_1^{R,B}}\right)}{\partial \delta}$ and when $d > \frac{1-c_1}{4}$, $\frac{\partial\left(\frac{q_1^{R,A}}{q_1^{R,B}}\right)}{\partial \delta} < 0$. Recall from Proposition 2 that when $d \leq \frac{1-c_1}{4}$, $\frac{q_1^{R,A}}{q_1^{R,B}} \geq 1$, otherwise $\frac{q_1^{R,A}}{q_1^{R,B}} < 1$. Hence, $\frac{q_1^{R,A}}{q_1^{R,B}}$ reaches its minimum when $\delta = 1$ and $d = d^{AV}$ and reaches its maximum when $\delta = 1$ and $d = 0$. Substituting $\delta = 1$ and $d = d^{AV}$ (respectively $d = 0$ for maxima), we find that $\min\{\frac{q_1^{R,A}}{q_1^{R,B}}\} = 0.8421$ and $\max\{\frac{q_1^{R,A}}{q_1^{R,B}}\} = 1.0909$, which is independent of c_1. Overall, depending on c_1, d, and δ, $\frac{q_1^{R,A}}{q_1^{R,B}} \in [0.8421, 1.0909]$. Next, by using the first order condition, we find that $\min\{\frac{\Pi^{S,A}}{\Pi^{S,B}}\} = 0.9248$ when $\delta = 1$ and $d = \frac{3(1-c_1)(\delta+4)}{2\left(14-\delta^2\right)}$ and $\max\{\frac{\Pi^{S,A}}{\Pi^{S,B}}\} = 1.5$ when $\delta = 0$ and $d = 0$. That is, depending on c_1, d, and δ, $\frac{\Pi^{S,A}}{\Pi^{S,B}} \in [0.9248, 1.5]$.

Comparing the retailer's first period sales quantity and the supplier's profit in the voluntarily mute encroachment and the no encroachment region gives $\frac{q_1^{R,V}}{q_1^{R,B}} = \frac{4\left(8-\delta^2\right)(5d-3(1-c_1))}{3(1-c_1)\delta(3\delta+8)}$ and $\frac{\Pi^{S,V}}{\Pi^{S,B}} = \frac{64\left(\delta^2-8\right)\left(d^2\left(50-14\delta^2\right)-3d(c_1-1)\left(2\delta^2-5\delta-20\right)+9(c_1-1)^2(\delta+2)\right)}{9(c_1-1)^2\delta^2\left(\delta^2+48\delta+128\right)}$. Following similar steps as above, we find that $\min\{\frac{q_1^{R,V}}{q_1^{R,B}}\} = 0.3561$ when $d = d^{AV}$ and $\delta = 1$ and $\max\{\frac{q_1^{R,V}}{q_1^{R,B}}\} = 1.6969$ when $d = d^{VF}$ and $\delta = 1$. Overall, depending on c_1, d, and δ, $\frac{q_1^{R,V}}{q_1^{R,B}} \in [0.3561, 1.6969]$. Similarly, we find that $\min\{\frac{\Pi^{S,V}}{\Pi^{S,B}}\} = 0.9427$ when $\delta = 1$ and $d = d^{AV}$ and $\max\{\frac{\Pi^{S,V}}{\Pi^{S,B}}\} = 1.704$ when $d = \frac{23(1-c_1)}{24}$ and $\delta = 1$. That is, depending on c_1, d, and δ, $\frac{\Pi^{S,V}}{\Pi^{S,B}} \in [0.9427, 1.704]$.

Finally, comparing the retailer's first period sales quantity and the supplier's profit in the forced mute encroachment and the no encroachment region, we find that $\frac{q_1^{R,F}}{q_1^{R,B}} = \frac{4\left(8-\delta^2\right)(1+d\delta-c_1)}{(1-c_1)(3\delta+8)\left(4-\delta^2\right)}$, $\frac{\Pi^{S,F}}{\Pi^{S,B}} = \frac{32\left(8-\delta^2\right)\left(4d^2\left(\delta^2-3\right)-2d(1-c_1)\left(2\delta^2-\delta-8\right)+(1-c_1)^2\left(\delta^2-3\right)\right)}{(1-c_1)^2\left(4-\delta^2\right)\left(\delta^2+48\delta+128\right)}$. Following the similar steps as above, we find that

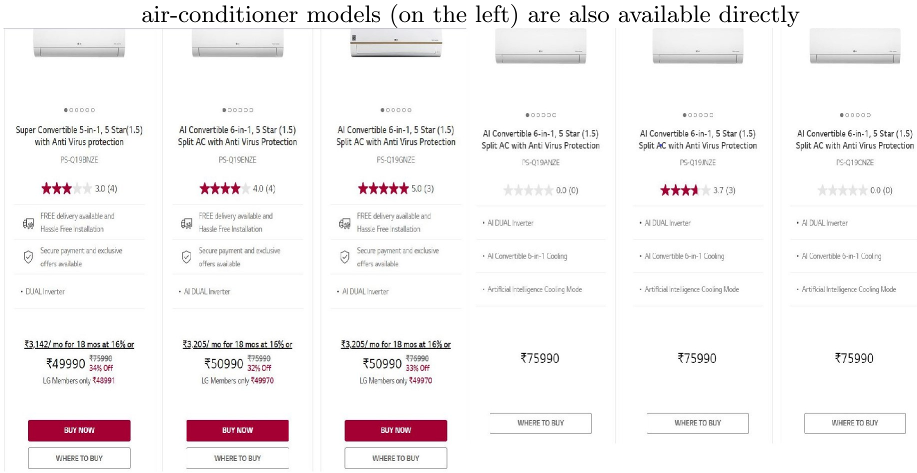
$\min\{\frac{q_1^{R,F}}{q_1^{R,B}}\} = 1$ when $\delta = 0$ and $d = d^{VF} = d^{AV}$ and $\max\{\frac{q_1^{R,F}}{q_1^{R,B}}\} = 1.988$ when $d = d^{FN}$ and $\delta = 1$. Further, $\min\{\frac{\Pi^{S,F}}{\Pi^{S,B}}\} = 1$ when $d = d^{FN}$ and $\max\{\frac{\Pi^{S,F}}{\Pi^{S,B}}\} = 1.6873$ when $d = d^{VF}$ and $\delta = 1$. That is, depending on c_1, d, and δ, $\frac{q_1^{R,F}}{q_1^{R,B}} \in [1, 1.988]$, $\frac{\Pi^{S,F}}{\Pi^{S,B}} \in [1, 1.6873]$.

