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On the importance of service parts when taking commonality and reliability decisions

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Competitive Original Equipment Manufacturers (OEMs) do not only sell equipment, but also provide service contracts that ensure proper functioning and uptime of equipment after the sale. This makes OEMs responsible for a large part of an equipment's life cycle. Therefore, OEMs aim to minimize the total life cycle costs of their equipment, in particular by commonality and reliability level decisions during the design phase. We consider these decisions for one component occurring in a family of systems. The commonality decision is about choosing a common component or dedicated components for the systems. The life cycle costs consist of design and production costs of all components, repair costs, inventory holding costs of service parts, and logistic downtime costs (i.e., downtime costs due to insufficient spare parts inventory). At many OEMs, the design department tends to exclude service parts considerations for the commonality and reliability level decisions. Excluding service parts leads to a simpler decision model, however, this may lead to non–optimal decisions.

We compare two approaches for the commonality and reliability decisions: the anticipating approach, which includes inventory holding costs of service parts and logistic downtime costs, and the non–anticipating approach, which excludes these costs. Since the cost function under the anticipating approach is intractable, we first derive a cost function that approximates the actual costs via an asymptotic analysis. Next, we show that the anticipating approach selects the common component more frequently in comparison to the non–anticipating approach. For many problem instances, the non–anticipating approach leads to decisions that have a much higher overall cost than using the anticipating approach. The relative cost difference can become arbitrarily large when the family of systems consists of many systems.

Key words: Commonality; Spare parts inventory; Life cycle costs; Reliability

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1. Introduction

Production and service companies use capital intensive systems to manufacture their products or render their services. For example, lithography systems are critical for operations in semiconductor manufacturing, jet engines and avionics are vital in the aviation industry, MRI scanners are critical in healthcare, and material handling systems are essential for distribution operations. The users of such capital intensive systems require a high system availability, as unavailability results in millions of dollars in production or service losses (Parent 2000, CNET News 2001, Kranenburg and Van Houtum 2009). However, realizing a high system availability at low costs is a major challenge without the help of the Original Equipment Manufacturers (OEMs). Therefore, the users of capital goods close service contracts (Cohen et al. 2006) with the OEMs of these systems (e.g., ASML, Philips, GE, Pratt & Whitney, Vanderlande, Bombardier). Under such a contract, the OEM becomes responsible for the system availability during the usage period and benefits (suffers) from the system to be operational (or malfunctioning). As a consequence, modern OEMs are responsible for the entire life cycle of systems, and are therefore primarily interested in minimizing the total life cycle costs (LCC). These are the total costs that are accrued over a system's life, from development and production to usage, maintenance and disposal; see Kumar et al. (2012). Notice that in some industries, it is more common to lease capital intensive systems. In those cases, the OEM is responsible for the system availability and it will aim to minimize the LCC.

To serve their markets as good as possible, OEMs want to offer a wide variety of systems, but the use of unique components in these systems typically increases the LCC. In an attempt to alleviate this burden, OEMs use common components in different systems of their product portfolio. For example, identical rotor blades are used in different aerospace engines or the same positioning sensors are used in various lithography systems. The main motivation for a variety of systems with commonality comes from a marketing and production perspective. It enables a firm to offer multiple systems with a relatively small increase in production costs. However, a commonality decision can also have a positive impact on after–sales services. In particular, commonality enables service parts pooling which reduces inventory costs. This cost benefit of common service parts can offset the potentially higher production costs. This is particularly the case in industries that use capital intensive systems. Therefore, one would expect that these benefits regarding service parts are considered in the decision to use common components instead of dedicated components. We will refer to this as the commonality decision. However, we have observed that design departments tend to omit service part considerations in the commonality decision due to the organizational structure in a company: the design department carries no responsibility for after–sales performance, but the after–sales department may suffer a lot from the decisions made in the design department.

We conceptualize commonality as follows. A system consists of components, and we say that components – from different systems – belong to the same *component family* when they fulfill the same functionality, but are not necessarily identical (Meyer and Lehnerd 1997). Therefore, the OEM can decide per component family whether to use a single *common component* for all systems or to use a *dedicated component* per system. Figure 1 illustrates two systems with dedicated components for component families I and II, and a common component for component family III. We do not consider partial commonality, where a common component is used for a subset of the systems. A component family may correspond to rotor blades in the case of aerospace engines, to positioning sensors in lithography systems, or to electric motors in MRI scanners.

Besides the commonality decision, an OEM also makes other important design decisions that affect after–sales performance and costs. One of these decisions is the reliability of a component since it determines how often a component might fail. We capture reliability in terms of the mean time between failures. An increased (reduced) reliability can reduce (increase) the required investment in service parts inventory. Therefore, there exists a trade–off between the cost of reliability

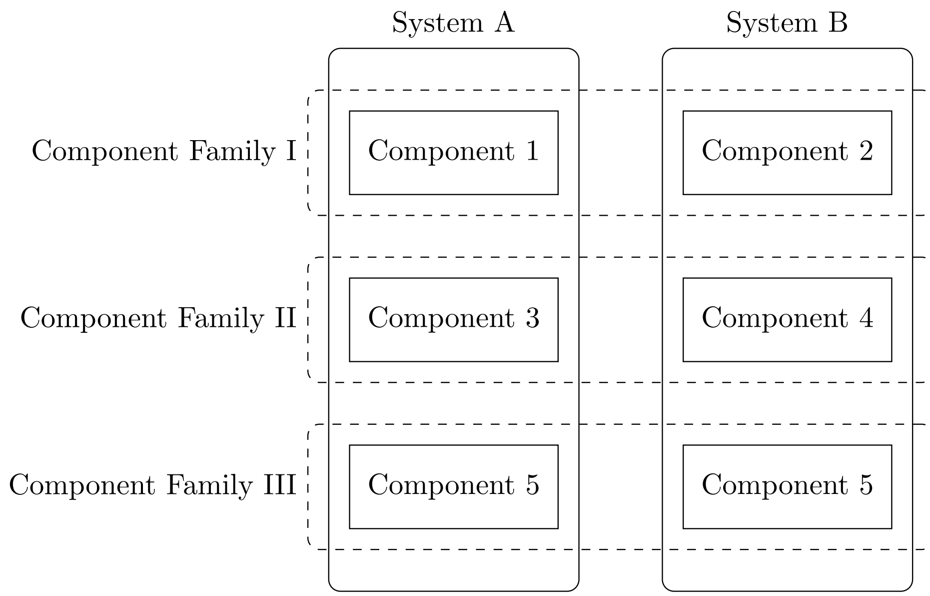

Figure 1 A schematic representation of our concepts

and the holding cost of service part inventory. The required number(s) of service parts will depend on the *total market size*, which we define as the total number of units that will be sold for all systems together. For the example in Figure 1, this is the number of units sold of System A plus the number of units sold of System B. Per system, the number of units sold is also called the *market segment*, and we refer to the division of the total market size into market segments as the *market segmentation*. When the OEM opts for dedicated components, the required number of service parts for each component will depend on the market segments. When the OEM opts for a common component, only service parts of the common component will be needed and the required number will only depend on the total market size.

Literature has studied the effects of service parts and costs on the two design decisions (commonality and reliability) separately. We study the decisions on commonality, reliability, and service parts in one model. We focus on circumstances under which service parts should be included in the life cycle cost calculations when taking commonality and reliability decisions into account. Our objective is to answer the following main research question for a single component family: *How does the commonality decision – under optimized reliability levels – differ between an OEM who considers service parts for its design decisions (commonality and reliability) and an OEM who does not consider this?* When studying this question, we look at the effect of various input parameters, and we pay extra attention to the effect of different market segmentations. This means that we study what happens to the difference for the commonality decision when relative market segment sizes vary, while the total market size remains constant; and we study the difference when the total market size increases, while the relative size of each segment remains constant. Choosing for a common component will give a portfolio effect for the spare parts inventory, which does not happen for dedicated components. Consequently, we expect that the total market size and the market segments have a strong effect on the commonality decision.

To study the main research question, we consider two approaches: one in which the OEM considers service parts, and one in which the OEM excludes service parts when deciding on the commonality and reliability of components. We refer to these approaches as the *anticipating approach* and the *non–anticipating approach*, respectively. For each approach, we develop two scenarios; one for the common component and one for the dedicated components. The goal of the OEM is to select the alternative – common or dedicated components – with corresponding optimal reliabilities and service part stock levels (if considered) such that the total LCC are minimized.

We make the following main contributions. (1) We show that simultaneously optimizing the reliability and service part stock levels under the anticipating approach becomes intractable. We propose to approximate the cost function, where the approximations are proven to be asymptotically equivalent to the original cost function as the cost of system downtime tends to infinity. This approximation of the LCC in the anticipating approach is very accurate and will be used to answer our main research question. (2) We characterize – for both approaches – a *switching threshold* for the unit production cost of a common component: below this threshold commonality yields lower life cycle costs than dedicated components. We show that the switching threshold under the anticipating approach is greater or equal to the switching threshold under the non-anticipating approach (see Theorem 2). (3) We study the special case in which the common and dedicated components have the same fixed reliability level. For this special case, we are able to obtain explicit analytical insights. In particular, we show that the (relative) difference for both the switching threshold and associated LCC, can become arbitrarily large. Furthermore, these differences decrease as a function of the total market size, and, in the case of equal relative unit cost factors for all dedicated components, they increase when market segments become more equal (see Propositions 1-4). (4) We execute a large computational experiment to investigate the difference between the switching thresholds and the cost difference under the two approaches. We show that the differences in switching thresholds and associated LCC are large for many problem instances (these differences range from 0% to 14% for the switching thresholds and from 0% to 20% for the associated LCC).

For the model that we develop for the anticipating approach, we use a similar modeling approach as Kim et al. (2017) and Öner et al. (2010). For the asymptotically equivalent cost functions as mentioned under main contribution (1), we build on asymptotic techniques similar to Huh et al. (2009) and Bijvank et al. (2014). The main contributions (2) to (4) are based on the model that we develop for the anticipating approach; this model is richer than any other model in the literature as we shall explain in Section 2 (see also Table 1).

The organization of this paper is as follows. In Section 2, we discuss related literature. In Section 3, we present the model and formulate the optimization problem under the anticipating approach. Next, in Section 4, we present the non-anticipating approach. In Section 5, we present the approximate cost functions for the anticipating approach, and we show how decisions are made under the anticipating approach. In Sections 6 and 7, we compare both approaches in an analytical way and with an extensive numerical study. We first obtain explicit analytical results for the special case when the reliability level is the same and fixed for all components; see Section 6. Next, we execute a large computational experiment for the general case in Section 7. We conclude this research in Section 8. Proofs of all lemmas, theorems and propositions are given in the appendices.

2. Literature

The first literature stream our work relates to is the stream that studies the combination of service parts and reliability. Huang et al. (2007) propose a profit maximization model that optimizes the reliability and considers sales revenues, production cost, and repair cost. Their objective is to support the reliability and warranty decisions. The authors take a life cycle approach, but do not consider inventory costs related to service parts. Kim et al. (2017)[1] study inefficiencies in traditional resource–based contracts (with payments based on the use of service parts, labor, and other resources) in contrast to performance–based contracts (PBC). Under a PBC, an OEM is paid for the realized system availability and hence has clear incentives to invest in reliability improvements. To obtain their results, they present a LCC minimization model that jointly optimizes reliability and service part stock levels. The LCC are comprised of design, production, service parts inventory, and backorder costs. Kim et al. (2017) find that reliability and service parts are substitutes for each other, and they derive analytical insights for different types of service contracts through the use of a game theoretical analysis. By contrast to Kim et al. (2017), Öner et al. (2010) do not take a game theoretic perspective, and focus on the after–sales services aspects of the LCC minimization problem. Their objective is to develop a quantitative model to support the decision on the reliability level of a critical component. The authors extend the cost function of Kim et al. (2017) by incorporating that demand during a stockout is satisfied via an emergency procedure. Öner et al. (2010) develop an efficient algorithm, and find that substantial cost savings (in the order of magnitude of 50%) can be realized by simultaneously optimizing reliability and the service part stock levels. Our work also incorporates this important trade–off between reliability and inventory costs for service parts. We do this by using a similar modeling approach as Kim et al. (2017) and Öner et al. (2010). However, our focus is on studying the commonality decision, for which we compare the life cycle cost function when using a common component to the life cycle cost function

[1] The work by Kim et al. (2017) was first published as a working paper, see Kim et al. (2007).

when using dedicated components. Both life cycle cost functions include the optimization of the reliability level(s) and spare parts inventory.

