A thermomechanical constitutive model for unsaturated clays

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A thermomechanical constitutive model for unsaturated clays is presented in this paper based on an existing model for saturated clays originally proposed by the authors. The saturated clay model was formulated in the framework of critical state soil mechanics and modified Cam-clay. The existing model has been generalised to simulate the experimentally observed behaviour of unsaturated clays by introducing Bishop's stress and suction as independent stress parameters and modifying the hardening rule and yield criterion to take into account the role of suction. The number of modelling parameters was kept to a minimum, and they all have clear physical interpretations, to facilitate the usefulness of model for practical applications. A step-by-step procedure for parameter calibration is also described. The model was finally evaluated using a comprehensive set of experimental data for the thermomechanical behaviour of unsaturated soils. Based on the results, the model is able to simulate the principle aspects of thermomechanical behaviour of unsaturated clays in a good manner.

Keywords: Constitutive model, Unsaturated clays, Plasticity, Cam-clay, Critical state, Thermomechanical behaviour

Introduction

Modelling of the thermomechanical behaviour of soils, particularly clays, has been the subject of many studies in the past. The possible reason for this attention is the thermal conditions faced in a number of high-priority applications such as geological storage of radioactive wastes, buried high-voltage cables and geothermal energy. Thermomechanical models that are able to simulate most of the observed behaviour characteristics of saturated clays at increased temperatures have been developed by several researchers.

Hueckel and Borsetto (1990) developed one of the first thermomechanical constitutive models by modifying the Cam-clay model to take into account the thermo-elastoplastic behaviour of clays. Also, thermal softening was considered in their model. The normally consolidated and overconsolidated conditions were modelled using thermoplasticity and thermoelasticity rules, respectively. They employed thermal evolution of the yield surface at constant plastic strain condition to model the irreversible effects induced by temperature.

Using the framework of modified Cam-clay model, Robinet *et al*. (1996) proposed a new model implementing two yield surfaces, one for the thermomechanical behaviour and the other one for thermal softening. Their model was capable of predicting expansive thermal strains correctly.

Modaressi and Laloui (1997) used non-associated flow rule and defined the rate of thermo-elastoplastic strains. Their model was non-isothermal and multi-mechanism considering isotropy and kinematic hardening.

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Using the framework of Hueckel and Borsetto (1990), Cui *et al*. (2000) proposed a thermal plastic mechanism that was capable of predicting thermal plastic strains in high overconsolidation ratios. Graham *et al*. (2001) proposed a non-isothermal model based on the modified Cam-clay model. Using a different approach, Laloui and Cekerevac (2003) proposed a model based on multi-surface placticity and a simple thermo-plastic mechanism including thermal hardening.

Abuel-Naga (2005) proposed a constitutive model in triaxial plane using the critical state framework. The model was comprised of an isotropic part to predict the volume changes of saturated clays in normally consolidated and overconsolidated states and a thermo-elastoplastic part for modelling the behaviour under deviatoric loading in high temperatures. One of the features of model was using two yield surfaces for mechanical and thermal loading conditions.

Liu and Xing (2009) proposed a thermo-mechanical model for overconsolidated clays by modification of a double hardening model using two extra parameters. The model was able to simulate important features of saturated clays with different overconsolidation ratios under different loading conditions.

Hamidi and Khazaei (2010) proposed a thermo-elastoplastic model based on the modified Cam-clay approach that was able to predict the mechanical behaviour of saturated clays in high temperatures in triaxial plane. The general idea behind their model was to consider the difference between normal consolidation lines (NCL) in the ambient and elevated temperatures. Delfan *et al*. (2015) applied the model in finite element code PISA, and simulated ATLAS experiment in one- and two-dimensional conditions.

Hueckel *et al*. (2011) studied temperature dependency of the internal friction angle in a boundary value problem. They

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simulated the impact of a cylindrical heat source on the soil mass surrounding it and showed that such temperature dependency may substantially affect the interpretation of thermal failure. Even if thermal increase of the internal friction was quite modest (less than 20%), its effect on the effective stress path near the heat source was quite significant.

Hong *et al*. (2013) evaluated the performance of several constitutive models in simulating the thermomechanical behaviour of saturated clays. Model evaluation was performed in terms of thermodynamics and elastoplastic requirements. It was shown that all the models would predict the thermomechanical behaviour of saturated clays reasonably. However, each constitutive model had its own limitations or unclear points from the theoretical point of view.

Using the general framework of critical state soil mechanics and modified Cam-clay formulation, Hamidi *et al*. (2015), modified the thermomechanical model originally proposed by Hamidi and Khazaei (2010) to improve the predictions by taking into account the isotropic thermal loading conditions as well as thermal dependency of yield surface and critical state line (CSL) in deviatoric plane that were neglected in the original model. Most of the characteristics of saturated clays in temperatures lower than boiling temperature of water were taken into account in their model.

Unlike some of the advanced models for the thermo-hydro-mechanical behaviour of saturated clays, the models for unsaturated clays are almost based on a Cam-clay elastoplastic approach. In the same manner, some thermo-hydro-mechanical models have been proposed for the behaviour of unsaturated clays. Philip and de Vries (1957) presented a model for coupled heat and moisture transfer in rigid porous media under the combined gradients of temperature and moisture. Also, de Vries (1958) extended this theory to include moisture and latent heat storage in the vapour phase, and the advection of sensible heat by water.

Modified versions of Philip-de Vries model were proposed by Milly (1982), Thomas and King (1991) and Thomas and Sansom (1995), using matric suction rather than volumetric moisture content as the primary variable.

The laboratory and field validation of Philip-de Vries theory have been reported by Ewen and Thomas (1989) and Thomas and He (1997), amongst others. Reasonable agreement has been found between the theoretical analyses and the laboratory/field results.

Furthermore, Geraminegad and Saxena (1986) presented a de-coupled flow deformation model, including the effect of matrix deformation on moisture, heat and gas flow through the porous media.

A coupled version of this formulation was later presented by Thomas and He (1995, 1997). They introduced the matrix displacement vector as a primary variable, and improved the coupling effects between the temperature and deformation. They also improved the energy balance equation by including moisture and latent heat storage in vapour phase, in addition to the advection of heat by water previously accounted for by de Vries (1958).

Similar formulations have also been given by Gawin *et al*. (1995) and Zhou *et al*. (1998). However, in Gawin *et al*. (1995), the constitutive laws of the solid phase were introduced through the concept of effective stress. Nevertheless, they used degree of saturation as the effective stress parameter, which is not fully supported by experimental evidences. They also retained the degree of saturation as the main coupling element between air and water flow fields resulting in the governing differential equations to be strongly non-linear.

In general, a major difficulty in the formulations discussed above is that they either completely ignored the matrix deformation (e.g. Philip and de Vries 1957; de Vries 1958; Milly 1982; Thomas and King 1991; Thomas and Sansom 1995) or used the theory of elasticity (Gawin *et al*. 1995) in conjunction with the 'state surfaces' approach (Thomas and He 1995; Zhou *et al.* 1998) to account for the strongly non-linear deformation behaviour of the soil matrix.

Such stress path dependency cannot be modelled using the theory of elasticity and/or the state surfaces approach. An appropriate plasticity model needs to be invoked, in order to take into account the variations of the yield surface with temperature and suction. Furthermore, in these formulations, effect of temperature on the state surfaces is not well defined, despite its importance in deformation response of the soil (e.g. Hueckel and Baldi 1990; Lingnau *et al*. 1995; Cui *et al*. 2000; Graham *et al*. 2001).

Moreover, some researchers simulated the consolidation process and pore water pressure around hot cylinders buried in saturated clay (e.g. Booker and Smith 1989; Britto *et al*. 1989; Lotfi *et al*. 2012). However, only the reversible volume change of the soil due to a change in temperature was considered in their models.

The elastoplastic constitutive model developed by Pastor *et al*. (1990), for fully saturated soils, has been extended to include partially saturated soil behaviour by Bolzon *et al*. (1996). The saturated soil model, formulated in the framework of generalised plasticity, considering volumetric and deviatoric strain hardenings and taking into account the memory of past stress history and possible limit states. The generalisation of the existing model to simulate the experimentally observed behaviour of partially saturated soils was obtained by introducing Bishop's stress and suction as independent stress parameters and modifying the hardening parameter and yield condition to take into account the role of suction.

Khalili and Loret (2001) presented an alternative theory for heat and mass transport through deformable unsaturated porous media. The work was an extension of theoretical developments of Loret and Khalili (2000) dealing with fully coupled isothermal flow and deformation in variably saturated porous media to include thermal coupling effects.

Wu *et al*. (2004) presented a thermo-hydro-mechanical constitutive model for unsaturated soils. The model was based on four component yield surfaces of the cap plasticity model and experimental results obtained for different types of soils. Extension of the model to include temperature effects was embodied through thermal softening and suction variation with respect to temperature, as well as thermal effects on hydraulic properties.

François and Laloui (2008) presented a constitutive model for soils based on a unified thermomechanical modelling approach for unsaturated conditions. The relevant temperature and suction effects were studied in the light of elastoplasticity. A generalised effective stress framework was adopted that would include a number of intrinsic thermo-hydro-mechanical connections to represent the state of stress in the soil. Two coupled constitutive aspects were used to fully describe the non-isothermal behaviour. The mechanical constitutive part was built on the concepts of bounding surface theory and multi-mechanisms plasticity, while water retention characteristics were described using elastoplasticity to reproduce the hysteretic response and the effect of temperature and dry density on retention properties.

