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Is it Alpha or Beta? Decomposing Hedge Fund Returns When Models are Misspecified

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Abstract

The decomposition of hedge fund returns is hampered by model misspecification. To address this issue, we develop a novel approach to compare models in a large population of funds. This comparison, which accounts for misspecification-driven estimation errors, sharpens the separation between alpha and beta. Our analysis reveals that: (i) prominent models are as misspecified as the CAPM, (ii) several factors—primarily time-series momentum, variance, carry—capture hedge fund strategies and lower performance, (iii) alpha and beta components correlate negatively and vary substantially across funds, consistent with equilibrium models featuring search costs, and (iv) fund valuation is sensitive to investor sophistication.

Keywords: Hedge fund returns, alpha, beta, model misspecification, large cross-section
JEL : C55, C58, G11, G12, G23

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I. Introduction

The average return that any hedge fund i delivers to investors is the sum of two components. The alpha component ac_i^* (or alpha) is based on private information—it captures the return that the fund earns by exploiting its unique investment abilities. The beta component bc_i^* is based on public information—it captures the return that the fund earns by following mechanical trading strategies. The economic importance of these two components is potentially large. A commonly held view is that hedge fund managers deliver positive alphas because they are more sophisticated, less constrained, and more incentivized than mutual fund managers. In addition, the literature consistently emphasizes that hedge funds increase their returns by using alternative strategies that are weakly correlated with the equity market (*e.g.*, Getmansky, Lee, and Lo, 2015).

Decomposing returns is key for evaluating the performance and risk profile of hedge funds. It is therefore important for researchers, investors, and policymakers alike. This decomposition is likely to exhibit substantial variation across funds as they rely on many investment strategies and private information signals (*e.g.*, Pedersen, 2015, ch. 3). As a result, averaging across funds provides limited information about the entire fund population. For example, it does not reveal how many funds deliver positive alphas—an important quantity for testing the predictions of equilibrium models of active asset management. Capturing fund heterogeneity is also crucial for hedge fund investors because they can only invest in a handful of funds (Bollen, Joenväärä, and Kauppila, 2021). To address this issue, we need to infer the entire distributions of the alpha and beta components characterized by the densities ϕ_{ac}^* and ϕ_{bc}^*.

The estimation of these distributions is hampered by misspecification. Capturing all the strategies followed by hedge funds is notoriously difficult, which implies that any chosen model k is surely misspecified—that is, it omits relevant factors that drive the beta components of individual funds. In this case, we cannot infer the true components ac_i^* and bc_i^*. Instead, we only observe the estimated components $\hat{ac}_i^k$ and $\hat{bc}_i^k$, from which we can compute the distributions $\hat{\phi}_{ac}^k$ and $\hat{\phi}_{bc}^k$. These distributions are imperfect and noisy versions of ϕ_{ac}^* and ϕ_{bc}^*. They are imperfect because misspecification bundles together the alpha and the beta components due to the omitted factors. Put simply, high alphas can be hidden betas. The distributions are also noisy because of the sampling variation of the omitted factors. This variation, which affects all funds, does not vanish even when the fund population grows large.

In this paper, we develop a novel approach to decompose hedge fund returns. We show how to estimate and compare the entire alpha and beta distributions ϕ^k_{ac} and ϕ^k_{bc} among models. This nonparametric approach has two key features to meet the challenges of misspecification. First, it provides a simple comparison framework to select models less prone to misspecification. Using these models sharpens the separation between alpha and beta as they improve the identification of the true distributions ϕ^*_{ac} and ϕ^*_{bc}. Second, it comes with a full-fledged asymptotic theory that incorporates the estimation noise due to misspecification. We can therefore (i) perform valid comparisons between models, and (ii) conduct formal tests to assess the performance of hedge funds and their economic exposures to specific strategies.

Our main methodological contributions are twofold. First, previous studies measure the distribution of fund alphas under the assumption of correct specification (*e.g.*, Barras, Gagliardini, and Scaillet, 2022; Chen, Cliff, and Zhao, 2017; Harvey and Liu, 2018). Here, we account for the impact of misspecification, which is essential for examining hedge fund returns. Second, a few studies formally compare misspecified models under the assumption of a small number of assets (*e.g.*, Kan and Robotti, 2009; Kan, Robotti, and Shanken, 2013). Here, we design comparison tests for a large number of funds. Previous tests cannot be applied because they require the inversion of the entire return covariance matrix—an operation that cannot be performed because the number of funds is larger than the number of return observations.

To demonstrate the benefits of our approach, we focus on a set of nine diverse models. In addition to the CAPM, we include four standard models commonly used in previous work: the models of Carhart (1997), Fama and French (2015), Fung and Hsieh (2004), and Asness, Moskowitz, and Pedersen (2013). Next, we examine the two sparse machine learning models of Kozak, Nagel, and Santosh (2020), which are trained on 50 characteristic-based equity portfolios. Finally, we consider two models formed with five alternative factors which plausibly capture hedge fund strategies: marketwide illiquidity, betting-against-beta (BAB), variance (short position), carry, and time-series (TS) momentum.[1] The first model proposed by Joenväärä et al. (2021) (JKKT) includes illiquidity, BAB, and TS momentum. Based on previous work by Carhart et al. (2014) and Pedersen (2015), we form another model (CP), which includes all five alternative factors.

[1] As discussed in Section IV.B, the illiquidity, BAB, carry, and TS momentum factors are constructed by Pástor and Stambaugh (2003), Frazzini and Pedersen (2014), Koijen et al. (2018), and Moskowitz, Ooi, and Pedersen (2012).

We conduct the empirical analysis between 1994 and 2020 on 5,231 hedge funds. To construct this sample, we follow Joenväärä et al. (2021) and carefully aggregate four different databases to mitigate the various biases that affect hedge fund reporting (backfill, selectivity, and survivorship). For each model k, the estimation of the alpha and beta distributions requires as inputs the estimated components $\hat{ac}_i^k$ and $\hat{bc}_i^k$ for each fund. To this end, we run a fund-by-fund regression of the monthly excess fund return (net of fees) on the factors included in model k (and a constant). We then compute the two components as $\hat{ac}_i^k = \hat{\alpha}_i^k$ and $\hat{bc}_i^k = \hat{\mu}_i - \hat{ac}_i^k$, where $\hat{\alpha}_i^k$ and $\hat{\mu}_i$ are the estimated alpha and the average return of the fund.

Our comparison tests uncover sharp differences in misspecification across models. We formally compare the entire alpha distribution of each model with that of the simplest benchmark—the CAPM. We find no statistically significant differences for the four standard models and the two machine learning models. It implies that these models are as misspecified as the CAPM and thus ill-equipped to capture hedge fund strategies beyond the market factor. In contrast, the JKKT and CP models are less prone to misspecification—including the five alternative factors produces highly significant differences in alphas relative to the CAPM. These differences hold when we consider the factor trading costs and alternative filters to mitigate hedge fund data biases.

The model comparisons have a strong impact on the decomposition of hedge fund returns. Consistent with previous studies, the standard models produce large alpha components—the alpha is equal to 2.7% per year on average and is positive for more than 70% of the funds.[2] At the same time, the beta component, which equals 2.8% per year on average, is only driven by the market. In short, hedge funds deliver superior performance while being immune to alternative sources of risk.

The JKKT and CP models reverse these conclusions. Under the CP model, the alpha component drops to 0.4% per year on average and is positive for only half of the funds. In contrast, the beta component increases substantially—its average value nearly doubles to 5.2% per year and represents 93% of the average return earned by funds. We observe the same patterns across all investment categories. The average alphas are equal to 0.6%, -0.4%, and 0.9% per year among equity, macro, and arbitrage funds (versus 2.4%, 3.7%, and 2.8% under the CAPM). These models

[2]A non-exhaustive list of papers that document positive average alphas under the standard models includes Avramov, Barras, and Kosowski (2013), Buraschi, Kosowski, and Trojani (2014), Capocci and Hübner (2004), Chen, Cliff, and Zhao (2017), Diez de los Rios and Garcia (2010), Duarte, Longstaff, and Yu (2006), Getmansky, Lee, and Lo (2015), Kosowski, Naik, and Teo (2007), and Patton and Ramadorai (2013).

also uncover notable time trends between 1994 and 2020. In particular, hedge funds converge towards mutual funds along two dimensions as (i) their alphas become increasingly similar, and (ii) their exposure to the equity market becomes a core driver of average returns.

The alternative factors included in the JKKT and CP models are economically important. A majority of funds load on them, which supports the view that hedge funds follow exotic strategies to boost their returns (*e.g.*, Carhart et al., 2014). We find that TS momentum, variance, and carry are the most relevant factors—their average return contributions are equal to 1.1%, 0.8%, and 0.4% per year. These results are in line with the ample anecdotal evidence that hedge funds take short option positions, buy cheap assets with high carry, and follow trends in asset prices (*e.g.*, Lhabitant, 2007). The analysis of investment styles is also consistent with economic intuition. For instance, carry matters for all categories as it is widely used by hedge fund managers (Pedersen, 2015). Trend-following funds load heavily on TS momentum, while arbitrage funds load on variance risk as they commonly follow option strategies (Duarte, Longstaff, and Yu, 2006).

Our analysis also reveals a large heterogeneity in the return components across funds. Measured over the nine models, the volatility of the alphas ranges between 6.8% and 9.2% per year. Similarly, the beta components vary substantially even within investment styles, which contradicts the common practice of benchmarking funds uniformly using style indices. We also find that the alpha and beta components are strongly negatively correlated. In other words, the worst funds load heavily on alternative strategies to boost their returns, possibly to hide their lack of skills.

This large dispersion has implications for the two popular models of active management proposed by Berk and Green (2004) and Gârleanu and Pedersen (2018). The first model predicts that all funds deliver zero alphas (after a learning adjustment period). Whereas this prediction holds relatively well for mutual funds (Barras, Gagliardini, and Scaillet, 2022), it is at odds with the dispersion observed here. In contrast, the model of Gârleanu and Pedersen (2018) naturally produces different fund alphas as investors need to be compensated for the costs of searching funds. This explanation fits well with the complex and leveraged nature of hedge fund investments. The hedge fund selection process is complex, which leads to high search costs (Lhabitant, 2007). These costs then force hedge funds to take on leverage (Stein, 2009).

An important question is how investors value hedge fund investments. Whereas sophisticated investors only value the alpha components, unsophisticated investors also value the beta components due to the factors they cannot replicate. Our model comparisons shed light on this issue by measuring the valuation of hypothetical investors with varying degrees of sophistication. Whereas the CAPM alpha gives the valuation of an investor who can only invest in the market, the CP alpha gives the valuation of an investor able to replicate multiple strategies. We find that the average valuation gap is large (2.6% per year) as the CAPM investor highly values the fund exposures to the alternative factors. Finally, we examine hedge fund flows to gauge the level of sophistication of actual hedge fund investors. The overall evidence suggests that they are closest to the CP investor. Decomposing returns among funds with low and high flows, we find that flows primarily respond to the alpha component obtained with the CP model (but not to the beta component).

The remainder of the paper is as follows. Section II presents the framework for decomposing hedge fund returns. Section III describes the methodology. Section IV presents the hedge fund dataset and the model selection. Section V contains the empirical analysis, and Section VI concludes. The appendix provides additional information on the methodology, the Monte Carlo simulations, the hedge fund dataset, and the empirical results.

II. Decomposing Hedge Fund Returns Under Misspecification

II.A. Fund Return Decomposition

II.A.1. The Alpha and Beta Components

We consider a population of n funds over T periods, where we denote each fund by the subscript i ($i = 1, \ldots, n$) and each period by the subscript t ($t = 1, \ldots, T$). We denote by $r_{i,t}$ the excess net-of-fee return of the fund and by f_t the excess return vector of the mechanical trading strategies that hedge funds follow. Our objective is to decompose the average fund return into its alpha and beta components:

$$E[r_{i,t}] = \alpha_i^* + b_i^{*\prime} E[f_t] = ac_i^* + bc_i^* \,. \tag{1}$$

The alpha component ac_i^* is equal to the alpha α_i^*. The beta component bc_i^* is equal to the average return of a benchmark portfolio with the same exposures to the trading strategies as the fund: $bc_i^* = b_i^{*\prime} E[f_t] = b_i^{*\prime}\lambda$, where b_i^* is the vector of fund betas and λ is the vector of factor premia.

The alpha component ac_i^* measures the average return that the fund delivers to investors by exploiting its private information. This component is therefore a measure of performance, not skill.

Whereas the two notions are commonly used interchangeably, they differ in important ways (Barras, Gagliardini, and Scaillet, 2022; Berk and van Binsbergen, 2015). Skill determines whether funds are able to extract value from capital markets. Performance determines whether investors hold sufficient bargaining power to receive some of the value created by the funds. Positive performance implies positive skill, but not vice-versa.

The beta component bc_i^* captures the average return that the fund produces by following mechanical strategies based on public information. In other words, bc_i^* captures the return that can be replicated using tradable assets. The formulation of the vector f_t is general. It can include the payoffs of traditional assets and their nonlinear transformations obtained with option trading strategies (Glosten and Jaganathan, 1994).[3] The assumption of constant betas b_i^* is generally not restrictive because f_t can include managed portfolios based on publicly available information such as past prices or business cycle indicators (see Cochrane, 2005, ch. 8).[4] The factor premia λ can be the outcome of systematic risk compensation, limits to arbitrage, or imperfect risk sharing driven by segmentation or behavioral biases (see Pedersen, 2015, ch. 3). We remain agnostic on this issue—the beta component simply controls for strategies that investors can replicate themselves.

II.A.2. Several Remarks About the Specification

The return decomposition in Equation (1) calls for several comments. First, it does not require that we model the determinants of ac_i^* and bc_i^* across funds. For instance, performance can vary if some funds have stronger managerial incentives (Agarwal, Daniel, and Naik, 2009), or if investors hold more bargaining power (Glode and Green, 2011; Pástor and Stambaugh, 2012). In this case, we can simply interpret ac_i^* and bc_i^* as fund-specific functions of the fund characteristics.[5]

Second, we do not model the short-term variations in alpha (around its average α_i^*) due to changing economic conditions, industry competition, or aggregate mispricing (*e.g.*, Avramov, Barras, and Kosowski, 2013; Pástor, Stambaugh, and Taylor, 2015, 2017). As discussed below, mea-

[3] A key requirement is that all elements in f_t are tradable factors (*i.e.*, excess returns with zero prices). Otherwise, the factor premia are not equal to the average factor returns ($\lambda \neq E[f_t]$), implying that the difference $E[r_{i,t}] - b_i^{*\prime} E[f_t]$ cannot be interpreted as the fund alpha α_i^* (see Ferson, 2013).

[4] To illustrate, consider a fund whose market beta $b_{im,t}^* = b_{im,0}^* + b_{im,1}^* z_{t-1}$ varies with a public signal z_{t-1} that predicts the market return $r_{m,t}$. As in Equation (1), we can write the benchmark return as $b_i^{*\prime} f_t$, where $b_i^* = (b_{im,0}^*, b_{im,1}^*)'$, and $f_t = (r_{m,t}, z_{t-1} r_{m,t})'$ includes the managed portfolio $z_{t-1} r_{m,t}$ (also known as scaled factor).

[5] Understanding the determinants of ac_i^* and bc_i^* is important for forming hedge fund portfolios with high alpha or high exposures to specific strategies. If we impose a common panel structure across funds, we can examine which characteristics explain the cross-sectional variation in ac_i^* and bc_i^* (see DeMiguel et al., 2021; Kaniel et al., 2021; Wu et al., 2021, for recent advances based on machine learning techniques).

suring α_i^* for hedge funds is a challenging task. Modelling its conditional variation via a proper choice of predictors and functional forms makes the estimation even more difficult (see, for instance, Bakalli, Guerrier, and Scaillet, 2021, in the context of individual stocks).

Third, the value that investors attach to ac_i^* and bc_i^* depends on their sophistication. Sophisticated investors can directly replicate all the mechanical strategies f_t and thus only value the alpha component. In contrast, less sophisticated investors also value the beta component as it captures exposures to factors they cannot replicate (*e.g.*, Agarwal, Green, and Ren, 2018). We discuss the differences in valuation between these two types of investors in Section V.E.