Second, our research strongly relates to commonality research, which is studied from multiple perspectives in the literature, e.g., marketing (Desai et al. 2001), new product development (Muffatto and Roveda 2000), and engineering (Fellini et al. 2004). Our work mostly relates to commonality studies from an operations management (OM) perspective. See Labro (2004) for an excellent review on this topic. Many of the OM studies focus on inventory implications of commonality in an Assemble–To–Order (ATO) setting (Baker et al. 1986, Hillier 2000, van Mieghem 2004, Song and Zhao 2009). Authors generally conclude that commonality allows for inventory pooling, which reduces the total costs. However, these results are derived in an ATO setting. In such a setting, a product demand is satisfied if *all* components are on stock (coupled demand). This differs from an after–sales service setting that we consider, in which demand typically occurs for each of the individual components. Commonality in such settings has been studied far less extensively. Kranenburg and van Houtum (2007) present a multi–item inventory model that focuses on the service part provisioning costs. These costs consist of inventory holding and transportation costs. They define a commonality percentage as the ratio of common items between so–called machine groups, and the effect of changes in this commonality percentage is studied. The authors confirm the fact that inventory pooling occurs in after–sales settings, and thus reduces total costs. However, their objective is limited to inventory related costs and the commonality decision is exogenous to the optimization problem. Thonemann and Brandeau (2000) endogenously determine the commonality decision for a major automobile manufacturer. The authors determine which features common components should have based on component requirements. They consider the after–sales service costs of service part inventories and production costs in their model, but they ignore the cost for system downtime or repairs. In a capital goods environment such as ours, the downtime and repair cost can be the majority of the life cycle costs.

None of the abovementioned works studies the effect that market characteristics have on the commonality decision in after–sales service problems. In fact, very few papers consider *how* market characteristics affect the attractiveness of commonality. Heese and Swaminathan (2006) take a marketing perspective when studying the commonality decision. They are also interested in the commonality decision under the consideration of an interdependency of two factors, cost–reduction effort and quality decisions. This feature superficially resembles ours, because we also study a commonality decision under the presence of an interdependency of the two factors. However, reliability and service part inventories are interdependent in our case. Heese and Swaminathan (2006) model the customer valuation of products with simple assumptions, which leads to a closed–form function for the total cost of components (random demand and after–sales aspects are excluded). They then focus on how the commonality decision depends on the size of two market segments. We consider a model that is closer to a capital goods setting, because we explicitly consider after–sales costs such as inventory costs, downtime costs and repair costs. Furthermore, we study in more detail how the commonality decision depends on the size of market segments and the total market size.

In addition to Heese and Swaminathan (2006), Fisher et al. (1999) also study how commonality decisions depend on market characteristics. The authors develop a highly stylized model in order to argue that commonality is less attractive when the various market segments differ more in size. They focus on the total costs that are incurred before the sale of a system (development and production costs) and do not consider the after–sales costs. The stylized model is then used to test their hypotheses that dedicated components are more attractive when the total market size increases, when the market segments differ less, and when the market is more segmented. Our work differs from Fisher et al. (1999) on two essential aspects. First, we focus on the after–sales costs as they constitute a large part of the life cycle costs in a capital goods setting. Second, we derive our insights for the dependence of the commonality decision on market characteristics from an analytical model, while the results of Fisher et al. (1999) are mainly driven by statistical methods to test the research hypotheses. Because our model and focus differ from Fisher et al. (1999) we also find different results for the market characteristics that determine when commonality is attractive.

We provide a comparison between our work and the most related research in Table 1.

Paper	Commonality	Reliability	Downtime	Repairs	Service parts	Life cycle costs	Market segmentation	Asymptotic analysis
Kranenburg and van Houtum (2007)	x		x	x	x			
Thonemann and Brandeau (2000)	x				x			
Huang, Liu, and Murthy (2007)		x		x		x		
Kim, Cohen, and Netessine (2017)		x			x	x		
Öner, Kiesmüller, and van Houtum (2010)		x	x	x	x	x		
Heese and Swaminathan (2006)	x						x	
Fisher, Ramdas, and Ulrich (1999)	x					x	x	
This research	x	x	x	x	x	x	x	x

Table 1 Overview of most related papers and the aspects that are included in each of them

3. Model

In this section, we present the model and optimization problems when the decision maker anticipates the costs related to after–sales services and the associated spare parts to provide these services as well as the procurement costs of the components.

The OEM offers her customers various systems $i \in J$, where J is the set of different systems. Furthermore, the OEM expects to sell $N_i > 0$ units of each system $i \in J$ at time $t = 0$ with a supplementary service contract. Upon the introduction of the systems, customers typically do not wait with purchasing a new system in order to be competitive. Furthermore, the customers buy a service contract upon the system purchase with a finite duration $T > 0$, because servicing is not their core business and oftentimes highly complex. Hence, we assume that all contracts are sold at time $t = 0$ and have an identical duration T (typically 10–15 years for capital intensive systems). Under such a contract, the OEM is penalized for downtime of the system and the OEM has to repair any failed parts during the contract duration T (notice that we use the word 'part' to refer to a physical unit of a component). After the contract duration, most customers either sell the system on a second hand market, outsource maintenance and after–sales services to a third party other than the OEM, or perform the after–sales services themselves.

Given the component structure for the systems (see Figure 1), the OEM has to determine for each component family whether to opt for a common component or dedicated components. We focus on a single component family that is critical for the system to function. The components are repairable upon failure. Furthermore, we assume that exactly one component of the family occurs in a system $i \in J$, e.g., one rotor blade assembly occurs in an aerospace engine or only one positioning sensor occurs in a lithography system. As a consequence, if the OEM opts for dedicated components, the number of parts of the dedicated component for system i that is installed in the field at time $t = 0$ is equal to the sales quantity N_i of system i . Furthermore, the component family can also be denoted by J. Each dedicated component $i \in J$ corresponds to the component used in system $i \in J$. We refer to N_i as the size of the market segment for system i. Hence, it is also the market segment for component i. In addition to the notation for the dedicated components, we denote the common component by index q. If the OEM opts for commonality, the number of parts of the common component installed in the field at time $t = 0$ is $N_q = \sum_{i \in J} N_i$. The set of all components is denoted by $I = J \cup \{q\}$.

At time $t = 0$, the OEM decides on the reliability level $\tau_i > 0$ for component i in terms of the Mean Time Between Failures (MTBF) and the initial on-hand stock level s_i of service parts. We assume that a base-stock policy with base-stock level s_i is followed after $t = 0$. This implies that upon the arrival of a demand for a service part, immediately a repair order is placed for one unit, so that the inventory position is equal to s_i at all times. We use the term reliability in the remainder instead of MTBF. At time $t = 0$, $N_i + s_i$ units of component i are produced with reliability τ_i (for all $i \in J$ if the OEM opts for dedicated components and for $i = q$ if the OEM opts for a common component). When choosing a higher reliability level, the unit production cost also increases, because higher quality materials are used or production steps are added. Moreover, there is an increasing marginal cost to increase the reliability level by one unit. This implies that the unit production cost is convex and increasing in the reliability level τ_i, see also Mettas (2000) and Öner et al. (2010). Hence, we introduce the function $c : [\tau_{\min}, \tau_{\max}] \to \mathbb{R}_+$, where $\mathbb{R}_+$ denotes the set of positive real numbers, i.e., $\mathbb{R}_+ = \{x \in \mathbb{R} \,|\, x > 0\}$, and $\tau_{\min}$ and $\tau_{\max}$ denote a given minimum and maximum reliability level, respectively, where $0 < \tau_{\min} \leq \tau_{\max}$. Furthermore, c is a continuous, twice differentiable, convex, and strictly increasing function. We remark that function c is identical for all components $i \in I$. However, in the case of dedicated components, the reliability level of the components in J may be different: there can be low–end, medium–end, and high–end components. As a result, the common component will be a high–end component. For instance, a low–end positioning sensor of a lithography system has a low resolution, while a high–end sensor has a high resolution. Increasing the reliability of a higher end component is typically expensive, because such components are typically more advanced (technology wise). As a consequence, it is more complex and thus more expensive to improve its reliability. We model this by multiplying the convex function c by a so–called relative unit cost factor $\beta_i > 0$ that enables us to differentiate between the various components in I, such that it is more expensive to increase the reliability for high–end components then it is for low-end components. Hence, the unit production cost for component $i \in I$ is given by $\beta_i c(\tau_i)$.

After the N_i parts of component i have been installed in the field at time $t = 0$, they operate independently with the same reliability τ_i. During operation, the parts fail and each failure triggers a demand at the stockpoint. We denote the stochastic demand for component i during the interval $[0, t]$ by $D(t, N_i, \tau_i)$ for any $t \geq 0$. Once an installed part fails, the total demand rate or total failure rate will decrease. However, for advanced capital goods, downtime is often in the order of hours, while the systems are used for multiple years. Therefore, using a stationary process to describe the total number of failures for a component works well, and it has the additional benefit that the model is further simplified; see Muckstadt (2005) and van Houtum and Kranenburg (2015). Moreover, we assume that the demand process for component i starts in stationarity at time $t = 0$ and it has independent and stationary increments. The demand process $D(t, N_i, \tau_i)$ is normally distributed with mean $\mathbb{E}[D(t, N_i, \tau_i)] = N_i t / \tau_i$ and standard deviation $\sqrt{\alpha N_i t / \tau_i}$, where the constant $\alpha > 0$ is the variance–to–mean ratio. Such a normally distributed demand process enables us to obtain a closed–form expression for the LCC under an optimal spare parts stock level. Our proposed demand process can approximate a (more conventional) Poisson process when $\mathbb{E}[D(t, N_i, \tau_i)]$ is sufficiently large and $\alpha = 1$, similar to Kim et al. (2017).

If a part fails, a service part is taken from stock (if possible) and it replaces the failed part. The failed part is sent to a repair shop, where it takes a lead time $L > 0$ time units to repair the failed part. In our problem, we consider components to perform similar functions and have similar technical complexity. As a result, the difference in repair lead time of the various parts is small and thus we assume L to be the same for all components $i \in I$. After repair, the part is forwarded to the service part stockpoint, see Figure 2. Note that this stockpoint corresponds to a stockpoint operating under a policy with base–stock level s_i and a deterministic lead time L.

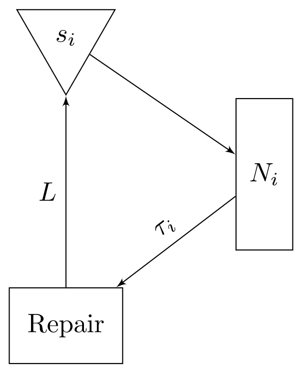

Figure 2 **The failure and repair process for the parts of component** $i \in I$

Furthermore, if a part fails and there is no service part available at the stockpoint, the replacement of the failed part in the field cannot occur. Consequently, the system in which the failed part was installed cannot operate until a new service part becomes available, i.e., we have a backorder. The OEM incurs a penalty cost $b > 0$ per unit time that the system cannot operate due to service part unavailability. This b is the penalty specified in the service contract for each time unit that a system is down.

Failed parts are repaired, and the cost per repair is related to the unit production cost of a component. More expensive components (in terms of the unit production cost) are also more expensive to repair, because more expensive materials may have been used or repairing the part requires extra steps. Hence, we assume that the cost per repair of component i is a linear scaling $r > 0$ of the unit production cost, i.e., $r\beta_i c(\tau_i)$. The expected number of repairs during the entire service contract period T can be derived from our demand process and the expected total repair cost is given by $r\beta_i c(\tau_i)\mathbb{E}[D(T, N_i, \tau_i)] = r\beta_i c(\tau_i) N_i T/\tau_i$.

The OEM owns all s_i service parts that are on stock or in repair during the components' life cycle $(0, T]$. Therefore, the OEM pays storage costs for all s_i service parts, either in repair or in stock. These storage costs represent the costs to keep the part on stock in the stockpoint or to have them in the repair shop (notice that the purchase costs are included in another cost term). The per time unit storage cost for one service part is a fraction $h \in (0,1)$ of its unit cost, i.e., $h\beta_i c(\tau_i)$. The total service parts storage costs over period $(0,T]$ are given by $hs_iT\beta_ic(\tau_i)$.