Mašín and Khalili (2011) presented a mechanical model for non-isothermal behaviour of unsaturated soils. The model was based on an incrementally non-linear hypoplastic model for saturated clays and could therefore simulate the nonlinear behaviour of overconsolidated soils. A hypoplastic model for non-isothermal behaviour of saturated soils was developed and combined with the existing hypoplastic model for unsaturated soils based on the effective stress principle.

In this paper, an isothermal constitutive model is presented for predicting the thermo-elastoplastic behaviour of unsaturated clays in triaxial plane. This model is an extension of the thermo-elastoplastic model for fully saturated clays proposed by Hamidi *et al*. (2015). Using a non-associated temperature-dependent flow rule, the present model simulates the mechanical behaviour of clays with respect to temperature and suction changes.

The reference thermomechanical model for saturated clays

The model presented in this research is based on the model proposed by Hamidi *et al*. (2015). They presented a thermo-mechanical constitutive model for predicting the isothermal behaviour of saturated clays in different temperatures based on the general framework of critical state soil mechanics and modified Cam-clay formulation. Different parts of the reference constitutive model are illustrated in the following sections.

Elastic behaviour

Assuming that the coefficient of thermal expansion is independent of the stress, elastic volumetric strain increments are the sum of two thermal and mechanical components. However, the elastic shear strains are completely mechanical with no thermal component (Graham *et al*. 2001). Elastic volumetric strains can be generally expressed as Equation (1):

$$\mathrm{d}\varepsilon_v^e = \mathrm{d}\varepsilon_{vT}^e + \mathrm{d}\varepsilon_{vp}^e \tag{1}$$

where $\mathrm{d}\varepsilon_{vT}^e$ is the elastic volumetric strain caused by temperature and $\mathrm{d}\varepsilon_{vp}^e$ is the elastic volumetric strain caused by stress changes. The elastic volumetric strain increment caused by temperature can be calculated by Equation (2):

$$\mathrm{d}_{vT}^e = 3\alpha_e \mathrm{d}T \tag{2}$$

where $\mathrm{d}T$ is the increment of temperature change and α_e is the thermal expansion coefficient of the solid skeleton. This coefficient is dependent on both temperature and pressure changes. However, for most practical purposes, it can be considered as a constant and equal to 10^{-5} °C^{-1} (Bolzon and Schrefler 2005). The elastic volumetric strain increment caused by stress can be calculated by Equation (3):

$$\mathrm{d}\varepsilon_{vp}^e = \frac{1}{K}\mathrm{d}p' \tag{3}$$

where $K = K_0\left(p'/p'_0\right)^a$ is the bulk modulus and $K_0 = v_0 p'_0/\kappa_T$ is its initial value. Also, v_0 is the initial specific volume and A is the modelling parameter for considering non-linear dependency of bulk modulus to the effective stress.

Elastic shear strains are generally consisted of two thermal and mechanical components:

$$\mathrm{d}\varepsilon_s^e = \mathrm{d}\varepsilon_{sT}^e + \mathrm{d}\varepsilon_{sp}^e \tag{4}$$

where $\mathrm{d}\varepsilon_{sT}^e$ and $\mathrm{d}\varepsilon_{sp}^e$ are the thermal elastic shear strain and mechanical elastic shear strain, respectively. Since in this study the thermal shear strains are assumed to be negligible, using the increment of deviatoric stress (dq) and temperature-dependent shear modulus (G_T), it can be written as follows:

$$\mathrm{d}\varepsilon_s^e = \mathrm{d}\varepsilon_{sp}^e = \frac{\mathrm{d}q}{G_T} \tag{5}$$

Plastic behaviour

Plastic volumetric strain ($\mathrm{d}\varepsilon_v^p$) is the result of changes in the pre-consolidation pressure or the so-called hardening parameter, therefore:

$$\mathrm{d}\varepsilon_v^p = \frac{1}{H_L}\frac{\mathrm{d}p'_{cT}}{p'_{cT}} \tag{6}$$

where p'_{cT} is the pre-consolidation pressure in temperature T and $\mathrm{d}p'_{cT}$ is the change in hardening parameter with temperature. In this equation, H_L is the hardening modulus and depends on seven parameters as shown by Equation (7):

$$H_L = \frac{v}{\lambda - \kappa_T - \left(n\Delta e_T\right)\left(\frac{M_T}{M_T-\eta}\right)} \tag{7}$$

Here v is the current specific volume $(1 + e_T)$, Δe_T is the thermal change in void ratio, M_T is the slope of CSL in elevated temperature, λ is the slope of NCL and η is the current stress ratio (q/p'). Also, n as a dimensionless parameter can be determined by calibration. If the slope of NCL is not temperature dependant, the value of n will be equal to zero.

Plastic shear strains ($\mathrm{d}\varepsilon_s^p$) can be obtained by Equation (8) in which ψ is the ratio of plastic volumetric strain to plastic shear strain.

$$\mathrm{d}\varepsilon_s^p = \frac{\mathrm{d}\varepsilon_v^p}{\psi} \tag{8}$$

An associated flow rule has been presented for the model as follows:

$$\psi = \frac{\alpha\left[M_T^2(\beta-1)-\eta^2\right]}{2\theta\eta} \tag{9}$$

Here α, β and θ are the modelling parameters.

Yield surface

The yield surface can be determined using the following equation:

$$q = M_T p'\left\{\frac{\alpha^2(\beta-1)}{2\theta-1}\left[\left(\frac{p'_{cT}}{p'}\right)^{\frac{1}{\Omega}}-1\right]\right\}^{0.5} \tag{10}$$

$$\Omega = \frac{\theta}{2\theta - 1} \tag{11}$$

It should be mentioned that if $\beta = 2$ and $\alpha = \theta = 1$, Equation (10) represents the yield surface of the modified Cam-clay model. The yield surface shrinks with increase in temperature according to Equation (10).

Mechanical behaviour and constitutive models for unsaturated soils

This section is subdivided into two main parts. The first part reviews the basic features of the mechanical behaviour of unsaturated soils. In the second part, constitutive modelling of the mechanical behaviour of these soils is reviewed.

Mechanical behaviour

Generalised effective stress expression of Bishop (1959) was proposed in order to include unsaturated soils into the conventional soil mechanics framework as follows:

$$p'_B = p - u_a + \chi\left(u_a - u_w\right) = \bar{p} + \chi s \tag{12}$$

where χ is a function of the degree of saturation, u_a is the pore air pressure and u_w is the pore water pressure. This approach was proved to be capable of reproducing some features of the behaviour of unsaturated soils such as the increase in shear strength due to suction, but could not explain the other aspects such as wetting induced collapse. It was first demonstrated by Jennings and Burland (1962), who performed a series of oedometer and isotropic compression and wetting/drying tests on unsaturated soils ranging from silty sands to silty clays.

However, collapse due to the increase in pore water pressure cannot be illustrated well using the generalised effective stress formulation. Fully saturated soils collapse when pore water pressure becomes equal to the total stress and therefore the effective stress is zero. However, collapse of unsaturated soils can take place even at high values of the generalised effective stress. For any value of the net stress $(p - u_a)$ other than zero, an effective stress approach would predict swelling of a partially saturated soil subjected to wetting or increase in pore water pressure.

The inability to explain collapse behaviour of partially saturated soils is not the only problem of the single effective stress approach. More importantly, as noted by Jennings and Burland (1962), Gens (1996) and Wheeler and Karube (1996), changes in suction have different effects on the soil structure than changes in applied stress.

It is now generally accepted that two independent stress variables are necessary to explain the behaviour of partially saturated soils. Bishop and Blight (1963) first used net stress $(p - u_a)$ and suction $(s = u_a - u_w)$ to investigate the strength and volume change behaviour of unsaturated soils and produced graphical representations of state surfaces and failure envelopes for these soils.

Fredlund and Morgenstern (1977) suggested that any pair of stress state variables amongst $(p - u_a)$, $(p - u_w)$ and $(s = u_a - u_w)$ should be adopted when describing unsaturated soil behaviour. The most commonly used pair is net total stress $(p - u_a)$ and suction $(s = u_a - u_w)$. Recently, other stress state variables have also been used successfully (Alonso *et al*. 1990; Wheeler and Sivakumar 1995; Cui and Delage 1996; Bolzon *et al*. 1996). Hence, the mechanical behaviour of unsaturated soils can be properly described using any pair of stress fields chosen from three mentioned independent variables, i.e. total stress in excess of pore air pressure, total stress in excess of pore water pressure and suction or pore air pressure in excess of pore water pressure (Fredlund and Morgenstern 1977).

Houlsby (1997) presented two new stress variables for unsaturated granular media as Bishop's effective stress (Equation (12)) with $\chi = S_r$ and a modified suction parameter (ns), where S_r is the degree of saturation and n is the porosity.

