Funnier arithmetic sequences

Joint work with the students of the Master in Secondary Education Mathematics, 2024.

QUIZ: Educators are encouraged to propose quizzes on any topic, and in particular on the topic of arithmetic sequences. Quiz questions can be presented in class or given as homework assignment with a digital tool (so that pupils are made aware of the correct answer and possibly receive small explanations; further explanations can be provided directly by the educators).

FILL THE GAP: There exist colourful programs (that can be easily produced, in case of no track/ad-free option) that have as task to fill in gaps of arithmetic sequences. One not immediate example: *Consider the arithmetic sequence*

$$?,90,?,\underline{?},111,\dots$$

and determine the marked unknown value. Answer: 104.

ROUNDED VALUES: This question is better open-ended, so that pupils engage in its resolution not relying on trial and error. *Can you find an arithmetic sequence whose initial values, rounded, are the following?*

$$1,1,2,3,3,4$$

Answer: The positive multiples of 2/3. The data allows to estimate the common difference of the sequence and then a suitable initial term can also be found. (Since there are only finitely many terms, if one allows the initial value or the common difference to be real numbers or rational numbers with unbounded denominators, then there are infinitely many solutions, as can be seen with tiny variations of a solution).

JUMPS: A classical but important exercise is determine the "time" at which an arithmetic sequence exceeds a certain given bound.

COLLISIONS: Pupils can determine an arithmetic sequence knowing some terms that are common to other sequences. For example: *What is the arithmetic sequence that has the seventh term in common with the arithmetic sequence 1,3,5,7,… and the tenth term in common with the arithmetic sequence 0,2,4,6,8,…?*

FIX THE MISTAKE: *In the following initial values of an arithmetic sequence one mistake has been made, can you fix it?*

$$1, 3, ?, ?, 13, ?, 19, \dots$$

Answer: The value 3 should be replaced by 4. Pupils should use various choices of two of the given terms to cook up an arithmetic sequence, and see if the other values are fitting.

COUNTING ANTS OR SHEEPS: Funny stories can be invented to describe arithmetic sequence problems (warning: some pupils may have phobias, be ready to adapt the setting). For example: *A child regularly counts sheets for falling asleep. From sheep 25 to sheep 85, 5 minutes have passed. If the child falls asleep at sheep 100, how long did it take to the child to fall asleep?* Answer: Counting 60 sheep took 5 minutes (5 seconds per sheep), so counting 100 sheep took 500 seconds, which is 8 minutes and 20 seconds.

WINNING ARITHMETIC SEQUENCE: Take-away games are combinatorial games that can often be studied with the so-called backward induction. The two-players simple game example from https://www.worldscientific.com/doi/pdf/10.1142/9789813227361_0001 (21 chips; one can remove 1,2,3 of them; the player removing the last chip wins) has winning positions that form an arithmetic sequence, namely they are the numbers 0,4,8,12,16,20. Pupils can be challenged to determine that winning arithmetic sequence.