SageMath for teachers

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Introduction

SageMath is a free open-source mathematics software that we recommend to mathematics teachers. Indeed, SageMath can be helpful to them because of personal use and because it can be presented in class or within additional activities that target selected pupils.

One advantage of SageMath is that its commands resemble those of the widely-used programming language Python. In particular, learning SageMath can be seen as a friendly way of learning some Python.

Getting started

To get started with SageMath, one can download it from the official website

https://www.sagemath.org/

(Pieces of advice: do not install the version for development; SageMath also needs an editor that has to be installed separately, one common choice being JupyterLab).

Any mathematics teacher has sufficient mathematical knowledge to understand the tutorials and learn the basic features of SageMath.

Food for thoughts

Many mistakes with SageMath lead to an error message. Pay particular attention to prevent those that do not lead to error messages. In short: Be aware of what you are doing!

We collect here some SageMath facts that captured our attention. (Disclaimer: we refer to the current software version of 2024, and commands may evolve with time).

Very strange.

- tan(pi/2) outputs Infinity (however, the tangent function is not defined at $\pi/2$)

Very good.

- For quadratic equations with integer coefficients, the square-root of the determinant will not be approximated but remains as a radical. Moreover, square factors will be taken out of the square-root automatically. For example, the square-root of the discriminant 12 is $2 * \text{sqrt}(3)$.

Be careful.

- Decimal fractions will be approximated, even allowing for a very high precision. The reason is that 1/10 when written in binary has infinitely many digits after the comma. To avoid this issue, write terminating decimals as fractions.

Pay attention.

- For the logarithm, both ln and log mean logarithm in base e. In general, SageMath occasionally has more ways to get to the very same thing.
- Don't forget that Sagemath uses the scientific notation when displaying many digits of a number.

Look closely.

- The notation for a closed interval resembles the union of two halflines. For example, the real numbers with absolute value greater than 1 are

$$[[x < -1], [x > 1]]$$

while the real numbers with absolute value less than 1 are

$$[[x > -1, x < 1]]$$

- When SageMath plots parametric plane curves it could choose by default different scales on the two coordinate axes. In practice, a circle may look like an ellipse with high eccentricity.

Unexpected.

- Although it is possible to give custom names to quantities and variables, Sagemath will not understand the command

$$\text{solve}([x^2 == 9], x)$$

if for example x is replaced with y.

Too many assumptions.

- To focus on some cases, it is possible to restrict the values for a variable, for example with

$$\text{assume}(x > 5)$$

There is a command forget() to stop such an assumption. Indeed, just writing a new assumption will not forget the previous ones, so SageMath can signal an error because of contradictory or simply redundant assumptions.

Under development.

- The function find_root determines one (approximated) root in a given interval. This will be updated in future versions of SageMath to find more roots automatically, and possibly (for some class of functions) to find all roots.

References

- Official SageMath Tutorials, https://www.sagemath.org/
- Michael O'Sullivan, David Monarres and Matteo Polimeno, *SDSU Sage Tutorial*, https://mosullivan.sdsu.edu/Teaching/sdsu-sage-tutorial/sagecalc.html
- Aaron Tresham, *Common mistakes with Sage*, https://cocalc.com/share/public_paths/f8df5b36830778dde7b3c3c4e68c542bbaeeefba