As mentioned, the weighting parameter χ is often assumed to be equal to the relative saturation, S_r being consistent with thermodynamic considerations (Gray and Hassanizadeh 1991). In present study, the following equation presented by Khalili and Khabbaz (1998) has been used to calculate the parameter χ:

$$\chi = \begin{cases} \left[s_e(T)/s\right]^{\Omega} & s > s_e \\ 1 & s \le s_e \end{cases} \tag{13}$$

where χ is a weighting parameter which equals to one for saturated soils and decreases with decrease in degree of saturation (s_r). Also, $s_e(T)$ is the bubbling pressure or the air entry value at temperature, T and $\Omega = 0.5$. The air entry value at temperature T can be determined by conducting temperature-controlled drying tests to obtain the soil–water characteristic curves at any particular temperature. Alternatively, air entry value at temperature T can be obtained from the following expression:

$$s_e(T) = \frac{\sigma_T}{\sigma_{T_0}} s_e\left(T_0\right) \tag{14}$$

where $s_e\left(T_0\right)$ is the air entry value at reference temperature, T_0 and σ_T is the surface tensile energy at temperature T which is approximated as follows (Edlefsen and Anderson 1943):

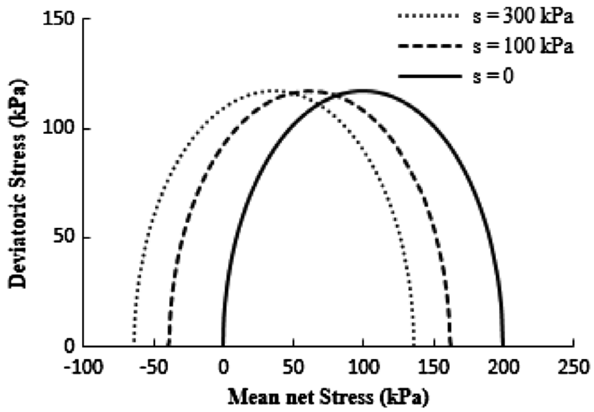
$$\sigma_T\left(\mathrm{J/m^2}\right) = 0.1171 - 0.0001516T \tag{15}$$

Therefore, dependency of the correlation between suction and mean effective stress (p') is considered via parameter χ.

Constitutive models for unsaturated soils

Elastic models are relatively easy to implement within numerical analysis and determining the relevant parameters, but have some major drawbacks. Most importantly, no distinction is made between reversible and irreversible strains. Elastoplastic constitutive models have been developed for both expansive and non-expansive soils. They all fall into two categories depending on the adopted stress variables, total stress models and effective stress models. Most elastoplastic models of unsaturated soils are extensions of models for saturated soils and are based on the concept of the loading-collapse yield surface. The following elements are usually defined for these models:

(a) A yield function, which represents the surface that separates elastic and elastoplastic behaviour. This surface expands with increase in suction to model the increase of shear strength and yield stress with suction. In this way, collapse due to wetting can also be reproduced.



1 Parametric representation of the yield surface in ($\bar{p}$, q, s) space

The expansion of the yield surface is defined by the increase of the isotropic yield stress and the apparent cohesion. The increase of the yield stress with suction is related to the variation of the position and shape of NCL in $e - \log p'$ plane. A number of models such as the Barcelona basic model (Alonso *et al.* 1990), Wheeler and Sivakumar (1995) and Bolzon *et al.* (1996) made an assumption about the position and shape of NCL, while the model of Josa *et al.* (1992) made an assumption about the variation of the isotropic yield stress with suction. The Barcelona basic model defines a second yield function which is a vertical 'wall' perpendicular to the suction axis in (q, p', s) plane.

(b) A plastic potential function, which determines the relative magnitudes of plastic strains at each point of the yield surface. The Barcelona basic model and the Josa *et al.* (1992) model assumed a non-associated flow rule such as K_0 condition with no lateral strain prediction. Wheeler and Sivakumar (1995) assumed an associated flow rule with predicted plastic strain vector normal to the yield surface. The plastic potential function also determines the position of the critical state line in (v, p', q) plane for each value of suction.

(c) A hardening/softening rule that determines the magnitude of the plastic volumetric strains. It is defined in terms of the equivalent isotropic yield stress for fully saturated state since it is assumed that collapse induced by wetting process is the same as isotropic compression beyond yield.

(d) Definition of the elastic behaviour within the yield surface. The loading/unloading coefficient (κ_T) and shear modulus (G_T) are usually assumed to be independent of suction.

Adaptation of the model to unsaturated condition

An unsaturated soil is a mixture of several phases. It is proposed that an unsaturated soil actually consists of four phases rather than the commonly referred to three phases. It is postulated that in addition to the solid, air, and water phases, there is the air–water interface that can be referred to as the contractile skin (Fredlund and Rahardjo 1993).

The major difference between unsaturated and saturated soil mechanics is the influence of matric suction on its behaviour, that is to say the mechanical and flow characteristics of unsaturated soil are affected by matric suction (Fredlund and Rahardjo 1993).

The simple introduction of Bishop's stress in the model proposed by Hamidi *et al.* (2015) allowed volume changes induced by suction to be taken into account, but the predictions obtained were not always in accordance with experimental observations. In particular, dependency of soil compressibility to suction was not properly reproduced. Also, the isotropic yield stress would decrease as the suction increased, which is in contrast with experimental observations of Fredlund *et al.* (1978). The largest amount of volumetric strains under increasing net mean stress was always predicted for fully saturated conditions, meaning that it was not possible to reproduce the maximum collapse on wetting side of CSL as a typical behaviour of loose soils (Josa *et al.* 1992).

In this paper, it is shown that a proper enhancement of the model developed by Hamidi *et al.* (2015) makes it possible to overcome these drawbacks and experimentally observed behaviour can be reproduced. Therefore, theoretical predictions of the enhanced model along different stress paths are presented and comparisons with experimental observations are shown. Since this model is an extension to the fully saturated model, the main features of saturated soils can also be captured by the enhanced model as explained above.

Substituting Bishop's definition of mean effective stress for unsaturated soils from Equation (12) into Equation (1) and Equation (6) implies dependency of volumetric strains to water content. However, this substitution does not result in a proper description of the behaviour of unsaturated soils, especially the volume change behaviour.

As pointed out by Fredlund and Morgenstern (1977), introduction of suction as an independent stress parameter is required to express plastic volumetric strains as bellow:

$$d\varepsilon_v^p = \frac{1}{H_L}\frac{d(\bar{p}_{cT} + \chi s)}{\bar{p}_{cT} + \chi s} \tag{16}$$

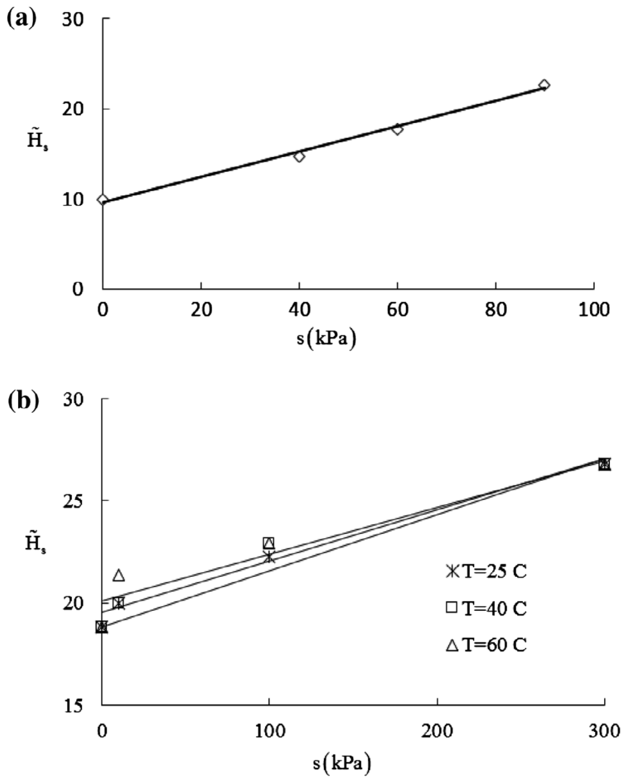
However, it is clear that the observed dependency of soil compressibility to suction is not considered in this equation. Thus, volumetric strains obtained from Equation (16) overestimate the experimental results. Therefore, plastic modulus in Equation (16) has been modified in order to consider the effect of suction.

Substituting Equation (12) into Equation (10) leads to the yield function defined in three-dimensional space ($\bar{p}$, s, q):

$$q = m_t(\bar{p} + \chi s)\left\{\frac{\alpha^2(\beta - 1)}{2\theta - 1}\left[\left(\frac{\bar{p}_{cT} + \chi s}{\bar{p} + \chi s}\right)^{\frac{1}{\Omega}} - 1\right]\right\}^{0.5} \tag{17}$$

The yield surfaces obtained by Equation (17) are plotted in Fig. 1 for $p'_{cT} = 200$ kPa, $M_T = 1.17$, $\beta = 2$, $\theta = 1$, $\chi = 1$ and three suction values of 0, 100 and 300 kPa. It can be observed that the yield surface shifts towards lower mean net stress values as suction increases, which is in contrast with experimental observations (Fredlund *et al.* 1978).

It was shown that the model proposed by Hamidi *et al.* (2015) was not able to properly explain the behaviour of unsaturated soils. However, the thermo-elastoplastic model under the framework of critical state soil mechanics can easily be improved as is described in the next sections.