II.A.3. The Importance of Measuring Heterogeneity Across Funds

A proper return decomposition requires that we capture hedge fund heterogeneity. Hedge funds follow a large range of alternative strategies and rely on multiple information signals to create value (Lhabitant, 2007; Pedersen, 2015). It is therefore likely that the alpha and beta components vary across funds. This heterogeneity cannot be captured with a simple average—instead, it requires that we estimate the entire cross-sectional distributions characterized by their densities ϕ_{ac}^* and ϕ_{bc}^*.

Measuring fund heterogeneity is important for several reasons. The alpha distribution ϕ_{ac}^* determines how many funds deliver positive alphas—a key quantity to test the equilibrium predictions of asset management models. It is also useful for hedge fund investors, who only select a handful of funds because of multiple frictions (Bollen, Joenväärä, and Kauppila, 2021). The alpha distribution allows these investors to determine the range of performance outcomes when selecting funds. In other words, ϕ_{ac}^* has a natural Bayesian interpretation as it provides prior information about individual fund alphas (*e.g.*, Jones and Shanken, 2005; Pástor and Stambaugh, 2002).

The distribution ϕ_{bc}^* measures the extent to which hedge funds follow mechanical strategies. It sheds light on their risk profile and preference for specific tradable factors. While this information obviously matters for researchers and investors, it is also relevant for regulators. As discussed by Brown, Lynch, and Petajisto (2010, ch. 12), hedge funds can contribute to systemic risk when they liquidate positions, reduce liquidity provision, or impose losses on counterparties. Contrary to a simple average, ϕ_{bc}^* can identify clusters of funds with strong exposures to similar factors.

II.B. The Dual Impact of Misspecification

II.B.1. The Misspecification of Hedge Fund Models

A key assumption for estimating the distributions ϕ^*_{ac} and ϕ^*_{bc} is that we use the correct model. However, the requirement that f_t is known is difficult to meet given the large number of hedge fund strategies. Hedge funds invest in a large cross-section of countries and asset classes. They can take nonlinear option positions that are difficult to capture with a limited set of option factors (Karehnke and de Roon, 2020). They can dynamically change their factor exposures in response to changing economic conditions (Avramov, Barras, and Kosowski, 2013; Bollen and Whaley, 2009). Hedge funds can also change leverage to increase performance fees or reduce liquidation costs (Buraschi, Kosowski, and Sritrakul, 2014; Lan, Wang, and Yang, 2013). These changes require proper modelling of the fund betas (using managed portfolios), which is particularly challenging when the trading frequency is higher than the reporting frequency (Patton and Ramadorai, 2013).

The conclusion of this analysis is that we are likely to use a misspecified model—that is, a model that omits some relevant factors contained in f_t.[6] As we explain below, misspecification has a dual impact on the return decomposition. It produces an estimation of the alpha and beta distributions ϕ^*_{ac} and ϕ^*_{bc} that is both imperfect and noisy.

II.B.2. Imperfect Estimation

The first impact of misspecification is to produce an imperfect separation between alpha and beta. To see this point, suppose that we decompose returns using a misspecified model k that only includes the factors $f^k_{I,t}$, but omits the factors $f^k_{O,t}$ (with $f_t = (f^{k\prime}_{I,t}, f^{k\prime}_{O,t})'$). From Equation (1), we have $E[r_{i,t}] = ac^*_i + bc^*_{i,I} + bc^*_{i,O}$, where $bc^*_{i,I} = b^{*\prime}_{i,I}\lambda^k_I$ and $bc^*_{i,O} = b^{*\prime}_{i,O}\lambda^k_O$ are the beta components due to the included and omitted factors under the true model (*i.e.*, $b^*_{i,I}$ and $b^*_{i,O}$ are the true fund exposures to $f^k_{I,t}$ and $f^k_{O,t}$). The key question is how the omitted beta component $bc^*_{i,O}$ affects the return decomposition obtained with the misspecified model, which we write as $E[r_{i,t}] = ac^k_i + bc^k_i$.

Regressing the omitted factors on the included factors, we have $f^k_{O,t} = \alpha^k_O + \Psi^k_{O,I} f^k_{I,t} + u^k_{O,t}$ and $\lambda^k_O = \alpha^k_O + \Psi^k_{O,I}\lambda^k_I$, where α^k_O is the vector of factor alphas, $\Psi^k_{O,I}$ is the matrix of slope coefficients, and $u^k_{O,t}$ is the vector of errors. We can split the omitted beta component into two

[6] One could use holdings-based portfolio measures to avoid specifying the factors included in the benchmark portfolio (*e.g.*, Ferson, 2013; Grinblatt and Titman, 1993; Lo, 2008). However, these measures are still subject to misspecification when the fund exhibits time-varying betas (Ferson and Khang, 2002). Another issue is that hedge funds generally do not disclose their portfolio weights.

parts: $bc^*_{i,O} = b^{*\prime}_{i,O}\alpha^k_O + b^{*\prime}_{i,O}\Psi^k_{O,I}\lambda^k_I$, where the split depends on the correlation between the included and omitted factors (captured by $\Psi^k_{O,I}$). The first part, which arises from the component of $f^k_{O,t}$ that is orthogonal to $f^k_{I,t}$, is absorbed by ac^k_i. The second part, which arises from the component of $f^k_{O,t}$ that is spanned by $f^k_{I,t}$, is absorbed by bc^k_i. As a result, we have:

$$ac^k_i = ac^*_i + b^{*\prime}_{i,O}\alpha^k_O = \alpha^k_i \,, \tag{2}$$

$$bc^k_i = bc^*_i - b^{*\prime}_{i,O}\alpha^k_O = b^{k\prime}_{i,I}\lambda^k_I \,, \tag{3}$$

where α^k_i and $b^k_{i,I} = b^*_{i,I} + \Psi^{k\prime}_{O,I} b^*_{i,O}$ are the coefficients from the regression of the fund return $r_{i,t}$ on the factors $f^k_{I,t}$ included in model k (and a constant). Similar to standard results on omitted variable bias, Equations (2)–(3) reveal that ac^k_i and bc^k_i are informative about the true components ac^*_i and bc^*_i. However, this information is polluted—if the fund loads on omitted factors that deliver positive alphas, $b^{*\prime}_{i,O}\alpha^k_O$ is positive and the model-implied alpha component is inflated (*i.e.*, $ac^k_i > ac^*_i$ and $bc^k_i < bc^*_i$). Put differently, misspecification allows us to infer the pseudo-true distributions ϕ^k_{ac} and ϕ^k_{bc}, but not the true ones ϕ^*_{ac} and ϕ^*_{bc} (Gourieroux, Monfort, and Trognon, 1984; White, 1982).

II.B.3. Noisy Estimation

The second impact of misspecification pertains to the estimation of the distributions ϕ^k_{ac} and ϕ^k_{bc}. When the model is misspecified, these densities are estimated with substantial noise. The simplest way to illustrate this point is to focus on the estimation of the average alpha given by $M^k_{1,ac} = \int_{-\infty}^{+\infty} x\phi^k_{ac}(x)dx$, where $\phi^k_{ac}(x)$ is the density evaluated at x. For each fund, we run the linear regression associated with model k,

$$r_{i,t} = \alpha^k_i + b^{k\prime}_{i,I} f^k_{I,t} + \varepsilon^k_{i,t} \,, \tag{4}$$

and compute the alpha component as $\hat{ac}^k_i = \hat{\alpha}^k_i$. We then average across funds to compute the cross-sectional mean $\hat{M}^k_{1,ac} = \frac{1}{n}\sum_i \hat{ac}^k_i$.

Contrary to the correctly specified case, the error terms $\varepsilon^k_{i,t}$ in Equation (4) are strongly cross-correlated because they all depend on the common omitted factors $f^k_{O,t}$. Formally, we have $\varepsilon^k_{i,t} = \varepsilon^*_{i,t} + b^{*k\prime}_{O,i} u^k_{O,t}$, where $u^k_{O,t}$ is the error vector of $f^k_{O,t}$. Even if the hedge fund population size n is large, the information is limited because the values of $\hat{ac}^k_i$ ($i = 1, ..., n$) are all impacted by the random realizations of the omitted component $u^k_{O,t}$. It implies that $\hat{M}^k_{1,ac}$ is estimated with substantial noise, as discussed in detail in Section III.

II.C. A New Approach for Decomposing Returns Under Misspecification

II.C.1. Overview of the Approach

We develop a novel approach for decomposing hedge fund returns. We consider a set of candidate models indexed by k, which are all allowed to be misspecified. We then show how to estimate and compare the distributions of the alpha and beta components ϕ^k_{ac} and ϕ^k_{bc} among models. A key feature of this approach is to explicitly account for the dual impact of misspecification.

First, we jointly examine multiple models to address the imperfect separation between alpha and beta. Given the diversity in hedge fund strategies, several models could reasonably be used for decomposing returns. Some of these models are likely less misspecified and more able to capture the true distributions ϕ^*_{ac} and ϕ^*_{bc}. To identify them, we derive formal comparison tests of the alpha distributions. We use the simplest model—the CAPM—as a natural benchmark. The CAPM is not equipped to capture hedge fund strategies because of their weak correlation with the market (Carhart et al., 2014). To formalize this intuition, we refer to the CAPM as model 0 and denote by $f^0_{I,t}$ the market factor and by $f^0_{O,t}$ the non-market strategies followed by hedge funds. If $\Psi^0_{O,I} = 0$ (no correlation), the CAPM alpha entirely absorbs the beta component due to these strategies:

$$ac^0_i = ac^*_i + b^{*\prime}_{i,O}\alpha^0_O = ac^*_i + b^{*\prime}_{i,O}\lambda^0_O = ac^*_i + bc^*_{i,O}\,. \tag{5}$$

Building on this insight, we obtain an intuitive measure of misspecification of any proposed model. If model k produces the same alpha distribution as the CAPM (ϕ^k_{ac} and ϕ^0_{ac} are the same), it behaves like the CAPM and is therefore unable to capture the alternative hedge fund strategies.

Second, our approach comes with a full-fledged inferential theory to incorporate the estimation noise caused by misspecification. We derive the asymptotic properties of the estimated distributions among models in a proper setting that accounts for the large population of hedge funds observed in the data (*i.e.*, we let n to grow large). Using these results, we obtain valid tests for comparing the differences between models (including the CAPM). We can also conduct statistical inference on each model separately to evaluate individual fund performance, or measure the economic importance of specific factors in driving hedge fund returns.

II.C.2. Contributions to the Existing Literature

Our approach contributes to two strands of the literature. First, several studies show how to estimate the alpha distribution under the assumption of correct specification (*e.g.*, Chen, Cliff, and Zhao, 2017; Harvey and Liu, 2018). In contrast, we account for model misspecification—an important feature given the difficulty in modelling hedge fund returns. Our approach also brings several benefits. First, it provides a framework for estimating both the alpha and beta distributions ϕ_{ac}^k and ϕ_{bc}^k. Second, it is flexible because it imposes no restrictions on the shape of the distributions (contrary to standard parametric approaches). Third, it is simple and fast—it does not rely on sophisticated and computer-intensive Gibbs sampling and expectation maximization methods. Last but not least, it relies on asymptotic theory to conduct statistical inference.

Second, previous studies (*e.g.*, Kan and Robotti, 2009; Kan, Robotti, and Shanken, 2013) derive comparison tests for misspecified models under the assumption of a fixed number of assets n (single asymptotics with n fixed and $T \to \infty$). These tests require the inversion of the entire covariance matrix of returns—an operation that cannot be performed here because we have thousands of hedge funds, but only hundreds of observations. To address this issue, we derive comparison tests in which the number of assets n is allowed to grow large (double asymptotics with n and T $\to \infty$). Another important difference is that previous comparison tests focus on a single aggregate measure of asset alphas (such as the Hansen-Jagannathan distance). In contrast, we focus on the entire (disaggregated) distribution of alphas to capture fund heterogeneity.

II.C.3. A Simple Illustrative Example

Before describing the methodology in more detail, we briefly illustrate the usefulness of our approach using a simple example. We assume that the correct hedge fund model includes the market and three uncorrelated alternative factors: $E[r_{i,t}] = \alpha_i^* + b_{i,m}^*\lambda_m + b_{i,1}^*\lambda_1 + b_{i,2}^*\lambda_2 + b_{i,3}^*\lambda_3$, where λ_m is the equity premium, and λ_j denotes the premium of each alternative factor j ($j = 1, 2, 3$), which we set equal to λ_m. For each fund, the alpha component α_i^* is drawn from a normal $N(\mu_\alpha^*, \sigma_\alpha^{*2})$, $b_{i,m}^*$ from a normal $N(\mu_b^*, \sigma_b^{*2})$, and $b_{i,j}^*$ from a normal $N(\mu_{b_j}^*, \sigma_b^{*2})$, where $\mu_{b_j}^*$ is positive to capture the view that hedge funds load on alternative strategies (each draw is mutually independent). We further assume that the first factor is a more important driver of hedge fund returns by setting $\mu_{b_1}^* = \mu_b^*$ and $\mu_{b_2}^* = \mu_{b_3}^* = \mu_b^*/3$.

Suppose that we use a candidate hedge fund (HF) model (model 1), which includes the market and two alternative factors (f_1 and f_2). For comparison purposes, we also consider the CAPM (model 0), which only includes the market. We assume that we observe the fund alpha and beta components without errors, leaving aside estimation issues for now. Given the above assumptions, the densities ϕ_{ac}^k and ϕ_{bc}^k under each model k are both normal. Under the CAPM, we have

$$ac_i^0 \sim N(\mu_\alpha^* + (\mu_{b_1}^* + \mu_{b_2}^* + \mu_{b_3}^*)\lambda, \sigma_\alpha^{*2} + 3\sigma_b^{*2}\lambda^2)\,, \tag{6}$$

$$bc_i^0 \sim N(\mu_b^*\lambda, \sigma_b^{*2}\lambda^2)\,, \tag{7}$$

while the HF model produces the following densities:

$$ac_i^1 \sim N(\mu_\alpha^* + \mu_{b_3}^*\lambda, \sigma_\alpha^{*2} + \sigma_b^{*2}\lambda^2)\,, \tag{8}$$

$$bc_i^1 \sim N((\mu_b^* + \mu_{b_1}^* + \mu_{b_2}^*)\lambda, 3\sigma_b^{*2}\lambda^2)\,. \tag{9}$$

To begin, we plot in Panel A of Figure 1 the two alpha densities using the following parameter values: $\mu_\alpha^* = 0\%$, $\sigma_\alpha^* = 1.4\%$, $\lambda = 7.5\%$, $\mu_b^* = 0.3$, and $\sigma_b^* = 0.4$.[7] The comparison of ϕ_{ac}^0 and ϕ_{ac}^1 reveals that the magnitude of the alpha components decreases substantially under the HF model (ϕ_{ac}^1 moves to the left towards zero). This difference arises because the HF model is less misspecified than the CAPM—by capturing two out of the three alternative strategies, it produces a sharper identification of the true components ac_i^* and bc_i^*. The implications for performance evaluation are economically important. Under the CAPM, the average alpha reaches 3.8% per year, and more than 75% of the funds deliver positive alphas. In contrast, the average alpha drops to 0.8% per year under the HF model and moves closer to the true average μ_α^* equal to zero.

In addition to reducing the magnitude of the alphas, the HF model also produces a lower dispersion as it absorbs the variation due to factors 1 and 2 (*i.e.*, the term $2\sigma_b^{*2}\lambda^2$). However, this result depends on the specific assumptions in our simple example—in particular, we assume that funds choose similar factor exposures regardless of their true alphas (*i.e.*, we set $corr[\alpha_i^*, b_{i,j}^*] = 0$). If we relax this assumption, a lower average does not necessarily come with a lower dispersion. In

[7] We express μ_α^*, σ_α^*, and λ in percent per year. For simplicity, we set μ_α^* equal to zero and calibrate σ_α^* using the the value reported by Barras, Gagliardini, and Scaillet (2022, Table VI). We further set λ equal to the average market return and μ_b, σ_b equal to the cross-sectional mean and volatility of the market betas in our sample of hedge funds.

the empirical analysis, we simply let the data speak—our approach is nonparametric and thus does not require any assumptions on the joint distributions of α_i^* and $b_{i,j}^*$.