Combining all cost components results into general LCC function $\tilde{\pi}(\cdot)$, which can be expressed for a component $i \in I$ as

$$\tilde{\pi}(\tau_i, s_i, N_i, \beta_i) = \beta_i c(\tau_i)(N_i + s_i) + hs_iT\beta_ic(\tau_i) + r\beta_ic(\tau_i)\frac{N_iT}{\tau_i} + bT\mathbb{E}[(D(L, N_i, \tau_i) - s_i)^+]. \qquad (1)$$

Note that our model allows for common components to be more expensive than dedicated components, i.e., $\beta_q \geq \beta_i$ for all $i \in J$, as argued by van Mieghem (2004); and vice versa, i.e., $\beta_q < \beta_i$ for one or more $i \in J$, as discussed by Krishnan and Gupta (2001). Furthermore, we can set $\beta_i = 1$ for a particular component $i \in I$ without loss of generality. When all components of the system $i \in J$ are dedicated, the OEM seeks to minimize the sum of all costs per dedicated component:

$$(DP) \qquad \min_{\boldsymbol{\tau} \in \mathbb{R}_+^{|J|}, \boldsymbol{s} \in \mathbb{R}^{|J|}} \left\{ \sum_{i \in J} \tilde{\pi}(\tau_i, s_i, N_i, \beta_i) \right\}, \qquad (2)$$

with $\boldsymbol{\tau}$ and $\boldsymbol{s}$ denoting the vector of τ_i and s_i, $i \in J$, respectively. When all systems $i \in J$ use a common component, the OEM seeks to minimize the common component costs:

$$(CP) \qquad \min_{\tau_q \in \mathbb{R}_+, s_q \in \mathbb{R}} \left\{ \tilde{\pi}(\tau_q, s_q, N_q, \beta_q) \right\}. \qquad (3)$$

After solving both scenarios, the OEM selects the alternative with the lowest minimal cost.

Before we continue with applying the non-anticipating and anticipating approach in the next sections, we introduce two assumptions that will be used.

Assumption 1. *For each component $i \in I$, we have $bT > 2\beta_i c(\tau_{max})(1 + hT)$.*

The interpretation of Assumption 1 is that having a system down over the horizon $(0,T]$ is more than twice as expensive as producing a new part with the maximum reliability and keeping this part on stock throughout the period $(0,T]$. Such an assumption is typically satisfied in practice, because downtime of capital goods is expensive.

The second assumption is about extra properties for the function $c(\tau)$.

Assumption 2. *$c(\tau)$ satisfies the following properties: $\frac{c(\tau)}{\tau}$ is convex and $\frac{c(\tau)}{\sqrt{\tau}}$ is convex.*

Also Assumption 2 is not very restrictive, because there exists a large class of functions that satisfies this assumption: polynomial functions with one constant term and all others terms being at least of order two, and exponential forms; see e.g. Mettas (2000) and Öner et al. (2010).

4. Non–anticipating approach

Before we start analyzing the Problems (DP) and (CP), we show which decisions are obtained under the non–anticipating approach. This approach ignores service part considerations from the design decision. Consequently, there are no storage or backorder costs in the LCC. This yields a simplified cost function for each component $i \in I$:

$$\hat{\pi}(\tau_i, N_i, \beta_i) = \beta_i N_i \left(c(\tau_i) + \frac{rc(\tau_i)T}{\tau_i} \right).$$

Hence, we obtain for the following minimization problems:

$$(\widehat{DP}) \qquad \min_{\boldsymbol{\tau} \in \mathbb{R}_+^{|J|}} \left\{ \sum_{i \in J} \hat{\pi}(\tau_i, N_i, \beta_i) \right\}.$$

$$(\widehat{CP}) \qquad \min_{\tau_q \in \mathbb{R}_+} \left\{ \hat{\pi}(\tau_q, N_q, \beta_q) \right\},$$

Let $C_{\widehat{DP}}$ and $C_{\widehat{CP}}$ denote the minimal cost of Problems $(\widehat{DP})$ and $(\widehat{CP})$, respectively. Under the non-anticipating approach, the OEM solves $(\widehat{CP})$ and $(\widehat{DP})$ and she selects the alternative with the lowest expected production and repair cost between the common and dedicated components.

Let $\hat{\tau}_i^*$ minimize $\hat{\pi}(\tau_i, N_i, \beta_i)$ for component $i \in I$. Observing that $(\widehat{DP})$ is separable in the components $i \in J$ and given Assumption 2, we find all $\hat{\tau}_i^*$ efficiently by the result of Lemma 1.

Lemma 1. *The function $f(\tau) = c(\tau) + \frac{rc(\tau)T}{\tau}$ is twice differentiable and convex on $[\tau_{min}, \tau_{max}]$, and it is minimized by a positive, finite $\hat{\tau}^*$. This $\hat{\tau}^*$ solves the first order condition, and it holds that $\hat{\tau}_i^* = \hat{\tau}^*$ minimizes $\hat{\pi}(\tau_i, N_i, \beta_i)$ for each $i \in I$.*

The value of $\hat{\tau}_i^*$ specified in Lemma 1 is independent of β_i and N_i. With these reliability levels $\hat{\tau}_i^*$, we obtain the associated minimal cost of Problems $(\widehat{DP})$ and $(\widehat{CP})$ as follows,

$$C_{\widehat{DP}} = \sum_{i \in J} \beta_i N_i \left(c(\hat{\tau}^*) + \frac{rc(\hat{\tau}^*)T}{\hat{\tau}^*} \right)$$
$$C_{\widehat{CP}} = \beta_q N_q \left(c(\hat{\tau}^*) + \frac{rc(\hat{\tau}^*)T}{\hat{\tau}^*} \right).$$

Hence, $C_{\widehat{CP}} \leq C_{\widehat{DP}}$ if and only if $\beta_q N_q \leq \sum_{i \in J} \beta_i N_i$. Thus, the largest relative unit cost factor β_q for the common component under which $C_{\widehat{CP}} \leq C_{\widehat{DP}}$, i.e., under which the non-anticipating approach selects the common component, is given by

$$\hat{\Theta}(\boldsymbol{N}, \boldsymbol{\beta}) = \sum_{i \in J} \frac{N_i}{N_q} \beta_i, \tag{4}$$

where $\boldsymbol{N}$ and $\boldsymbol{\beta}$ denote the vector of all N_i and β_i, $i \in J$, respectively. We call $\hat{\Theta}(\boldsymbol{N}, \boldsymbol{\beta})$ the switching threshold for β_q (under the non-anticipating approach). This threshold is a convex combination of the relative unit cost factors β_i, $i \in J$, of the dedicated components, and the weights of this convex combination are equal to the relative sizes of the market segments. Notice that this threshold is at most equal to the largest relative unit cost factor of the dedicated components and this threshold is independent of the total market size.

To obtain the actual expected cost under the non-anticipating approach, the commonality and reliability decisions have to be complemented with the base-stock level decisions (for this purpose, Eq. (5) can be used; this formula gives the optimal base-stock level under a given reliability level for a component). Next, the actual expected cost is obtained via the function $\tilde{\pi}(\cdot)$ (cf. Eq. (1)).

5. Anticipating approach

In this section, we analyze the Problems (DP) and (CP). In Section 5.1, we study the optimization of the reliability and base-stock levels. Next, in Section 5.2, we consider the commonality decision.

5.1. Optimal reliability and base-stock levels

We first look at the minimization of each function $\tilde{\pi}(\tau_i, s_i, N_i, \beta_i)$, $i \in I$, for a given τ_i, i.e., as a function of the base-stock level s_i for the service parts. We obtain the result given in Lemma 2. Its proof is given in Appendix B and uses Assumption 1, which implies that the factor $\Phi^{-1}\left(\frac{bT-\beta_i c(\tau_i)(1+hT)}{bT}\right)$ in the optimal base-stock level is strictly positive.

Lemma 2. *For each component $i \in I$ and reliability $\tau_i \in [\tau_{min}, \tau_{max}]$, $\tilde{\pi}(\tau_i, s_i, N_i, \beta_i)$ is twice differentiable and strictly convex in s_i. The function $\tilde{\pi}(\tau_i, s_i, N_i, \beta_i)$ is minimized by a strictly positive, unique, finite value of $s_i^*(\tau_i)$ that solves the first order condition, and is given by*

$$s_i^*(\tau_i) = \mathbb{E}[D(L, N_i, \tau_i)] + \sigma[D(L, N_i, \tau_i)]\Phi^{-1}\left(\frac{bT - \beta_i c(\tau_i)(1+hT)}{bT}\right), \tag{5}$$

where $\Phi^{-1}(\cdot)$ denotes the inverse of the standard normal distribution.

When we insert the optimal stock level $s_i^*(\tau_i)$ into the cost functions of the Problems (DP) and (CP) (see Eq. (2) and Eq. (3), respectively), we obtain a complex embedding of $c(\tau_i)$ in $\tilde{\pi}(\tau_i, s_i^*(\tau_i), N_i, \beta_i)$: the inverse of the normal cumulative distribution includes the reliability τ_i through $c(\tau_i)$, which makes it intractable to determine the optimal reliability levels. However, the backorder cost rate b is high for capital goods, sometimes even in the order of tens of thousands of U.S. dollars per hour (Parent 2000, CNET News 2001). Therefore, we study the asymptotic behavior of the cost function in Eq. (2) and formulate an alternative cost function that is asymptotically equivalent as b approaches infinity. We follow the same procedure for the cost function in Eq. (3).

Because we now start using cost functions with different values for the backorder cost rate, we add b as an argument to our cost functions: $\tilde{\pi}(\tau_i, s_i^*(\tau_i, b), N_i, \beta_i \mid b)$. Because $s_i^*(\tau_i)$ also depends on b, we include b also as an argument in $s_i^*(\cdot)$. We prove that the cost function of dedicated components with b substituted by $b\beta_i c(\tau_i)$ is asymptotically equivalent to the original cost function of dedicated components as b tends to infinity, and we do the same for the cost function of the common component; see Theorem 1. Our proof is similar to Huh et al. (2009) and Bijvank et al. (2014), and we use some of their results (see Appendix C).

THEOREM 1. *For given* $\tau_i \in [\tau_{min}, \tau_{max}]$, *the functions* $\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)), N_i, \beta_i \mid b\beta_i c(\tau_i))$ *and* $\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b), N_i, \beta_i \mid b)$ *are asymptotically equivalent as* $b \to \infty$. *That is,*

$$\lim_{b\to\infty} \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)), N_i, \beta_i \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b), N_i, \beta_i \mid b)} = 1.$$

Furthermore, $\tilde{\pi}(\tau_q, s_q^*(\tau_q, b\beta_q c(\tau_q)), N_q, \beta_q | b\beta_q c(\tau_q))$ *and* $\tilde{\pi}(\tau_q, s_q^*(\tau_q, b), N_q, \beta_q \mid b)$ *are asymptotically equivalent as* $b \to \infty$. *That is,*

$$\lim_{b\to\infty} \frac{\tilde{\pi}(\tau_q, s_q^*(\tau_q, b\beta_q c(\tau_q)), N_q, \beta_q \mid b\beta_q c(\tau_q))}{\tilde{\pi}(\tau_q, s_q^*(\tau_q, b), N_q, \beta_q \mid b)} = 1.$$

Theorem 1 yields directly the following corollary.

COROLLARY 1. *For each* $i \in I$, *the optimal base-stock level* $s_i^*(\tau_i)$ *only depends on the reliability level* τ_i *through the mean lead time demand* $\mathbb{E}[D(L, N_i, \tau_i)] = \frac{N_i L}{\tau_i}$ *and through the lead time demand's standard deviation* $\sigma[D(L, N_i, \tau_i)] = \sqrt{\frac{N_i L}{\tau_i}}$, *for a large backorder cost* b.

Corollary 1 informs us that the optimal stock level of service parts is determined by the reliability level through the mean and standard deviation of the lead time demand for costly downtime. This is generally the case for capital goods, and thus the safety factor $\Phi^{-1}\left(\frac{bT-\beta_i c(\tau)(1+hT)}{bT}\right)$ can be substituted by the reliability–independent counterpart $\Phi^{-1}\left(\frac{bT-(1+hT)}{bT}\right)$.

5.1.1. Asymptotics as an approximation Next, we explore how well the approximations for the total LCC (i.e., when b is substituted by $b\beta_i c(\tau_i)$) represent the actual total LCC. We introduce Testbed 1 to provide this comparison and we use this testbed also in the remainder of this work.

TESTBED 1. We consider a full factorial testbed with data based on interviews held at a large OEM of lithography systems. We consider months as our time unit, i.e., τ_i, T, and L are in months; b is the cost per system down for one month; and h is a fraction per part per month. We use a modified version of a well-established unit cost function for $c(\tau)$, see Mettas (2000) and Öner et al. (2010):

$$c(\tau) = p_1 + p_2 \exp\left(k\frac{\tau}{\overline{\tau} - \tau}\right), \quad p_1, p_2, k > 0, \quad \tau_{\min} \le \tau \le \tau_{\max},$$

with $\overline{\tau} = 600$ months, $\tau_{\min} = 1$ month, and $\tau_{\max} = 300$ months. We generate a large testbed that considers 387,099 instances for dedicated components and 72,171 instances for the common component. We vary the following parameters on three levels, see Table 2.

	h	r	b	T	L	p_1	p_2	k
low	0.015	0.1	10,000	90	2	500	100	1.0
medium	0.030	0.2	100,000	180	3	1,500	300	1.5
high	0.050	0.3	1,000,000	360	4	5,000	1,000	2.0

Table 2 Parameter values for Testbed 1

The values for the downtime cost b range from low to high. The low value $b = \$10,000$ corresponds to a downtime cost of only \$333 per day or \$13.9 per hour. The high value $b = \$1,000,000$ corresponds to a downtime cost of \$33,333 per day or \$1,389 per hour. The unit production cost $c(\tau)$ ranges from \$600.2 to \$771.8 for $\tau \in [\tau_{\min}, \tau_{\max}]$ when the parameters p_1, p_2, and k are equal to their lowest values, and they range from \$6,003 to \$12,389 when p_1, p_2, and k are equal to their highest values.

