

The improbable outcome

Suppose that there is a bag with three balls, each ball being either red or blue. The colors are at random (more precisely, the color of each ball is decided by the toss of a fair coin). Suppose that you draw one ball from the bag, and it is red. Is it more likely, less likely, or equally likely that the other two balls are both red or both blue?

If you think about it, the sample you took is in fact the only possibility if all balls were red, and it's not very probable if there were one red ball and two blue balls. So, you would bet that the other two balls are red.

To make this more extreme, suppose that there are ninety-nine further balls and that you are told that the other balls all have the same color. It would be so unlikely to have drawn the only red ball out of one hundred balls, so you would bet that the balls are all red.

In fact, the above reasoning is misleading. The problem is that you evaluate the probability of a situation (about the colors of the unknown balls) making use of the probability of a different situation (picking one red ball, supposing that the ball colors are known).

To understand that the correct answer is „equally likely“, you may reformulate the problem as follows. You toss a fair coin three times. The first time you get heads. Then you have no information on the other two tosses, so it's equally likely that the second and third toss are two heads or two tails.

We may tend to believe in some „instinctive axiom“ that *what happens must be rather likely than unlikely*. And indeed, even in the absence of sufficient information, we would tend to make a theory for which the observed data are most fitting and likely. Say, if an alien would meet only one person on Earth and this person comes from Luxembourg, it may believe that Luxembourg is the largest country on Earth.