EC.3 Illustration from LG India Website

In this section, we demonstrate the impact of cost and cost learning on the supplier's encroachment decisions through the LG India Example. First, we use the ACs example (Figure EC.1) and then use the TVs example (Figures EC.2-EC.4) to illustrate the impact of cost learning on the encroachment decisions.

LG India sells only its less expensive models through both the direct and the indirect channels, while redirecting the consumers to the nearby retailer (indirect channel) for costlier models. This is illustrated for the air-conditioner category in Figure EC.1, which shows how LG abstains from selling its more expensive models (the right most 3 models) directly, although it could, as is evident from the "buy now" button available for the other models.

Figure EC.1 [Color online] Snippet from LG India's website (www.lg.com/in) showing that the cheaper air-conditioner models (on the left) are also available directly



In Figures EC.2-EC.4, LG India primarily sells established technologies like UHD (Figure EC.2)— whose cost have dropped over time due to cost learning—through both the direct and the indirect channels. However, the latest TV technologies like OLED (Figure EC.3) and NanoCell (Figure EC.4), which are relatively costly and yet to realize cost learning, are sold primarily through the indirect channel.

EC.4 Encroachment in the Presence of Supplier's Inventory

In this section, the supplier has the option to produce additional q_i units in the first period, which he can carry as inventory while incurring a unit holding cost h. Consequently, the second-period

Figure EC.2 [Color online] Snippet from LG India's direct channel highlighting that most UHD TVs (one of the well-established technologies) are sold through both direct and indirect (retail) channels, Size: 65".

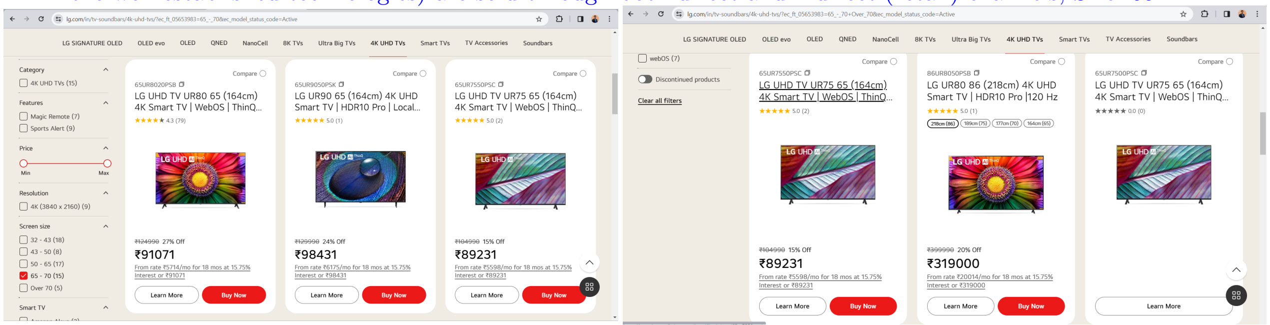


Figure EC.3 [Color online] Snippet from LG India's direct channel highlighting that most OLED TVs (one of the latest technologies) are sold only through the indirect (retail) channel, Size: 65".

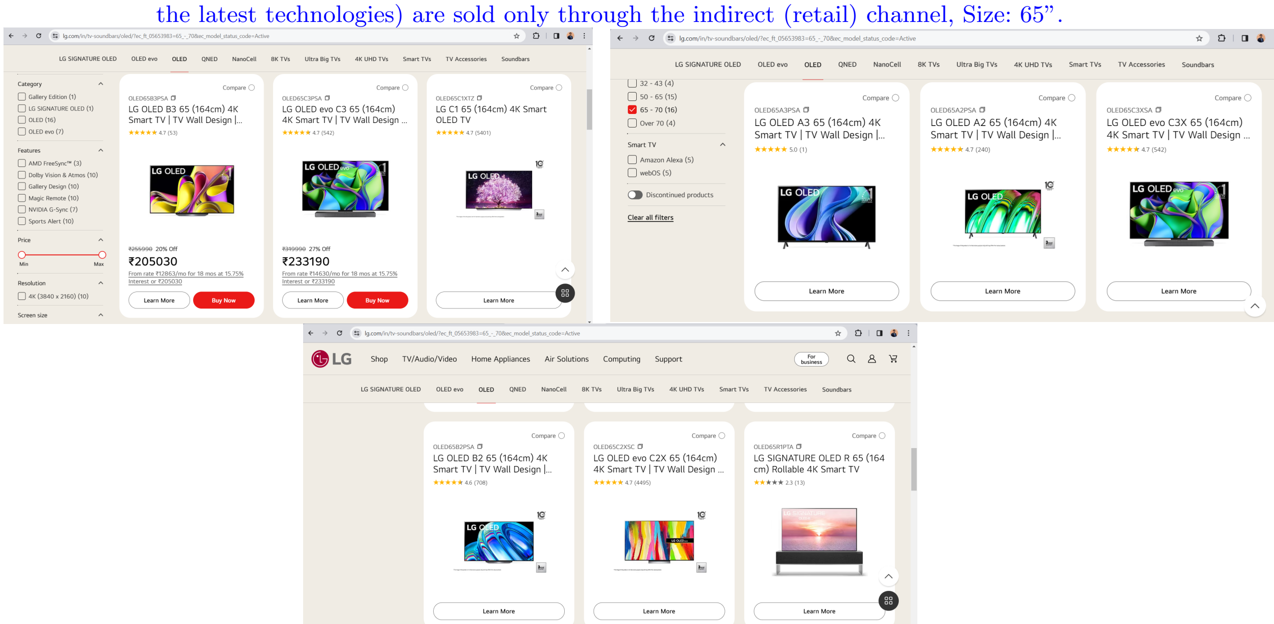
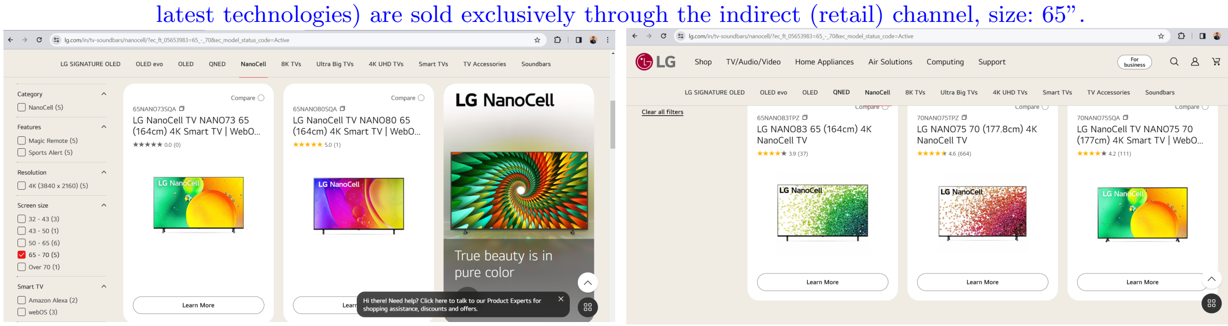


