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Thermo-mechanical constitutive modeling of unsaturated clays based on the critical state concepts

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A thermo-mechanical constitutive model for unsaturated clays is constructed based on the existing model for saturated clays originally proposed by the authors. The saturated clays model was formulated in the framework of critical state soil mechanics and modified Cam-clay model. The existing model has been generalized to simulate the experimentally observed behavior of unsaturated clays by introducing Bishop's stress and suction as independent stress parameters and modifying the hardening rule and yield criterion to take into account the role of suction. Also, according to previous studies, an increase in temperature causes a reduction in specific volume. A reduction in suction (wetting) for a given confining stress may induce an irreversible volumetric compression (collapse). Thus an increase in suction (drying) raises a specific volume i.e. the movement of normal consolidation line (NCL) to higher values of void ratio. However, some experimental data confirm the assumption that this reduction is dependent on the stress level of soil element. A generalized approach considering the effect of stress level on the magnitude of clays thermal dependency in compression plane is proposed in this study. The number of modeling parameters is kept to a minimum, and they all have clear physical interpretations, to facilitate the usefulness of model for practical applications. A step-by-step procedure used for parameter calibration is also described. The model is finally evaluated using a comprehensive set of experimental data for the thermo-mechanical behavior of unsaturated soils.

1. Introduction

Modeling of the thermo-mechanical behavior of soils, particularly clays, has been the subject of many studies in the past. The possible reason accounting for this attention is the thermal conditions faced in a number of high-priority applications, such as geological storage of nuclear waste, buried high-voltage cables, pavements and geothermal energy.

Geotechnical applications to these problems require a comprehension of the thermo-mechanical behavior of clays. Notwithstanding there is the practical relevance of the thermo-mechanical applications, the effect of high temperatures on soil behavior is not yet completely understood, due to the intricate influence of temperature on the behavior of soils.

Thermo-mechanical models able to simulate most of the observed behaviors of saturated clays at increased temperatures have been developed by several researchers. Unlike some of the advanced models for the thermo-hydromechanical behavior of saturated clays, the models for unsaturated clays are almost based on a Cam-clay elastoplastic approach. Some thermo-hydromechanical models have been proposed for the behavior of unsaturated clays. Philip and de Vries (1957) presented a model for coupled heat and moisture transfer in rigid porous media under the combined gradients of temperature and moisture. Also, de Vries (1958) extended this theory to include moisture and latent heat storage in the vapor phase, and the advection of sensible heat by water. Modified versions of Philip–de Vries model were proposed by Milly (1982), Thomas and King (1991) and Thomas and Sansom (1995), using matric suction rather than volumetric moisture content as the primary variable. The laboratory and field validations of Philip–de Vries model have been reported by Ewen and Thomas (1989) and Thomas and He (1997), amongst others. Reasonable agreement has been found between the theoretical analyses and the laboratory/field results. Furthermore, Geraminegad and Saxena (1986) presented a de-coupled flow deformation model, including the effect of matrix deformation on moisture, heat and gas flow through porous media. A coupled version of this formulation was later presented by Thomas and He (1995, 1997). They introduced the matrix displacement vector as a primary variable, and improved the coupling effects between the temperature and deformation. They also improved the

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energy balance equation by including moisture and latent heat storage in the vapor phase, in addition to the advection of heat by water previously proposed by de Vries (1958). Similar formulations have also been given by Gawin et al. (1995) and Zhou et al. (1998). However, in Gawin et al. (1995), the constitutive laws of the solid phase were introduced in terms of the concept of effective stress. Nevertheless, they used the degree of saturation as the effective stress parameter, which is not fully supported by the experimental evidence. They also retained the degree of saturation as the main coupling element between the air and water flow fields, resulting in the governing differential equations to be strongly non-linear.

In general, a major difficulty in the formulations discussed above is that they either completely ignore the matrix deformation (e.g. Philip and de Vries, 1957; de Vries, 1958; Milly, 1982; Thomas and King, 1991; Thomas and Sansom, 1995) or use the theory of elasticity (Gawin et al., 1995) in conjunction with the "state surfaces" approach (Thomas and He, 1995; Zhou et al., 1998) to account for the strongly non-linear deformation behavior of the soil matrix. Such stress path dependency cannot be modeled using the theory of elasticity and/or the state surfaces approach. An appropriate plasticity model needs to be invoked, in order to take into account the variations of the yield surface with temperature and suction. Furthermore, in these formulations, effect of temperature on state surfaces is not well defined, despite its importance on response of soil (e.g. Hueckel and Baldi, 1990; Lingnau et al., 1995; Hueckel et al., 1998; Cui et al., 2000; Graham et al., 2001).