Furthermore, we consider two component families (i.e., $|J| = 2$) and we vary the size of the market segments and the relative unit cost factors. We set $\beta_1 = 1$ and vary β_2 as follows: $\beta_2 \in \{1; 1.05; 1.1; 1.15; 1.2; 1.25; 1.3\}$. For N_i, we let $\sum_{i\in J} N_i = N_1 + N_2 = 400$ for all instances, and vary the size of one market segment on nine levels: $N_1 \in \{40, 80, 120, 160, 200, 240, 280, 320, 360\}$ and have $N_2 = 400 - N_1$. This yields $59 \times 3^8 = 387,099$ possible instances (due to duplicates when $\beta_1 = \beta_2$, we obtain 59 instead of 63 combinations for $(N_1, N_2, \beta_1, \beta_2)$) for each parameter combination.

For the common component we let $\beta_q \in \{1; 1.05; 1.1; \ldots; 1.5\}$ and $N_q = 400$. This results in $11 \times 3^8 = 72,171$ instances for commonality. ◇

We use Testbed 1 to see how well the approximation of the total LCC of dedicated components compares to the original total LCC function of dedicated components. We also compare the approximation and original total LCC function of common components. For these comparisons, we enumerate the reliability levels over $\{1, 2, \ldots, 300\}$, and we let $\tilde{\tau}_i^*$ and τ_i^* correspond to the optimal reliability level for component $i \in I$ when using the original LCC function $\tilde{\pi}(\tau_i, s_i^*(\tau_i, b), N_i, \beta_i \mid b)$ and the approximation $\tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)), N_i, \beta_i \mid b\beta_i c(\tau_i))$, respectively. Subsequently, we are interested in the relative cost differences for dedicated and common components

$$\Delta_J = \left(\frac{\sum_{i\in J} \tilde{\pi}(\tau_i^*, s_i^*(\tau_i^*, b\beta_i c(\tau_i^*)), N_i, \beta_i \mid b\beta_i c(\tau_i^*))}{\sum_{i\in J} \tilde{\pi}(\tilde{\tau}_i^*, s_i^*(\tilde{\tau}_i^*, b), N_i, \beta_i \mid b)} - 1 \right) \times 100\%, \quad \text{and}$$
$$\Delta_q = \left(\frac{\tilde{\pi}(\tau_q, s_q^*(\tau_q^*, b\beta_q c(\tau_q^*)), N_q, \beta_q \mid b\beta_q c(\tau_q^*))}{\tilde{\pi}(\tilde{\tau}_q^*, s_q^*(\tilde{\tau}_q^*, b), N_q, \beta_q \mid b)} - 1 \right) \times 100\%,$$

respectively. In particular, our interest goes out to the average, maximum, and minimum value of Δ_J and Δ_q. For Δ_J we find an average, maximum, and minimum value of 0.0091%, 0.0871%, and 0.0000%, respectively; and for Δ_q we find an average, maximum and minimum value of 0.0059%, 0.0543%, and 0.0000%. We observe that the relative cost differences are very low. Therefore, we conclude that the function $\pi()$ that approximates the LCC accurately.

For the remainder of this paper, we propose to study $\sum_{i\in J}\tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)), N_i, \beta_i | b\beta_i c(\tau_i))$ and $\tilde{\pi}(\tau_q, s_q^*(\tau_q, b\beta_q c(\tau_q)), N_q, \beta_q | b\beta_q c(\tau_q))$ in light of Theorem 1 and the results from Testbed 1. We do this to come to very accurate approximations for the optimal cost C_{DP} and C_{CP} when choosing for dedicated components and a common component, respectively. These approximate total LCC functions can be optimized easily over τ_i and τ_q, respectively. This results from the fact that the inverse of the standard normal cdf $\Phi^{-1}(\cdot)$ no longer depends on τ_i and thus is a constant. For brevity, we define $\pi(\tau_i, N_i, \beta_i) = \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)), N_i, \beta_i \mid b\beta_i c(\tau_i))$, of which the simplified expression is given in Lemma 3 with $\phi(\cdot)$ and $\Phi^{-1}(\cdot)$ the standard normal pdf and the inverse of the standard normal cdf, respectively (for the proof, see Appendix D). In this simplified expression, we have a factor $\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)$ and to ensure that this factor is well defined, we first make an additional assumption; notice that this assumption is generally satisfied.

Assumption 3. $bT > 1 + hT$

Lemma 3. *The function $\pi(\tau_i, N_i, \beta_i)$ can be rewritten as*

$$\pi(\tau_i, N_i, \beta_i) = \beta_i c(\tau_i)\left(1 + \frac{rT + L(1+hT)}{\tau_i}\right)N_i + b\beta_i c(\tau_i)T\sqrt{\frac{\alpha N_i L}{\tau_i}}\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right). \quad (6)$$

We use Eq. (6) to derive the following cost minimization problems based on our proposed approximation functions for the common component and the dedicated components:

$$(DP') \quad \min_{\boldsymbol{\tau}\in\mathbb{R}^{|J|}}\left\{\sum_{i\in J}\pi(\tau_i, N_i, \beta_i)\right\}.$$

$$(CP') \quad \min_{\tau_q\in\mathbb{R}}\left\{\pi(\tau_q, N_q, \beta_q)\right\}, \text{ and}$$

The minimal cost of Problems (DP') and (CP') is denoted by $C_{DP'}$ and $C_{CP'}$, respectively.

Just like Problem (DP), the dedicated components problem (DP') is separable in a problem per component $i \in J$. Hence, for each component $i \in J$, we can solve the optimization problem separately. We can rewrite the function $\pi(\tau_i, N_i, \beta_i)$ as

$$\pi(\tau_i, N_i, \beta_i) = \beta_i N_i c(\tau_i) + \beta_i(rT + L(1+hT))N_i\frac{c(\tau_i)}{\tau_i} + b\beta_i T\sqrt{\alpha N_i L}\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right)\frac{c(\tau_i)}{\sqrt{\tau_i}}.$$

We have assumed that $c(\tau)$ is convex. By Assumption 2, $\frac{c(\tau)}{\tau}$ and $\frac{c(\tau)}{\sqrt{\tau}}$ are convex. This results in the following corollary.

Corollary 2. *For each component $i \in I$, the function $\pi(\tau_i, N_i, \beta_i)$ is twice differentiable and convex, and it is minimized by a positive τ_i^*.*

Furthermore, we note that the reliability levels τ_i^* are independent of β_i, because the cost function $\pi(\tau_i, N_i, \beta_i)$ is proportional to the relative unit cost factor β_i.

5.2. Commonality decision

As the optimal reliability level is independent of β_i for each $i \in I$, it holds that

$$\pi(\tau_i^*, N_i, \beta_i) = \beta_i \pi(\tau_i^*, N_i, 1), \quad i \in I.$$

Hence, we find that

$$\begin{aligned} C_{DP'} &= \sum_{i \in J} \pi(\tau_i^*, N_i, \beta_i) = \sum_{i \in J} \beta_i \pi(\tau_i^*, N_i, 1), \\ C_{CP'} &= \pi(\tau_q^*, N_q, \beta_q) = \beta_q \pi(\tau_q^*, N_q, 1). \end{aligned}$$

It holds that $C_{CP'} \leq C_{DP'}$ if and only if $\beta_q \pi(\tau_q^*, N_q, 1) \leq \sum_{i \in J} \beta_i \pi(\tau_i^*, N_i, 1)$. So, the largest relative unit cost factor β_q under which the OEM will choose for a common component is

$$\Theta(\boldsymbol{N}, \boldsymbol{\beta}) = \frac{\sum_{i \in J} \beta_i \pi(\tau_i^*, N_i, 1)}{\pi(\tau_q^*, N_q, 1)}. \tag{7}$$

The value of $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$ is the switching threshold under the anticipating approach. When we consider $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$ as a function of the market segments $\boldsymbol{N}$ and/or the relative unit cost factors $\boldsymbol{\beta}$, we also refer to this function as the switching curve.

Earlier, in Section 4, we derived the switching threshold $\hat{\Theta}(\boldsymbol{N}, \boldsymbol{\beta})$ that is obtained under the non-anticipating approach. The following result states that the switching threshold under the anticipating approach is larger than or equal to the value under the non–anticipating approach.

Theorem 2. *The anticipating approach selects commonality more than the non–anticipating approach, i.e.,* $\Theta(\boldsymbol{N}, \boldsymbol{\beta}) \geq \hat{\Theta}(\boldsymbol{N}, \boldsymbol{\beta})$.

6. Comparison for a fixed reliability level

In this section, we compare the anticipating and non-anticipating approach for the special case that we have a fixed reliability level that is used for both the dedicated and common components. In that case, we obtain simpler cost functions and we are able to obtain interesting analytical insights for both the difference between the switching thresholds under both approaches and the difference in LCC.

6.1. Commonality decision

We assume that the interval $[\tau_{\min}, \tau_{\max}]$ for the reliability levels of the common and dedicated components consists of a single point denoted by τ_0. In that case, there is no longer a reliability decision and we get simpler cost functions. The function $c(\tau)$ reduces to a single point as well; let $c_0 := c(\tau_0)$.

In this section, we study the absolute difference $\Delta_\Theta(\boldsymbol{N}, \boldsymbol{\beta}) = \Theta(\boldsymbol{N}, \boldsymbol{\beta}) - \hat{\Theta}(\boldsymbol{N}, \boldsymbol{\beta})$ between the switching thresholds of the anticipating and non-anticipating approach. Notice that we know, by Theorem 2, that $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$ is larger than $\hat{\Theta}(\boldsymbol{N}, \boldsymbol{\beta})$ (i.e., $\Delta_\Theta(\boldsymbol{N}, \boldsymbol{\beta}) \geq 0$).

We are able to derive a simple expression for the difference $\Delta_\Theta(\boldsymbol{N},\boldsymbol{\beta})$. The functions $\pi(\tau_i^*,N_i,1)$ in Eq. (7) for $\Theta(\boldsymbol{N},\boldsymbol{\beta})$ can be simplified to

$$\begin{aligned}
\pi(\tau_i^*,N_i,1) &= \pi(\tau_0,N_i,1)\\
&= c_0\left(1+\frac{rT+L(1+hT)}{\tau_0}\right)N_i + bc_0T\sqrt{\frac{\alpha N_i L}{\tau_0}}\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right)\\
&= c_0\sqrt{N_q}\left[A_1\sqrt{N_q}\zeta_i + A_2\sqrt{\zeta_i}\right],\quad i\in I, \qquad (8)
\end{aligned}$$

where

$$\begin{aligned}
A_1 &= \left(1+\frac{rT+L(1+hT)}{\tau_0}\right),\\
A_2 &= bT\sqrt{\frac{\alpha L}{\tau_0}}\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right),\\
\zeta_i &= N_i/N_q,\ i\in I.
\end{aligned}$$

Notice that the factors A_1 and A_2 are independent of the market segments $\boldsymbol{N}$ and relative unit cost factors $\boldsymbol{\beta}$, and they are both strictly positive. Each ζ_i, $i\in I$, denotes the market share of component i; $0<\zeta_i<1$ for each dedicated component $i\in J$ and $\zeta_q=1$ for the common component. For $\Theta(\boldsymbol{N},\boldsymbol{\beta})$, we now obtain the following formula (using Eq. (7)):

$$\begin{aligned}
\Theta(\boldsymbol{N},\boldsymbol{\beta}) &= \frac{\sum_{i\in J}\beta_i c_0\sqrt{N_q}\left[A_1\sqrt{N_q}\zeta_i + A_2\sqrt{\zeta_i}\right]}{c_0\sqrt{N_q}\left[A_1\sqrt{N_q}+A_2\right]}\\
&= \frac{A_1\sqrt{N_q}\sum_{i\in J}\zeta_i\beta_i + A_2\sum_{i\in J}\sqrt{\zeta_i}\beta_i}{A_1\sqrt{N_q}+A_2}. \qquad (9)
\end{aligned}$$

Next, the formula for the switching threshold $\hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$ can be rewritten as follows by using Eq. (4)

$$\hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta}) = \sum_{i\in J}\zeta_i\beta_i\ .$$

Hence, for the absolute difference between both switching thresholds, we obtain

$$\begin{aligned}
\Delta_\Theta(\boldsymbol{N},\boldsymbol{\beta}) &= \frac{A_2(\sum_{i\in J}\sqrt{\zeta_i}\beta_i - \sum_{i\in J}\zeta_i\beta_i)}{A_1\sqrt{N_q}+A_2} \qquad (10)\\
&= \frac{\sum_{i\in J}\sqrt{\zeta_i}(1-\sqrt{\zeta_i})\beta_i}{1+\frac{A_1}{A_2}\sqrt{N_q}}\ . \qquad (11)
\end{aligned}$$

Eq. (11) allows to show how $\Delta_\Theta(\boldsymbol{N},\boldsymbol{\beta})$ depends on various input parameters (see Proposition 1) and how this difference depends on the market sizes under equal relative unit cost factors for all dedicated components (see Proposition 2).