Figure EC.4 [Color online] Snippet from LG India's direct channel highlighting that OLED TVs (one of the latest technologies) are sold exclusively through the indirect (retail) channel, size: 65".



manufacturing cost, $c_2 = c_1 - \delta(q_1^R + q_i)$. As in Li et al. (2015), in the first period, the supplier decides his manufacturing quantity and the first-period wholesale price simultaneously, followed by the retailer deciding her sales quantity. In the second period, the supplier sets his second-period wholesale price, then the retailer decides her second-period sales quantity, and finally, the supplier decides his direct channel sales quantity. The supplier's profit function is given as, $\Pi^S = q_1^R(w_1 - c_1) + q_2^R w_2 + q_2^S(p_2^S - d) - (q_2^R + q_2^S - q_i)c_2 - q_i(c_1 + h)$ and the retailer's profit function is given as, $\Pi^R = q_1^R(p_1^R - w_1) + q_2^R(p_2^R - w_2)$. We solve the game using backward induction to find SPNE given in Table EC.1. The proof of the equilibrium solution proceeds similar to that of Proposition 1.

Table EC.1 Equilibrium solution in the presence of potential supplier encroachment

	Region A: Active encroachment		Region V: Voluntarily mute encroachment
	$d \in [0,\, d_i^{AI})$	$d \in [d_i^{AI}, d_i^{AV}]$	$d \in (d_i^{AV},\, d_i^{VI}]$
w_1^j	$\frac{4c_1-2h+4-\delta(3-d+-h)}{8-3\delta}$	$\frac{4+4c_1-2(1-c_1-d)\delta-\delta^2}{8-\delta^2}$	$\frac{6-6c_1-10d+3\delta+2d\delta^2}{3\delta}$
q_i	$\frac{2\delta(1-2d-c_1)+\delta^2(d+h)-8h}{2\delta(8-3\delta)}$	0	0
$q_1^{R,j}$	$\frac{1-w_1^j}{2}$	$\frac{1-w_1^j}{2}$	$\frac{5d+3c_1-3}{3\delta}$
w_2^j	$\frac{3+3c_2-d}{6}$	$\frac{3+3c_2-d}{6}$	$\frac{3+3c_2-d}{6}$
$q_2^{R,j}$	$\frac{2d}{3}$	$\frac{2d}{3}$	$\frac{2d}{3}$
$q_2^{S,j}$	$\frac{1-d-c_2+-q_2^{R,j}}{2}$	$\frac{3-3c_1-5d+3\delta q_1^{R,j}}{6}$	0
	Region V: Voluntarily mute encroachment	Region F: Forced mute encroachment	Region N: No encroachment
	(d_i^{VI}, d_i^{VF})	$d \in [d_i^{VF},\, d_i^{FN})$	$d \in [d_i^{FN},\, 1]$
w_1^j	$\frac{6-d(5-2\delta)-3h}{6}$	$\frac{2(2-h)+4c_1(1-\delta)+\delta(7-8d-4h)-\delta^2(2-3d-h)-\delta^3(1-d-h)}{8+3\delta-2\delta^2-\delta^3}$	$\frac{32c_1(8-\delta)-\delta^3(1-h)-8\delta^2(2-h)-16\delta(6-h)+128(2-h)}{512-128\delta-16\delta^2-\delta^3}$
q_i^j	$\frac{d\left(20-5\delta-2\delta^2\right)-3(4-4c_1+\delta h)}{12\delta}$	$\frac{\delta^3(1-2d-c_1)-\delta(5-8d-5c_1)+\delta^2(h-d)-4h}{\delta\left(8+3\delta-2\delta^2-\delta^3\right)}$	$\frac{(1-c_1)\delta^3+32\delta^2 h-256h}{\delta\left(512-128\delta-16\delta^2-\delta^3\right)}$
$q_1^{R,j}$	$\frac{d(2\delta+5)+3h}{12}$	$\frac{\delta(1-d-c_1)+\delta^2 q_i^j+1-w_1^j}{2-\delta^2}$	$\frac{8+\delta(1-c_1)+\delta^2 q_i^j-8w_1^j}{16-\delta^2}$
w_2^j	$\frac{3+3c_2-d}{6}$	$\frac{3(c_2+d)-1}{2}$	$\frac{1+c_2-\delta}{2}$
$q_2^{R,j}$	$\frac{2d}{3}$	$1-(c_2+d)$	$\frac{1-w_2^j}{2}$
$q_2^{S,j}$	0	0	0

Note: $j \in \{A, V, F, N\}$, where $A =$Active encroachment, $V =$ Voluntarily mute encroachment, $F =$ Forced mute encroachment, $N =$No encroachment.