2 Variation of the hardening modulus with suction in different temperatures (25, 40 and 60 °C). ***a*** **Experimental data after Josa (1988).** ***b*** **Experimental data after Uchaipichat and Khalili (2009)**

Modification of the hardening modulus

As the first step, experimentally observed variation of the plastic behaviour of soils with suction was taken into account as displayed in Fig. 2 (Uchapichat and Khalili 2009; Josa 1988). Based on this figure, hardening modulus can be simply modified for unsaturated condition through the introduction of a multiplicative function H_{ws} which increases almost linearly with increase in matric suction (drying) as shown in Equations (18) to (20):

$$\tilde{H}_s = H_L(1 + \Lambda s) \tag{18}$$

where $\tilde{H}_s$ is the hardening modulus in suction s and Λ is a material parameter. The above relation can be rewritten as Equation (19):

$$\tilde{H}_s = H_L H_{ws} \tag{19}$$

Therefore, Equation (16) for calculation of the plastic volumetric strains is changed to Equation (20):

$$\mathrm{d}\varepsilon_v^p = \frac{1}{\tilde{H}_s} \frac{\mathrm{d}\left(\bar{p}_{cT} + \chi s\right)}{\bar{p}_{cT} + \chi s} \tag{20}$$

Modification of the yield surface

As discussed earlier, a decrease in yield stress was observed with increase in suction by substitution of Bishop's mean effective stress in the yield surface equation, which is in contrast with the experimental results. Considering the experimental data reported by Josa (1988) and Uchapichat and Khalili (2009) that show variations of the yield stress in different suction values at elevated temperatures in Fig. 3, it is evident that pre-consolidation stress increases almost linearly by increase in suction. Also, it can be concluded that the rate of changes would decrease with increase in temperature. Therefore, it is proposed to consider a linear relationship to show variations of pre-consolidation stress ($p'_{cT(us)}$) with suction as follows:

$$p'_{cT(us)} = p'_{cT} + \varpi s \tag{21}$$

where $p'_{cT(us)}$ is the pre-consolidation stress at matric suction s and temperature T. The dimensionless parameter ϖ is considered to model linear variations of overconsolidation stress at temperature T with suction. Substituting the yield stress from Equation (21) into Equation (17), the equation of yield surface can be rewritten according to Equation (22):

$$\left(\frac{q}{M_T(\bar{p} + \chi s)}\right)^2 = \frac{\alpha^2(\beta - 1)}{2\theta - 1}\left[\left(\frac{\bar{p}_{cT(us)} + \chi s}{\bar{p} + \chi s}\right)^{\frac{1}{\Omega}} - 1\right] \tag{22}$$

Contrary to Equation (17), this equation shows contraction of the yield surface with increase in temperature at constant suction, which is in agreement with experimental data. On the other hand, an increase in suction at constant temperature causes expansion of the yield surface. The rate of expansion decreases in high temperatures. On the basis of Equation (17), the elasticity domain is thought of as temperature dependent, shrinking when soil is heated and expanding during cooling as shown in Fig. 4.

Suction dependency of the critical state line

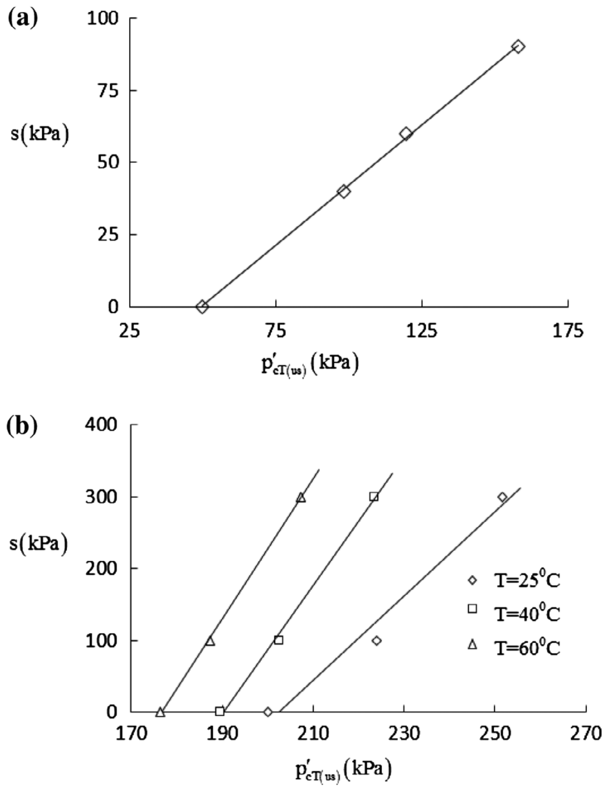
As the shape and characteristic dimensions of the yield locus are controlled by two independent functions of suction, the outcome may be quite different, as shown in Fig. 5. The peak strength on a given stress path may vary non-monotonically, first increasing and then decreasing. Second, if the peak strength does increase monotonically, then at some suction scrit it reaches a value at a current critical line, at q_{crit} (100 kPa in Fig. 5).

However, as suction increases and the yield locus continues to expand, say to 300 kPa, the yield point changes from the strain-hardening side to the strain-softening part of the locus. Hence, subsequent triaxial loading at that suction produces yielding at $f_{(s=300\ \mathrm{kPa})}$, but strain hardening allows the locus to shrink until it reaches the current critical line at $M_{(s=300\ \mathrm{kPa})}$, and hence possibly at a much lower $\tilde{q} < q_{crit}$. On the other hand, it was shown by Hamidi *et al.* (2015) that the slope of CSL is temperature dependant and therefore, Equations (23) and (24) are proposed to calculate it in different temperatures and suctions:

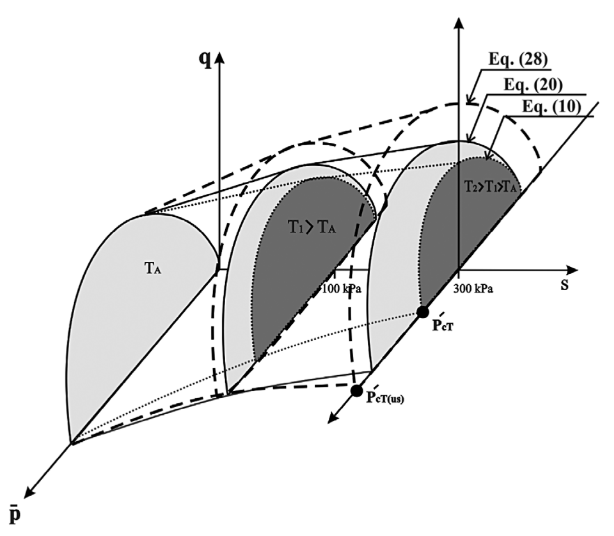
$$M_T = M_A + (-1)^{(x+1)} \zeta \ln\left(t/T_0\right) \tag{23}$$

where x is equal to 1 or 0 if the slope of CSL increases or decreases with the increase in temperature, respectively. Equation (23) would reduce to $M_T = M_A$ in the ambient temperature.

For unsaturated soil, it is necessary to modify Equation (23) to take into account the effect of matric suction. Equation (24) is presented to predict variations of the slope of CSL in temperature T with regard to the matric suction:



3 **Yield surfaces at (p', s, T) plane in different temperatures (25, 40 and 60 °C).** ***a*** **Experimental data after Josa (1988).** ***b*** **Experimental data after Uchaipichat and Khalili (2009)**



4 **Thermal evolution of the proposed yield surface**

$$M_{Ts} = M_A + (-1)^{(x+1)}\zeta \ln\left(T/T_0\right) + (-1)^{y+1} \ln\left(\frac{\chi s}{s_0 + 1}\right) \quad (24)$$

where y is equal to 1 if $S < S_{cr}$ and 0 for $S > S_{cr}$.

Therefore, elastic volumetric strains for an unsaturated soil can be calculated by Equation (25):

$$\mathrm{d}\varepsilon_v^e = 3\alpha_e \mathrm{d}T + 1/K_B \mathrm{d}p'_B \quad (25)$$

Here, K_B shows the elastic modulus for the unsaturated condition. In the model proposed by Hamidi *et al*. (2015), shear modulus was assumed to be dependent to both temperature and current state of stress. Thus, substituting the Bishop's stress in shear modulus equation, elastic shear strains can be obtained using Equation (26):

$$\mathrm{d}\varepsilon_{sp}^e = \mathrm{d}q/G_{T(us)} \quad (26)$$

where $G_{T(us)}$ is the shear modulus for soil with matric suction s. For fully saturated cases, Equation (26) will reduce to its original form proposed by Hamidi *et al*. (2015). Also, as mentioned in previous sections, substituting the Bishop's stress and modifying the hardening modulus, plastic volumetric strains can be calculated using Equation (27):

$$\mathrm{d}\varepsilon_v^p = \frac{1}{\tilde{H}_s}\frac{\mathrm{d}p'_{cTB}}{p'_{cTB}} \quad (27)$$

where p'_{cTB} shows the pre-consolidation stress for unsaturated condition. For fully saturated conditions, p'_{cTB} would turn into p'_{cT} as used in the original formulation (Hamidi et al. 2015). Finally, applying all the modifications explained in previous sections, the yield surface is obtained using Equation (28):

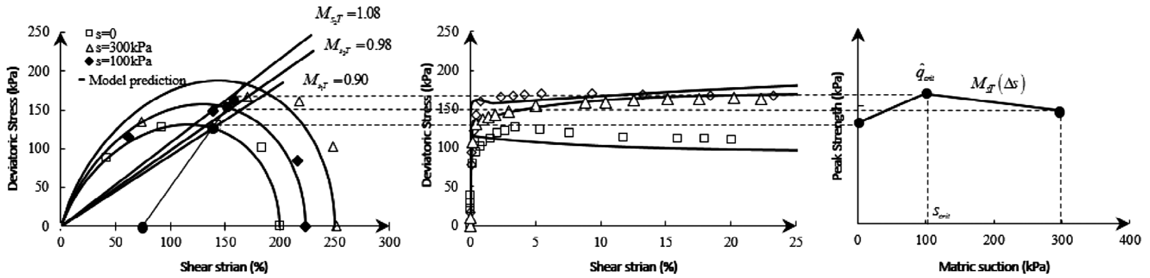
$$f_b = \left(\frac{q}{M_{Ts}p'_B}\right)^2 - \frac{\alpha^2(\beta - 1)}{2\theta - 1}\left[\left(\frac{p'_{cTB}}{p'_B}\right)^{1/\Omega} - 1\right] \quad (28)$$

where f_b is the equation of yield surface in matric suction s and temperature T. Again, this equation reduces to its original form for the fully saturated condition.