Also, some researchers simulated the consolidation process and pore-water pressure around hot cylinders buried in saturated clay (e.g. Smith and Booker, 1989; Britto et al., 1989). However, only the reversible volume change of the soil due to a change in temperature was considered in their models. Khalili and Loret (2001) presented an alternative theory for heat and mass transport through deformable unsaturated porous media. The work was an extension of the theoretical developments of Loret and Khalili (2000) dealing with fully coupled isothermal flow and deformation in variably saturated porous media to include thermal coupling effects. Wu et al. (2004) presented a thermo-hydromechanical model for unsaturated soils. The model was based on the four component yield surfaces of the cap plasticity model and on the experimental results obtained for different types of soils. The extension of the model to include temperature effects was embodied through the thermal softening and the suction variation with respect to temperature, as well as the thermal effects to the hydraulic properties. François and Laloui (2008) presented a constitutive model for soils based on a unified thermo-mechanical model for unsaturated conditions. The relevant temperature and suction effects were studied in the light of elastoplasticity. A generalized effective stress framework was adopted that would include a number of intrinsic thermo-hydromechanical connections to represent the state of stress in the soil. Two coupled constitutive aspects were used to fully describe the non-isothermal behavior. The mechanical constitutive part was built on the concepts of bounding surface theory and multi-mechanism plasticity, while water retention characteristics were described using elastoplasticity to reproduce the hysteretic response and the effect of temperature and dry density on retention properties. Mašín and Khalili (2012) presented a model for non-isothermal behavior of unsaturated soils. The model was based on an incrementally non-linear hypoplastic model for saturated clays and could therefore simulate the non-linear behavior of overconsolidated soils. A hypoplastic model for non-isothermal behavior of saturated soils was developed and combined with the existing hypoplastic model for unsaturated soils based on the effective stress principle. The elastoplastic constitutive model developed by Pastor et al. (1990), for fully saturated soils, has been extended to include partially saturated soil behavior by Bolzon et al. (1996). The saturated soil model, formulated in the framework of generalized plasticity, considers volumetric and deviatoric strain hardening and takes into account past stress history and possible limit states.

The generalization of the existing model to simulate the experimentally observed behavior of partially saturated soils is obtained by introducing Bishop's stress and suction as independent stress parameters and by modifying the hardening parameter and the yield condition to take into account the role of suction. Using a similar approach, an isothermal constitutive model is presented for predicting the thermo-elastoplastic behavior of unsaturated clays in triaxial stress space. This model is an extension of the thermo-elastoplastic model for fully saturated clays proposed by Hamidi et al. (2015). Using a non-associated temperature dependent flow rule, the present model can simulate the mechanical behavior of clays with respect to temperature and suction changes.

2. Mechanical behavior and constitutive models for unsaturated soils

The first part in this section reviews the basic features of the mechanical behavior of unsaturated soils. In the second part, constitutive modeling of the mechanical behavior is reviewed.

2.1. Mechanical behavior of unsaturated soils

Generalized effective stress expressions were proposed in order to include unsaturated soils into the conventional soil mechanics framework, the best known being proposed by Bishop (1959):

$$p'_{\rm B} = p - u_{\rm a} + X(u_{\rm a} - u_{\rm w}) = \overline{p} + Xs \tag{1}$$

where $p'_{\rm B}$ is the Bishop's mean effective stress, p is the mean total stress, $\overline{p}$ is the mean effective stress in dry soil, X is a function of the degree of saturation, $u_{\rm a}$ is the pore air pressure, $u_{\rm w}$ is the pore water pressure, and s is the matric suction. In this research, the following equation presented by Khalili and Khabbaz (1998) has been used to calculate the parameter X:

$$X = \begin{cases} [s_{\rm e}(T)/s]^{\Omega} & (s \geq s_{\rm e}(T)) \\ 1 & (s \leq s_{\rm e}(T)) \end{cases} \tag{2}$$

where $s_{\rm e}(T)$ is the bubbling pressure or the air entry value at the temperature T, and $\Omega = 0.5$.