PROPOSITION 1. *Let* $\tau_{min}=\tau_{max}=\tau_0$ *and* $c_0 := c(\tau_0)$. *Then* $\Delta_\Theta(\boldsymbol{N},\boldsymbol{\beta})$
(a) increases as a function of A_1 *and decreases as a function of* A_2*;*

(b) increases as a function of each relative unit cost factor β_i, $i \in J$, and the variance-to-mean ratio α;

(c) decreases as a function of the market size N_q and the repair lead time r.

PROPOSITION 2. *Let $\tau_{min} = \tau_{max} = \tau_0$ and $c_0 := c(\tau_0)$ and suppose that the relative unit cost factors of all dedicated components are equal, i.e., $\beta_i = \beta_1$ for all $i \in J \setminus \{1\}$. Then:*

(a) More equal market segments ζ_i, $i \in J$, give a higher $\Delta_\Theta(\boldsymbol{N}, \boldsymbol{\beta})$;

(b) If all market segments are equal, then $\Delta_\Theta(\boldsymbol{N}, \boldsymbol{\beta}) = \beta_1(\sqrt{|J|} - 1)\left[1 + \frac{A_1}{A_2}\sqrt{N_q}\right]^{-1}$. This difference is increasing as a function of the number of market segments $|J|$ and can grow to an arbitrarily large number.

Of particular interest are the results for how the difference $\Delta_\Theta(\boldsymbol{N}, \boldsymbol{\beta})$ depends on the total market size and the market segments. Part (c) of Proposition 1 shows that the difference decreases as a function of the total market size. Part (a) of Proposition 2 says that more equal market segments lead to a larger difference in the case of equal relative unit cost factors. Part (b) of that proposition shows that the absolute difference can become very large.

6.2. Difference in cost

In this section, we continue with the analysis of the previous section and compare the cost of the two approaches. Both approaches result in selecting dedicated components when $\beta_q \leq \hat{\Theta}(\boldsymbol{N}, \boldsymbol{\beta})$ and common components are selected when $\beta_q > \Theta(\boldsymbol{N}, \boldsymbol{\beta})$. Because all reliability levels are equal to τ_0, we obtain the same optimal cost for both approaches in these cases. Hence, for studying the difference in optimal cost, we can limit ourselves to values of $\beta_q \in (\hat{\Theta}(\boldsymbol{N}, \boldsymbol{\beta}), \Theta(\boldsymbol{N}, \boldsymbol{\beta})]$.

For component i, the LCC can be rewritten similar to Eq. (8):

$$\pi(\tau_0, N_i, \beta_i) = \beta_i c_0 \sqrt{N_q}\left[A_1\sqrt{N_q}\zeta_i + A_2\sqrt{\zeta_i}\right], \quad i \in I.$$

For $\beta_q \in (\hat{\Theta}(\boldsymbol{N}, \boldsymbol{\beta}), \Theta(\boldsymbol{N}, \boldsymbol{\beta})]$, the non-anticipating approach selects dedicated components, which results in the following expected cost:

$$\sum_{i\in J}\pi(\tau_0, N_i, \beta_i) = c_0\sqrt{N_q}\left[A_1\sqrt{N_q}\sum_{i\in J}\zeta_i\beta_i + A_2\sum_{i\in J}\sqrt{\zeta_i}\beta_i\right].$$

Obviously, this cost is independent of β_q. The anticipating approach selects the common component for $\beta_q \in (\hat{\Theta}(\boldsymbol{N}, \boldsymbol{\beta}), \Theta(\boldsymbol{N}, \boldsymbol{\beta})]$, which results in the following expected cost:

$$\pi(\tau_0, N_q, \beta_q) = \beta_q c_0\sqrt{N_q}\left[A_1\sqrt{N_q} + A_2\right].$$

This cost is linearly increasing as a function of β_q. For $\beta_q = \Theta(\boldsymbol{N}, \boldsymbol{\beta})$, this cost is equal to the cost of the non-anticipating approach (because choosing dedicated components is equally expensive at that point). Further this cost is lowest when $\beta_q \downarrow \hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$ and the relative cost difference is largest in that case. In the rest of this section, we focus on the limiting value of the relative cost difference.

For $\beta_q \in (\hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta}), \Theta(\boldsymbol{N},\boldsymbol{\beta})]$, the relative cost increase when using the non-anticipating approach instead of the anticipating approach is

$$\begin{aligned}\Delta\pi(\boldsymbol{N},\boldsymbol{\beta},\beta_q) &= \left(\frac{\sum_{i\in J}\pi(\tau_0,N_i,\beta_i)}{\pi(\tau_0,N_q,\beta_q)} - 1\right)\\ &= \frac{A_1\sqrt{N_q}\sum_{i\in J}\zeta_i\beta_i + A_2\sum_{i\in J}\sqrt{\zeta_i}\beta_i}{\beta_q\left[A_1\sqrt{N_q}+A_2\right]} - 1\ , \quad \hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta}) < \beta_q \le \Theta(\boldsymbol{N},\boldsymbol{\beta}).\end{aligned}$$

For $\beta_q \downarrow \hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$, this relative cost difference goes to a value that we denote by $\Delta\pi^{\sup}(\boldsymbol{N},\boldsymbol{\beta})$ and equals

$$\Delta\pi^{\sup}(\boldsymbol{N},\boldsymbol{\beta}) = \frac{A_2\left(\sum_{i\in J}\sqrt{\zeta_i}\beta_i - \hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta})\right)}{\hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta})\left[A_1\sqrt{N_q}+A_2\right]} = \frac{\Delta_\Theta(\boldsymbol{N},\boldsymbol{\beta})}{\hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta})}, \tag{12}$$

where the second equality results from Eq. (10). This means that, when taking the supremum of the relative cost difference for the non-anticipating approach in comparison to the anticipating approach, this relative difference is precisely the same as the relative difference in the switching thresholds of both approaches. Based on Eq. (12), we can obtain similar results for this cost difference supremum as the results given in Propositions 1 and 2 for the difference between the switching thresholds.

Proposition 3. *Let $\tau_{min} = \tau_{max} = \tau_0$ and $c_0 := c(\tau_0)$. Then $\Delta\pi^{\sup}(\boldsymbol{N},\boldsymbol{\beta})$*
(a) increases as a function of A_1 and decreases as a function of A_2;
(b) increases as a function of the variance to mean ratio α;
(c) decreases as a function of the market size N_q and the repair lead time r.

Proposition 4. *Let $\tau_{min} = \tau_{max} = \tau_0$ and $c_0 := c(\tau_0)$ and suppose that the relative unit cost factors of all dedicated components are equal, i.e., $\beta_i = \beta_1$ for all $i \in J \setminus \{1\}$. Then:*
(a) The formula for $\Delta\pi^{\sup}(\boldsymbol{N},\boldsymbol{\beta})$ simplifies to $\Delta\pi^{\sup}(\boldsymbol{N},\boldsymbol{\beta}) = \sum_{i\in J}\sqrt{\zeta_i}(1-\sqrt{\zeta_i})\left[1+\frac{A_1}{A_2}\sqrt{N_q}\right]^{-1}$;
(b) More equal market segments ζ_i, $i\in J$, give a higher $\Delta\pi^{\sup}(\boldsymbol{N},\boldsymbol{\beta})$;
(c) If all market segments are equal, then $\Delta\pi^{\sup}(\boldsymbol{N},\boldsymbol{\beta}) = (\sqrt{|J|}-1)\left[1+\frac{A_1}{A_2}\sqrt{N_q}\right]^{-1}$. This difference is increasing as a function of the number of market segments $|J|$ and can grow to an arbitrarily large number.

The results in Proposition 3 follow directly from Proposition 1. Notice that we cannot conclude that $\Delta\pi^{\sup}(\boldsymbol{N},\boldsymbol{\beta})$ is increasing as a function of each relative unit cost factor β_i, $i\in J$, because $\hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$ also depends on this factor.

The results in Proposition 4 follow from Proposition 2. Part (b) says that more equal market segments lead to a larger supremum for the relative cost difference in the case of equal relative unit cost factors. Part (c) shows that this supremum can become very large.

7. Comparison for the general case

In this section, we give insights in the differences between the anticipating and non-anticipating approach with respect to the commonality decision (see Section 7.1) and the LCC (see Section 7.2) for the general case with optimized reliability levels.

7.1. Commonality decision

Let us start with studying the switching curves for both approaches in the following example.

Example 1. We consider a representative parameter setting for an OEM in the semiconductor industry. Let $|J| = 2$, $h = 0.03$ per part per month, $r = 0.2$ per repair, $L = 3$ months, $T = 360$ months, $b = \$1 \times 10^6$ per month per system down, and $\alpha = 1$. Moreover, we consider the size of the market segment $N_1 \in \{1, \ldots, 399\}$ and let $N_1 + N_2 = 400$. Furthermore, we use $c(\tau) = 5,000 + 1,000 \exp\left(\frac{\tau}{600-\tau}\right)$ in \$ per unit, where $\tau \in [1, 300]$. In Figures 3 and 4, both switching curves are given for equal relative unit cost factors ($\beta_1 = \beta_2 = 1$) and for unequal relative unit cost factors ($\beta_1 = 1$, $\beta_2 = 1.1$), respectively.

Figures 3 and 4 clearly show that the switching threshold $\hat{\Theta}(\boldsymbol{N}, \boldsymbol{\beta})$ under the non-anticipating approach is a convex combination of the relative unit cost factors β_1 and β_2 with the relative size of the market segments ($N_1/400$ and $N_2/400$) as weights. The switching threshold $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$ is greater than $\hat{\Theta}(\boldsymbol{N}, \boldsymbol{\beta})$. Under equal relative unit cost factors, we see that the maximum of $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$ is largest when both market segments are equal in size (this matches with the insight given in Proposition 2a). When the relative unit cost factor of the second market segment is larger than the relative unit cost factor of the first market segment, we see that the maximum level of $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$ is obtained when the second market segment is larger in size than the first market segment.

In this example with optimized reliability levels, we obtain a noticeable difference with what we found in Proposition 1b for a single fixed reliability level. In this example, we find that $\beta_2 = 1.1$ gives smaller differences $\Delta_{\Theta}(\boldsymbol{N}, \boldsymbol{\beta})$ than $\beta_2 = 1$ by comparing Figures 5 and 6. When reliability levels can be optimized, a higher reliability level will be chosen for an expensive component (component 2 in Example 1) than for a cheaper component (component 1). Also for the common component, a different level can be chosen. This leads apparently to the smaller differences, while we would have obtained larger differences in the case with a single fixed reliability level (see Proposition 1b). ◇

Let us now consider the difference between the switching thresholds $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$ and $\hat{\Theta}(\boldsymbol{N}, \boldsymbol{\beta})$ for all instances in Testbed 1 (see Section 5.1.1; we use the 387,099 instances for which $(N_1, N_2, \beta_1 \beta_2)$ have been specified). We find that $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$ is on average 5.10% larger than $\hat{\Theta}(\boldsymbol{N}, \boldsymbol{\beta})$; the corresponding minimum and maximum values are equal to 0.86% and 14.05%, respectively. Note that this is a large difference. It means that, under the use of the anticipating approach, the production (or purchasing) cost for a common component may be 5.10% more expensive than a dedicated component and be still attractive.