Where,

$$d_i^{AI} = \frac{2(1-c_1)\delta+\delta^2 h-8h}{(4-\delta)\delta}$$

$$d_i^{VI} = \frac{3(4-4c_1+\delta h)}{20-5\delta-2\delta^2}$$

$$d_i^{AV} = \frac{3(1-c_1)\left(\left(\delta^4-\delta^3-12\delta^2+20\delta+80\right)-\sqrt{\delta^3\left(\delta^5-2\delta^4-34\delta^3-24\delta^2+208\delta+320\right)}\right)}{11\delta^4-120\delta^2+400}$$

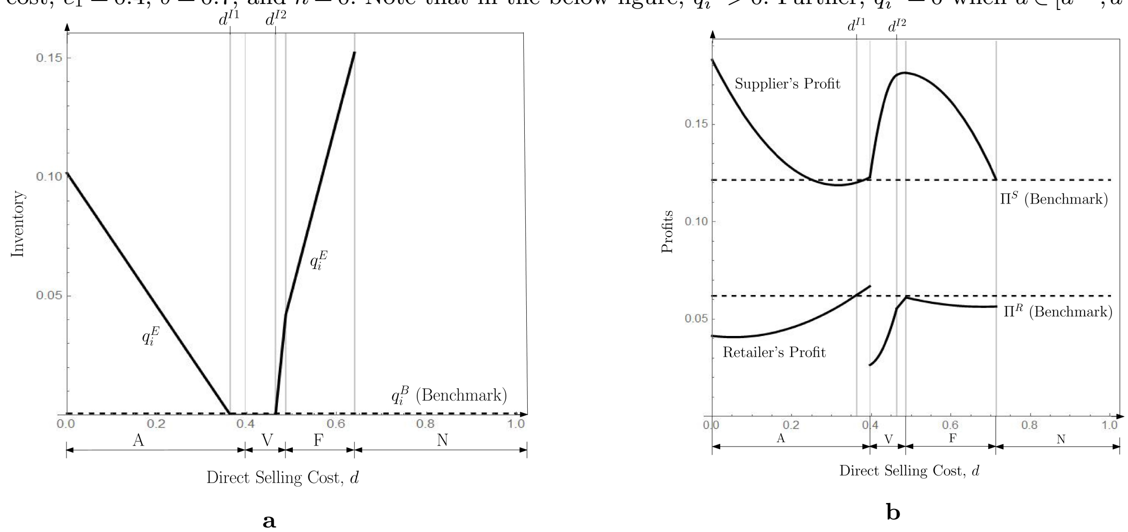
$$d_i^{VF} = \frac{2(h+2)+6\left(-4c_1(1-\delta)-\delta^3 h-\delta^2 h-4\delta(1-h)\right)}{2\delta^4+5\delta^3+2\delta^2-49\delta+40}$$

$$d_i^{FN} = \frac{-2c_1\delta^5-34c_1\delta^4-272c_1\delta^3+1024c_1\delta^2+3072c_1\delta-8192c_1-2\delta^5 h+2\delta^5-33\delta^4 h+34\delta^4-264\delta^3 h+272\delta^3+1024\delta^2 h-1024\delta^2+1536\delta h-3072\delta-4096h+8192}{4\delta^5+71\delta^4+600\delta^3-1536\delta^2-6656\delta+12288}$$
$$+\frac{2\sqrt{\left(\delta^6+18\delta^5+157\delta^4-312\delta^3-1536\delta^2+512\delta+4096\right)\left(c_1^2\left(\delta^4+16\delta^3+116\delta^2+320\delta+256\right)+c_1\left(2\delta^4(h-1)+\delta^3(3h-32)-4\delta^2(5h+58)-64\delta(h+10)+256(h-2)\right)+g\right)}}{4\delta^5+71\delta^4+600\delta^3-1536\delta^2-6656\delta+12288}$$

$$g := \delta^4(h-1)^2+\delta^3\left(-16h^2-3h+16\right)+\delta^2\left(69h^2+20h+116\right)-16\delta\left(7h^2-4h-20\right)+64(h-2)^2$$

Figure EC.5(a) illustrates the supplier's inventory as a function of the direct selling cost, and Figure EC.5(b) presents the players' profits as a function of the direct selling cost. Figure EC.5(a) shows that when the supplier cannot encroach in the second period (Benchmark setting) and the inventory holding cost is sufficiently low, i.e., $h < \frac{(1-c_1)\delta^3}{32\left(8-\delta^2\right)}$, the supplier over-produces in the first period and barely carries and inventory to the next period. The reason is simple. A very high

Figure EC.5 Inventory carried by the supplier (a) and the players' profits (b) as a function of the direct selling cost; $c_1 = 0.4$, $\delta = 0.7$, and $h = 0$. Note that in the below figure, $q_i^B > 0$. Further, $q_i^E = 0$ when $d \in [d^{AI}, d^{VI}]$



inventory level will reduce the second-period manufacturing quantity, diminishing the benefit of cost learning.

By contrast, if the supplier can encroach, he does carry a significant level of inventory for some range of the direct selling cost. In the active encroachment region, the supplier's inventory decreases in the direct selling cost, and it drops below the benchmark setting when $d > d^{I1}$ (the threshold d^{I1} is given in the Proof of Proposition EC.1). When the direct selling cost is low, the supplier sells a higher quantity through the direct channel. So, he has a greater incentive to carry inventory to accelerate cost learning. As this cost increases, the supplier's benefit from faster cost learning diminishes, so he decreases his inventory level. The decrease in the supplier's inventory, q_i, and the retailer's first-period sales quantity, q_1^R, result in slower cost learning, leading to a relatively higher manufacturing cost in the second period than the benchmark setting when the direct selling is sufficiently high. This relative increase in the manufacturing cost due to encroachment renders encroachment detrimental to the supplier, as is evident from Figure EC.5(b).