Next, we repeat the analysis for the two beta densities ϕ_{bc}^0 and ϕ_{bc}^1. Consistent with intuition, Panel B is the mirror image of Panel A as the magnitude of the beta components rises under the HF model (ϕ_{bc}^1 moves to the right away from zero). We can go one step further and determine the economic importance of the additional factors included in the HF model. For each factor j ($j = 1, 2$), we measure its contribution to the beta component as $bc_{i,j}^1 = b_{i,j}^1 \lambda_j$.[8] We can then infer the entire density $\phi_{bc_j}^1$ and examine if it is located away from zero. In our example, $\phi_{bc_j}^1$ is normally distributed: $bc_{i,j}^1 \sim N(\mu_{b_j}^* \lambda, \sigma_b^{*2} \lambda^2)$. This analysis identifies factor 1 as the most relevant factor in the cross-section of hedge funds. Its contribution equals 2.3% per year on average and is positive for 77% of the funds. In contrast, these values are only equal to 0.8% and 60% for factor 2.

Please insert Figure 1 here

III. Methodology

III.A. Estimation Procedure

We now describe the methodology for decomposing hedge fund returns using K models indexed by k ($k = 0, \ldots, K-1$). Each model k is allowed to be misspecified as it includes the factors $f_{I,t}^k$ but omits the factors $f_{O,t}^k$. We are interested in (i) the density ϕ_{ac}^k of the alpha component, (ii) the density ϕ_{bc}^k of the beta component, and (iii) the density $\phi_{bc,j}^k$ of the beta component due to each factor j included in model k. We summarize the shape of each of these densities with the following key characteristics: (i) the cross-sectional mean and standard deviation, denoted by M_1^k and M_2^k, (ii) the proportion of funds with a return component below a given value a, denoted by $P^k(a)$, and (iii) the quantile at a given percentile level u, denoted by $Q^k(u) = (P^k)^{-1}(u)$.

To estimate the distribution characteristics of each density, we need to estimate the fund components $ac_i^k = \alpha_i^k$, $bc_i^k = b_{i,I}^{k\prime} \lambda_I^k$, and $bc_{i,j}^k = b_{i,I,j}^k \lambda_{I,j}^k$. For each fund, we compute these values by running the time-series regression in Equation (4). We interpret this regression as a random coefficient model (*e.g.*, Hsiao, 2003) in which α_i^k and $b_{i,I}^k$ are not fixed parameters, but random

[8] By definition, we can decompose the (total) beta component as $bc_i^1 = bc_{i,m}^1 + \sum_j bc_{i,j}^1$, where $bc_{i,m}^1 = b_{i,m}^1 \lambda_m$ is the beta component due to the market, and $\sum_j bc_{i,j}^1$ is the beta component due to the additional factors 1 and 2.

realizations from a continuum of funds in order to invoke cross-sectional limits.[9] We also assume that at least one omitted factor is strong in the sense that it has a pervasive impact on the cross-section of fund returns.[10] This mild assumption, which is satisfied by all the models we examine, delivers a well-defined convergence rate for the distribution characteristics. The least square estimate $\hat{\gamma}_i^k = (\hat{\alpha}_i^k, \hat{b}_{i,I}^{k\prime})'$ is given by

$$\hat{\gamma}_i^k = (\hat{Q}_{x,i}^k)^{-1} \frac{1}{T_i} \sum_t I_{i,t} x_t^k r_{i,t} \,, \tag{10}$$

where $I_{i,t}$ is an indicator variable equal to one if $r_{i,t}$ is observable, $T_i = \sum_t I_{i,t}$, $x_t^k = (1, f_{I,t}^{k\prime})'$, and $\hat{Q}_{x,i}^k = \frac{1}{T_i} \sum_t I_{i,t} x_t^k x_t^{k\prime}$. We then compute the alpha and beta components for each fund as

$$\hat{ac}_i^k = \hat{\alpha}_i^k \,, \tag{11}$$

$$\hat{bc}_i^k = \hat{\mu}_i - \hat{\alpha}_i^k \,, \tag{12}$$

$$\hat{bc}_{i,j}^k = \hat{b}_{i,I,j}^k \hat{\lambda}_{I,j}^k \,, \tag{13}$$

where $\hat{bc}_i^k$ is the empirical counterpart of $bc_i^k = E[r_{it}] - ac_i^k$, $\hat{\mu}_i = \frac{1}{T_i} \sum_t I_{i,t} r_{i,t}$ is the average fund return, and $\hat{\lambda}_{I,j}^k = \frac{1}{T_i} \sum_t I_{i,t} f_{I,j,t}^k$ is the empirical average of $f_{I,j,t}^k$.

Next, we account for the unbalanced nature of the hedge fund sample. Following Barras, Gagliardini, and Scaillet (2022) and Gagliardini, Ossola, and Scaillet (2016), we introduce a formal selection rule $\mathbf{1}_i^\chi$ equal to one if the following conditions are met: $\mathbf{1}_i^\chi = \mathbf{1}\left\{ CN_i \leq \chi_{1,T}, \tau_{i,T} \leq \chi_{2,T} \right\}$, where $CN_i = \sqrt{eig_{\max}\left(\hat{Q}_{x,i}^k\right) / eig_{\min}\left(\hat{Q}_{x,i}^k\right)}$ is the condition number of $\hat{Q}_{x,i}^k$, $\tau_{i,T} = T/T_i$, and $\chi_{1,T}$, $\chi_{2,T}$ denote the two threshold values. The first condition $CN_i \leq \chi_{1,T}$ excludes funds for which the time-series regression is subject to multicollinearity problems (*e.g.*, Belsley, Kuh, and Welsch, 2004). The second condition $\tau_{i,T} \leq \chi_{2,T}$ excludes funds for which the sample size is too small. Both thresholds $\chi_{1,T}$ and $\chi_{2,T}$ increase with the sample size T—with more return observations, we estimate the fund coefficients with greater accuracy, which allows for a less stringent selection rule. Applying this selection rule, we work with a population size equal to $n_\chi = \sum_{i=1}^n \mathbf{1}_i^\chi$.

[9] Barras, Gagliardini, and Scaillet (2022) and Gagliardini, Ossola, and Scaillet (2016) use the same sampling scheme to measure mutual fund performance and test the arbitrage pricing theory in a large cross-section of assets.

[10] More formally, an omitted factor is strong if the largest eigenvalue of the residual covariance matrix of hedge fund returns does not vanish as the population size n grows large. In contrast, a factor is weak if its loading vanishes at a rate $1/\sqrt{T}$ (Gagliardini, Ossola, and Scaillet, 2019). Our formulation allows for weak factors in both $f_{I,t}^k$ and $f_{O,t}^k$.

The final step is to compute the distribution characteristics using the vector of estimated components for the set of n_χ selected funds. We compute the mean, standard deviation, proportion, and quantile of the distribution of the alpha components as $\hat{M}_1^k = \frac{1}{n_\chi}\sum_i \mathbf{1}_i^\chi \hat{ac}_i^k$, $\hat{M}_2^k = \left(\frac{1}{n_\chi}\sum_i \mathbf{1}_i^\chi (\hat{ac}_i^k)^2 - (\frac{1}{n_\chi}\sum_i \mathbf{1}_i^\chi \hat{ac}_i^k)^2\right)^{1/2}$, $\hat{P}^k(a) = \frac{1}{n_\chi}\sum_i \mathbf{1}_i^\chi \mathbf{1}\{\hat{ac}_i^k \leq a\}$, and $\hat{Q}^k(u) = (\hat{P}^k)^{-1}(u)$. To obtain the characteristics of the distributions of the beta components, we use the same formulas after replacing $\hat{ac}_i^k$ with $\hat{bc}_i^k$ or $\hat{bc}_{i,j}^k$.

III.B. Properties of the Distribution Characteristics

We begin our theoretical analysis by examining the properties of the distribution characteristics $\hat{M}_1^k$, $\hat{M}_2^k$, $\hat{P}^k(a)$, and $\hat{Q}^k(u)$. The following proposition derives the asymptotic distribution of the estimated characteristics of the alpha distribution ϕ_{ac}^k as the numbers of funds n and observations T grow large. To capture the large cross-sectional dimension of the hedge fund population observed in the data, we require that n is larger than T.

Proposition 1. *As $n, T \to \infty$, such that $T/n \to 0$, we obtain the following properties for the estimated characteristics of ϕ_{ac}^k under the misspecified model k:*

$$\sqrt{T}\left(\hat{M}_s^k - M_s^k\right) \to_d N(0, V[M_s^k])\,, \tag{14}$$

$$\sqrt{T}\left(\hat{P}^k(a) - P^k(a)\right) \to_d N(0, V[P^k(a)])\,, \tag{15}$$

$$\sqrt{T}\left(\hat{Q}^k(u) - Q^k(u)\right) \to_d N(0, V[Q^k(u)])\,, \tag{16}$$

where $s \in \{1, 2\}$ and $\to_d$ denotes convergence in distribution. The variance terms are given by

$$V[M_s^k] = \left(\eta_{M_s}^{k\prime} \otimes E_1'(Q_x^k)^{-1}\right)\Omega_{ux}^k\left(\eta_{M_s}^k \otimes (Q_x^k)^{-1}E_1\right)\,, \tag{17}$$

$$V[P^k(a)] = \left(\eta_{P(a)}^{k\prime} \otimes E_1'(Q_x^k)^{-1}\right)\Omega_{ux}^k\left(\eta_{P(a)}^k \otimes (Q_x^k)^{-1}E_1\right)\,, \tag{18}$$

$$V[Q^k(u)] = V[P^k(Q^k(u))]/\phi_{ac}^k(Q^k(u))^2\,, \tag{19}$$

where $\eta_{M_s}^k = E\left[\left(\frac{\partial M_s^k}{\partial E[g_i^k]}\right)' \frac{\partial g_i^k}{\partial ac_i^k} b_{i,O}^*\right]$, $E[g_i^k]$ is the vector of uncentered moments with $g_i^k = (ac_i^k, (ac_i^k)^2)'$, $\otimes$ denotes the Kronecker product, $Q_x^k = E[x_t^k x_t^{k\prime}]$, $\Omega_{ux}^k = \lim_{T\to\infty} V\left[\frac{1}{\sqrt{T}}\sum_t u_{O,t}^k \otimes x_t^k\right]$, $\eta_{P(a)}^k = E[b_{i,O}^* | ac_i^k = a]\phi_{ac}^k(a)$, $b_{i,O}^*$ and $u_{O,t}^k$ denote the vectors of betas and residuals associated with the omitted factors $f_{O,t}^k$, and $\phi_{ac}^k(a)$ is the probability density evaluated at a.

Proof. *See the appendix.*

To save space, we refer the reader to the appendix for the theoretical analysis of the distributions ϕ_{bc}^k and $\phi_{bc,j}^k$, whose estimated characteristics have the same properties as in Proposition 1. These results allow for formal tests on the shape of the alpha and beta distributions. Denoting the generic estimated characteristic by $\hat{C}^k \in \{\hat{M}_s^k, \hat{P}^k(a), \hat{Q}^k(u)\}$, we can test the null hypothesis:

$$H_0 : C^k = \upsilon, \tag{20}$$

where υ is a given scalar. For instance, we can test whether the proportion of positive-alpha funds equals 50% ($\upsilon = 0.5$), or whether the average beta component due to any factor j is null ($\upsilon = 0$).[11]

Proposition 1 reveals two important insights about inference under misspecification. First, the estimated characteristics converge towards their respective parameter values. Asymptotically, we are able to estimate the alpha distribution ϕ_{ac}^k under model k without bias, even though we use as inputs noisy versions of the fund components (*i.e.*, we use $\hat{ac}_i^k$ instead of ac_i^k). Under misspecification, Proposition 1 therefore provides a theoretical justification for the common practice of reporting cross-sectional summary statistics (*e.g.*, boxplots) based on estimated coefficients without any bias adjustment.[12] Second, the variance of the estimators is large because the convergence rate is equal to $1/\sqrt{T}$ (and not $1/\sqrt{n}$)—a result that formalizes our previous point that misspecification amplifies estimation noise. This result is a priori surprising because the characteristics are all computed as cross-sectional averages, that is, we sum across n funds, not across T periods.

None of these properties hold when the model is correctly specified—a setting examined in detail by Barras, Gagliardini, and Scaillet (2022). In this case, the estimated distribution characteristics must be adjusted for the error-in-variable (EIV) bias that arises because we use as inputs noisy versions of the fund coefficients. In addition, the characteristics are estimated with greater precision because the convergence rate is equal to $1/\sqrt{n}$ (instead of $1/\sqrt{T}$).

The strong impact of misspecification on inference stems from the properties of the fund error terms. The estimation error on the alpha component $\hat{ac}_i^k$ involves the term $\bar{\varepsilon}_i^k = \bar{\varepsilon}_i^* + b_{i,O}^{*\prime}\bar{u}_O$, where $\bar{\varepsilon}_i^k$, $\bar{\varepsilon}_i^*$, and $\bar{u}_O$ denote the time-series averages of $\varepsilon_{i,t}^k$, $\varepsilon_{i,t}^*$, and $u_{O,t}$. The average $\bar{\varepsilon}_i^*$ obtained with the correct model is weakly correlated across funds, which implies that its impact on the estimated characteristics vanishes with the population size n. It is not the case for the error term $\bar{u}_O$ due

[11] As in Barras, Gagliardini, and Scaillet (2022), we can also design a kernel estimator $\hat{\phi}^k(a)$ for each density evaluated at a. Similar to Proposition 1, misspecification implies that $\sqrt{T}\left(\hat{\phi}^k(a) - \phi^k(a)\right) \to_d N(0, V[\phi^k(a)])$.

[12] For instance, Almeida, Ardison, and Garcia (2020), Capocci and Hübner (2004), and Kosowski, Naik, and Teo (2007) follow this practice when reporting the characteristics of the distribution of hedge fund alphas.

to the omitted factors because it affects all funds simultaneously. This term is noisy because it converges to zero at the rate equal to $1/\sqrt{T}$. As a result, the noise contained in $\bar{u}_O$ (i) slows down the convergence rate of the estimated characteristics from $1/\sqrt{n}$ to $1/\sqrt{T}$, and (ii) is bigger in magnitude than the EIV bias, which makes any bias adjustment unnecessary.[13]

III.C. Comparison Tests Between Models

We now turn to the comparison tests. We only focus on the alpha component because the comparison of the beta components provide similar insights (as shown in our illustrative example in Figure 1). We compare the alpha distributions ϕ^k_{ac} and ϕ^l_{ac} between two misspecified models k and l ($k, l = 0, ..., K-1$). As discussed in the appendix, the two models can be nested or not as long as some regularity conditions on the asymptotic variance hold (models are nested if one is included in the other). When we set $l = 0$, the comparison is made with respect to the CAPM.

We compute the differences in distribution characteristics between the two models as $\Delta\hat{M}_1 = \hat{M}^k_1 - \hat{M}^l_1$, $\Delta\hat{M}_2 = \hat{M}^k_2 - \hat{M}^l_2$, $\Delta\hat{P}(a) = \hat{P}^k(a) - \hat{P}^l(a)$, and $\Delta\hat{Q}(u) = \hat{Q}^k(u) - \hat{Q}^l(u)$. Proposition 2 derives the asymptotic distribution of each estimated difference as the numbers of funds n and observations T grow large.