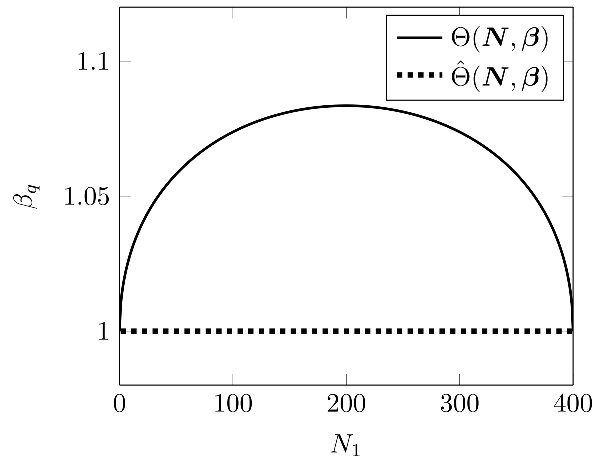

Figure 3 **Illustration of $\Theta(\boldsymbol{N},\boldsymbol{\beta})$ and $\hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$ for Example 1, with** $\beta_1 = \beta_2 = 1$

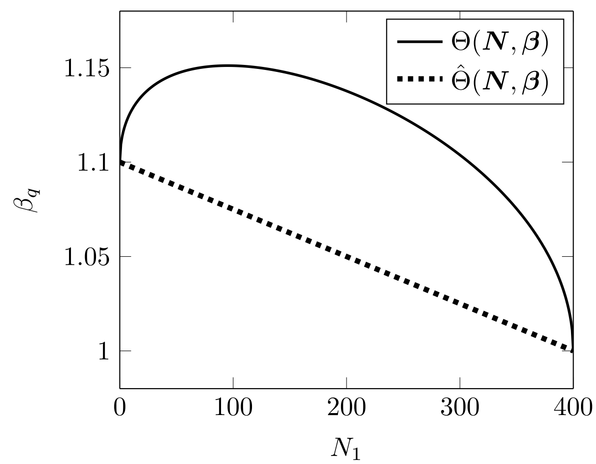

Figure 4 **Illustration of $\Theta(\boldsymbol{N},\boldsymbol{\beta})$ and $\hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$ for Example 1, with** $\beta_1 = 1, \beta_2 = 1.1$

7.2. Difference in cost

The objective of this section is to explore the main motivation of an OEM to consider the anticipating approach over the non–anticipating approach, and study the reduction in LCC. We perform an extensive numerical study to present insights in the magnitude of the cost savings.

We compare the optimal cost obtained under the anticipating approach to the cost of the non–anticipating approach. For the anticipating approach, the optimal cost is given by $\min\{\pi(\tau_q^*, N_q, \beta_q), \sum_{i\in J} \pi(\tau_i^*, N_i, \beta_i)\}$, where τ_q^* and τ_i^*, $i \in J$, are the optimal reliability levels. Recall that the choice for commonality is made if and only if $\pi(\tau_q^*, N_q, \beta_q) \leq \sum_{i\in J} \pi(\tau_i^*, N_i, \beta_i)$, which is equivalent to the condition $\beta_q \leq \Theta(\boldsymbol{N},\boldsymbol{\beta})$. The non-anticipating approach uses approximate cost functions $\hat{\pi}(\tau_q, N_q, \beta_q)$ and $\sum_{i\in J} \hat{\pi}(\tau_i, N_i, \beta_i)$ for common and dedicated components, respectively (see Section 4). These functions are all optimized by the same reliability level, denoted by $\hat{\tau}^*$, and the choice for commonality is made if and only if $\hat{\pi}(\tau^*, N_q, \beta_q) \leq \sum_{i\in J} \hat{\pi}(\tau^*, N_i, \beta_i)$, which is equivalent to the condition $\beta_q \leq \hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$. In general, the non-anticipating approach will result in suboptimal reliability levels, and it will lead to a suboptimal commonality decision if and only if the relative unit cost factor for the common component β_q lies in the interval $(\hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta}), \Theta(\boldsymbol{N},\boldsymbol{\beta})]$. We are interested in the relative increase in LCC when the non-anticipating approach is used instead of the anticipating approach. This relative cost increase is given by

$$\Delta\pi(\boldsymbol{N},\boldsymbol{\beta},\beta_q) = \left(\frac{\gamma\pi(\hat{\tau}^*, N_q, \beta_q) + (1-\gamma)\sum_{i\in J}\pi(\hat{\tau}^*, N_i, \beta_i)}{\min\{\pi(\tau_q^*, N_q, \beta_q), \sum_{i\in J}\pi(\tau_i^*, N_i, \beta_i)\}} - 1\right) \times 100\%,$$

where γ is a binary variable such that $\gamma = 1$ if $\hat{\pi}(\hat{\tau}_q^*, N_q, \beta_q) \le \sum_{i\in J}\hat{\pi}(\hat{\tau}_i^*, N_i, \beta_i)$ and $\gamma = 0$ otherwise. We also refer to this difference as the *LCC difference.*

In Figures 5 and 6, the LCC difference $\Delta\pi(\boldsymbol{N},\boldsymbol{\beta},\beta_q)$ is depicted for Example 1. As we see in these figures, the LCC difference can be large (up to 10%) for problem instances where the non-anticipating approach gives a suboptimal commonality decision. For other instances, the LCC difference is small. The largest differences are obtained when the relative unit cost factor β_q of the common component is just above the switching threshold $\hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$ of the non-anticipating approach (notice that we obtained this property analytically in Section 6.2 for the case with a fixed reliability level).

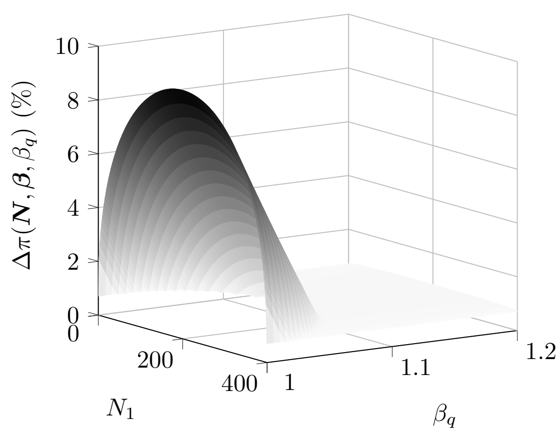

Figure 5 **Numerical illustration of** $\Delta\pi(\boldsymbol{N},\boldsymbol{\beta},\beta_q)$ **for Example 1 with** $\beta_1 = \beta_2 = 1$

Next, we study the LCC difference $\Delta\pi(\boldsymbol{N},\boldsymbol{\beta},\beta_q)$ for the instances in Testbed 2.

Testbed 2. For this testbed, we reuse the 387,099 instances of Testbed 1 for the dedicated components. For each of these instances, we consider $\beta_q \in \{1; 1.05; 1.1; \ldots; 1.5\}$ in order to determine $\Delta\pi(\boldsymbol{N},\boldsymbol{\beta},\beta_q)$. This gives a total of $387,099 \times 11 = 4,258,089$ instances. ◇

Based on these instances, we observe an average, maximum and minimum LCC difference of 1.65%, 19.58%, and 0.04%, respectively. In 3,850,809 instances the anticipating and non–anticipating approach make the same commonality decision. For these instances, we observe an average, maximum and minimum LCC difference of 1.35%, 19.53%, and 0.04%, respectively. Similarly, the average, maximum and minimum LCC difference for the remaining 407,280 instances

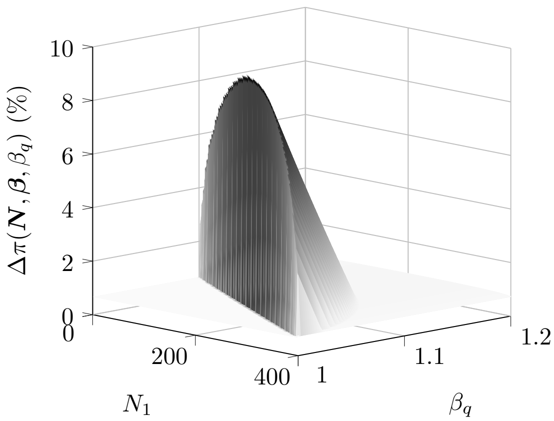

Figure 6 **Numerical illustration of $\Delta\pi(\boldsymbol{N}, \boldsymbol{\beta}, \beta_q)$ for Example 1 with $\beta_1 = 1, \beta_2 = 1.1$**

with a different commonality decision correspond to 4.49%, 20.05%, and 0.04%, respectively. These numbers indicate that making the commonality and reliability decisions based on the suboptimal non-anticipating approach (i.e., by excluding the effect on service parts) can substantially increase the LCC.

Finally, we also studied the importance of the various model parameters by performing a sensitivity analysis. We show how the LCC difference depends on each of the parameters that is varied in Testbed 2. That shows that the LCC difference is most sensitive for the repair rate r, the holding cost rate h and the repair leadtime L. The details of this analysis and further explanations are included in Appendix H.

8. Conclusion

In this research, we studied an OEM who is responsible for its equipment's life cycle and is therefore interested in minimizing the life cycle costs (LCC). It can choose to either use a single common component or dedicated components in its equipment and simultaneously should decide upon the components' reliability levels. We make four major contributions in this work: (1) we show that optimizing reliability levels and service part stock levels simultaneously is intractable and we propose an asymptotically equivalent and tractable cost function; (2) we define an approach considering service parts in decision making and an approach that does not. For each approach, we characterize a switching threshold for the cost of a common component such that commonality yields lower LCC; (3) We show that the commonality decision itself can substantially differ between both approaches, and the cost effects between both approaches are often substantial; (4) Lastly, we present analytical insights into the commonality decision and LCC difference for the special case in which the reliability levels are fixed.

This work illustrates the cost effects and underlines the importance of service part considerations early in an equipment's design phase. However, making these considerations in practice is far from trivial as there exists a high amount of uncertainty during the design phase. Therefore, future research may address this, e.g., uncertainty in the reliability estimates. Moreover, the benefits of considering service parts is not restricted to questions of commonality and reliability, but its impact spans a wider range of (design) topics, for instance the definition of what constitutes a service part (Parada Puig and Basten 2015).

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References

K. R. Baker, M. J. Magazine, and H. L. W. Nuttle. The effect of commonality on safety stock in a simple inventory model. *Management Science*, 32(8):982–988, 1986.

M. Bijvank, W. T. Huh, G. Janakiraman, and W. Kang. Robustness of order-up-to policies in lost-sales inventory systems. *Operations Research*, 62(5):1040–1047, 2014.

CNET News. California power outages suspended – for now, 2001. URL `https://www.cnet.com/news/california-power-outages-suspended-for-now/`.

M. A. Cohen, N. Agrawal, and V. Agrawal. Winning in the aftermarket. *Harvard Business Review*, 84(5): 129–138, 2006.

P. Desai, S. Kekre, S. Radhakrishnan, and K. Srinivasan. Product differentiation and commonality in design: Balancing revenue and cost drivers. *Management Science*, 47(1):37–51, 2001.

R. Fellini, M. Kokkolaras, N. Michelena, P. Papalambros, A. Perez-Duarte, K. Saitou, and P. Fenyes. A sensitivity-based commonality strategy for family products of mild variation, with application to automotive body structures. *Structural and Multidisciplinary Optimization*, 27(1-2):89–96, 2004.

M. Fisher, K. Ramdas, and K. Ulrich. Component sharing in the management of product variety: A study of automotive braking systems. *Management Science*, 45(3):297–315, 1999.

H. S. Heese and J. M. Swaminathan. Product line design with component commonality and cost-reduction effort. *Manufacturing & Service Operations Management*, 8(2):206–219, 2006.

M. S. Hillier. Component commonality in multiple-period, assemble-to-order systems. *IIE Transactions*, 32 (8):755–766, 2000.

H. Z. Huang, Z. J. Liu, and D. N. P. Murthy. Optimal reliability, warranty and price for new products. *IIE Transactions*, 39(8):819–827, 2007.

W. T. Huh, G. Janakiraman, J. A. Muckstadt, and P. Rusmevichientong. Asymptotic optimality of order-up-to policies in lost sales inventory systems. *Management Science*, 55(3):404–420, 2009.

S. H. Kim, M. A. Cohen, and S. Netessine. Reliability or inventory? contracting strategies for after-sales product support. In *Proceedings of 2007 International Conference on Manufacturing & Service Operations Management*, 2007.

S. H. Kim, M. A. Cohen, and S. Netessine. Reliability or Inventory? An Analysis of Performance-Based Contracts for Product Support Services. In A. Y. Ha and C. S. Tang, editors, *Handbook of Information Exchange in Supply Chain Management*, pages 65–88. Springer International Publishing, 2017.

A. A. Kranenburg and G. J. van Houtum. Effect of commonality on spare parts provisioning costs for capital goods. *International Journal of Production Economics*, 108(1):221–227, 2007.

A. A. Kranenburg and G. J. Van Houtum. A new partial pooling structure for spare parts networks. *European Journal of Operational Research*, 199(3):908–921, 2009.