Model predictions by experimental results of Uchaipichat and Khalili (2009)

Uchaipichat and Khalili (2009) conducted triaxial tests on Bourke silt in three different temperatures of 25, 40 and 60 °C. A full description of modelling parameters for saturated soils has been reported in Hamidi *et al*. (2015). For example, n was introduced for modelling the variations of NCL slope in $e - \ln p'$ space at temperatures higher than the ambient temperature. Since the slope of NCL did not change with temperature in these tests, n was assumed equal to zero. Moreover, considering that the slope of CSL changes with matric suction in the tests, the values of M_{Ts} in matric suctions of 0, 100, 300 kPa were obtained as 0.9, 1.075 and 0.98, respectively. Also, ζ that controls the variations of CSL slope with temperature was considered to be zero.

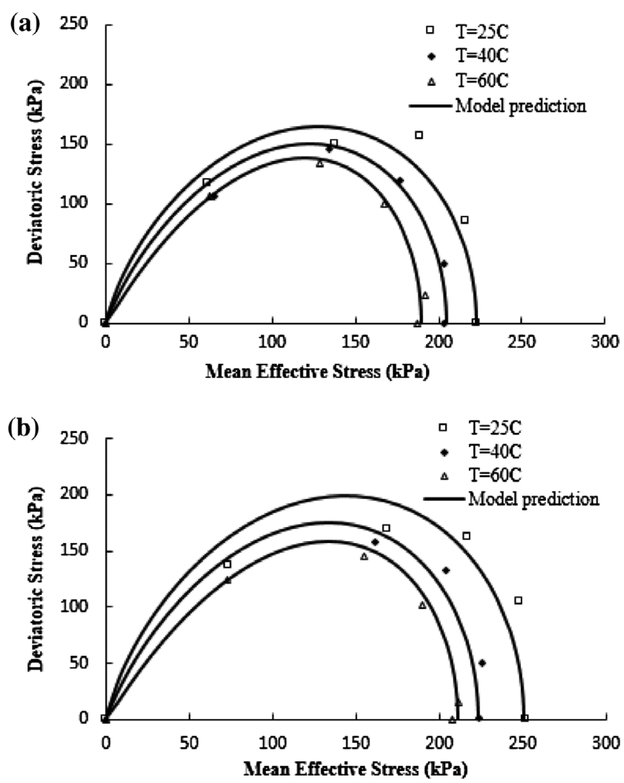
Two newly defined parameters for extending the scope of model to unsaturated conditions are Λ and ϖ for considering the relationship of hardening modulus and yield criterion with suction, respectively. Parameter Λ can be obtained by performing suction controlled tests and plotting plastic modulus versus matric suction (according to Fig. 2). Parameter ϖ can also be estimated from the experimental results of suction-controlled tests and drawing matric suction variations versus yield stress (same as Fig. 3). These two parameters are calibrated equal to 0.0015 and 0.167, respectively, using the experimental results of Uchaipichat and Khalili (2009). Table 1 summarises the modelling parameters used for this set of data.



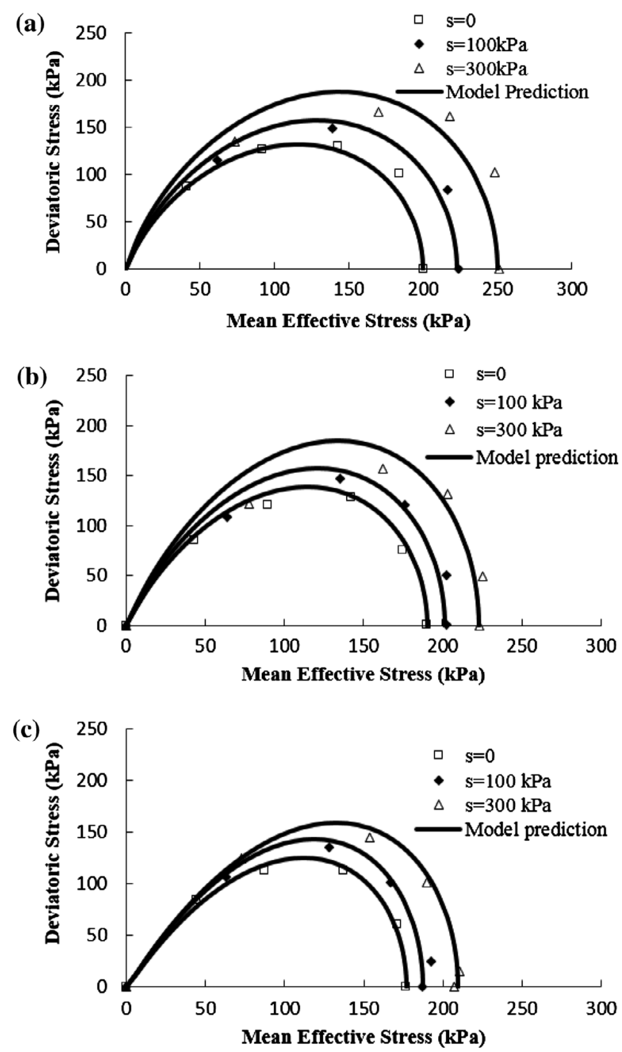
5 Evolution of peak shear strength with matric suction at constant temperature

Table 1 Constitutive model parameters for experimental data of Uchaipichat and Khalili (2009)

Model parameter	M_T	λ_A	κ_A	n	f	ϖ	Λ	$c(\alpha)$	$\beta(f)$	$s_{e(T_0)}$
Value	1.17	0.09	0.006	0	0.234	0.167	0.0015	1.09	0.24	18

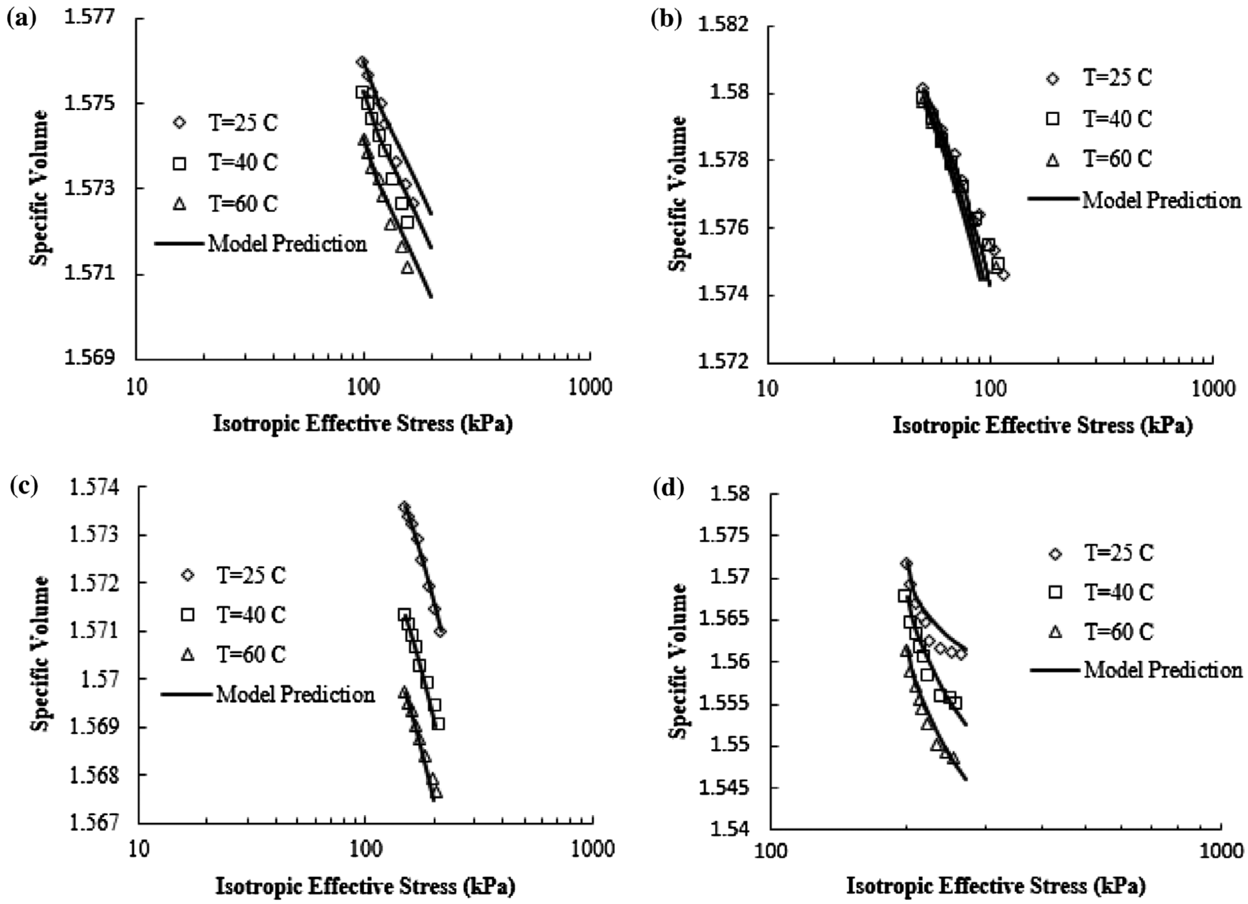


6 Results of model predictions for the yield surface at different temperatures and matric suctions. (Experimental data after Uchaipichat and Khalili, 2009). *a* 100 kPa, *b* 300 kPa