In the mute encroachment region, the supplier's inventory increases in the direct selling cost, and it surpasses the level of inventory he carries in the absence of encroachment when $d > d^{I2}$ (the threshold d^{I2} is given in the Proof of Proposition EC.1). As the inventory helps the supplier to accelerate cost learning, it reduces the supplier's reliance on the retailer's order quantity. Consequently, instead of dropping the first-period wholesale price to encourage the retailer to sell a higher quantity, the supplier can over-produce in the first period and charge a relatively higher first-period wholesale price. We summarize the above discussion in the following statement:

PROPOSITION EC.1. *If $d \in [0, d^{I1}] \cup [d^{I2}, d^{FN})$, then supplier encroachment increases the supplier's inventory level, i.e., $q_i^E \geq q_i^B$; otherwise, it decreases the supplier's inventory level, $q_i^E < q_i^B$.*

Proof of Proposition EC.1: Comparing q_i^A and q_i^B, we find that $q_i^A - q_i^B \geq 0$ when $d \leq d^{I1} = \frac{512(2-h)-4c_1\left(\delta^3-12\delta^2-64\delta+256\right)-\delta^4 h+\delta^3(4-16h)-24\delta^2(2-3h)-128\delta(2-h)}{(4-\delta)\left(512-128\delta-16\delta^2-\delta^3\right)}$. Similarly, Comparing q_i^V and q_i^B, we find that $q_i^V - q_i^B \geq$ when $d \geq d^{I2} = \frac{192\left(16(2-h)-c_1\left(32-8\delta-\delta^2\right)-\delta^2(1-2h)-8\delta\right)}{\left(20-5\delta-2\delta^2\right)\left(512-128\delta-16\delta^2-\delta^3\right)}$. □

EC.5 Should the Supplier Encroach in Both Periods?

This section considers the case where the supplier can encroach in both periods. The timeline of the model is as follows. At the start of Period t, $t \in \{1,2\}$, the supplier decides whether to encroach in Period t and accordingly decides the Period t wholesale price. Then the retailer decides her Period t sales quantity, followed by the supplier deciding his Period t direct channel sales quantity (if he decides to encroach in Period t). The profit of the supplier and the retailer in period t, $t \in \{1,2\}$, is given by $\pi_t^S = (p_t - c_t - d)q_t^S + (w_t - c_t)q_t^R$ and $\pi_t^R = (p_t - w_t)q_t^R$. We solve the game using backward induction to find SPNE. The proof of the equilibrium solutions is similar to that of the proof of Proposition 1. The equilibrium solution is given in Table EC.2.[15]

Figure EC.6 illustrates the supplier's and the retailer's total profits as a function of the direct selling cost, d. We find that the supplier sells through the direct channel in the first period if $c_1 < c_1^A = Min\{1 - \frac{d(10-2\delta-2\delta^2+\delta^3)}{6}, c_1^{AN}\}$ (the threshold c_1^{AN} is given WHERE?!!!), and in the second period if $c_2 < 1 - \frac{5d}{3}$. Note that when $\delta = 0$, then the above two thresholds reduce to the same value $\left(1 - \frac{5d}{3}\right)$. That is, in the absence of cost learning, the supplier's encroachment decision in the second period is an exact replica of his encroachment decision in the first period. However, this is no longer true in the presence of cost learning as evident from Figure EC.6. In the presence of cost learning (i.e., when $\delta > 0$), in the first period, as the direct selling cost increases, the equilibrium transitions from active encroachment to no encroachment, then surprisingly to forced mute encroachment and, finally, to no encroachment. While in the second period, similar to Proposition 1, the equilibrium transitions from active encroachment to voluntarily mute encroachment to forced mute encroachment, and finally to no encroachment.

As the direct selling becomes less profitable with an increase in d, we find that the supplier drops his first-period direct channel sales quantity, resulting in an increase in the second-period manufacturing cost. That is, when the direct selling cost is high enough, direct selling in the first period is not able to compensate for the drop in the retailer's first-period sales quantity, rendering encroachment detrimental (as compared to the benchmark setting) for the supplier (see Figure EC.6).

[15] The threshold d_1, d_2, d_3, and d_4 can be found by solving $\Pi^{S,AA} - \Pi^{S,NV} = 0$, $\Pi^{S,NV} - \Pi^{NF} = 0$ for d, $\Pi^{NF} - \Pi^{S,FF} = 0$, and $\Pi^{S,FF} - \Pi^{NN} = 0$, respectively, for d.

Table EC.2 Equilibrium solution if the supplier can encroach in both the periods

	Region AA $d \in [0,\, d_1]$	Region NV $d \in (d_1,\, d_2]$
w_1^{ij}	$\frac{2d\delta-d+3c_1-3\delta+3}{6-3\delta}$	$\frac{6-6c_1-10d+3\delta+2d\delta^2}{3\delta}$
$q_1^{R,ij}$	$-\frac{d\left(4-\delta^2\right)}{6}$	$\frac{5d+3c_1-3}{3\delta}$
$q_1^{S,ij}$	$\frac{6(1-c_1)-d\left(\delta^3-2\delta^2-2\delta+10\right)}{6(2-\delta)}$	0
p_1^{ij}	$\frac{d\delta+d+3c_1-3\delta+3}{6-3\delta}$	$\frac{3\delta-5d-3c_1+3}{3\delta}$
w_2^{ij}	$\frac{d\delta^2+2d\delta-2d+6c_1-6\delta+6}{6(2-\delta)}$	$\frac{3+3c_2-d}{6}$
$q_2^{R,ij}$	$\frac{2d}{3}$	$\frac{2d}{3}$
$q_2^{S,ij}$	$\frac{6(1-c_1)-d\left(\delta^2-4\delta+10\right)}{6(2-\delta)}$	0
p_2^{i}	$\frac{d\delta^2+2d+6c_1-6\delta+6}{6(2-\delta)}$	$\frac{3-2d}{3}$