2.2. Constitutive models for unsaturated soils

Elastic models are relatively easy to implement within numerical analysis and to obtain the relevant parameters, but have some major drawbacks. Most importantly, no distinction is made between reversible and irreversible strains. Elastoplastic constitutive models have been developed for both expansive and non-expansive soils. They all fall into two categories depending on the adopted stress variables, i.e. total stress models and effective stress models. Most elastoplastic models are extensions of models for fully saturated soils and are based on the concept of the loading-collapse yield surface. The following elements are usually defined:

(1) A yield function to represent the surface that separates fully elastic from elastoplastic behavior. This surface expands with increasing suction in order to model the increase of shear strength and yield stress with suction. In this way, collapse due to wetting can also be reproduced. The expansion of the yield

surface is defined by the increase of the isotropic yield stress and the apparent cohesion. The increase of the yield stress with suction is related to the variation of the position and shape of the isotropic compression line in e-$\log_{10} p'$ space, where e is the void ratio and p' is the mean effective stress. Models such as the Barcelona basic model (Alonso et al., 1990), Wheeler and Sivakumar (1995) model and Bolzon et al. (1996) model make an assumption about the position and shape of the isotropic compression line, while the Josa et al. (1992) model makes an assumption about the variation of the isotropic yield stress with suction. The Barcelona basic model defines a second yield function, which in (q, p', s) space is a vertical "wall" perpendicular to the suction axis, where q is the deviatoric stress.

(2) A plastic potential function to determine the relative magnitudes of the plastic strains at each point of the yield surface. The Barcelona basic model and the Josa et al. (1992) model assume a non-associated flow rule such that, for the at rest earth pressure coefficient K_0 conditions, no lateral strains are predicted. Wheeler and Sivakumar (1995) assume an associated flow rule, giving plastic strain increments normal to the yield surface. The plastic potential function also determines the position of the critical state line (CSL) in the (v, p', q) space for each value of suction, where v is the specific volume.
(3) A hardening/softening rule to determine the magnitude of the plastic volumetric strains. This is defined in terms of the equivalent fully saturated isotropic yield stress, since it is assumed that the process of wetting induced collapse is the same as isotropic compression beyond yield.
(4) Definition of the elastic behavior within the yield surface. The elastic compression coefficient, k, and the elastic shear modulus, G, are usually assumed to be independent of suction.

3. Adaptation of the model to unsaturated state

The simple introduction of Bishop's stress in the model proposed by Hamidi et al. (2015) allowed volume changes induced by suction to be considered, but the predictions obtained were not always in accordance with experimental observations. In particular, dependency of soil compressibility to suction was not properly reproduced. Also, the isotropic yield stress would decrease as the suction increased, which is in contrast with experimental observations of Fredlund et al. (1978). The largest amount of volumetric strains under increasing net mean stress was always predicted for fully saturated conditions, indicating that it was not possible to reproduce the maximum collapse on wetting side of CSL as a typical behavior of loose soils (Josa et al., 1992).

In this study, it is shown that a proper enhancement of the model developed by Hamidi et al. (2015) makes it possible to overcome these drawbacks, and the experimentally observed behavior can be reproduced. Therefore, theoretical predictions of the enhanced model along different stress paths are presented in comparison with experimental observations. Since this model is an extension to the fully saturated model, the main features of saturated soils can also be captured by the enhanced model as described above.

As pointed out by Fredlund and Morgenstern (1977), introduction of suction as an independent stress parameter is required to express plastic volumetric strain increment $d\varepsilon_v^p$ as fallows:

$$d\varepsilon_v^p = \frac{1}{H_L}\frac{d(\overline{p}_{cT}+Xs)}{\overline{p}_{cT}+Xs} \tag{3}$$

where $\overline{p}_{cT}$ is the preconsolidation pressure at temperature T, and H_L is the plastic modulus. However, it is clear that the observed dependency of soil compressibility on suction is not considered in this equation. Thus, the plastic volumetric strains obtained from Eq. (3) overestimate the experimental results. Therefore, the plastic modulus (H_L) in Eq. (3) has been modified in order to consider the effect of suction. Yield function can be defined in the three-dimensional (3D) space ($\overline{p}$, s, q) as bellow:

$$q = M_T(\overline{p}+Xs)\left\{\frac{\alpha^2(\beta-1)}{2\theta-1}\left[\left(\frac{\overline{p}_{cT}+Xs}{\overline{p}+Xs}\right)^{\frac{1}{\alpha}}-1\right]\right\}^{0.5} \tag{4}$$

where M_T is the slope of CSL at elevated temperature; α, β and θ are the model parameters. It can be concluded from Eq. (4) that the yield surface shifts towards lower mean net stress values as the suction increases, which is in contrast with experimental observations (Fredlund et al., 1978). It was shown in the previous section that the model proposed by Hamidi et al. (2015) in its present form was not able to properly explain the behavior of unsaturated soils. However, the thermo-elastoplastic model under the framework of critical state soil mechanics can easily be improved as is described in the following sections.