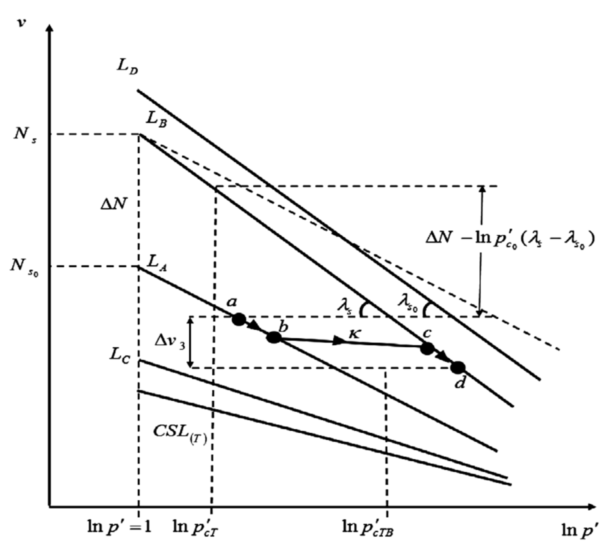


7 Results of model predictions for the yield surface at different matric suctions and temperatures. (Experimental data after Uchaipichat and Khalili 2009). *a* 25 °C, *b* 40 °C, *c* 60 °C

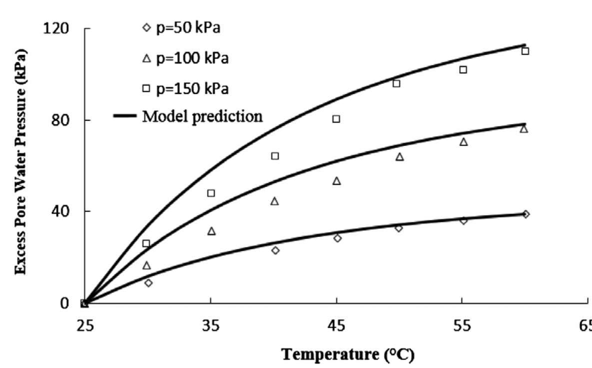
The evolution of the yield locus with temperature and matric suction is shown in Figs. 6 and 7. At a constant suction, an increase in temperature caused a reduction in the size of yield locus. This effect was most prominent at higher matric suctions. On the other hand, at a constant temperature, an increase in matric suction caused an expansion of the yield locus. This effect was less noticeable at higher temperatures.



8 **Comparison of model predictions and experimental results for specific volume-isotropic effective stress curves. (Experimental data after Uchaipichat and Khalili 2009).** *a* $\bar{p}$ = **50 kPa,** *b* $\bar{p}$ = **100 kPa,** *c* $\bar{p}$ = **150 kPa,** *d* $\bar{p}$ = **200 kPa**

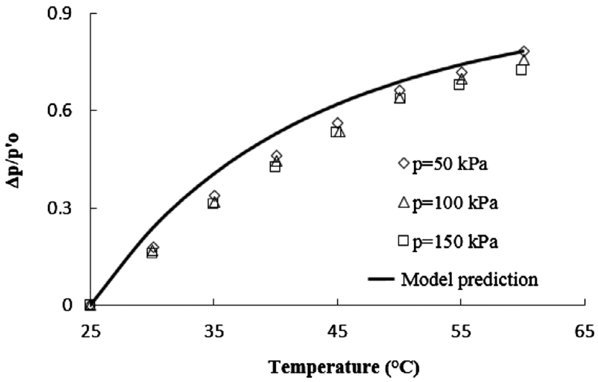


9 **Thermal and suction dependency of normal consolidation line**

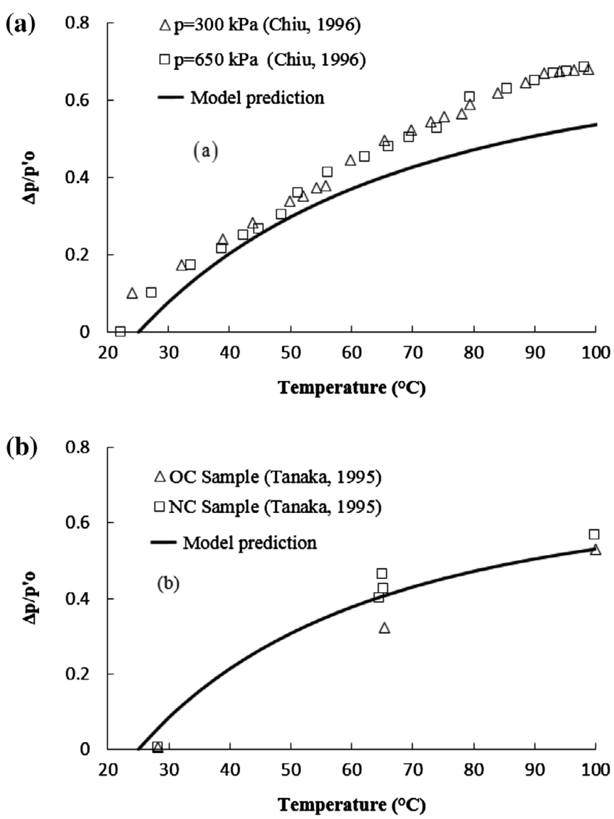


10 **Prediction of pore water pressure changes for saturated samples with different initial mean effective stresses during constant water content heating tests (Experimental data after Uchaipichat and Khalili 2009)**

In this section, model simulations are investigated for 12 temperature-controlled desaturation tests by Uchaipichat and Khalili (2009). Both overconsolidated and normally consolidated samples were used in the comparisons. The testing temperatures varied from 25 to 60 °C, and the net stresses ranged from 50 to 200 kPa. The matric suction within the samples ranged from 0 to 300 kPa which was applied at a constant temperature.



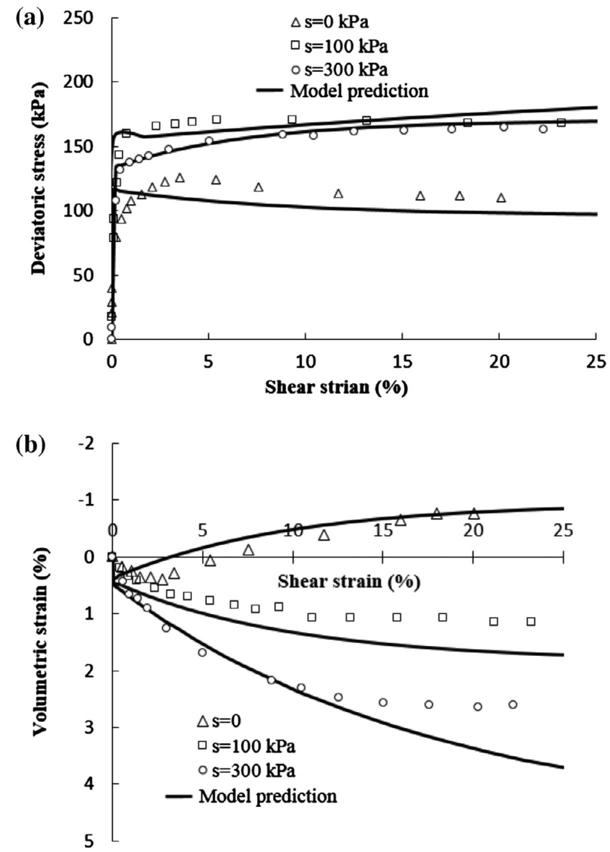
11 Predicted normalised pore water pressure changes of saturated samples during constant water content heating (Experimental data after Uchaipichat and Khalili 2009)



12 Prediction of the normalised pore water pressure changes during heating under constant water content condition. *a* Kaolinite samples tested by Chiu (1996), *b* Illite samples tested by Tanaka (1995)

Temperature-controlled desaturation tests on overconsolidated samples

For these tests, the samples were loaded to an isotropic effective stress of 200 kPa and then unloaded to different overconsolidation ratios before applying the matric suction. The change



13 Model predictions for overconsolidated samples (OCR = 4) in matric suctions of 0, 100 and 300 kPa at 25 °C. (Experimental data after Uchaipichat and Khalili 2009). *a* Deviatoric stress–shear strain, *b* volumetric strain–shear strain