Proposition 2. *As $n, T \to \infty$ such that $T/n \to 0$, we obtain the following properties for the differences in the estimated characteristics of ϕ^k_{ac} and ϕ^l_{ac} under the misspecified models k and l:*

$$\sqrt{T}\left(\Delta\hat{M}_s - \Delta M_s\right) \to_d N\left(0, V[\Delta M_s]\right), \tag{21}$$

$$\sqrt{T}\left(\Delta\hat{P}(a) - \Delta P(a)\right) \to_d N\left(0, V[\Delta P(a)]\right), \tag{22}$$

$$\sqrt{T}\left(\Delta\hat{Q}(u) - \Delta Q(u)\right) \to_d N\left(0, V[\Delta Q(u)]\right), \tag{23}$$

where $s \in \{1, 2\}$. The variance of the characteristic differences are equal to

$$V[\Delta M_s] = V[M^k_s] + V[M^l_s] - 2Cov[M^k_s, M^l_s], \tag{24}$$

$$V[\Delta P(a)] = V[P^k(a)] + V[P^l(a)] - 2Cov[P^k(a), P^l(a)], \tag{25}$$

$$V[\Delta Q(u)] = V[Q^k(u)] + V[Q^l(u)] - 2Cov[Q^k(u), Q^l(u)], \tag{26}$$

where $V[M^k_s]$, $V[M^l_s]$, $V[P^k(a)]$, $V[P^l(a)]$, $V[Q^k(u)]$, and $V[Q^l(u)]$ are obtained from Proposi-

[13] Our Monte Carlo simulations calibrated on our sample of funds confirm these results (see the appendix). When the model is misspecified, the mean squared error (MSE) of each characteristic estimator (i) is primarily driven by the variance (and not by the finite-sample bias), and (ii) decreases with T, but not with n.

tion 1. The covariance terms are given by

$$Cov[M_s^k, M_s^l] = \left((\eta_{M_s}^{k\prime} \otimes E_1'(Q_x^k)^{-1}\right) \Omega_{ux}^{kl} \left(\eta_{M_s}^{l} \otimes (Q_x^l)^{-1} E_1\right) , \tag{27}$$

$$Cov[P^k(a), P^l(a)] = \left(\eta_{P(a)}^{k\prime} \otimes E_1'(Q_x^k)^{-1}\right) \Omega_{ux}^{kl} \left(\eta_{P(a)}^{l} \otimes (Q_x^l)^{-1} E_1\right) , \tag{28}$$

$$Cov[Q^k(u), Q^l(u)] = \frac{Cov[P^k(Q^k(u)), P^l(Q^l(u))]}{\phi_{ac}^k(Q^k(u))\phi_{ac}^l(Q^l(u))} , \tag{29}$$

where $\Omega_{ux}^{kl} = \lim_{T\to\infty} Cov\left[\frac{1}{\sqrt{T}}\sum_t u_{O,t}^k \otimes x_t^k, \frac{1}{\sqrt{T}}\sum_t u_{O,t}^l \otimes x_t^l\right]$.

***Proof**. See the appendix.*

The results in Proposition 2 provide simple comparison tests for each pair of models k and l. We denote the generic estimated characteristic difference by $\Delta\hat{C} \in \{\Delta\hat{M}_s, \Delta\hat{P}(a), \Delta\hat{Q}(u)\}$. We can then test the null hypothesis that each characteristic difference equals zero:

$$H_0 : \Delta C = 0. \tag{30}$$

Misspecification arises naturally in pairwise comparisons because competing models cannot be all correct. Therefore, the impact of misspecification discussed in Proposition 1 carries over to model comparisons. Proposition 2 shows that the comparison tests inherit the high estimation noise caused by misspecification as each characteristic difference converges at a slow rate equal to $1/\sqrt{T}$.[14] In other words, the bar for detecting significant differences between models is considerably higher when the tests are properly adjusted for misspecification.

III.D. Estimation of the Asymptotic Variance Terms

Applying our methodology requires consistent estimators of the variance terms in Propositions 1 and 2. For each distribution characteristic, the variance V depends on the error term $u_{O,t}$ and betas $b_{i,O}^*$ associated with the omitted factors. For instance, the estimated average $\hat{M}_1^k$ is more volatile when the variance of the factor residuals $V[u_{O,t}]$ and the magnitude of the average betas $E[b_{i,O}^{k*}]$ increase. Because $u_{O,t}$ and $b_{i,O}^*$ are not observable, estimating V is not trivial.

To address this issue, we derive a consistent variance estimator based on the observed fund residuals of each model $\hat{\varepsilon}_{i,t}^k = r_{i,t} - x_t^{k\prime}\hat{\gamma}_i^k$. The estimators of the asymptotic variances of $\sqrt{T}(\hat{C}^k -$

[14]If one of the models is correct, the sampling variability is entirely driven by the misspecified model (*i.e.*, we can treat the estimated characteristic under the correct model as known). Therefore, the convergence rate of the characteristic difference remains equal to $1/\sqrt{T}$.

C^k) and $\sqrt{T}(\Delta\hat{C} - \Delta C)$ are given by

$$\hat{V}[\hat{C}^k] = \frac{1}{n_\chi^2 T} \sum_i \sum_j \sum_t \mathbf{1}_i^\chi \tau_{i,T} I_{i,t} \mathbf{1}_j^\chi \tau_{j,T} I_{j,t} \hat{a}_{i,t}(\hat{C}^k) \hat{a}_{j,t}(\hat{C}^k)\,, \tag{31}$$

$$\hat{V}[\Delta\hat{C}] = \frac{1}{n_\chi^2 T} \sum_i \sum_j \sum_t \mathbf{1}_i^\chi \tau_{i,T} I_{i,t} \mathbf{1}_j^\chi \tau_{j,T} I_{j,t} \hat{a}_{i,t}^\Delta(\Delta\hat{C}) \hat{a}_{j,t}^\Delta(\Delta\hat{C})\,, \tag{32}$$

where the terms $\hat{a}_{i,t}(\hat{C}^k)$ and $\hat{a}_{i,t}^\Delta(\Delta\hat{C})$ are functions of $\hat{C}^k$ and $\Delta\hat{C}$ (see the appendix). The following proposition shows that $\hat{V}[\hat{C}^k]$ and $\hat{V}[\Delta\hat{C}]$ are consistent variance estimators as the numbers of funds n and observations T grow large.

Proposition 3. *As $n, T \to \infty$ such that $T/n \to 0$, we have*

$$\hat{V}[\hat{C^k}] \to_p V[\hat{C^k}]\,, \tag{33}$$

$$\hat{V}[\Delta\hat{C}] \to_p V[\Delta\hat{C}]\,, \tag{34}$$

where $\to_p$ denotes convergence in probability.
Proof*. See the appendix.*

IV. Data and Model Construction

IV.A. Hedge Fund Dataset

We collect the monthly net-of-fee returns of hedge funds between January 1994 and December 2020. We combine four databases—Barclayhedge, HFR, Morningstar, and TASS. This aggregation mitigates the selection bias that arises from the voluntary nature of information disclosure by hedge funds. In particular, it largely improves the coverage of underperforming funds, which typically report to only one database (see Joenväärä et al., 2021). We remove the first 12 months of data for each fund to control for backfill bias. The appendix provides more detail on the construction of the dataset, which largely follows Joenväärä et al. (2021). For comparison purposes, we also collect the monthly net-of-fee returns of open-end, actively managed U.S. equity funds over the same period. Here, we follow Barras, Gagliardini, and Scaillet (2022), who provide detailed information on the construction of the mutual fund database.

Table I reports summary statistics for the equal-weighted portfolio of all hedge funds in our sample, as well as three categories: (i) equity funds (long-short and market neutral), which rely on discretionary or quantitative analysis to detect mispriced stocks, (ii) macro funds (global macro and CTA/managed futures), which take directional bets across asset classes using broad economic and financial indicators, and (iii) arbitrage funds (relative value and event driven), which exploit various sources of mispricing primarily in the debt market. Overall, the results are similar to those reported by Getmansky, Lee, and Lo (2015) between 1996 and 2014.

To obtain reliable estimates of ac_i^k, bc_i^k, and $bc_{i,j}^k$ in the unbalanced panel of hedge funds, we apply the selection rule of Section III.A. Taking the same thresholds as Barras, Gagliardini, and Scaillet (2022), we set the minimum condition number of $\hat{Q}_{x,i}^k$ equal to 15 and the minimum number of return observations equal to 60.[15] This selection leaves us with a total number of $5{,}231$ funds ($n_\chi = 5{,}231$). To address the concern that the evaluation of models depends on the construction of the hedge fund database, we consider alternative filters in the appendix.[16] Consistent with intuition, we find that these changes have the same impact across models. As a result, they leave the model comparison analysis largely unchanged.

Please insert Table I here

IV.B. Hedge Fund Models

We apply our methodology to a diverse set of nine models examined in previous work. All the models include tradable factors in order to estimate the alpha and beta components using a fund-by-fund regression approach (as per Equation (4)). To make the comparisons more relevant, we focus on models that aim at explaining the return of any given fund. We therefore do not include models designed for capturing specific investment styles (*e.g.*, Agarwal and Naik, 2004; Duarte, Longstaff, and Yu, 2006; Mitchell and Pulvino, 2001), and fund-specific models whose factors vary with each individual fund (*e.g.*, Bollen and Whaley, 2009; O'Doherty, Savin, and Tiwari, 2016; Shu and Tiwari, 2022).

The first model is the CAPM, which is used as a reference for some of our comparison tests. We then consider a set of four standard models. The chosen set is by no means exhaustive, but provides a good representation of the models commonly used for performance evaluation (*e.g.*, Cremers, Petajisto, and Zitzewitz, 2013; Getmansky, Lee, and Lo, 2015). We select the Carhart (1997) model

[15]Empirically, the condition number has no impact on fund selection after imposing a minimum number of observations. Therefore, the comparison of models is based on the same sample of funds.

[16]We change the fund selection rule by imposing 36 or 84 minimum return observations. We also apply the more stringent backfill bias correction of Joenväärä et al. (2021), eliminating all the observations before the fund listing date. Finally, we use the five filters proposed by Straumann (2009) to remove errors in reported fund returns.

and the Five-Factor model of Fama and French (2015), which includes the market, size, value, momentum, profitability, and investment factors. We examine the well-known model of Fung and Hsieh (2004), which includes two equity factors (market and size), two bond factors (term and default), and three option straddles (bond, commodity, and currency). Finally, we consider the model of Asness, Moskowitz, and Pedersen (2013), which adds to the CAPM two global value and momentum factors across international asset classes.

Next, we consider the models proposed by Kozak, Nagel, and Santosh (2020) based on machine learning techniques. These authors apply lasso and ridge penalizations to form models with the highest out-of-sample ability to explain the average excess returns of 50 characteristic-based equity portfolios.[17] We use the two models presented in their Table 4, which impose sparsity by selecting five factors only. The first one is formed with the equity portfolios themselves, while the second one includes their principal components. We also add the market return to each set of factors to maintain consistency across models.

Finally, we consider two models that include five alternative factors: illiquidity, betting against beta (BAB), variance (short position), carry, and time-series (TS) momentum. These factors, based on economic intuition, plausibly capture several strategies followed by hedge funds.[18] We consider the model of Joenväärä et al. (2021, JKKT), which extends the Carhart (1997) model by including the illiquidity, BAB, and TS momentum factors. Building on the work of Carhart et al. (2014) and Pedersen (2015), we examine another model (CP) which replaces the five non-equity factors of Fung and Hsieh (2004) (bond factors and straddles) with the five alternative factors.[19] Measuring the costs of trading these factors is difficult because they require timely price information on a wide range of assets beyond the equity market. Given this uncertainty, we conduct our baseline analysis using the original factor returns and then discuss the impact of trading costs.

[17]We thank Serhiy Kozak for providing us with the data and code. In principle, we could expand the set of candidate factors using machine learning techniques. However, these techniques require a considerably large number of return observations (Gu, Kelly, and Xiu, 2020), which is not the case for hedge fund datasets.

[18]The illiquidity factor of Pástor and Stambaugh (2003) captures marketwide changes in market liquidity. The BAB strategy of Frazzini and Pedersen (2014) exploits the price distortions caused by leverage-constrained investors on low- and high-beta stocks. The variance factor tracks the realized variance of the S&P 500. The global carry and TS momentum factors of Koijen et al. (2018) and Moskowitz, Ooi, and Pedersen (2012) invest in assets with high carry and positive 12-month returns across international asset classes. TS momentum departs from traditional cross-sectional momentum, which only invests in assets with past returns higher than the cross-sectional average.

[19]In addition to the five alternative factors, Carhart et al. (2014) and Pedersen (2015) also consider factors based on real assets, quality, credit, and catastrophe bonds. We do not include these factors either because they are closely related to other factors (*e.g.*, quality is similar to profitability) or difficult to construct (*e.g.*, catastrophe bonds).

Table II reports summary statistics for the excess returns of the factors.[20] We find that all but one factor (bond term) deliver positive premia. It implies that hedge funds increase their average returns when their factor betas are positive. Unreported results also show that the factors capture distinct strategies—only 11 pairwise correlations out of 135 are above 0.5 (in absolute value). Table III provides a complete list of the nine models with the factors they include (see the appendix for additional details on the data sources of the factors).

Please insert Tables II and III here

V. Empirical Results

V.A. Analysis of Model Misspecification

V.A.1. Misspecification Diagnostic

To begin the empirical analysis, we examine the misspecification of the nine proposed models. An intuitive misspecification statistic is the adjusted R^2 of the time-series regression of Equation (4). If the R^2 is high, the omitted factors must have implausibly high premia to blur the separation between alpha and beta (see Cochrane, 2005, ch. 9). Therefore, a high R^2 signals that misspecification is not a major concern for the hedge fund return decomposition.

Table IV shows that the average R^2 is relatively low across all models—the values range between 20.4% and 31.0%, leaving plenty of room for omitted factors. This result is consistent with the analysis of Bollen (2013) on the Fung-Hsieh model and its extensions. In theory, it is possible for a correctly specified model to deliver a low R^2. If hedge funds take concentrated positions to exploit their private information, the idiosyncratic risk is high and the R^2 is low. In this case, a low R^2 should be interpreted as a measure of skill, not misspecification (Titman and Tiu, 2011).

To address this issue, we also compute the misspecification diagnostic criterion of Gagliardini, Ossola, and Scaillet (2019, GOS). As n and T converge to infinity, this criterion is positive with probability one if (i) the model is misspecified, and (ii) at least one of the omitted factors is strong (see the appendix). We find that the GOS criterion is always positive in the population and across

[20] We define the variance and straddle factors as short positions to obtain positive premia. As shown, among others, by Bakshi and Kapadia (2003), short variance swaps and short (delta-hedged) option positions deliver positive premia because they perform poorly in bad times when realized market variance is high.

investment categories. This result confirms that all the models are misspecified and validates our estimation assumption that each model omits at least one strong factor.

While misspecified, the nine models are likely to produce different return decompositions. Table IV provides preliminary evidence of such variations. For each model k, we report the relative importance of each component of the average fund return: (i) the alpha component $\hat{ac}_i^k$, (ii) the beta component due to the market computed as $\hat{bc}_{i,m}^k = \hat{b}_{i,m}^k \hat{\lambda}_m$, and (iii) the beta component due to the non-market factors computed as $\hat{bc}_{i,I_{nm}}^k = \hat{b}_{i,I_{nm}}^{k\prime} \hat{\lambda}_{I_{nm}}^k$. The results show that the alpha-beta decomposition is sensitive to the choice of model. For instance, the relative importance of the alpha component is equal to 47.1% on average under the Carhart model, but drops to 18.0% under the JKKT model. Interpreting these differences requires a proper setting to conduct formal comparison tests—a point we examine in detail below.

These results depart from those obtained with the universe of US mutual funds. Consistent with intuition, Table IV shows that the average R^2 is substantially higher (above 80%). In addition, there is little variation in the alpha-beta decomposition across models. They all lead to the same conclusion that average returns are driven by a single component—the equity market. In short, our analysis confirms that misspecification matters for hedge funds, but not for mutual funds.

Please insert Table IV here

V.A.2. Comparisons With the CAPM

Next, we measure the degree of misspecification of each model by comparing it with the simplest benchmark—the CAPM. If the two alpha distributions are the same, the proposed model is no better than the CAPM at capturing hedge fund strategies. Applying the methodology in Section III, we summarize the distribution differences using the mean and standard deviation, the proportions of negative- and positive-alpha funds, and the quantiles at 10% and 90%. To compute the standard deviation of the estimated differences, we replace T with $T_\chi = \frac{1}{n_\chi} \sum_{i=1}^n \mathbf{1}_i^\chi T_i$, where T and T_χ are equal to 324 and 125 observations. As a result, we account for the increased estimation imprecision due to the unbalanced nature of the hedge fund panel.

The comparison tests reported in Table V reveal several new insights. We observe a striking similarity between the CAPM and the four standard models (Carhart, Five-Factor, Fung-Hsieh, and Asness-Moskowitz-Pedersen). Only one of the 24 characteristic differences is statistically significant. These results are not an artifact of data aggregation—as shown in the appendix, we observe the same patterns across all three investment categories. Similar to the CAPM, the standard models are therefore ill-equipped to capture the strategies followed by hedge funds.

Our formal comparison resonates with previous studies that examine multiple models. For instance, Getmansky, Lee, and Lo (2015) and Joenväärä et al. (2021) report differences in average alphas between the Fung-Hsieh model and the CAPM. We show that accounting for misspecification is key when interpreting these differences. Suppose that we naively use the convergence rate of $1/\sqrt{n_\chi}$, instead of the appropriate rate of $1/\sqrt{T_\chi}$ under misspecification. In this case, we find that 75% of the 24 characteristic differences are significant at the 5% level. Applying the correct procedure, we conclude that the standard models and the CAPM are not statistically different.