3.1. Modification of the hardening modulus

As the first step, experimentally observed variation of the plastic behavior of soils with suction (Josa, 1988; Uchaipichat and Khalili, 2009) is taken into account, as displayed in Fig. 1.

Based on this figure, hardening (plastic) modulus can be simply modified for unsaturated conditions through the introduction of a multiplicative function H_{ws}, which increases almost linearly with increase in matric suction (drying):

$$\tilde{H}_s = H_L(1+\Lambda s) = H_L H_{ws} \tag{5}$$

where $\tilde{H}_s$ is the plastic modulus at suction s, and Λ is a material parameter. Therefore, Eq. (3) for calculation of the plastic volumetric strains is changed to

$$d\varepsilon_v^p = \frac{1}{\tilde{H}_s}\frac{d(\overline{p}_{cT}+Xs)}{\overline{p}_{cT}+Xs} \tag{6}$$

3.2. Modification of the yield surface

As discussed earlier, a decrease in yield stress was observed with increase in suction by substitution of Bishop's mean effective stress in the yield surface equation, which is in contrast with the experimental results. Considering the experimental data reported by Uchaipichat and Khalili (2009) which show variations of the yield

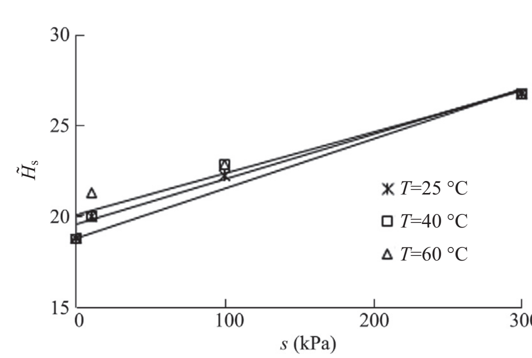

Fig. 1. Variation of the hardening modulus with suction. Experimental data after Uchaipichat and Khalili (2009) at temperatures of 25 °C, 40 °C and 60 °C, respectively.

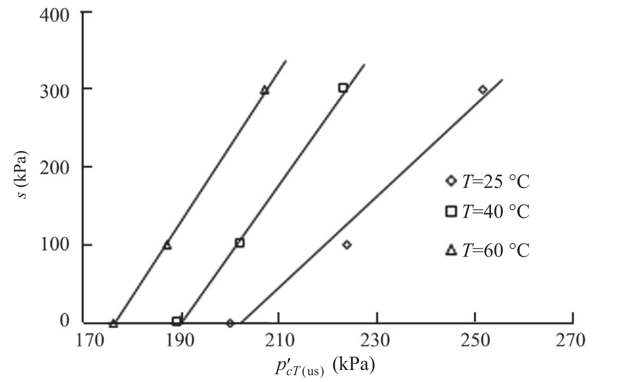

Fig. 2. Yield surfaces in the (p', s, T) space. Experimental data after Uchaipichat and Khalili (2009) at different temperatures of 25 °C, 40 °C and 60 °C, respectively.

stress in different suction values at elevated temperatures (as shown in Fig. 2), it is evident that preconsolidation stress increases almost linearly with an increase in suction. Also, it can be concluded that the rate of changes would decrease if the temperature increased.

Therefore, it is proposed to consider a linear relationship (continuous lines in Fig. 2) to show variations of yield stress for unsaturated soils with suction:

$$p'_{cT(us)} = p'_{cT(us)} + \varpi s \tag{7}$$

where $p'_{cT(us)}$ is the overconsolidation stress at matric suction s and temperature T. The dimensionless parameter ϖ is considered to model linear variations of overconsolidation stress at temperature T with suction. Substituting the yield stress from Eq. (7) into Eq. (4), the equation of yield surface can be rewritten as

$$\left[\frac{q}{M_T(\bar{p}+Xs)}\right]^2 = \frac{\alpha^2(\beta-1)}{2\theta-1}\left\{\left[\frac{\bar{p}_{cT(us)}+Xs}{\bar{p}+Xs}\right]^{\frac{1}{\Omega}}-1\right\} \tag{8}$$

Contrary to Eq. (4), Eq. (8) shows contraction of the yield surface with increase in temperature at constant suction, which is in agreement with experimental data. On the other hand, an increase in suction at constant temperature causes expansion of the yield surface. The rate of expansion decreases at high temperatures.

