FUZZY TRUE/FALSE QUIZ

We are all very familiar with true/false quizzes, in which one has to determine correctly whether a statement is true or false. In mathematics, there is a bias on the false. Namely, *if true=white and false=black, then any shade of grey is declared to be black*. Something that is not always true, in all cases true, without exceptions true, is false. There is zero tolerance, a piece of truth is an absolute truth.

The above convention is practical and reassuring for mathematicians that can safely rely on true statements, however it is expectedly annoying and unfair for pupils. This is because "false" can mean "almost true" or "blatantly false" alike, and pupils loose points in a test for a false answer even if the answer is almost correct. Moreover, they also loose (sometimes too many) points in a test if they forget the tiny one trivial and special case where the given property fails to hold, while concentrating on the general case where the key idea lies…
Surely, one has to be 100% correct, however there are different shades of being incorrect. *A plausible answer is better than an implausible one.* For example, suppose that an exercise asks to compute the weight of 12 apples, the correct answer being 3.2 kilos. A pupil that computes 2.9 kilos gives a wrong answer, but a plausible one. One could mark that answering 3.2 tons is a worse answer (because it's a useful life skill distinguishing realistic from unrealistic, and pupils should train to make the plausibility check in every calculation).

In any case, to point out that false has different shades of false, pupils could be given "fuzzy true/false questions" of the following form:

Consider the following statement (…..). What is correct?

A: The statement is true.
B: The statement is false: it is true in most cases, but not all.
C: The statement is false: it is true in several cases, and it is false in several cases.
D: The statement is false: it is in very few cases true.
E: The statement is false: in no case it is true.

Examples of passing statements are:

A prime number is positive. (A)
A prime number is odd. (B)
A natural number is odd. (C)
A prime number is even. (D)
A prime number is negative. (E)

Let me point out one very important aspect. There are relatively few mathematical statements that are suitable as true/false questions in the above form. For example, consider the statement

"Three points in the plane are not on the same line".

We can safely exclude A, E and D. Since such exercise is meant to inspire critical thinking, one should accept both answers B and C. Indeed, from a probabilistic point of view it is unlikely that thee points picked at random are on the same line (so B is

acceptable), however there are infinitely many choices for which the property is true (so C is also acceptable). For such statements, it is appropriate to use a rougher scale of fuzzy true/false:

A: The statement is true.
B: The statement is false: it is in some cases true and in some cases false.
C: The statement is false: it no case it is true.

Logically speaking C here means that the contrary is true (meaning that the negation of the property holds for all cases).

Provided that pupils are repeatedly told that, mathematically, “slightly not true is already false”, it could be funny to make fuzzy quizzes for pupils orally, in class, with the following spontaneous answers:

True.
Possibly true.
Blatantly false!

For example: *A quadratic equation has only one solution* is “possibly true”. However, *A quadratic equation has 3 solutions* is “blatantly false”. The pupils can also be challenged to invent their own quizzes for their peers. The teacher could also spice up the above list with the category “outrageous” (for example, in my course of arithmetics, the worse possible mistake for me was having a negative remainder in an Euclidean division).

To wrap up, a teacher should always say that “almost true” is mathematically false as well as “very much not true”. However, at the appropriate occasion, it would be interesting to discuss with the pupils the “shades of false”. Indeed, this stimulates critical thinking and a deeper understanding, and it also avoids certain answers that are *correct for the wrong reason* (for example, if a person believes that all prime numbers are even, they would correctly say that “all prime numbers are odd” is false).

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