The machine learning models are also similar to the CAPM as they leave the magnitude of the fund alphas largely unchanged. Whereas these models do a great job at explaining the average returns of characteristic-based portfolios (Kozak, Nagel, and Santosh, 2020), they are not trained on strategies beyond the equity space. Their limited success in the hedge fund population highlights the importance of accounting for other asset classes, such as bonds, currencies, or commodities.

In contrast, the JKKT and CP models are less prone to misspecification than the CAPM. The comparisons tests imply strong rejections of the null hypothesis that the alpha distributions are identical. In line with our illustrative example in Figure 1, both models capture the returns of hedge fund strategies and thus produce a strong shift of the alpha distribution towards zero. The average alpha drops by 1.9% and 2.6% per year under the JKKT and CP models. As discussed below, we can therefore use these models to sharpen the decomposition of hedge fund returns.

Please insert Table V here

V.B. Decomposing Hedge Fund Returns

V.B.1. Magnitude of the Alpha and Beta Components

We now turn to the decomposition of the average returns of hedge funds. For each model k, we apply the methodology in Section III to compute the main characteristics of (i) the distribution of the alpha component $\hat{\phi}^k_{ac}$, and (ii) the distribution of the beta component $\hat{\phi}^k_{bc}$. We report these results for the entire hedge fund population in Table VI.

Both the CAPM and the standard models produce large alpha components. Panel A shows that the average alpha clusters around 2.8% per year, and the proportion of positive-alpha funds is always above 70%. These results are in line with the previous literature, which, by and large, finds that hedge funds deliver superior performance (*e.g.*, Diez de los Rios and Garcia, 2010; Duarte, Longstaff, and Yu, 2006; Kosowski, Naik, and Teo, 2007). Another takeaway from these models is that hedge funds are only exposed to the market factor. Panel B shows that the beta components remain largely unchanged as we include bond, value, momentum, or straddle factors. In short, hedge funds deliver high alphas to investors while being immune to alternative sources of risk.

The JKKT and CP models reverse these conclusions as the beta component dominates the return decomposition. For instance, the CP model delivers an average alpha of 0.4% per year and a proportion of positive-alpha funds close to 50%. At the same time, the average beta component rises to 5.2% per year (versus 2.6% under the CAPM). To visualize the differences between models, we plot in Figure 2 the densities of the two components for (i) the Fung-Hsieh model, (ii) the JKKT model, and (iii) the CP model. These differences stem from the ability of the alternative factors to capture hedge fund strategies. This ability is consistent with the ample anecdotal evidence that hedge funds hold illiquid assets, take levered positions, trade equity options, buy cheap assets with high carry, and follow trends in asset prices (*e.g.* Lhabitant, 2007; Pedersen, 2015).

Please insert Table VI and Figure 2 here

Table VII reports similar patterns for each investment style.[21] In the equity and arbitrage categories, the average alphas equal 2.4% and 2.8% per year under the CAPM but drop to 0.6% and 0.9% under the CP model. The difference is even more striking for macro funds. The average CAPM alpha equals 3.7% per year (which represents 78% of the average return). Under the CP model, the average alpha drops to -0.4% per year as the majority of funds underperform (51.8%). In the appendix, we also examine the returns of multi-strategy funds and funds of funds, which could be more difficult to capture given their diverse strategies. In both categories, the JKKT and CP models still deliver a sharp reduction in the alpha components.

The similarity between the JKKT and CP models is not surprising, given that they have five factors in common (market, size, illiquidity, BAB, TS momentum). Yet, the overall evidence gives

[21]For brevity, we only report the distribution characteristics for the the JKKT model, the CP model, and the CAPM (the results for the standard models and the two machine learning models are similar to the CAPM).

an edge to the CP model. It consistently delivers lower average alphas and lower proportions of positive-alpha funds. In addition, the appendix shows that the CP model produces a statistically significant reduction in average alphas relative to the CAPM in all three categories (versus only one for the JKKT model). These differences arise because the CP model includes variance and carry. Both factors, which rest on solid economic intuition, are particularly useful for capturing alternative hedge fund strategies—a point we examine in more detail in Section V.D.

Please insert Table VII here

V.B.2. Return Decomposition Over Time

As noted by Getmansky, Lee, and Lo (2015), the universe of hedge funds has expanded substantially since 1994. As a result of this expansion, the average return decomposition may be subject to notable time trends. To examine this issue, we use the CP model to track the evolution of (i) the average alpha component $\hat{M}^k_{1,ac} = \frac{1}{n_\chi}\sum_i^{n_\chi} \hat{ac}^k_i$, (ii) the average beta component due to the market $\hat{M}^k_{1,bc,m} = \frac{1}{n_\chi}\sum_i^{n_\chi} \hat{bc}^k_{i,m}$, and (iii) the average beta component due to the non-market factors $\hat{M}^k_{1,bc,I_{nm}} = \frac{1}{n_\chi}\sum_i^{n_\chi} \hat{bc}^k_{i,I_{nm}}$ (size, illiquidity, BAB, variance, carry, TS momentum). We start the analysis in 2004 and estimate each cross-sectional average using the entire return history for each fund up to that point in time. As we move forward in time, we expand the set of return observations and add new hedge funds once they satisfy the fund selection rule. The final estimates correspond to the averages shown in Table VI. For comparison purposes, we conduct the same analysis for mutual funds using the traditional Carhart model.

Figure 3 identifies two sources of convergence between hedge funds and mutual funds. First, performance becomes increasingly similar—at the end of 2020, the gap in average alphas drops to 1.6%. One intuitive explanation is the presence of scalability constraints. As a result of the growth of the hedge fund industry, it becomes increasingly difficult to maintain the same performance level. Bollen, Joenväärä, and Kauppila (2021) find support for this explanation but also suggest that central bank interventions might have reduced the profitability of hedge fund strategies. Second, hedge funds rely increasingly on the equity market to generate returns. At the end of 2020, $\hat{M}^k_{1,bc,m}$ reaches its highest level at 2.3% per year. Whereas it remains smaller than for mutual funds (7.8%), it follows the same trend in the aftermath of the 2008 crisis.

We also find that one key difference remains—hedge funds consistently use non-market factors to boost their returns. These factors contribute to at least 52% of the average hedge fund return during the sample period. We find no evidence that hedge funds reduce their exposure to these factors over time. Instead, the moderate decrease in $\hat{M}^k_{1,bc,I_{nm}}$ from 4.4% to 2.9% is caused by the reduction in the factor premia over time.

Please insert Figure 3 here

V.B.3. Impact of Factor Trading Costs

Our baseline specification does not include factor trading costs. As a result, the JKKT and CP models do not include the diversification services offered by funds in replicating the factors (Berk and van Binsbergen, 2015). Whereas trading costs are generally modest for standard factors such as size or value, they could be much higher for the alternative factors. For instance, Novy-Marx and Velikov (2022) estimate that the costs of trading the BAB strategy reach 60 bps per month.

To address this issue, we build on previous studies and approximate the costs of trading the alternative factors. Consistent with intuition, we show in the appendix that accounting for trading costs increases the alpha components under the JKKT and CP models. However, this increase is modest (0.5% per year on average), which implies that the differences relative to the CAPM remain significant both economically and statistically.

V.C. Heterogeneity Across Funds

V.C.1. Cross-Sectional Variation in the Alpha and Beta Components

A striking observation in Table VI is the large heterogeneity across funds. To illustrate, the difference between the two quantiles of the alpha distribution ranges between 13.8% and 16.8% per year across models. This heterogeneity is not captured by investment styles. As shown in Table VII, the dispersion in performance and risk profile remains large within each fund group. In other words, forming style groups is not sufficient to absorb the differences between funds. This result is at odds with the common practice of benchmarking funds with style indices as it imposes that the beta component is constant within each style group.

Another commonality among models is the negative correlation between the alpha and beta components (it ranges between -35% and -70%). This negative relation is particularly strong under the CP model, as shown by the left tail of the alpha distribution. In the bottom decile, funds deliver alphas below -8.2% per year (versus -4.0% for the CAPM). These funds rely heavily on the five alternative factors to boost their returns (possibly to hide their lack of skills)—on average, their beta component is three times larger than in the population (16.7% versus 5.2% per year as shown in the appendix). By controlling for these factor exposures, the CP model uncovers the strong underperformance achieved by the worst funds.

This strong negative correlation explains why the CP model produces a higher dispersion in alphas than the CAPM. A priori, the CAPM should generate a higher cross-sectional variance because its alpha absorbs the dispersion due to the omitted factors (the term $2\sigma_b^{*2}\lambda^2$ in our example in Figure 1). Whereas this effect is at play, it is more than offset by the reduction in variance due to the negative correlation between the alpha and beta components.

V.C.2. Implications for Models of Active Management

The observed heterogeneity has implications for the models of Berk and Green (2004) and Gârleanu and Pedersen (2018)—two popular models of active management. In the model of Berk and Green (2004), skilled funds have bargaining power because they are in short supply. As investors compete for performance, the alphas of all funds are null, and the heterogeneity disappears (after an adjustment period due to learning). This prediction holds quite well for mutual funds—Barras, Gagliardini, and Scaillet (2022) find that the standard deviation of the alpha distribution only equals 1.4% per year. However, it is at odds with the volatility observed for hedge funds.

In contrast, the dispersion in alphas is consistent with the model of Gârleanu and Pedersen (2018). In this model, skilled funds deliver positive alphas because they need to compensate investors for their search costs. At the same time, unskilled funds deliver negative alphas as they charge fees to unsophisticated investors. While other economic mechanisms could produce fund heterogeneity, search costs are consistent with two well-known features of the hedge fund industry. First, the process for evaluating hedge funds is more complex than for mutual funds (*e.g.*, Lhabitant, 2007). Second, hedge funds commonly use leverage. When search costs are high, taking leverage is actually the only strategy that allows hedge funds to compensate investors—a point made by Stein (2009) in the context of systemic risk.

V.D. A Closer Look at Alternative Hedge Fund Strategies

V.D.1. The Determinants of Beta Component

Our previous analysis shows the importance of the five alternative factors for measuring the beta component. Motivated by these results, we measure the importance of each factor in shaping the risk profile of the funds. We compute the fund beta component due to each factor j included in the CP model (market, size, illiquidity, BAB, variance, carry, TS momentum). Using these estimated quantities, we then apply our methodology to infer the main characteristics of the cross-sectional beta distribution $\phi_{bc,j}^{k}$.

Table VIII shows that the market is the most prevalent source of risk as 78% of the funds have positive market betas. On average, the contribution due to the market is equal to 2.3% per year, which represents 44% of the total beta component (5.2% per year). There is also a notable variation in market exposures between the two quantiles equal to 7.6% per year. As discussed by Bali, Brown, and Caglayan (2012), this variation translates into different levels of systematic risk and average returns across individual funds.

Examining the alternative factors, we find that a majority of funds load positively on each of them. This result provides support to the view that hedge funds follow exotic strategies to boost their returns (*e.g.*, Carhart et al., 2014). Interestingly, individual funds do not load on all the alternative factors simultaneously. As shown in Panel B, the pairwise correlations between the beta contributions of the factors only range between -18% and 14%.

TS momentum, variance, and carry are the most important alternative factors. Their average contributions, which are all statistically significant, are equal to 1.1%, 0.8%, and 0.4% per year. In the top decile of funds with the highest exposures, these contributions reach 4.5%, 4.5%, and 2.7% per year. The beta contributions due to these factors is partly due to their Sharpe ratios—an observation made by Dew-Becker et al. (2017), Koijen et al. (2018), and Moskowitz, Ooi, and Pedersen (2012). Therefore, the CP model captures average fund returns without a large increase in the time-series R^2 (as shown in Table IV).

Please insert Table VIII here

V.D.2. Alternative Strategies Across Investment Styles

In Table IX, we repeat the analysis for each investment category (equity, macro, arbitrage). Carry strategies consist in investing in cheap assets with high carry—that is, assets with a high difference between spot and forward prices. As discussed by Pedersen (2015, ch.9,11,14), hedge funds routinely implement carry in their equity, currency, commodity, yield curve, and credit trades. Consistent with this view, we find that global carry matters for all three categories.

TS momentum plays a key role among macro funds, which rely on past returns to determine their asset allocation and exploit trends in asset prices caused by behavioural biases, frictions, or slow-moving capital. The appendix further shows that this positive exposure is primarily due to CTA funds, consistent with Pedersen (2015, ch.12), who finds that the alphas of CTA indices turn negative after controlling for TS momentum. In contrast, this factor should be irrelevant for arbitrage funds, which is confirmed by the average beta component of -0.3% per year.

Equity and arbitrage funds commonly take on variance risk—on average, the beta component is equal to 1.1% per year and rises above 4.5% in the top decile of funds with the highest variance exposure. There are several possible explanations for these results. First, the variance factor captures the risk associated with their option positions. For instance, mortgage, fixed income volatility, and merger arbitrage activities involve taking short option positions (Duarte, Longstaff, and Yu, 2006; Mitchell and Pulvino, 2001). Second, realized variance captures unexpected increases in the correlation between stocks (Driessen, Maenhout, and Vilkov, 2009). As such, it signals crisis times when equity and arbitrage funds suffer from less effective hedging strategies and tighter funding constraints (Buraschi, Kosowski, and Trojani, 2014).

Finally, arbitrage funds have the highest exposure to BAB as they extensively use leverage to exploit the price distortions observed in capital markets (Ang, Gorovyy, and van Inwegen, 2011). It is plausible that some of these distortions originate from the leverage constraints faced by traditional investors (mutual and pension funds). As a result, we expect a positive correlation with the return of the BAB strategy (Frazzini and Pedersen, 2014).

Please insert Table IX here

V.E. The Sophistication of Hedge Fund Investors

V.E.1. Implications for Hedge Fund Valuation

Another benefit of comparing models is to measure how hypothetical investors with different sophistication levels value hedge fund investments. If a sophisticated investor can replicate all the alternative factors, his valuation is given by the alpha under the CP model. We can formalize this intuition using the stochastic discount factor (SDF) framework. Writing the investor SDF m_t^k as a linear function of CP factors, we have $\alpha_i^k = (1 + r_f)E[m_t^k r_{i,t}]$, where r_f is the risk-free rate. A positive α_i^k signals that the investor can increase his overall utility by investing in the fund (*e.g.*, Chen and Knez, 1996; Ferson, 2013).[22] Next, consider a less sophisticated investor who can only invest in the equity market. His hedge fund valuation is then given by the CAPM alpha: $\alpha_i^0 = (1 + r_f)E[m_t^0 r_{i,t}]$, where m_t^0 is a linear function of the market.

Table VI shows that the average valuation is close to zero for the CP investor (0.4% per year), but substantially higher for the CAPM investor (2.9%). This valuation gap is consistent with intuition. Like the CP investor, the CAPM investor values the alpha component ac_i^*. In addition, he values the beta component $bc_{i,O}^*$ due to the alternative factors because he cannot replicate it. This point is well summarized by Cochrane (2011): "I tried telling a hedge fund manager, 'You don't have alpha. Your returns can be replicated with a value-growth, momentum, currency and term carry, and short-vol strategy.' He said, 'Exotic beta is my alpha. I understand those systematic factors and know how to trade them. My clients don't.'"

Regardless of their sophistication, hedge fund investors face substantial performance uncertainty when selecting individual funds from the underlying population. This result provides a strong rationale for conducting due diligence to avoid the worst funds. In the appendix, we find that several fund characteristics can be used as initial filters in the fund selection process. The best funds under the CP model focus more on equity strategies and have stronger managerial incentives as proxied by high watermark provisions. They also have more managerial flexibility (longer lockup and notice periods), which allows them to invest in illiquid assets and exploit arbitrage opportunities that take time to be profitable. Overall, these results are in line with those documented by Agarwal, Daniel, and Naik (2009), Aragon (2007), and Joenväärä et al. (2021).

[22] See also Almeida, Ardison, and Garcia (2020) and Karehnke and de Roon (2020) for a recent application of the SDF framework in which investors have nonlinear preferences (*i.e.*, m_t is a nonlinear function of the factors).