Fig. 3 depicts the thermal evolution of the yield surface in 3D stress space, including deviatoric stress, mean net stress and suction.

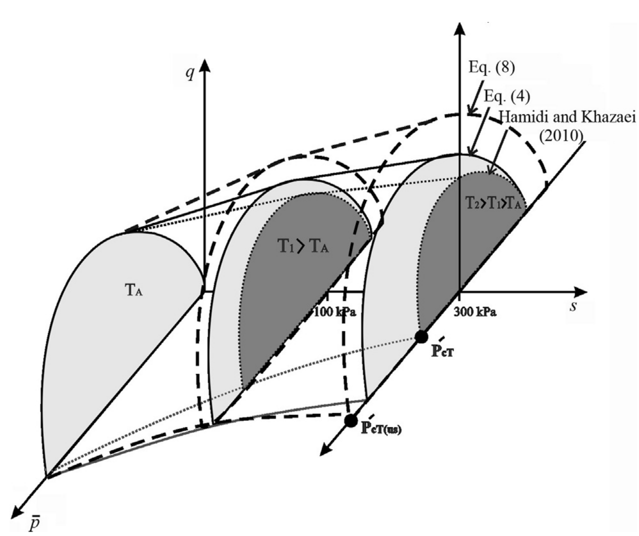

Fig. 3. Thermal evolution of the proposed yield surface. T_A is the ambient temperature, T_1 and T_2 are the elevated temperatures.

Table 1
Constitutive model parameters determined for experimental data of Uchaipichat and Khalili (2009).

M_T	λ_A	κ_A	n	f	ϖ	Λ	$c(\alpha)$	$\beta(f)$	$s_{e(To)}$
1.17	0.09	0.006	0	0.234	0.167	0.0015	1.09	0.24	18

Note: λ_A is the slope of NCL at ambient temperature; κ_A is the slope of reloading-unloading line in ambient temperature; $c(\alpha)$ is the parameter that controls thermal dependency of α; f is the parameter that controls thermal dependency of β; α and β are the model parameters for modification of flow rule; n is the parameter that controls thermal evolution of NCL; $s_{e(To)}$ is the air entry value in ambient temperature (Hamidi et al., 2015).

4. Evaluation of model predictions

Uchaipichat and Khalili (2009) conducted triaxial tests on Bourke silt at temperatures of 25 °C, 40 °C and 60 °C. The full description of modeling parameters for saturated soils was reported in Hamidi et al. (2015). For example, n was introduced to model variations in the slope of normal consolidation line (NCL) in space at temperatures higher than ambient temperature. Since the slope of NCL did not change with temperature in these tests, value of n was equal to zero.

Hamidi et al. (2015) proposed an equation for the variation of the CSL slope with suction. In order to consider the variation of M_T with suction s, M_{Ts} can be used instead; its values at matric suctions of 0 kPa, 100 kPa, 300 kPa are obtained to be 0.9, 1.075 and 0.98,

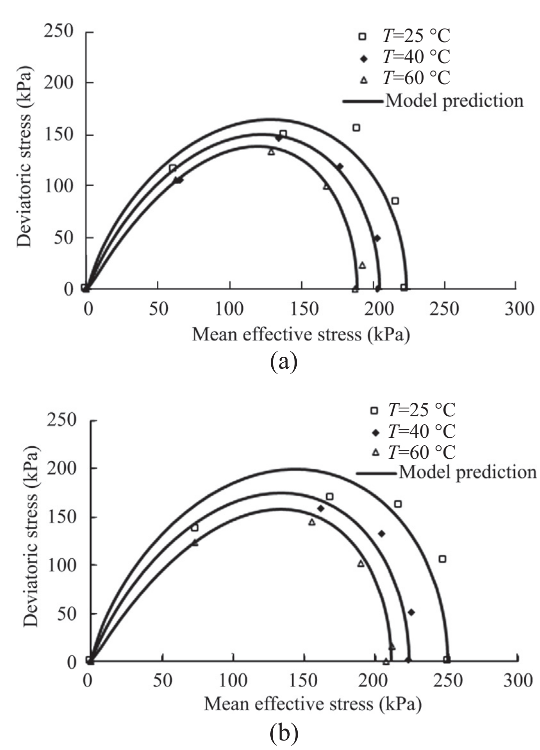

Fig. 4. Results of model prediction for the yield surface at different temperatures and matric suctions of (a) 100 kPa and (b) 300 kPa. Experimental data after Uchaipichat and Khalili (2009).