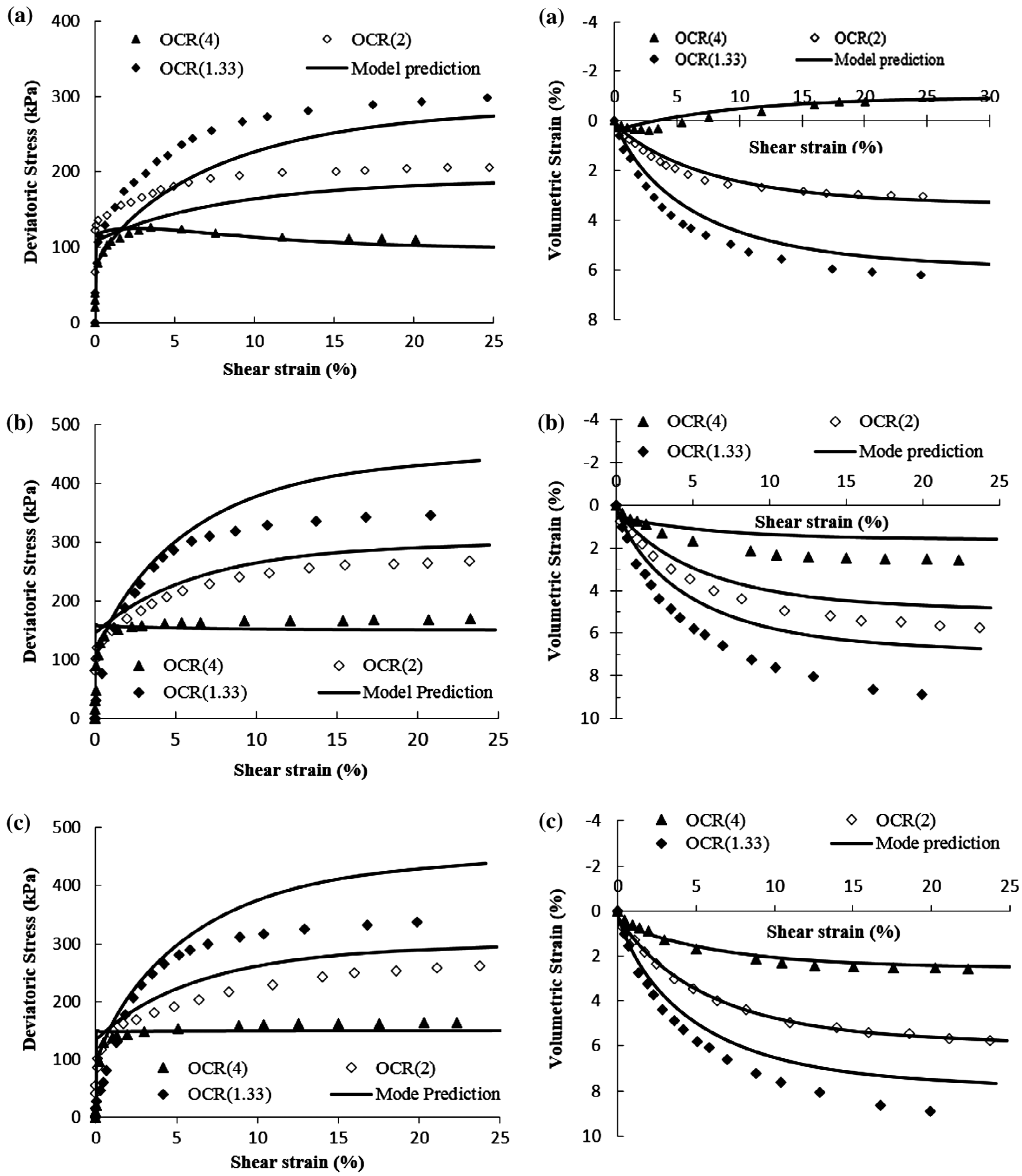
in the specific volume due to the change in matric suction is therefore in the elastic range and thus, integrating Equation (1) for volumetric elastic strains results in the following equation:

$$\Delta v_{B1} = 1/K_B \ln\left(\frac{p'_0 + \Delta(\chi s)}{p'_0}\right) \tag{29}$$

where p'_0 is the initial mean effective stress, Δv_B is the change in the specific volume and K_B is the elastic modulus (for unsaturated condition in temperature T). The effective stress parameter χ was obtained from Equation (13) as suggested by Khalili and Khabbaz (1998). Figure 8(a) and (b) compares the plot of predicted volume change in different matric suctions with the experimental results obtained from desaturation tests. As it can be seen, good agreement exists between predicted and measured experimental data in various temperatures.

Temperature-controlled desaturation tests on normally consolidated samples

For matric suctions lower than the air entry value ($s \leq s_e$), the soil is quasi-saturated, and specific volume changes with increase in matric suction. The normally consolidated samples would follow the isotropic normal consolidation line and



14 Prediction of deviatoric stress–shear strain curves in triaxial tests on saturated samples at different temperatures. (Experimental data after Uchaipichat and Khalili 2009). ***a*** **25 °C,** ***b*** **40 °C,** ***c*** **60 °C**

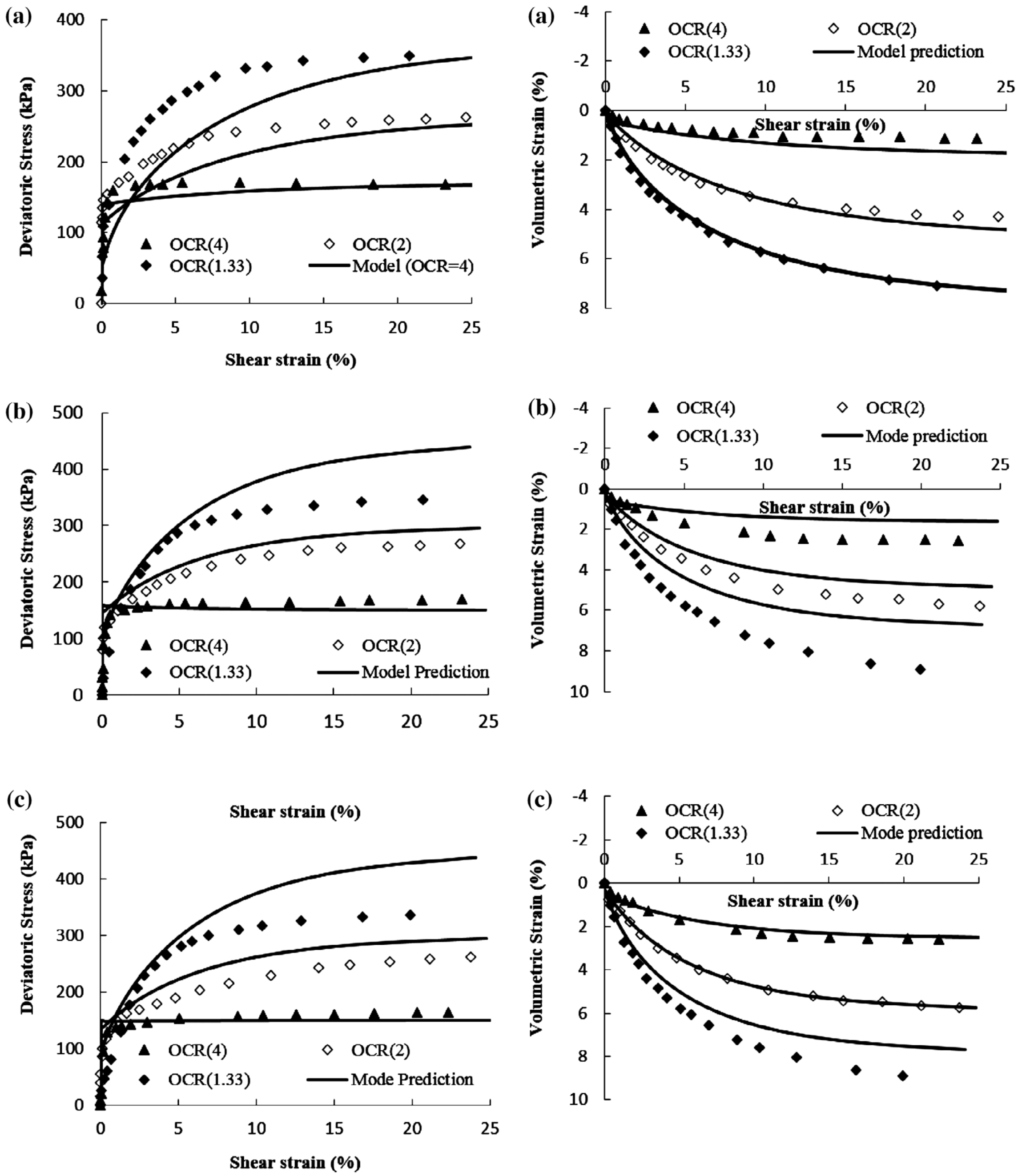
full saturation would not lead to creation of capillary forces required to generate negative pore water pressures.

Considering that plastic deformations are only due to the change in matric suction, the only unknown variable in Equation (27) would be the matric suction. Therefore, the change in specific volume at any temperature can be determined by integrating Equation (27) with respect to the matric suction. In this case, change in specific volume (Δv_B) is expressed as follows:

$$\delta v_{B2} = \frac{1}{H_{sw}} \ln\left(\frac{p'_{cT(us)} + s}{p'_{cT}}\right) \quad \text{for} \;\; s \le s_e \qquad (30)$$

For matric suctions greater than the air entry value ($s > s_e$), the soil is in the unsaturated state and increasing matric suction will increase both the effective stress and the effective pre-consolidation pressure ($\Delta p'_B > 0$, $\Delta p'_{cTB} > 0$).

If the increase in the effective stress is smaller than corresponding increase in the isotropic pre-consolidation pressure ($\Delta p'_B \le \Delta p'_{cTB}$), deformations are consisted of two parts. The first part includes deformations associated with the fully saturated condition until the air entry value (s_e) is reached. This part means the moving along NCL for the normally consolidated soils. Increasing the suction more than s_e, the soil would be in



15 Prediction of deviatoric stress–shear strain curves in triaxial tests at 100 kPa suction and various temperatures. (Experimental data after Uchaipichat and Khalili 2009). ***a*** **25 °C,** ***b*** **40 °C,** ***c*** **60 °C**

unsaturated state and its condition projects on '*b–c*' path in Fig. 9. Variations in specific volume can be obtained by summing up the integration of Equations (25) and (27):

$$\Delta v_{B3} = \frac{1}{H_{sw}} \ln\left(\frac{p'_{cT(us)} + s_e}{p'_{cT(us)}}\right) + 1/K_B \ln\left(\frac{p'_{cTB}}{p'_{cT(us)} + s}\right) \quad (31)$$

$$\text{for } s > s_e \text{ and } \Delta p'_B \leq \Delta p'_{cTB}$$

However if, $\Delta p'_B > \delta p'_{cTB}$, change in the specific volume will include both elastic and plastic volumetric changes. To quantify it, the stress path '*a–b–c–d*' in Fig. 9 should be considered. In this figure, L_A shows the isotropic consolidation line for the saturated soil with zero matric suction in the ambient temperature. Based on the assumptions in the model proposed by Hamidi *et al*. (2015), the isotropic consolidation line approaches to L_C line in temperatures more than the ambient one and eventually it would be coincided with the critical state line. On the other hand, according to the experiments reported by Alonso *et al*. (1990), Josa (1988) and Uchaipichat and Khalili (2009), increase in suction causes the L_A line to get closer to the L_B line at a constant temperature.