V.E.2. Measuring Investor Sophistication Using Flows

As a final exercise, we study the sophistication of actual hedge fund investors. Our analysis builds on the premise that investors learn about funds by observing past returns. As they update their valuation over time, they reallocate capital accordingly. If this mechanism is at play, fund flows contain information about investor preference for alpha and beta. With misspecified models, the learning process is likely to be noisier than for mutual funds. To address this issue, we compare the entire distributions of the alpha and beta components between low- and high-flow funds.[23]

We proceed in three steps. First, we follow Barras, Scaillet, and Wermers (2010) and partition our data into non-overlapping subperiods of five years, beginning with 1996 to 2000 and ending with 2016 to 2020. For each subperiod, we include all funds that pass the fund selection rule and compute their average monthly flows and return decomposition obtained with the CP model ($\hat{ac}_i^k$, $\hat{bc}_{i,m}^k$, $\hat{bc}_{i,I_{nm}}^k$). Second, we sort funds into flow quintiles (from low to high) and pool these five-year records together across all time periods. Third, we apply our methodology to compute the distributions of the alpha and beta components for each (pooled) flow quintile.

Table X suggests that actual investors are relatively sophisticated. Panel A shows that flows are primarily directed into funds with positive contemporaneous alphas. In the high-flow group, the alpha is equal to 3.3% per year on average and positive for 70.4% of the funds. On the contrary, the average alpha is negative in the low-flow group (-0.6%). At the same time, we observe a large distribution overlap between the two groups, as measured by the differences between quintiles. This result is consistent with our premise that the learning process is quite noisy.

Next, we turn to the analysis of the beta components. If actual investors chase past or market-adjusted returns, they should direct capital into funds that load aggressively on market and/or non-market factors (*i.e.*, funds with high $\hat{bc}_{i,m}^k$ and $\hat{bc}_{i,I_{nm}}^k$). Panels B and C do not support this interpretation. For the two sets of factors, the average beta components are larger in the low-flow group than in the high-flow group. In terms of sophistication, actual hedge fund investors are therefore closer to the CP investor than the CAPM investor.

Please insert Table X here

[23] An alternative approach, which does not capture fund heterogeneity, imposes a panel structure in which fund flows are regressed on fund return components (Barber, Huang, and Odean, 2016). Ben-David et al. (2022) argue that this approach produces spurious results because it overestimates the importance of alpha in driving flows.

VI. Conclusion

Decomposing hedge fund returns is challenging because factor models are likely misspecified—that is, they omit relevant factors for capturing the strategies followed by hedge funds. Model misspecification makes the estimation of the alpha and beta components both imperfect and noisy. To mitigate these challenges, we develop a new approach to estimate and compare the distributions of the alpha and beta components across models. Our approach improves the imperfect separation between alpha and beta by identifying models less prone to misspecification. It also explicitly accounts for estimation noise based on a full-fledged asymptotic theory in a large cross-section of funds.

Our comparison analysis yields several insights. We find that the standard models produce the same return decomposition as the CAPM, which implies that these models are ill-equipped for separating alpha and beta. In contrast, several economically motivated factors such as TS momentum, variance, and carry do a good job at capturing alternative hedge fund strategies. Including these factors increases the relative importance of the beta components, and uncovers a gradual convergence in performance between hedge funds and mutual funds. Regardless of the chosen model, we also observe a large dispersion in the alpha and beta components across funds. This large heterogeneity is consistent with an equilibrium model in which a fraction of the fund population delivers positive alphas to compensate investors for search costs.

Our methodology is flexible and can be applied in other situations where models are misspecified. For instance, it can be used to further improve the decomposition of hedge fund returns across specific investment categories. It could also be applied to international mutual funds for which the set of trading strategies is substantially larger than for traditional US equity funds.

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TABLE I. Summary Statistics for the Equal-Weighted Portfolio of Hedge Funds

This table provides summary statistics for the equal-weighted portfolio of all existing funds at the start of each month for the entire population and the three investment categories (equity, macro, and arbitrage). We report the mean (annualized), standard deviation (annualized), skewness, kurtosis, and quantiles at 10% and 90% of the portfolio excess return. Figures in parentheses denote the total number of selected funds for the entire population and each category. The statistics are computed using monthly data between January 1994 and December 2020.

	Moments				Quantiles	
	Mean	Std Dev.	Skewness	Kurtosis	10%	90%
All Funds	5.45	5.49	-0.31	4.51	-1.58	2.34
Equity	6.73	8.70	-0.47	5.32	-2.56	3.30
Long-Short	7.24	9.68	-0.44	5.31	-2.90	3.68
Market Neutral	3.07	2.72	-0.58	5.75	-0.66	1.13
Macro	5.05	5.79	0.24	3.30	-1.66	2.50
Global Macro	4.40	6.12	0.40	3.57	-1.68	2.59
CTA/Managed Futures	4.09	6.73	0.47	3.74	-1.79	2.92
Arbitrage	5.40	4.96	-2.16	14.57	-0.97	1.83
Relative Value	4.90	4.47	-2.46	18.03	-0.78	1.60
Event Driven	6.08	6.11	-1.74	11.79	-1.30	2.35

TABLE II. Summary Statistics for the Hedge Fund Factors

This table provides summary statistics for the factors used in the construction of the nine hedge fund models. The market factor is the excess return of the US equity market. The size, value, momentum, investment, and profitability factors are computed using US equity data. The global value and momentum factors are constructed across multiple international asset classes. The term and default factors are computed using US bond data. The bond, commodity, and currency straddles are computed using option data on bonds, commodities, and currencies. The illiquidity and betting-against-beta (BAB) factors are computed using US equity data. The variance factor is computed using index option data on the S&P500. The carry and time-series (TS) momentum factors are constructed across multiple international asset classes. We define the straddle and variance factors as short positions to obtain positive premia. We report the mean (annualized), standard deviation (annualized), skewness, kurtosis, and quantiles at 10% and 90% of the excess returns of the factors. The statistics are computed using monthly data between January 1994 and December 2020.

Panel A: Market and Standard Factors

	Moments				Quantiles	
	Mean	Std Dev.	Skewness	Kurtosis	10%	90%
Market	8.81	15.48	-0.64	4.26	-5.15	6.02
Size	1.61	10.83	0.38	7.54	-3.49	3.69
Value	0.20	10.88	0.05	5.84	-3.30	3.47
Momentum	4.67	17.15	-1.42	12.88	-5.07	5.50
Investment	2.11	7.04	0.67	5.09	-2.14	2.70
Profitability	3.77	9.38	-0.47	13.49	-2.03	2.90
Global Value	1.30	6.19	-0.64	12.25	-1.72	1.76
Global Momentum	3.26	7.55	-0.30	5.46	-2.25	2.69
Term	-0.18	0.89	-0.03	4.22	-0.35	0.29
Default	0.01	0.77	1.90	17.66	-0.20	0.19
Straddle on Bonds	16.00	57.86	-1.85	8.99	-19.68	17.73
Straddle on Commodities	2.99	50.38	-1.32	6.07	-19.71	15.74
Straddle on Currencies	11.27	69.02	-1.56	6.64	-23.36	20.15

Panel B: Alternative Factors

	Moments				Quantiles	
	Mean	Std Dev.	Skewness	Kurtosis	10%	90%
Iliquidity	6.52	12.97	-0.27	4.49	-3.60	4.99
BAB	8.58	13.77	-0.36	6.10	-3.58	5.21
Variance	38.10	23.80	-4.57	29.76	-1.03	7.61
Carry	6.85	4.96	0.01	3.98	-1.10	2.34
TS Momentum	11.11	12.56	0.16	3.16	-3.54	5.71

TABLE III. The Set of Hedge Fund Models

This table summarizes the set of nine hedge fund models chosen for the empirical analysis. This set includes the CAPM as well as three distinct groups. The first group includes the models formed with standard factors, namely the Carhart, Five-Factor, Fung-Hsieh, and Asness-Moskowitz-Pedersen (AMP) models. The second group includes the two machine learning models of Kozak, Nagel, and Shantosh (KNS) formed with either five characteristic-based equity portfolios, or five principal components of these portfolios. The final group includes two models formed with alternative factors. The first one is the model of Joenväärä et al. (2021) (JKKT). The second one combines the alternative factors proposed by Carhart (1997) and Pedersen (2018) (CP).

Model	List of Included Factors
CAPM	Market
Standard Models	
Carhart	Market, Size, Value, Momentum
Five-Factor	Market, Size, Value, Investment, Profitability
Fung-Hsieh	Market, Size, Term, Default, Straddles (Bonds, Commodities, Currencies)
AMP	Market, Global Value and Momentum
Machine-Learning Models	
KNS1	Market, Five Characteristic-Based Equity Factors
KNS2	Market, Five Principal Component Equity Factors
alternativeModels	
JKKT	Market, Size, Value, Momentum, Illiquidity, BAB, TS Momentum
CP	Market, Size, Illiquidity, BAB, Variance, Carry, TS Momentum

TABLE IV. Misspecification Diagnostic for the Hedge Fund Models
This table provides misspecification statistics for the CAPM, the four standard models (Carhart, Five-Factor, Fung-Hsieh, AMP), the two machine learning models (KNS1, KNS2), and the two alternative models (JKKT and CP). For each model, we measure the relative importance of the three components of the average fund return: (i) the alpha component (Alpha), (ii) the beta component due to the market (Beta Mkt), and the beta component due to the non-market factors (Beta Non-Mkt), *i.e.*, the other factors included in the model. These proportions, which are averaged across all funds, sum up to 100%. We also compute the average adjusted R^2 of the time-series regression of the fund return on the factors. We conduct this analysis for the entire hedge fund population (first four columns) and for the entire mutual fund population (last four columns).

	Hedge Funds				Mutual Funds			
	Relative Importance (%)				Relative Importance (%)			
	Alpha	Beta Mkt	Beta Non-Mkt	R^2	Alpha	Beta Mkt	Beta Non-Mkt	R^2
CAPM	52.75	47.25	0.00	20.43	-15.23	115.23	0.00	81.34
Carhart	47.14	46.60	6.26	25.28	-17.16	110.57	6.58	88.98
Five-Factor	49.04	46.65	4.31	24.85	-16.38	110.27	6.11	88.99
Fung-Hsieh	54.10	40.56	5.34	30.24	-14.96	107.78	7.18	85.88
AMP	47.08	47.37	5.55	24.36	-15.37	115.01	0.36	84.55
KNS1	52.52	48.24	-0.76	24.47	-17.41	114.58	2.84	83.40
KNS2	64.01	51.03	-15.04	26.39	-6.27	115.99	-9.73	87.09
JKKT	17.97	44.28	37.75	31.01	-20.01	110.20	9.81	89.65
CP	6.64	41.19	52.17	30.26	-24.33	109.06	15.27	87.15

TABLE V. Model Comparisons Relative to the CAPM

The table measures the degree of misspecification of the standard models (Carhart, Five-Factor, Fung-Hsieh, AMP), the two machine learning models (KNS1, KNS2), and the two alternative models (JKKT and CP). For each model, we examine whether the distribution of the alpha components departs from the one obtained with the CAPM. Lack of differences signals that the model is no better than the CAPM at capturing hedge fund strategies. We report the differences in the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated differences. ***, **, * indicate that the null hypothesis of equal characteristics is rejected at the 1%, 5%, and 10% levels.

	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Carhart	-0.31 (0.40)	-0.19 (0.31)	2.27 (2.50)	-2.27 (2.50)	-0.02 (0.40)	-0.60* (0.34)
Five-Factor	-0.21 (0.46)	0.02 (0.34)	2.41 (3.01)	-2.41 (3.01)	-0.01 (0.50)	-0.18 (0.40)
Fung-Hsieh	0.08 (0.60)	-0.20 (0.35)	0.52 (3.88)	-0.52 (3.88)	0.28 (0.56)	-0.15 (0.47)
AMP	-0.31 (0.42)	0.06 (0.37)	2.50 (2.44)	-2.50 (2.44)	-0.17 (0.37)	-0.34 (0.45)
KNS1	-0.01 (0.39)	0.43 (0.29)	1.59 (1.78)	-1.59 (1.78)	-0.26 (0.29)	0.19 (0.38)
KNS2	0.63 (0.49)	0.20 (0.31)	-2.47 (2.65)	2.47 (2.65)	0.58 (0.41)	0.65 (0.47)
JKKT	-1.93*** (0.67)	0.41 (0.50)	14.76*** (4.01)	-14.76*** (4.01)	-2.32*** (0.66)	-1.98*** (0.68)
CP	-2.56*** (0.76)	2.13*** (0.50)	19.82*** (4.27)	-19.82*** (4.27)	-4.22*** (0.73)	-1.42* (0.76)

TABLE VI. Decomposition of Average Fund Returns – Entire Population

This table shows the decomposition of average fund returns under the CAPM, the four standard models (Carhart, Five-Factor, Fung-Hsieh, AMP), the two machine learning models (KNS1, KNS2), and the two alternative models (JKKT and CP). Panel A reports the characteristics of the cross-sectional distribution of the alpha components under each model. We report the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%.Figures in parentheses denote the standard deviation of the estimated characteristics. Panel B reports the characteristics of the cross-sectional distribution of beta components under each model.

Panel A: Distribution of the Alpha Components						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
CAPM	2.93 (0.94)	7.01 (0.48)	27.15 (5.49)	72.85 (5.49)	-3.95 (0.67)	10.06 (0.67)
Carhart	2.62 (0.84)	6.82 (0.31)	29.42 (5.08)	70.58 (5.08)	-3.97 (0.57)	9.46 (0.55)
Five-Factor	2.72 (0.89)	7.03 (0.31)	29.55 (5.23)	70.45 (5.23)	-3.96 (0.63)	9.87 (0.54)
Fung-Hsieh	3.01 (0.74)	6.81 (0.29)	27.66 (3.60)	72.34 (3.60)	-3.67 (0.47)	9.90 (0.51)
AMP	2.62 (0.92)	7.08 (0.28)	29.65 (5.62)	70.35 (5.62)	-4.12 (0.59)	9.71 (0.54)
KNS1	2.92 (0.87)	7.44 (0.42)	28.73 (5.13)	71.27 (5.13)	-4.21 (0.63)	10.24 (0.60)
KNS2	3.56 (0.86)	7.21 (0.37)	24.68 (4.33)	75.32 (4.33)	-3.37 (0.55)	10.70 (0.62)
JKKT	1.00 (0.73)	7.42 (0.31)	41.90 (4.80)	58.10 (4.80)	-6.27 (0.60)	8.07 (0.38)
CP	0.37 (0.86)	9.15 (0.40)	46.97 (5.31)	53.03 (5.31)	-8.17 (0.77)	8.64 (0.44)

Panel B: Distribution of the Beta Components						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
CAPM	2.62 (0.83)	4.37 (0.67)	22.54 (9.57)	77.46 (9.57)	-0.71 (0.61)	8.01 (0.73)
Carhart	2.94 (0.81)	4.30 (0.52)	17.99 (5.68)	82.01 (5.68)	-0.65 (0.42)	8.20 (0.72)
Five-Factor	2.83 (0.84)	4.48 (0.51)	20.61 (5.62)	79.39 (5.62)	-1.06 (0.41)	8.32 (0.75)
Fung-Hsieh	2.55 (0.78)	4.39 (0.52)	22.58 (5.98)	77.42 (5.98)	-1.23 (0.44)	7.96 (0.67)
AMP	2.94 (0.84)	4.66 (0.49)	18.98 (5.41)	81.02 (5.41)	-0.96 (0.43)	8.56 (0.73)
KNS1	2.64 (0.80)	5.14 (0.57)	23.04 (5.71)	76.96 (5.71)	-1.37 (0.56)	8.38 (0.86)
KNS2	2.00 (0.82)	4.47 (0.49)	27.53 (7.96)	72.47 (7.96)	-1.61 (0.59)	7.28 (0.69)
JKKT	4.56 (0.83)	5.90 (0.42)	13.90 (2.20)	86.10 (2.20)	-0.50 (0.28)	11.12 (0.74)
CP	5.19 (0.93)	7.64 (0.49)	15.91 (2.17)	84.09 (2.17)	-1.17 (0.35)	12.78 (1.11)

TABLE VII. Decomposition of Average Fund Returns – Investment Categories

This table shows the decomposition of average fund returns under the CAPM and the two alternative models (JKKT and CP) across investment styles. Panel A reports the characteristics of the cross-sectional distribution of the alpha and beta components for equity funds. We report the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics. Panels B and C repeat the analysis for macro and arbitrage funds.