V. Krishnan and S. Gupta. Appropriateness and impact of platform-based product development. *Management Science*, 47(1):52–68, 2001.

U. D. Kumar, J. Crocker, J. Knezevic, and M. El-Haram. *Reliability, maintenance and logistic support: A life cycle approach*. Springer Science & Business Media, 2012.

E. Labro. The cost effects of component commonality: A literature review through a management-accounting lens. *Manufacturing & Service Operations Management*, 6(4):358–367, 2004.

A. Mettas. Reliability allocation and optimization for complex systems. In *Reliability and Maintainability Symposium, 2000. Proceedings. Annual*, pages 216–221. IEEE, 2000.

M. H. Meyer and A. P. Lehnerd. *The power of product platforms*. Simon and Schuster, 1997.

J. A. Muckstadt. *Analysis and Algorithms for Service Parts Supply Chains*. Springer, New York, 2005.

M. Muffatto and M. Roveda. Developing product platforms: Analysis of the development process. *Technovation*, 20(11):617–630, 2000.

K. B. Öner, G. P. Kiesmüller, and G. J. van Houtum. Optimization of component reliability in the design phase of capital goods. *European Journal of Operational Research*, 205(3):615–624, 2010.

J. E. Parada Puig and R. J. I. Basten. Defining line replaceable units. *European Journal of Operational Research*, 247(1):310–320, 2015.

K. Parent. Avantcom moves on pilot project, 2000. URL `http://www.edn.com/electronics-news/4362659/AvantCom-Moves-on-Pilot-Project`.

J. S. Song and Y. Zhao. The value of component commonality in a dynamic inventory system with lead times. *Manufacturing & Service Operations Management*, 11(3):493–508, 2009.

U. W. Thonemann and M. L. Brandeau. Optimal commonality in component design. *Operations Research*, 48(1):1–19, 2000.

G. J. van Houtum and A. A. Kranenburg. *Spare parts inventory control under system availability constraints*, volume 227. Springer, 2015.

J. A. van Mieghem. Commonality strategies: Value drivers and equivalence with flexible capacity and inventory substitution. *Management Science*, 50(3):419–424, 2004.

Appendix A: Proof of Lemma 1

The function $f(\tau)$ is twice differentiable and convex on $[\tau_{\min}, \tau_{\max}]$ by the assumptions for $c(\tau)$ and Assumption 2. Hence, there exists a positive, finite minimizer $\hat{\tau}^*$ of $f(\tau)$.

Next, for each $i \in I$, the cost expression is $\hat{\pi}(\tau_i, N_i, \beta_i) = \beta_i N_i \left(c(\tau_i) + \frac{rc(\tau_i)T}{\tau_i}\right)$. The minimum of $\hat{\pi}(\tau_i, N_i, \beta_i)$ over τ_i is solely determined by $c(\tau_i) + \frac{rc(\tau_i)T}{\tau_i} = f(\tau_i)$. This latter function is minimized by $\hat{\tau}^*$. So, $\hat{\tau}_i^* = \hat{\tau}^*$ minimizes $\hat{\pi}(\tau_i, N_i, \beta_i)$.

Appendix B: Proof of Lemma 2

Let us first write:

$$\begin{aligned}\tilde{\pi}(\tau_i, s_i, N_i, \beta_i) &= \beta_i c(\tau_i)(N_i + s_i) + h s_i T \beta_i c(\tau_i) + r\beta_i c(\tau_i)\frac{N_i T}{\tau_i} + bT\int_{s_i}^{\infty}(x - s_i) f_i(x)dx \\ &= \beta_i c(\tau_i)(N_i + s_i) + h s_i T \beta_i c(\tau_i) + r\beta_i c(\tau_i)\frac{N_i T}{\tau_i} \\ &\quad + bT\mathbb{E}[D(L, N_i, \tau_i)] - bTs_i + bT\int_0^{s_i}(s_i - x) f_i(x)dx,\end{aligned} \tag{13}$$

where $f_i(x)$ is defined as the pdf of $D(L, N_i, \tau_i)$. From Leibniz' rule, we obtain

$$\begin{aligned}\frac{\partial \tilde{\pi}(\tau_i, s_i, N_i, \beta_i)}{\partial s_i} &= \beta_i c(\tau_i) + \beta_i c(\tau_i) hT - bT + bT\int_0^{s_i} f_i(x)dx \\ &= \beta_i c(\tau_i) + \beta_i c(\tau_i) hT - bT + bTF_i(s_i),\end{aligned} \tag{14}$$

where $F_i(x)$ is the cdf of $D(L, N_i, \tau_i)$. Applying Leibniz' rule again, we obtain the second order derivative $\frac{\partial^2 \tilde{\pi}(\tau_i, s_i, N_i, \beta_i)}{\partial {s_i}^2} = bTf_i(s_i) > 0$, as $b, T > 0$, and $f_i(s_i) > 0$ by definition of the pdf. Hence, $\tilde{\pi}(\tau_i, s_i, N_i, \beta_i)$ is twice differentiable and strictly convex in s_i.

The minimal point of $\tilde{\pi}(\tau_i, s_i, N_i, \beta_i)$ satisfies (use (14))

$$F_i(s_i) = \frac{bT - \beta_i c(\tau_i)(1 + hT)}{bT}$$

which gives the following formula for a unique optimal $s_i^*(\tau_i)$:

$$s_i^*(\tau_i) = \mathbb{E}[D(L, N_i, \tau_i)] + \sigma[D(L, N_i, \tau_i)]\Phi^{-1}\left(\frac{bT - \beta_i c(\tau_i)(1 + hT)}{bT}\right),$$

where $\Phi(\cdot)$ denotes the standard normal cdf. By Assumption 1,

$$\frac{bT - \beta_i c(\tau_i)(1 + hT)}{bT} \geq 1 - \frac{\beta_i c(\tau_{\max})(1 + hT)}{bT} > 1 - \frac{1}{2} = \frac{1}{2}.$$

Hence, $\Phi^{-1}\left(\frac{bT - \beta_i c(\tau_i)(1+hT)}{bT}\right) > 0$ and thus we know that $s_i^*(\tau_i)$ is strictly positive.

Appendix C: Proof of Theorem 1

To prove Theorem 1 we use the following lemma.

LEMMA 4. *For each component $i \in I$, the optimal base-stock level approaches infinity as b tends to infinity, i.e.,* $\lim_{b\to\infty} s_i^*(\tau_i, b) = \infty$. *Moreover, we have for each component $i \in I$,* $\lim_{b\to\infty} \frac{b\beta_i c(\tau_i)\mathbb{E}[(D(L,N_i,\tau_i) - s_i^*(\tau_i, b\beta_i c(\tau_i)))^+]}{s_i^*(\tau_i, b\beta_i c(\tau_i))} = 0$ *and* $\lim_{b\to\infty} \frac{b\mathbb{E}[(D(L,N_i,\tau_i) - s_i^*(\tau_i, b))^+]}{s_i^*(\tau_i, b)} = 0$.

Proof. First, we show that for each component $i \in I$, we have $\lim_{b\to\infty} s_i^*(\tau_i, b) = \infty$, or in shorthand notation $s_i^*(\tau_i, b) \to \infty$ as $b \to \infty$. The optimal service part stock level is given in Eq. (5), where $\mathbb{E}[D(L, N_i, \tau_i)]$ and $\sigma[D(L, N_i, \tau_i)]$ are independent of b. Furthermore, $\lim_{b\to\infty} \frac{bT - \beta_i c(\tau_i)(1+hT)}{bT} = 1$ for given a given reliability τ_i. Consequently, $\lim_{b\to\infty} \Phi^{-1}\left(\frac{bT-\beta_i c(\tau_i)(1+hT)}{bT}\right) = \infty$ by definition of the inverse of the standard normal distribution. Hence, $\lim_{b\to\infty} s_i^*(\tau_i, b) = \infty$.

Let $\tau_i \in [\tau_{\min}, \tau_{\max}]$. For each $i \in I$, we have

$$\begin{aligned} 0 &\le \frac{b\mathbb{E}[(D(L,N_i,\tau_i) - s_i^*(\tau_i,b))^+]}{s_i^*(\tau_i,b)} \\ &= \frac{b\mathbb{P}\left[D(L,N_i,\tau_i) > s_i^*(\tau_i,b)\right]\mathbb{E}\left[D(L,N_i,\tau_i) - s_i^*(\tau_i,b) \mid D(L,N_i,\tau_i) > s_i^*(\tau_i,b)\right]}{s_i^*(\tau_i,b)} \\ &= \frac{\beta_i c(\tau_i)(1+hT)}{T} \times \frac{\mathbb{E}\left[D(L,N_i,\tau_i) - s_i^*(\tau_i,b) \mid D(L,N_i,\tau_i) > s_i^*(\tau_i,b)\right]}{s_i^*(\tau_i,b)}, \end{aligned}$$

where the first inequality follows from Eq. (13) in the proof of Lemma 2. The second equality follows from the definition of the optimal base-stock level. That is, $s_i^*(\tau_i, b)$ satisfies $\frac{\partial \tilde{\pi}(\tau_i, s_i, N_i, \beta_i)}{\partial s_i} = 0$, which implies $\mathbb{P}[D(L, N_i, \tau_i) > s_i^*(\tau_i, b)] = \frac{\beta_i c(\tau_i)(1+hT)}{bT}$ by the right continuity of the distribution function (Huh et al. 2009, p. 409). Then, for the limit of $b \to \infty$, Assumption 1 is satisfied for any finite $\tau_i \in [\tau_{\min}, \tau_{\max}]$ and thus we obtain

$$\begin{aligned} 0 &\le \lim_{b\to\infty} \frac{b\mathbb{E}[(D(L,N_i,\tau_i) - s_i^*(\tau_i,b))^+]}{s_i^*(\tau_i,b)} \\ &\le \lim_{b\to\infty} \frac{\beta_i c(\tau_i)(1+hT)}{T} \times \frac{\mathbb{E}\left[D(L,N_i,\tau_i) - s_i^*(\tau_i,b) \mid D(L,N,\tau_i) > s_i^*(\tau_i,b)\right]}{s_i^*(\tau_i,b)} = 0. \end{aligned}$$

The equality follows from the fact that $s_i^*(\tau_i, b) \to \infty$ as $b \to \infty$ and because we have that $\frac{\mathbb{E}[D(L,N_i,\tau_i) - s_i^*(\tau_i,b) \mid D(L,N_i,\tau_i) > s_i^*(\tau_i,b)]}{s_i^*(\tau_i,b)} \to 0$ as $b \to \infty$ due to the increasing failure rate of a normal distribution, see Huh et al. (2009, p. 409). □

Using Lemma 4 enables us to prove Theorem 1.

Proof of Theorem 1. Fix $\tau_i \in [\tau_{\min}, \tau_{\max}]$ for all $i \in I$. By definition of $s_i^*(\tau_i, b)$, we find the following bounds by inserting suboptimal base-stock levels:

$$\begin{aligned} \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)), N_i, \beta_i \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)), N_i, \beta_i \mid b)} &\le \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)), N_i, \beta_i \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b), N_i, \beta_i \mid b)} \\ &\le \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b), N_i, \beta_i \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b), N_i, \beta_i \mid b)}. \end{aligned}$$

We define $\kappa = \arg\max_{i\in J}\{s_i^*(\tau_i, b\beta_i c(\tau_i))\}$, and we rewrite the lower bound to

$$\begin{aligned} &\frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)), N_i, \beta_i \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)), N_i, \beta_i \mid b)} \\ &= \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)), N_i, \beta_i \mid b\beta_i c(\tau_i))/s_\kappa^*(\tau_\kappa, b\beta_\kappa c(\tau_\kappa))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)), N_i, \beta_i \mid b)/s_\kappa^*(\tau_\kappa, b\beta_\kappa c(\tau_\kappa))}. \end{aligned}$$

Taking the limit as $b \to \infty$ implies that $s_i^*(\tau_i, b\beta_i c(\tau_i)) \to \infty$ by Lemma 4, and that any $\tau_i \in [\tau_{\min}, \tau_{\max}]$ satisfies Assumption 1. Each cost term in the numerator and denominator has a finite limit. This follows by using Lemma 4 and concluding that $\lim_{b\to\infty} \frac{b\beta_i c(\tau_i)\mathbb{E}[(D(L,N_i,\tau_i)-s_i^*(\tau_i,b\beta_i c(\tau_i)))^+]}{s_\kappa^*(\tau_\kappa,b\beta_\kappa c(\tau_\kappa))} = 0$, because $s_\kappa^*(\tau_\kappa, b\beta_\kappa c(\tau_\kappa)) \geq s_i^*(\tau_i, b\beta_i c(\tau_i))$ for all components $i \in J$. Hence, we obtain

$$\lim_{b\to\infty} \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)), N_i, \beta_i \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)), N_i, \beta_i \mid b)} = 1.$$

Now, we redefine $\kappa = \arg\max_{i\in J}\{s_i^*(\tau_i, b)\}$ with a slight abuse of notation, and we rewrite the upper bound to

$$\frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b), N_i, \beta_i \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b), N_i, \beta_i \mid b)} = \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b), N_i, \beta_i \mid b\beta_i c(\tau_i))/s_\kappa^*(\tau_\kappa, b)}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b), N_i, \beta_i \mid b)/s_\kappa^*(\tau_\kappa, b)}.$$

Again, taking the limit as $b \to \infty$ and applying Lemma 4, we obtain

$$\lim_{b\to\infty} \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b), N_i, \beta_i \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b), N_i, \beta_i \mid b)} = 1.$$

By the sandwich theorem, we have $\lim_{b\to\infty} \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)), N_i, \beta_i \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b), N_i, \beta_i \mid b)} = 1$.