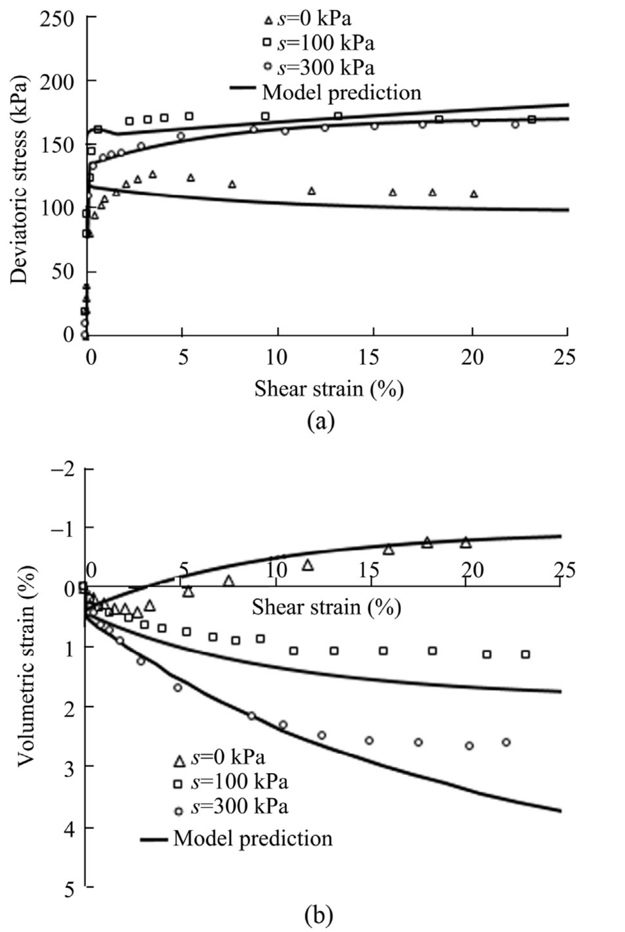

Fig. 5. Model predictions in overconsolidated samples with over-consolidation ratio of 4. Experimental data after Uchaipichat and Khalili (2009). (a) Deviatoric stress, (b) Volume change behavior.

respectively. In that equation, ζ that controls the variations of CSL slope with temperature was considered to be zero.

Parameter Λ can be obtained by performing suction-controlled tests and plotting plastic modulus versus matric suction (Fig. 1). Parameter ϖ can also be estimated from the experimental results of suction-controlled tests and plotting matric suction variations versus yield stress (Fig. 2). These two parameters are calibrated to be 0.0015 and 0.167, respectively, using the results of Uchaipichat and Khalili (2009). Table 1 summarizes the modeling parameters used for this set of data.

The modeling results for the yield surface at matric suctions of 100 kPa and 300 kPa are displayed in Fig. 4. At a constant suction, an increase in temperature has caused contraction of the yield surface, which is in agreement with the experimental evidence. This phenomenon was clearer at higher matric suctions. On the other hand, at constant temperature, increase in suction caused expansion of the yield surface. Indeed, increase in matric suction increased apparent viscosity (at constant internal friction angle), which increased shear strength of soil accordingly.

As shown in Fig. 5a, shear strength of overconsolidated samples increased with drying (increase in matric suction) to a specific limit; afterwards, increase of matric suction decreased its shear strength. This phenomenon was well predicted by the model. Also, based on the experimental data in Fig. 5b, higher volumetric strains were observed at higher matric suctions, which have been simulated well by the model.

5. Conclusions

In this research, a simple model is presented for modeling thermo-elastoplastic behavior of unsaturated clays in triaxial stress plane. The proposed model is developed within the framework of the critical state soil mechanics by adopting the modified Cam-clay model. Bishop's stress and suction have been introduced as independent parameters to describe the behavior of unsaturated soils under isothermal conditions. For fully saturated states, when the suction is equal to zero, Bishop's stress reduces to the total stress in excess of pore water pressure, which is the stress measure considered in the original saturated model. In order to generalize the base model to unsaturated states, only two parameters with a clear physical interpretation have been used. These parameters can easily be calibrated using triaxial shear tests. Finally, it was shown that an acceptable agreement exists between the model predictions and the experimental data obtained from unsaturated samples. This proves the reliability of the model.

Conflict of interest

The authors wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

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