MatheMAGICAL Guess the number..<u>INSTRUCTIONS</u>

One person chooses a number from 1 to 15, and the mathemagician figures it out with four questions.
Namely, there are four lists of numbers and the person has to tell whether the chosen number is on the list.
The key feature behind the trick is that different numbers would lead to different answers.
Let's consider a simplified version of this trick with the three letters A,B,C and two questions: Is the letter in the list AB? Is the letter in the list AC? If the answers are (Yes,Yes), then it's the letter A. If the answers are (Yes,No), then it's the letter B. If the answers are (No, Yes), then it's the letter C.

It could be an exercise to figure out in the actual proposed trick how the lists have been chosen. The first list of numbers consists of the odd numbers from 1 to 15. The fourth list consists of the numbers from 1 to 15 that are greater than or equal to 8. Funnily, the third and fourth list look mysterious. In fact, they are quite regular if one draws them on the number line.
The hint that one could give is "binary system". Indeed, considering the numbers in the binary system, the four lists display the numbers from 1 to 15 that, respectively, have the following property: the last digit is 1; the penultimate digit is 1; the third digit from the right is 1; the fourth digit from the right is 1.
So the mathemagician has enough information to determine the number from 1 to 15 in the binary system. The mental calculations are in fact even easier than that, because a number from 1 to 15 in the binary system is decomposed as a sum of the integers 1,2,4,8. For example, 11=1+2+8.
Thus, an affirmative answer to the first (respectively, second third or fourth) question gives a contribution of +1 (respectively, +2 +4 or +8) and the mathemagician only has to compute a small sum. For example, suppose that the first and third questions were the ones with an affirmative answer (namely, the number was on the first and third list, and not on the other lists). Then the number is 1+4=5.
A mathematical exercise could be extending the lists for the numbers from 1 to 30. More generally, understanding how the lists are build in practice (the representation on the number line can help) so that one can compute them without transforming each integer into the binary system.

MatheMAGICAL

Guess the number..

Think of a number

From 1 to 15

3 9 11 7

13 1 5 15

Is your number here?

3

6

2

10

15

11

7

14

Is your number here?

4

5

13

7

12

15

6

14

Is your number here?

9

13

12

10

15

11

8

14

Is your number here?

Your number is ...