	Region NF $d \in [d_2,\, d_3)$	Region FF $d \in [d_3,\, d_4)$	Region NN $d \in [d_4,\, 1]$
w_1^{ij}	$\frac{2(1+c_1)+(4-6d-4c_1)\delta-c_1\delta^2+(2d+c_1-1)\delta^3}{4-\delta^2}$	$\frac{d\left(3-\delta^3-7\delta\right)+c_1\left(3-\delta^3-4\delta\right)+\delta^4+\delta^3+3\delta^2+4\delta-1}{\delta^4+3\delta^2+2}$	$\frac{128(1+c_1)-16(1-c_1)\delta-8(3+c_1)\delta^2-(1-c_1)\delta^3}{32\left(8-\delta^2\right)}$
$q_1^{R,ij}$	$\frac{1+d\delta-c_1}{4-\delta^2}$	$\frac{(1-c_1)(1-\delta)-d(1-2\delta)}{\delta^2+1}$	$\frac{(1-c_1)\delta+8(1-w_1^i)}{16-\delta^2}$
$q_1^{S,ij}$	0	0	0
p_1^{ij}	$\frac{3+c_1-d\delta-\delta^2}{4-\delta^2}$	$\frac{d(1-2\delta)+c_1(1-\delta)+\delta^2+\delta}{\delta^2+1}$	$\frac{24+8c_1-3(1-c_1)\delta-4\delta^2}{32-4\delta^2}$
w_2^{ij}	$\frac{3(c_2+d)-1}{2}$	$\frac{3d\left(1-\delta^2+\delta\right)+3c_1(\delta+1)+2\delta^2-3\delta-1}{2\left(\delta^2+1\right)}$	$\frac{1+c_2}{2}$
$q_2^{R,ij}$	$\frac{(1-c_1)\left(4-\delta^2+\delta\right)-2d\left(2-\delta^2\right)}{4-\delta^2}$	$\frac{(1-c_1)(\delta+1)-d\left(1-\delta^2+\delta\right)}{\delta^2+1}$	$\frac{1-w_2^i}{2}$
$q_2^{S,ij}$	0	0	0
p_2^{ij}	$\frac{4(c_1+d)-(1-c_1)\delta-(c_1+2d)\delta^2}{4-\delta^2}$	$\frac{d\delta(1-\delta)+d+c_1(1+\delta)-\delta(1-\delta)}{\delta^2+1}$	$\frac{32(3+c_1)-8(1-c_1)\delta-(15+c_1)\delta^2}{16\left(8-\delta^2\right)}$

Note: Region ij represents encroachment strategy i in Period 1 and encroachment strategy j in Period 2. Where $i \in A, F, , N$ and $j \in \{A, V, F, N\}$. $A =$Active encroachment, $V =$ Voluntarily mute encroachment, $F =$ Forced mute encroachment, $N =$No encroachment.

EC.6 Stochastic Learning Rate

In this section, we extend our analysis to the case where the learning rate is stochastic. We consider a two point distribution, where $\delta = \delta_l$ with probability P and $\delta = \delta_h$ with probability $(1-P)$. The players' profit functions and the sequence of decisions are the same as discussed in Section 3. We solve the game using backward induction to arrive at SPNE. We present the supplier's and the retailer's profit as a function of probability P in Figure EC.7 for $c_1 = 04$, $d = 0.38$, $\delta_l = 0.15$, and $\delta_h = 0.8$. We select the specific value of $d = 0.38$, $\delta_l = 0.15$, and $\delta_h = 0.8$ to consider the transition between the forced mute and active encroachment regions depending on the realized δ value. If the realized $\delta = \delta_l = 0.15$, then the equilibrium in the second period is forced mute encroachment, and if the realized $\delta = \delta_h = 0.8$, then the equilibrium in the second period is active encroachment.

For the given set of parameter values, when P is sufficiently low, encroachment is detrimental for the supplier. By contrast, when P is sufficiently high, encroachment benefits the supplier (refer to Figure EC.7). The intuition is straightforward. When the P is low (i.e., the probability of $\delta = \delta_h$ is high), it is more likely that the final equilibrium solution will be active encroachment in the

Figure EC.6 The players' profits as a function of direct selling cost, d; $c_1 = 0.4$ and $\delta = 0.9$