Therefore, considering these two reverse effects on NCL due to the variations in suction and temperature, it can be concluded that for the suction s and temperature T, the line L_B is indeed the normal consolidation line and if the desaturation test is conducted in ambient temperature, the line L_D is the normal consolidation line.

For matric suction values beyond the air entry value, volume change of the sample 'A' follows the stress path a–b–c–d and the stresses at points a and d will be p'_{cT} and p'_{cTB}, respectively. It can be observed that the change in the specific volume from a to d is equal to the difference between the volume change from a to d equal to $\Delta N - \ln p'_{c0}\left(\lambda_s - \lambda_{s0}\right)$ Thus, the change in specific volume, in its most general form, is expressed as follows:

$$\delta v_{B3} = \frac{1}{H_{sw}} \ln\left(\frac{p'_{cT(us)} + \delta(\chi s)}{p'_{cT(us)}}\right) - \left[\Delta N - \ln p'_{c_0}(\lambda_s - \lambda_{s_0})\right] \quad (32)$$

For the tests conducted at 25 °C, increase in matric suction increased the isotropic effective stress less than the pre-consolidation pressure. Thus, the volume changes for these tests can be predicted using Equations (29) and (30).

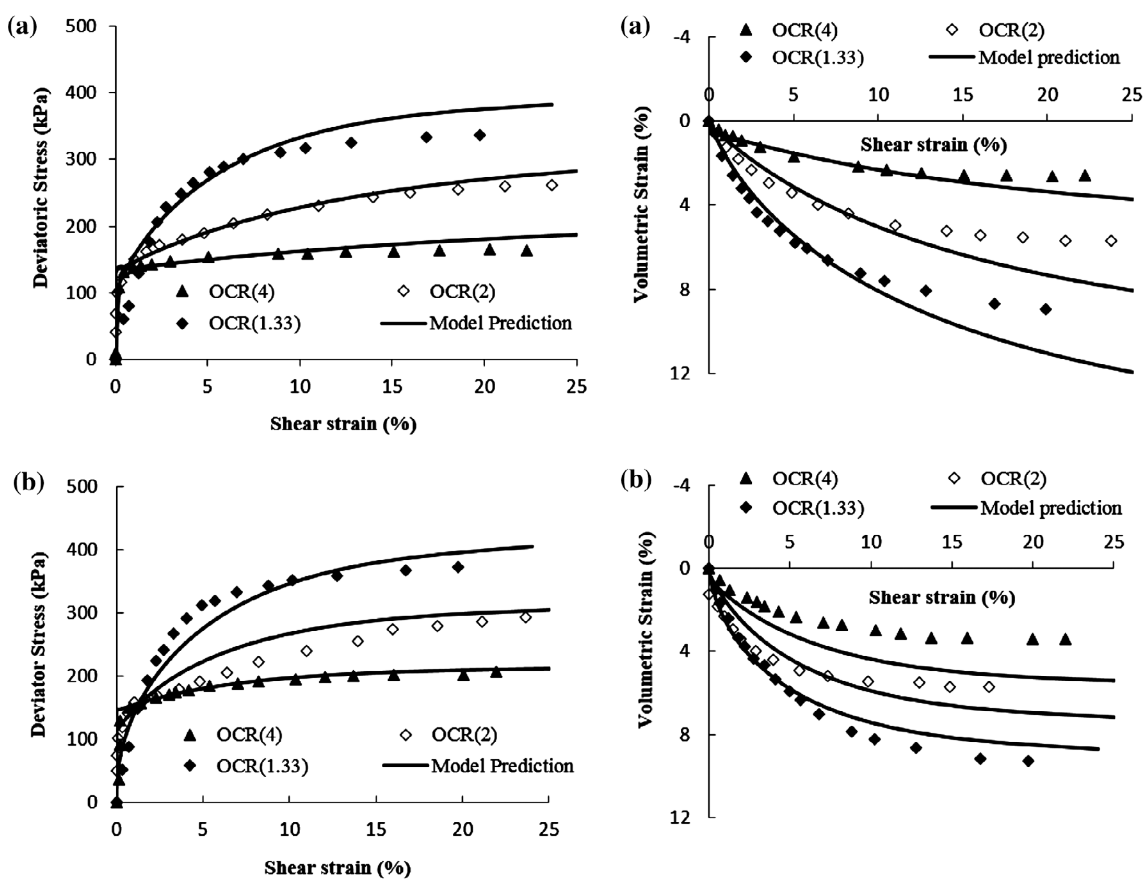
However, for the tests conducted at temperatures of 40 and 60 °C, increase in matric suction increased the isotropic effective stress more than pre-consolidation pressure. In this regard, the volume changes for these tests were predicted using Equations (31) and (32).

Figure 8(d) shows a plot between predicted volume change with increasing matric suction and experimental results obtained from desaturation tests. As it can be seen in the figure, a good agreement exists between predicted and measured experimental data for all testing temperatures.

Thermal loading tests with constant water content

In this section, model predictions and the experimental results for thermal loading tests in constant water content are considered. Uchaipichat and Khalili (2009) performed a number of constant water content thermal loading tests with initial matric suctions ranging from 0 to 300 kPa in a range of temperatures between 25 and 60 °C. For saturated samples ($s = 0$), the tests were performed in mean effective stresses of 50, 100 and 150 kPa. For unsaturated samples ($s = 100$ and 300 kPa), the tests were conducted in mean net stresses of 50 and 150 kPa. During heating, only the excess pore air pressure was allowed to dissipate. Thus, increase in pore water pressure was equal to the decrease in matric suction.

Since deformations are within the elastic range, Equation (25) was used to plot the variations of excess pore water pressure versus temperature. By integrating Equation (25), it can be rewritten as follows:



16 Prediction of deviatoric stress–shear strain curves in triaxial tests at 300 kPa suction and various temperatures. (Experimental data after Uchaipichat and Khalili 2009). ***a*** **25 °C,** ***b*** **60 °C**

$$\Delta p' = p'_0 \exp\left[K_B\left(\varepsilon_v^e - 3\alpha_e \Delta T\right)\right] \quad (33)$$

where $\Delta p'$ is the excess pore water pressure equal to the decrease in matric suction and p'_0 is the initial mean effective stress.

Figure 10 indicates that the excess pore water pressure can be predicted well using the model formulation for initial effective stresses of 50 and 100 kPa. The sample with initial effective stress of 150 kPa is lightly overconsolidated and its actual compressibility may have been greater than that is obtained from the unloading path. Therefore, the pore water pressure is slightly overpredicted in this case.

Figure 11 displays a plot of predicted excess pore water pressure normalised by the initial effective stress versus temperature where good agreement between predictions and experimental results can be observed. The slight differences are due to the variations in thermal expansion coefficient of water which was ranged from 4.3×10^{-4} to 8×10^{-4} °C^{-1}.

Figure 12(a) shows the results of undrained heating tests reported by Chiu (1996) on kaolinite samples. It can be observed that model predictions are in good agreement with experimental results. The model predictions also agreed well with the undrained heating test results reported by Tanaka (1995) on illite samples as shown in Fig. 12(b).

Temperature- and suction-controlled triaxial shear tests

Uchaipichat and Khalili (2009) conducted triaxial shear tests under matric suctions ranging from 0 to 300 kPa and temperatures ranging between 25 and 60 °C. All triaxial tests were conducted in drained condition. The samples were sheared under constant cell pressure, temperature and matric suction with increase in deviatoric stress.

As shown in Fig. 13(a), shear strength of overconsolidated samples increased in drying phase (with increase in matric suction) to a specific limit; afterwards, increase of matric suction decreased the shear strength. This phenomenon was well predicted by the model. Also, based on the experimental data in Fig. 13(b), higher volumetric strains were observed at higher matric suctions, that have been simulated well by the model.

Figures 14–16 show the results of drained triaxial shear tests conducted at different temperatures and overconsolidation ratios for suction levels of 0, 100 and 300 kPa, respectively. According to the experimental data, the soil dilated upon shear and showed a pronounced peak in the stress–strain curve which increased with decrease in overconsolidation ratio (OCR). Also, increase in temperature caused the decrease in peak shear strength and induced less dilative response, whereas an increase in suction had the reverse effect. All these aspects have been captured successfully by the model which resulted in a good agreement between experimental data and predictions.

Conclusion

In this research, a simple model was presented for modelling the thermo-elastoplastic behaviour of unsaturated clays in triaxial stress plane. The proposed model was developed within the framework of critical state soil mechanics by adopting the modified Cam-clay model. Bishop's stress and suction have been introduced as independent parameters to describe the behaviour of unsaturated soils under isothermal conditions. For fully saturated states, when suction was zero, Bishop's stress reduced to the total stress in excess of pore water pressure equal to the stress parameter considered in the original saturated model. In order to generalise the basic model to unsaturated states, only two parameters with clear physical interpretation have been used. These parameters can easily be calibrated using triaxial shear tests. Finally, it was shown that a good agreement existed between model predictions and experimental data which proved reliability of the model for application in numerical studies.

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