Panel A: Equity Funds						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Distribution of the Alpha Components						
CAPM	2.40 (1.06)	7.09 (0.48)	30.23 (6.31)	69.77 (6.31)	-4.52 (0.83)	9.54 (0.96)
JKKT	1.12 (0.81)	6.98 (0.31)	42.98 (5.48)	57.02 (5.48)	-5.73 (0.75)	7.77 (0.66)
CP	0.58 (0.96)	9.08 (0.49)	47.38 (5.46)	52.62 (5.46)	-7.84 (1.02)	8.53 (0.79)
Distribution of the Beta Components						
CAPM	4.18 (1.16)	5.19 (0.70)	12.94 (8.25)	87.06 (8.25)	-0.14 (0.74)	10.66 (1.39)
JKKT	5.46 (1.18)	5.75 (0.54)	10.59 (2.37)	89.41 (2.37)	-0.10 (0.43)	12.19 (1.41)
CP	6.00 (1.31)	7.58 (0.61)	13.69 (2.77)	86.31 (2.77)	-0.75 (0.56)	13.71 (1.77)

Panel B: Macro Funds						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Distribution of the Alpha Components						
CAPM	3.71 (1.72)	8.04 (0.78)	25.73 (6.47)	74.27 (6.47)	-4.26 (0.89)	12.41 (2.14)
JKKT	-0.14 (1.42)	9.14 (0.65)	51.52 (7.66)	48.48 (7.66)	-9.25 (1.43)	8.70 (0.75)
CP	-0.39 (1.65)	11.04 (0.70)	51.83 (7.02)	48.17 (7.02)	-10.91 (1.58)	10.26 (1.00)
Distribution of the Beta Components						
CAPM	1.01 (1.08)	3.70 (0.68)	43.44 (21.78)	56.56 (21.78)	-1.76 (1.29)	5.44 (0.66)
JKKT	4.87 (1.11)	7.49 (0.74)	20.32 (3.33)	79.68 (3.33)	-1.57 (0.44)	12.86 (1.30)
CP	5.11 (1.33)	9.62 (0.79)	23.74 (3.89)	76.26 (3.89)	-2.60 (0.63)	14.18 (1.35)

Panel C: Arbitrage Funds						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Distribution of the Alpha Components						
CAPM	2.81 (1.24)	5.61 (0.44)	24.74 (9.71)	75.26 (9.71)	-2.79 (1.31)	8.67 (0.63)
JKKT	1.97 (0.89)	5.70 (0.29)	31.03 (8.00)	68.97 (8.00)	-3.66 (0.92)	7.60 (0.50)
CP	0.87 (1.06)	6.81 (0.44)	41.64 (9.30)	58.36 (9.30)	-5.48 (1.20)	7.62 (0.49)
Distribution of the Beta Components						
CAPM	2.31 (0.98)	3.02 (0.66)	13.63 (8.51)	86.37 (8.51)	-0.08 (0.45)	5.88 (1.22)
JKKT	3.14 (0.81)	3.57 (0.42)	11.60 (3.04)	88.40 (3.04)	-0.09 (0.32)	7.28 (1.03)
CP	4.25 (0.98)	4.90 (0.56)	10.86 (2.29)	89.14 (2.29)	-0.10 (0.33)	9.42 (1.37)

TABLE VIII. Decomposition of the Beta Components – Entire Population

This table shows the decomposition of the beta components obtained with the CP model. Panel A reports the characteristics of the cross-sectional distribution of the beta components associated with each factor (market, size, illiquidity, BAB, variance, carry, TS momentum). For each fund, the beta component associated with a given factor is defined as the product between the fund beta and the factor premium. We report the annualized mean and standard deviation, the proportions of funds with negative and positive contributions, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics. Panel B reports the cross-sectional correlation between the beta components for each pair of factors.

Panel A: Distribution of the Beta Components for Each Factor

	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Market	2.29 (1.27)	3.84 (1.19)	21.62 (6.13)	78.38 (6.13)	-0.55 (0.15)	6.99 (2.05)
Size	0.23 (0.26)	1.01 (0.32)	38.23 (4.26)	61.77 (4.26)	-0.31 (0.03)	1.02 (0.53)
Illiquidity	0.05 (0.07)	1.26 (0.29)	44.94 (3.25)	55.06 (3.25)	-0.75 (0.24)	0.85 (0.38)
Betting Against Beta	0.35 (0.25)	2.22 (0.69)	35.14 (3.80)	64.86 (3.80)	-1.30 (0.44)	2.23 (0.90)
Variance	0.78 (0.25)	5.17 (0.70)	36.74 (1.89)	63.26 (1.89)	-3.08 (0.54)	4.53 (0.87)
Carry	0.43 (0.17)	2.56 (0.46)	39.21 (2.38)	60.79 (2.38)	-1.89 (0.37)	2.73 (0.51)
Time-Series Momentum	1.06 (0.32)	4.03 (0.78)	42.86 (1.51)	57.14 (1.51)	-1.35 (0.34)	4.49 (0.72)

Panel B: Correlations Between the Beta Components

	Size	Illiquidity	BAB	Variance	Carry	TS Mom
Market	-0.08	-0.01	-0.01	-0.03	-0.02	-0.18
Size		-0.07	-0.00	-0.06	0.06	0.05
Illiquidity			-0.09	0.14	-0.02	0.08
Betting Against Beta				-0.07	0.03	-0.05
Variance					0.04	-0.09
Carry						-0.13

TABLE IX. Decomposition of the Beta Components – Investment Categories

This table shows the decomposition of the beta components obtained with the CP model across investment styles. Panel A reports the characteristics of the cross-sectional distribution of the beta components of equity funds associated with each factor (market, size, illiquidity, BAB, variance, carry, TS momentum). For each fund, the beta component associated with a given factor is defined as the product between the fund beta and the factor premium. We report the annualized mean and standard deviation, the proportions of funds with negative and positive contributions, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics. Panel B and C repeat the analysis for macro and arbitrage funds.

Panel A: Equity Funds						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Market	3.67 (2.07)	4.53 (1.22)	14.54 (8.16)	85.46 (8.16)	-0.19 (0.61)	9.33 (3.18)
Size	0.42 (0.57)	1.32 (0.42)	32.68 (11.70)	67.32 (11.70)	-0.35 (0.11)	1.65 (1.04)
Illiquidity	0.14 (0.10)	1.49 (0.35)	44.13 (2.66)	55.87 (2.66)	-0.85 (0.25)	1.27 (0.48)
Betting Against Beta	0.25 (0.21)	2.66 (0.80)	38.48 (3.90)	61.52 (3.90)	-1.71 (0.55)	2.55 (1.05)
Variance	0.67 (0.25)	5.06 (0.70)	38.58 (2.19)	61.42 (2.19)	-3.12 (0.54)	4.46 (0.85)
Carry	0.34 (0.17)	2.80 (0.46)	44.78 (2.67)	55.22 (2.67)	-2.11 (0.41)	2.73 (0.52)
Time-Series Momentum	0.51 (0.17)	2.83 (0.55)	42.08 (2.04)	57.92 (2.04)	-1.56 (0.46)	3.05 (0.74)

Panel B: Macro Funds						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Market	1.18 (0.62)	3.47 (1.10)	35.11 (4.40)	64.89 (4.40)	-1.40 (0.35)	4.87 (1.62)
Size	0.07 (0.09)	0.83 (0.20)	50.59 (9.02)	49.41 (9.02)	-0.43 (0.28)	0.64 (0.20)
Illiquidity	-0.08 (0.10)	1.26 (0.31)	54.07 (3.37)	45.93 (3.37)	-0.90 (0.37)	0.75 (0.28)
Betting Against Beta	0.21 (0.19)	1.99 (0.65)	40.83 (4.63)	59.17 (4.63)	-1.30 (0.47)	1.84 (0.76)
Variance	0.22 (0.35)	6.56 (0.87)	49.41 (3.11)	50.59 (3.11)	-4.73 (0.92)	4.55 (0.76)
Carry	0.41 (0.26)	2.79 (0.57)	41.89 (3.33)	58.11 (3.33)	-2.43 (0.58)	3.23 (0.64)
Time-Series Momentum	3.10 (1.11)	5.83 (1.23)	24.43 (3.26)	75.57 (3.26)	-0.73 (0.12)	10.22 (2.20)

Panel C: Arbitrage Funds						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Market	1.68 (0.90)	2.52 (0.75)	16.96 (6.38)	83.04 (6.38)	-0.19 (0.21)	4.40 (1.44)
Size	0.16 (0.20)	0.60 (0.17)	32.82 (8.47)	67.18 (8.47)	-0.11 (0.04)	0.56 (0.45)
Illiquidity	0.07 (0.12)	0.87 (0.18)	36.89 (7.20)	63.11 (7.20)	-0.40 (0.06)	0.60 (0.32)
Betting Against Beta	0.61 (0.39)	1.76 (0.58)	25.35 (4.61)	74.65 (4.61)	-0.68 (0.14)	2.15 (0.95)
Variance	1.47 (0.42)	3.34 (0.55)	21.90 (2.39)	78.10 (2.39)	-0.78 (0.08)	4.59 (0.97)
Carry	0.56 (0.20)	1.91 (0.34)	29.67 (3.66)	70.33 (3.66)	-0.86 (0.17)	2.37 (0.52)
Time-Series Momentum	-0.30 (0.19)	1.53 (0.37)	62.12 (4.12)	37.88 (4.12)	-1.64 (0.57)	0.82 (0.17)

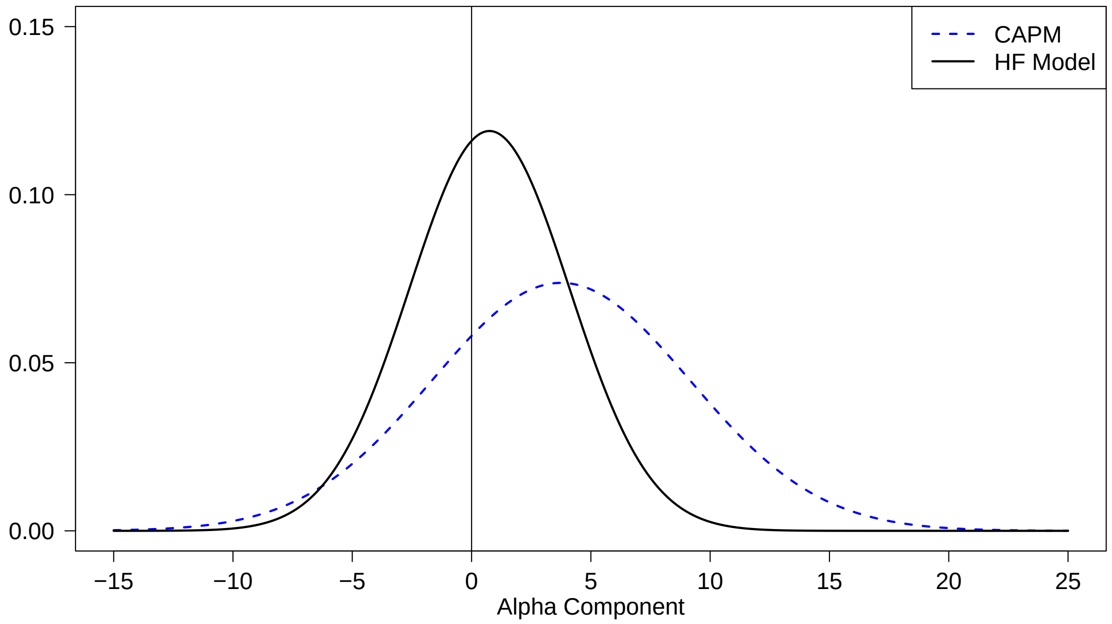
TABLE X. Fund Flows and Average Return Components

This table examines the relation between fund flows and the three components of the average fund return under the CP model: (i) the alpha component, (ii) the beta component due to the market, and (iii) the beta component due to the non-market factors (size, illiquidity, BAB, variance, carry, TS momentum). For each of the non-overlapping five-year periods between 1996 and 2020, we measure the three components for all funds. We then rank them according to their average monthly net flows and group them into quintiles (low, 2, 3, 4, high). Panel A reports the characteristics of the cross-sectional distribution of the alpha components (pooled over all five-year periods) for the five flow-based groups. We report the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics. Panels B and C repeat the analysis for the beta components due to the market and the beta components due to the non-market factors.

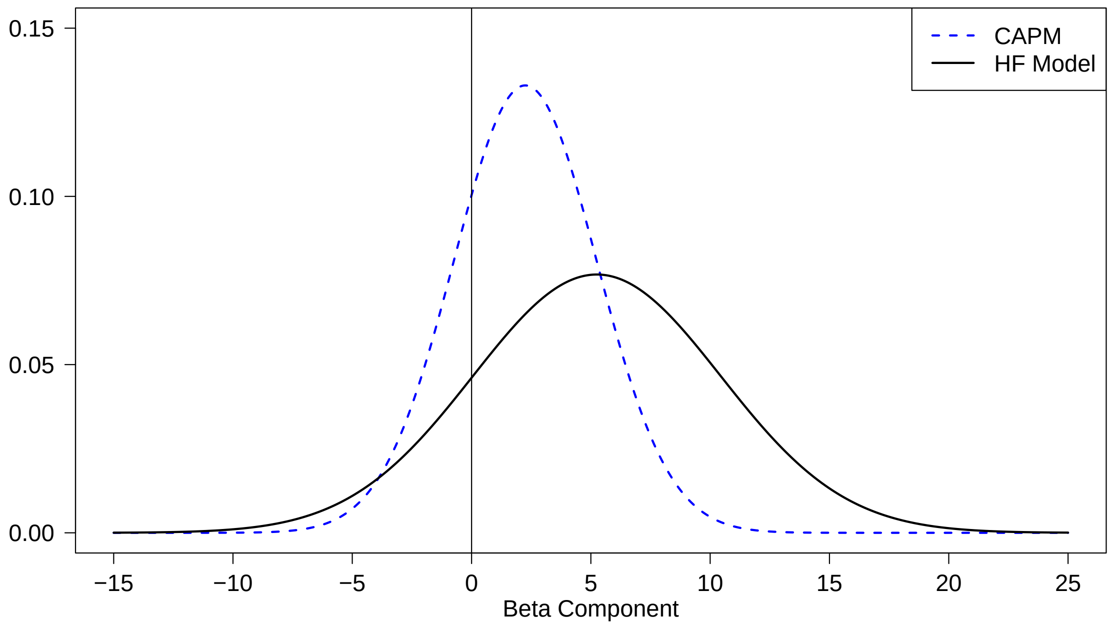
Panel A: Distribution of the Alpha Components						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Low	-0.57 (1.54)	10.71 (0.79)	54.84 (5.88)	45.16 (5.88)	-10.53 (1.33)	9.55 (0.74)
2	0.39 (1.33)	9.03 (0.72)	47.82 (5.66)	52.18 (5.66)	-9.01 (0.98)	9.15 (0.83)
3	1.52 (1.32)	8.42 (0.76)	41.49 (6.10)	58.51 (6.10)	-7.18 (0.90)	10.33 (0.65)
4	2.92 (1.44)	8.85 (0.82)	34.57 (6.90)	65.43 (6.90)	-5.97 (1.18)	13.00 (0.56)
High	3.25 (1.24)	9.87 (0.61)	29.61 (5.31)	70.39 (5.31)	-4.92 (0.98)	13.18 (0.74)
Panel B: Distribution of the Beta Components (Market Factor)						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Low	2.32 (1.98)	4.19 (1.75)	18.19 (19.41)	81.81 (19.41)	-0.53 (0.84)	8.37 (3.00)
2	3.15 (2.45)	5.09 (1.75)	14.25 (18.87)	85.75 (18.87)	-0.17 (1.19)	10.34 (4.27)
3	2.91 (2.26)	4.94 (1.84)	15.85 (18.60)	84.15 (18.60)	-0.17 (1.13)	9.48 (4.06)
4	2.40 (1.84)	4.42 (1.74)	19.96 (15.33)	80.04 (15.33)	-0.33 (0.71)	8.29 (3.30)
High	1.95 (1.45)	3.90 (1.61)	23.07 (13.06)	76.93 (13.06)	-0.46 (0.45)	7.31 (2.63)
Panel C: Distribution of the Beta Components (Non-Market Factors)						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Low	2.38 (0.90)	8.18 (3.58)	32.13 (8.54)	67.87 (8.54)	-4.43 (1.93)	10.44 (3.66)
2	2.13 (0.83)	7.36 (3.03)	35.93 (12.43)	64.07 (12.43)	-4.26 (1.59)	9.60 (3.57)
3	2.14 (0.81)	6.94 (3.28)	32.07 (11.95)	67.93 (11.95)	-4.23 (2.02)	9.34 (3.59)
4	2.30 (0.78)	7.16 (3.19)	34.30 (8.29)	65.70 (8.29)	-3.88 (1.79)	9.63 (3.59)
High	2.37 (0.81)	7.82 (3.09)	33.70 (6.23)	66.30 (6.23)	-3.81 (1.70)	8.65 (3.46)

Figure 1. Distributions of the Alpha and Beta Components – A Simple Example

Panel A compares the average return decomposition obtained with a candidate hedge fund (HF) model and the CAPM. In this simple example, the average fund returns are explained by four factors (the market and three alternative factors 1, 2, and 3 with similar premia), and hedge funds load more aggressively on factor 1 than on factors 2 and 3. Whereas the CAPM omits factors 1, 2, and 3, the HF model only omits factor 3. Panel A plots the cross-sectional distributions of the alpha components (annualized) under both models. Panel B repeats the analysis for the beta components.