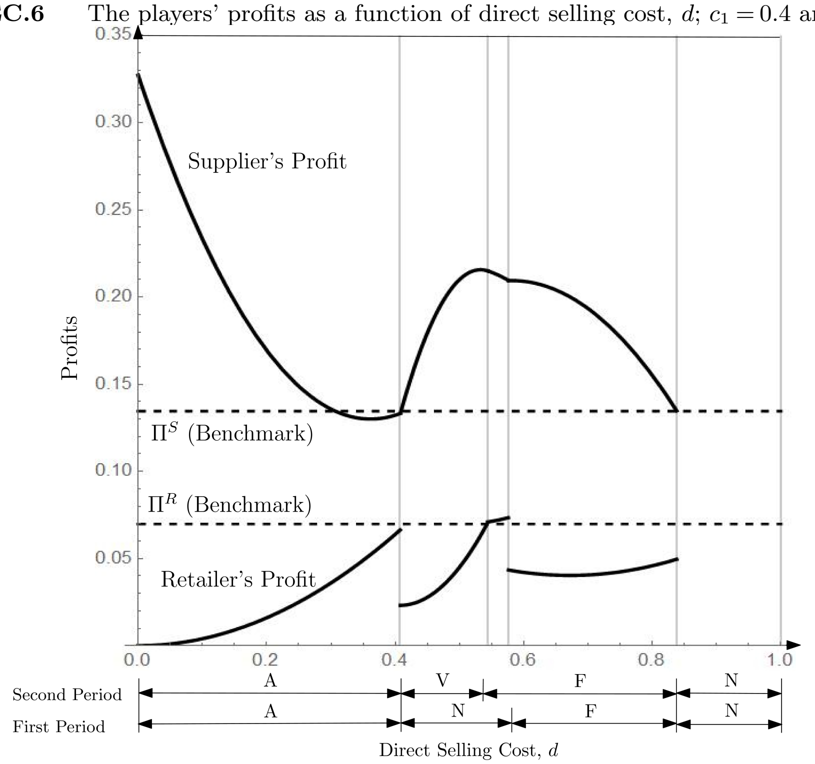
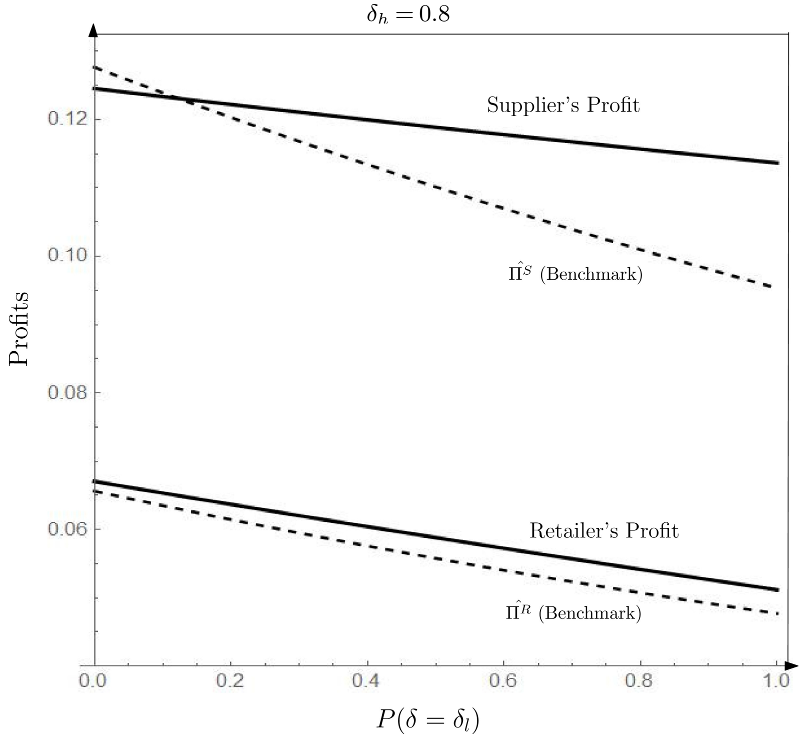


Figure EC.7 The players' expected profits as a function of probability P, when $c_1 = 0.4$, $d = 0.38$, $\delta_l = 0.15$, and $\delta_h = 0.8$



second period. Similar to Proposition 5, for the given set of parameters, active encroachment hurts the supplier. As a result, the supplier's expected profit is lower than the benchmark setting if

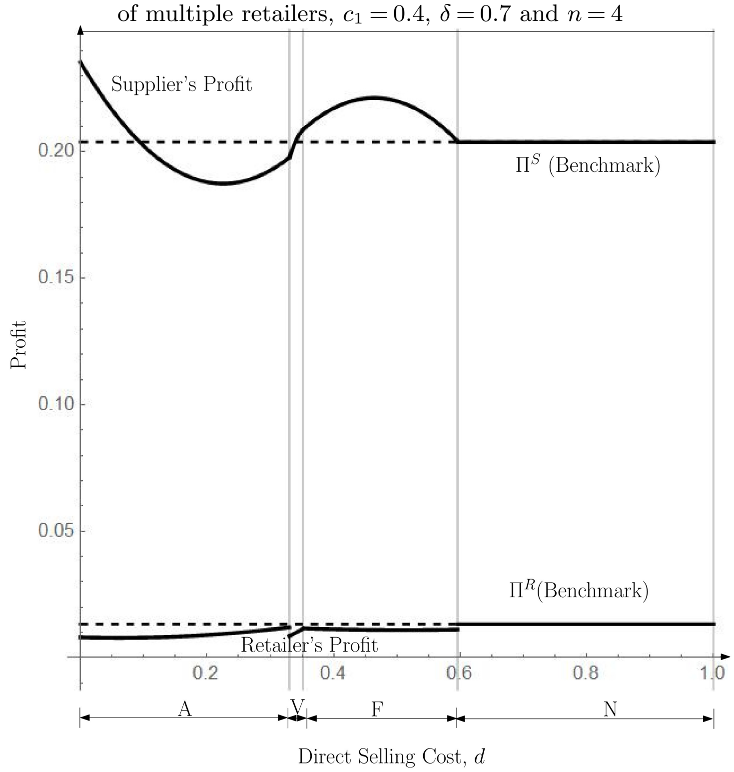
P is sufficiently low. By contrast, when P is sufficiently high (i.e., the probability of $\delta = \delta_l$ is high), it is more likely that the final equilibrium solution in the second period will be forced mute encroachment. As the forced mute encroachment always benefits the supplier (Proposition 6), the supplier's expected profit is higher than the benchmark setting when P is sufficiently high. Overall, consistent with our finding from Section 5, when the direct selling cost is intermediate ($d = 0.38$ in the given example), the supplier should encroach only if the learning rate is sufficiently low or the probability that the learning rate is low is sufficiently high.

EC.7 Multiple Retailers

In this section, we extend our base model to consider n retailers who sell the goods to the consumers. This is a generalization of our base model (Section 3), in which $n = 1$. The timeline of the model is similar to Section 3. First, the supplier decides the first-period wholesale price, followed by the retailers deciding their order quantity simultaneously. At the start of the second period, the supplier again decides his wholesale price, then the retailers decide their order quantity simultaneously, and finally, the supplier decides his direct channel sales quantity.