The proof of the asymptotic equivalence for the common component is analogous to the foregoing; we just have to omit the summations and replace all i and all κ by q. □

Appendix D: Proof of Lemma 3

Let us write $\pi(\tau_i, N_i, \beta_i)$ in terms of the normalized loss function of the normal distribution, with $\hat{s}_i = s_i^*(\tau_i, b\beta_i c(\tau_i))$.

$$\begin{aligned}
\pi(\tau_i, N_i, \beta_i) &= \beta_i c(\tau_i)(N_i + \hat{s}_i) + h\hat{s}_i T\beta_i c(\tau_i) + r\beta_i c(\tau_i)\frac{N_i T}{\tau} \\
&\quad + b\beta_i c(\tau_i) T\sigma[D(L, N_i, \tau_i)] \left\{ \phi\left(\frac{\hat{s}_i - \mathbb{E}[D(L, N_i, \tau_i)]}{\sigma[D(L, N_i, \tau_i)]} \right) \right. \\
&\quad \left. - \frac{\hat{s}_i - \mathbb{E}[D(L, N_i, \tau_i)]}{\sigma[D(L, N_i, \tau_i)]} \left(1 - \Phi\left(\frac{\hat{s}_i - \mathbb{E}[D(L, N_i, \tau_i)]}{\sigma[D(L, N_i, \tau_i)]} \right) \right) \right\},
\end{aligned}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal pdf and cdf, respectively. Next, we substitute $\hat{s}_i$ by $s_i^*(\tau_i, b\beta_i c(\tau_i))$ and simplify:

$$\begin{aligned}
&\pi(\tau_i, N_i, \beta_i) \\
&= \beta_i c(\tau_i) N_i + \beta_i c(\tau)(1 + hT) \left(\mathbb{E}[D(L, N_i, \tau_i)] + \sigma[D(L, N_i, \tau_i)]\Phi^{-1}\left(\frac{bT - 1 - hT}{bT} \right) \right) \\
&\quad + r\beta_i c(\tau_i)\frac{N_i T}{\tau} + b\beta_i c(\tau_i) T\sigma[D(L, N_i, \tau_i)]\phi\left(\Phi^{-1}\left(\frac{bT - 1 - hT}{bT} \right) \right) \\
&\quad - b\beta_i c(\tau_i) T\sigma[D(L, N_i, \tau_i)]\Phi^{-1}\left(\frac{bT - 1 - hT}{bT} \right) \left(1 - \Phi\left(\Phi^{-1}\left(\frac{bT - 1 - hT}{bT} \right) \right) \right) \\
&= \beta_i c(\tau_i) N_i + \beta_i c(\tau_i)(1 + hT) \left(\mathbb{E}[D(L, N_i, \tau_i)] + \sigma[D(L, N_i, \tau_i)]\Phi^{-1}\left(\frac{bT - 1 - hT}{bT} \right) \right) \\
&\quad + r\beta_i c(\tau_i)\frac{N_i T}{\tau_i} + b\beta_i c(\tau_i) T\sigma[D(L, N_i, \tau_i)]\phi\left(\Phi^{-1}\left(\frac{bT - 1 - hT}{bT} \right) \right) \\
&\quad - \beta_i c(\tau_i)(1 + hT)\sigma[D(L, N_i, \tau_i)]\Phi^{-1}\left(\frac{bT - 1 - hT}{bT} \right) \\
&= \beta_i c(\tau_i) N_i + \beta c(\tau_i)(1 + hT)\mathbb{E}[D(L, N_i, \tau_i)] + r\beta_i c(\tau_i)\frac{N_i T}{\tau_i} \\
&\quad + b\beta_i c(\tau_i) T\sigma[D(L, N_i, \tau_i)]\phi\left(\Phi^{-1}\left(\frac{bT - 1 - hT}{bT} \right) \right)
\end{aligned}$$

$$= \beta_i c(\tau_i)\left(1+\frac{rT+L(1+hT)}{\tau_i}\right)N_i + b\beta_i c(\tau_i)T\sigma[D(L,N_i,\tau_i)]\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right)$$
$$= \beta_i c(\tau_i)\left(1+\frac{rT+L(1+hT)}{\tau_i}\right)N_i + b\beta_i c(\tau_i)T\sqrt{\frac{\alpha N_i L}{\tau_i}}\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right).$$

Appendix E: Proof of Theorem 2

Let $\boldsymbol{N}$ and $\boldsymbol{\beta}$ be given. Define $\overline{\beta}_q = \sum_{i\in J}\frac{N_i\beta_i}{N_q}$. Notice that $\hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta}) = \overline{\beta}_q$. We obtain:

$$\begin{aligned}
&\pi(\tau_q^*, N_q, \overline{\beta}_q)\\
&= \sum_{i\in J}\frac{N_i}{N_q}\beta_i c(\tau_q^*)\left(1+\frac{rT+L(1+hT)}{\tau_q^*}\right)N_q + \sum_{i\in J}\frac{N_i}{N_q}b\beta_i c(\tau_q^*)T\sqrt{\frac{\alpha N_q L}{\tau_q^*}}\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right)\\
&= \sum_{i\in J}\left[\beta_i c(\tau_q^*)\left(1+\frac{rT+L(1+hT)}{\tau_q^*}\right)N_i + \sqrt{\frac{N_i}{N_q}}b\beta_i c(\tau_q^*)T\sqrt{\frac{\alpha N_i L}{\tau_q^*}}\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right)\right]\\
&\le \sum_{i\in J}\left[\beta_i c(\tau_q^*)\left(1+\frac{rT+L(1+hT)}{\tau_q^*}\right)N_i + b\beta_i c(\tau_q^*)T\sqrt{\frac{\alpha N_i L}{\tau_q^*}}\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right)\right]\\
&= \sum_{i\in J}\pi(\tau_q^*, N_i, \beta_i)\\
&\le \sum_{i\in J}\pi(\tau_i^*, N_i, \beta_i),
\end{aligned}$$

where the second inequality follows from inserting optimal values τ_i^* instead of the possibly suboptimal τ_q^* for the cost function for component $i\in I$. This implies that $\Theta(\boldsymbol{N},\boldsymbol{\beta}) \ge \overline{\beta}_q = \hat{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$.

Appendix F: Proof of Proposition 1

Part (a): By Eq. (11), we see immediately that $\Delta_\Theta(\boldsymbol{N},\boldsymbol{\beta})$ decreases as a function of A_1 and increases as a function of A_2.

Part (b): By Eq. (11), we see immediately that $\Delta_\Theta(\boldsymbol{N},\boldsymbol{\beta})$ increases as a function of each β_i, $i\in J$. For an increasing α, it holds that A_1 is constant and A_2 increases, and hence $\Delta_\Theta(\boldsymbol{N},\boldsymbol{\beta})$ increases.

Part (c): By Eq. (11), we see immediately that $\Delta_\Theta(\boldsymbol{N},\boldsymbol{\beta})$ decreases as a function of the total market size N_q. For an increasing r, it holds that A_1 increases and A_2 is constant, and hence $\Delta_\Theta(\boldsymbol{N},\boldsymbol{\beta})$ decreases.

Appendix G: Proof of Proposition 2

Suppose that the relative unit cost factors of all dedicated components are equal, i.e., $\beta_i = \beta_1$ for all $i\in J\setminus\{1\}$.

Part (a): Eq. (11) simplifies to

$$\Delta_\Theta(\boldsymbol{N},\boldsymbol{\beta}) = \frac{\beta_1\sum_{i\in J}\sqrt{\zeta_i}(1-\sqrt{\zeta_i})}{1+\frac{A_1}{A_2}\sqrt{N_q}}\ . \tag{15}$$

The sum $\sum_{i\in J}\sqrt{\zeta_i}(1-\sqrt{\zeta_i})$ is concave as a function of $(\zeta_1,\ldots,\zeta_{|J|})$ on the domain $\{(\zeta_1,\ldots,\zeta_{|J|})|\zeta_i\in(0,1)$ for all $i\in J,\ \sum_{i\in J}\zeta_i = 1\}$.

Part (b): If we choose $\zeta_i = 1/(|J|)$, $i\in J$, then formula (15) further simplifies to the formula given in this part (b).

Appendix H: Sensitivity analysis of cost comparison from Section 7.2

In this appendix, we explore for what instances it is particularly important to consider service parts in design decisions. For each input parameter, we cluster the instances of Testbed 2 based on the possible values of that parameter and we gives statistics for $\Delta\pi(\boldsymbol{N}, \boldsymbol{\beta}, \beta_q)$; see Table 3. We observe that the repair cost fraction r , the holding cost fraction h, and the repair lead time L have a significant effect on the importance of considering service parts for the design decisions. Furthermore, we see that considering service parts is important when the unit production costs are more strongly affected by the reliability level (in terms of p_1, p_2).

		min $\Delta\pi(\boldsymbol{N}, \boldsymbol{\beta}, \beta_q)(\%)$	avg $\Delta\pi(\boldsymbol{N}, \boldsymbol{\beta}, \beta_q)(\%)$	max $\Delta\pi(\boldsymbol{N}, \boldsymbol{\beta}, \beta_q)(\%)$
r	0.1	0.29	2.90	19.58
	0.2	0.10	1.28	14.64
	0.3	0.04	0.75	12.63
h	0.015	0.04	0.73	9.38
	0.03	0.10	1.55	14.62
	0.05	0.18	2.66	19.58
T	90	0.08	1.76	16.58
	180	0.06	1.65	18.31
	360	0.04	1.53	19.58
b	1000	0.04	1.55	17.85
	100,000	0.04	1.65	18.77
	1,000,000	0.04	1.74	19.58
L	2	0.04	1.12	15.53
	3	0.07	1.65	17.83
	4	0.11	2.16	19.58
p_1	500	0.04	1.86	19.58
	1,500	0.04	1.68	19.44
	5,000	0.04	1.40	18.62
p_2	100	0.04	1.41	18.81
	300	0.04	1.66	19.41
	1,000	0.04	1.86	19.58
k	1.0	0.04	1.62	19.58
	1.5	0.04	1.66	19.53
	2.0	0.04	1.66	19.10

Table 3 Numerical study results

The cost difference between both approaches decreases as the repair cost fraction r increases. A higher value for r results in a larger contribution of the repair costs to the LCC. Consequently, the difference in the objective function between both approaches decreases and we observe smaller cost differences. Hence, if components are very expensive to repair, the harm of using the non–anticipating approach is low. For cheap and medium priced repairs, OEMs would do well to consider service parts for the design decisions.

For two other aspects of after–sales service – increasing holding costs h and increasing repair leadtimes L – the LCC difference increases. As service part provisioning is costly (high h and high L), it is important to make smart decisions regarding these parts. This is better achieved under the anticipating approach as it considers service parts, reliability and commonality simultaneously. For the case in which service part provisioning is relatively cheap, after–sales services play a less dominant role in determining LCC, and thus using the non–anticipating approach is less harmful.

Service parts are important to consider in design decisions when the unit production cost is sensitive to the reliability level. That is, for low values of p_1 and high values for p_2 the unit production costs are more strongly influenced by the reliability level. If the effect of reliability on the life cycle costs is strong, considering service parts in the design decisions offers a benefit as the OEM can compensate a high (low) reliability level with fewer (more) service parts on stock. A small effect of reliability on the unit production costs opens the door for the non–anticipating approach, which becomes attractive in these cases.

In the semiconductor industry that motivates our research, we observe a relatively high repair costs, and very high costs for service part provisioning, and unit production costs are significantly affected by the reliability level. Therefore, OEMs in this industry can benefit significantly by considering service part aspect in their design decisions to obtain low life cycle costs.