In Figure EC.8, we present the supplier's and the retailer's profit as a function of the direct selling cost. We find that our key results, namely, (i) the presence of a voluntarily mute encroachment region, (ii) the retailers not benefiting from the supplier's cost learning, and (iii) the supplier becoming worse off due to encroachment, carry through even when multiple retailers are present. Furthermore, we find that when n is high, the supplier becomes worse off due to encroachment, even for lower direct selling cost values. For instance, when $c_1 = 0.4$, $\delta = 0.7$, and $n = 1$, the supplier becomes worse off due to encroachment if $d \in (0.24, 0.38)$ and if $n = 4$, the supplier becomes worse off due to encroachment if $d \in (0.09, 0.33)$. This result echoes the observation made by Liu et al. (2021), who suggest that encroachment in the presence of a higher number of retailers may be detrimental to the supplier. Our analysis shows that in the presence of cost learning, the negative effect of encroachment with multiple retailers becomes even more pronounced. The reason is similar to the one discussed in our main model. In the benchmark setting, all the retailers benefit from the supplier's cost learning. Hence, they all support the supplier in moving along the learning curve by ordering a higher quantity in the first period. However, in case the supplier can sell directly, then no retailer benefits from the supplier's cost learning in the second period. Hence, no retailer supports the supplier in accelerating his cost learning. Accordingly, all the retailers order as if cost learning does not exist, resulting in a sharp drop in the first-period sales and the cost learning as compared to the benchmark setting, which hurts the supplier.

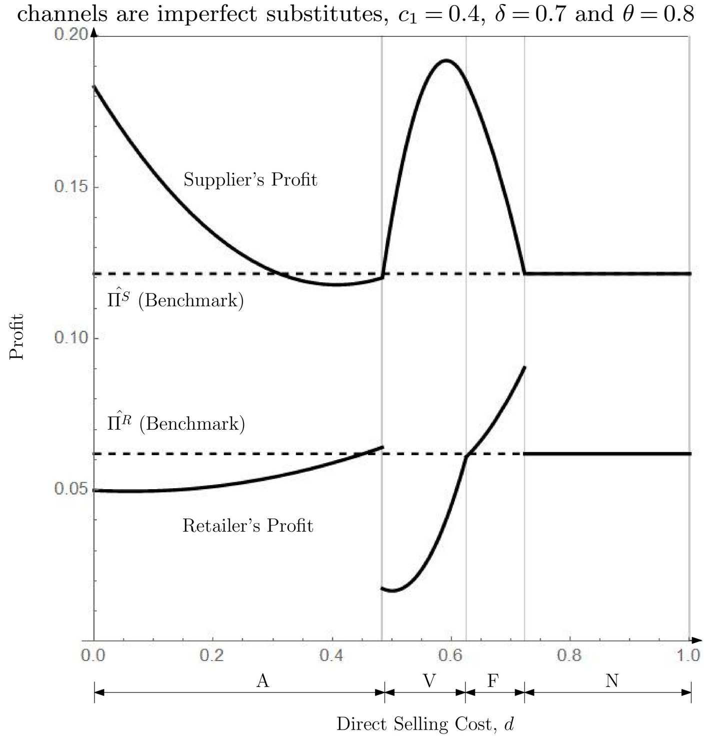
Figure EC.8 The players' profits in comparison with the benchmark (no encroachment) profits in the presence of multiple retailers, $c_1 = 0.4$, $\delta = 0.7$ and $n = 4$



EC.8 Imperfect Substitution between the two channels

In our base model in Section 3, we assumed that consumers are indifferent between the direct and the indirect channel. We relax this assumption in this section. We recognize that, in practice, some consumers may have a preference for one channel over the other. For example, some consumers might prefer the indirect channel for the physical experience of the product before buying it, whereas others might prefer the direct channel for the ease of access offered by this channel. We capture substitutability between the two channels by θ, where $\theta \in [0, 1]$. $\theta = 1$ represents perfect substitution between the two channels, as assumed in our base model, while $\theta = 0$ represents complete independence between the two channels. The inverse demand function for products sold through the indirect channel in the first period is $p_1^R = 1 - q_1^R$. The inverse demand functions for the products sold through the indirect and the direct channels in second-period are given by $p_2^R = 1 - q_2^R - \theta q_2^S$ and $p_2^S = 1 - q_2^S - \theta q_2^R$, respectively. Accordingly, the supplier's profit function is given as $\Pi^S = q_1^R(w_1 - c_1) + q_2^R(w_2 - c_2) + q_2^S(p_2^S - c_2 - d)$ and the retailer's profit function is given as $\Pi^R = q_1^R(p_1^R - w_1) + q_2^R(p_2^R - w_2)$. The sequence of decisions is the same as discussed in Section 3. We solve the game using backward induction to find SPNE. The proof of the equilibrium solution is similar to that of Proposition 1. However, the analysis of the impact of supplier encroachment on the players' profits when the two channels are imperfect substitutes ($\theta < 1$) is challenging for a general θ. We, therefore, limit our analysis to a fixed value of $\theta = 0.8$.

Figure EC.9 The players' profits in comparison with the benchmark (no encroachment) profits when the channels are imperfect substitutes, $c_1 = 0.4$, $\delta = 0.7$ and $\theta = 0.8$



Our analysis reveals that when the channels are sufficiently less substitutable, i.e., θ is sufficiently low, the supplier raises the second-period wholesale price while dropping the first-period wholesale price. By contrast, when the channels are perfect substitutes, i.e., $\theta = 1$, the cannibalization among the channels is high; hence the supplier drops the second-period wholesale price to ascertain that the direct channel does not unduly harm the indirect channel. The reason is simple. A lower θ value alleviates the fear of cannibalization between the two channels; hence the supplier sets a higher wholesale price in the second period. At the same time, a decrease in θ results in market expansion due to lower competition between the channels. This raises the benefit of cost reduction, motivating the supplier to further drop the first-period wholesale price to enhance cost learning. Due to the reduced cannibalization and faster cost learning, when $\theta = 0.8$, the supplier encroaches and sells directly for larger direct selling cost values. Specifically, when $\theta = 0.8$, he encroaches if $d < 0.723$ and sells directly if $d \leq 0.484$, as evident from Figure EC.9. By contrast, when $\theta = 1$, the supplier encroaches if $d < 0.656$ and sells directly if $d < 0.396$. Consistent with Proposition 5, we find that encroachment can be detrimental for the supplier for intermediate direct selling cost values, even when the two channels are not perfect substitutes, leading to either win-win, win-lose, lose-lose, or lose-win outcomes.