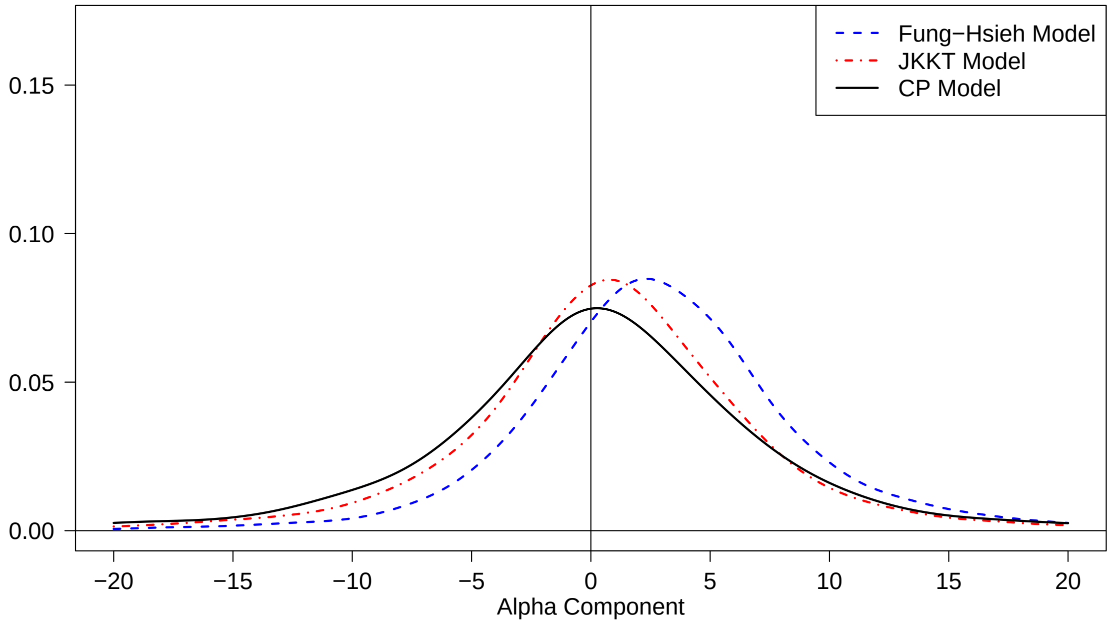
(a) Distribution of the Alpha Components



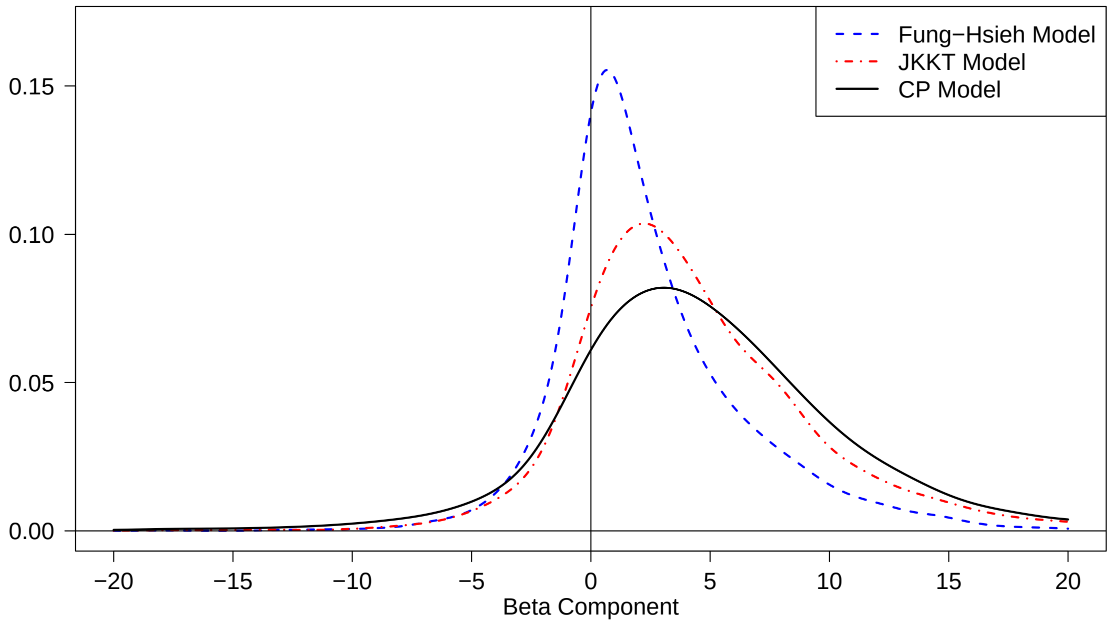
(b) Distribution of the Beta Components

Figure 2. Distributions of the Alpha and Beta Components

This figure provides a visual comparison of the Fung-Hsieh model and the two alternative models (JKKT and CP). Panel A plots the cross-sectional distributions of the alpha components (annualized) under the three models. Panel B repeats the analysis for the beta components.



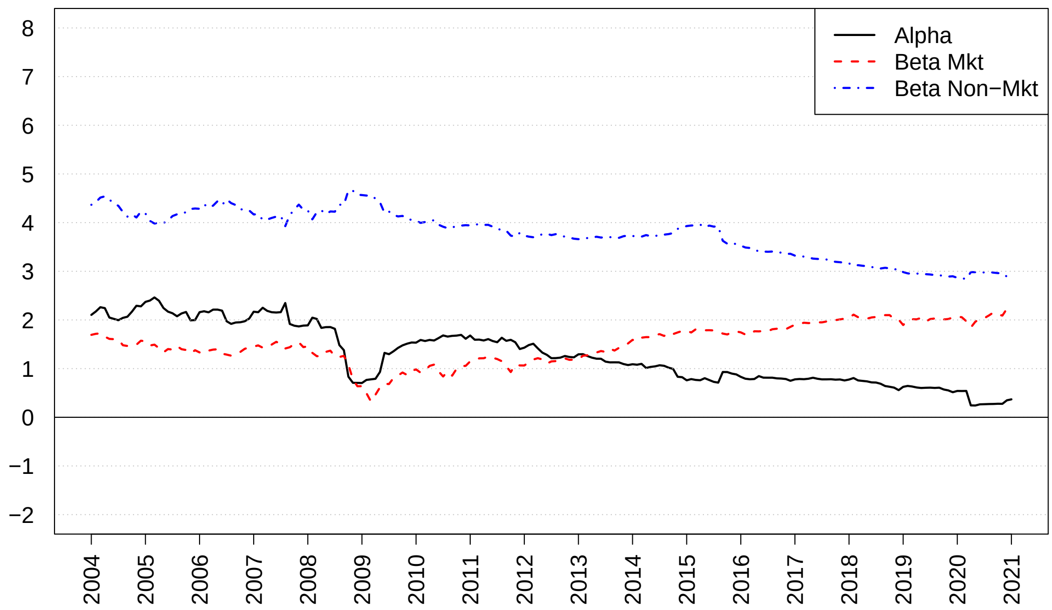
(a) Distributions of the Alpha Components



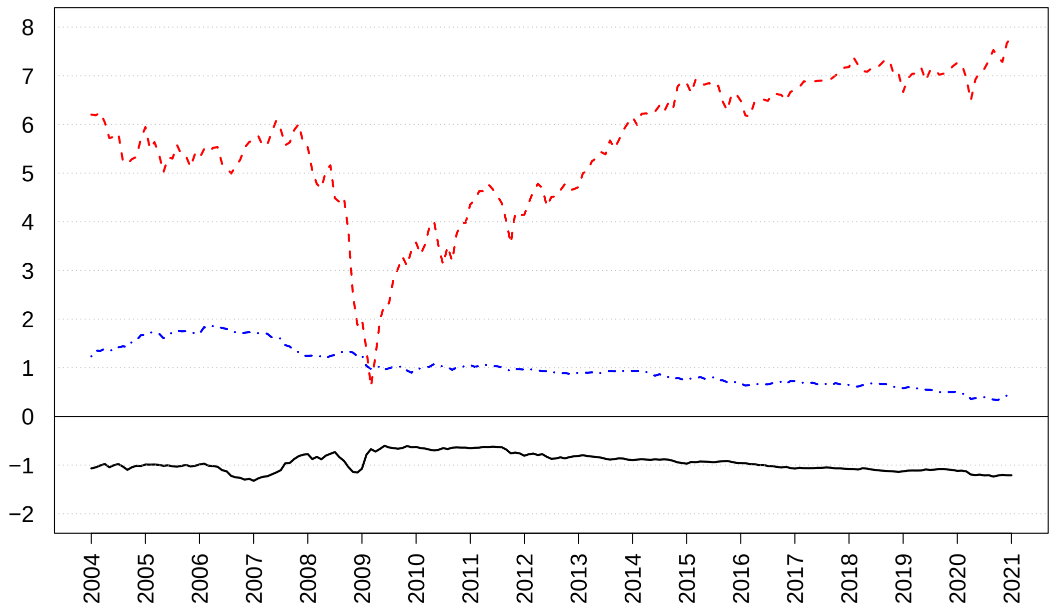
(b) Distributions of the Beta Components

Figure 3. Time-Variation in the Average Return Components

This figure plots the evolution of the average return components between 2004 and 2020, which are (i) the alpha component (Alpha), (ii) the beta component due to the market (Beta Mkt), and the beta component due to the non-market factors (Beta Non-Mkt). At the end of each year, we measure the three components for each fund using its entire return history up to that point and take cross-sectional averages across all existing funds. The initial estimates cover the period 1994 to 2004, while the last ones cover the period 1994 to 2020. Panel A shows the results for the hedge fund industry using the CP model for the return decomposition. Panel B repeats the analysis for the mutual fund industry using the traditional Carhart (1997) model.



(a) Hedge Funds



(b) Mutual Funds

Is it Alpha or Beta? Decomposing Hedge Fund Returns When Models are Misspecified

Internet Appendix

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This appendix is divided into four sections. Section I contains the proofs of the propositions discussed in the paper (including the regularity assumptions), provides the list of the terms for computing the asymptotic variance of the different estimators, and shows how to extend the methodology from the distribution of the alpha components to the distribution of the beta components. Section II presents the Monte-Carlo analysis for examining the properties of the estimators. Section III describes the construction of the hedge fund dataset and the different factors. Section IV reports additional empirical results on (i) the misspecification diagnostic criterion, (ii) the analysis of model comparisons across data filters and investment categories, (iii) the impact of factor trading costs, (iv) the return decomposition for multi-strategy funds and funds of funds, (v) the economic importance of the alternative factors within investment categories, and (vii) the characteristics of the worst and best funds under the CP model.

I. Methodology

I.A. Regularity Assumptions

To begin, we list the required assumptions underlying the results of Propositions 1 to 3. In particular, we need assumption A.7 to obtain non-zero asymptotic variances for the different estimators and guarantee that the limiting Gaussian distributions are all well-defined. We use a generic univariate function $g = g(\alpha_i)$ to simplify the presentation and avoid vectorial notations, and apply the compact notation $g^{(1)}$ and $g^{(2)}$ for its first- and second-order derivatives. We also omit the superscript k to lighten the notation when clarity permits.

Assumption A.1. *The individual effects* $\gamma_i^* = \left(\alpha_i^*, b_{i,I}^{*\prime}, b_{i,O}^{*\prime}\right)'$*, with* $i = 1, ..., n$*, are i.i.d. with continuous distribution,* $E[\|b_{i,O}^*\|^2] < \infty$*, and are independent of the factors and the errors.*

Assumption A.2. *The observability indicator processes* $(I_{i,t})$*, with* $i = 1, ..., n$*, are i.i.d., such that* $(I_{i,t})$ *is strictly stationary with mean* τ_i^{-1} *for any given* i*, and independent of the individual effects, the factors, and the error processes.*

Assumption A.3. *The factor process* $f_t = (f_{I,t}', f_{O,t}')'$ *is strictly stationary, such that* $E[\|f_{I,t}\|^8] < \infty$ *and satisfies the central limit theorem (CLT):* $\frac{1}{\sqrt{T}} \sum_t (u_{O,t} \otimes x_t) \to_d N(0, \Omega_{ux})$*, as* $T \to \infty$*,*

where $\Omega_{ux} = \lim_{T\to\infty} V\left[\frac{1}{\sqrt{T}} \sum_t u_{O,t} \otimes x_t\right]$.

Assumption A.4. *The error process* $(\varepsilon_{i,t}^*)$ *is such that* $\frac{1}{nT} \sum_{i,j} \sum_{t,s} E\left[E[\varepsilon_{i,t}^* \varepsilon_{j,s}^* | f_{\underline{T}}, \gamma_i^*, \gamma_j^*]^2\right]^{1/2} \leq$ C*, for a constant* C *and all* n, T *where* $f_{\underline{t}} = \{f_t, f_{t-1}, \ldots\}$.

Assumption A.5. *The trimming sequences $\chi_{1,T}$ and $\chi_{2,T}$ are such that $\chi_{1,T} = O((\log T)^{\kappa_1})$ and $\chi_{2,T} = O((\log T)^{\kappa_2})$, for $\kappa_1, \kappa_2 > 0$.*

Assumption A.6. *The function g is twice differentiable, and such that $E[|g(\alpha_i)|^2] < \infty$, $E[|g^{(1)}(\alpha_i)|^8] < \infty$, and $E[|g^{(2)}(\alpha_i)|^4] < \infty$.*

Assumption A.7. *For any pair of models k and l ($k, l = 0, ..., K-1$), we have: (1) $\eta_{M_s}^k \neq 0$ and $\eta_{M_s}^l \neq 0$, (2) $\eta_P^k(a) \neq 0$ and $\eta_P^l(a) \neq 0$, (3) $\eta_{M_s}^k(u_{O,t}^k \otimes x_t^k) - \eta_{M_s}^l(u_{O,t}^l \otimes x_t^l)$ is not the zero process, and (4) $\eta_P^k(a)(u_{O,t}^k \otimes x_t^k) - \eta_P^l(a)(u_{O,t}^l \otimes x_t^l)$ is not the zero process.*

I.B. Proofs of Propositions 1 and 2

We now prove Proposition 1 on the estimated characteristics of the alpha distribution under a given model k. To simplify notation, we drop the superscript k for the proof. We have

$$\begin{aligned}\hat{\alpha}_i &= E_1'\hat{Q}_{x,i}^{-1}\frac{1}{T_i}\sum_t I_{i,t}x_t r_{i,t} = \alpha_i + E_1'\hat{Q}_{x,i}^{-1}\frac{1}{T_i}\sum_t I_{i,t}x_t\varepsilon_{i,t} \\ &= \alpha_i + \frac{\tau_{i,T}}{\sqrt{T}}E_1'\hat{Q}_{x,i}^{-1}\left(\frac{1}{\sqrt{T}}\sum_t I_{i,t}x_t\varepsilon_{i,t}^*\right) + \frac{\tau_{i,T}}{\sqrt{T}}E_1'\hat{Q}_{x,i}^{-1}\left(\frac{1}{\sqrt{T}}\sum_t I_{i,t}x_t u_t'\right)b_{i,O}^* \\ &=: \alpha_i + \frac{1}{\sqrt{T}}\eta_{i,T}. \qquad \text{(A1)}\end{aligned}$$

By a second-order Taylor expansion,

$$g(\hat{\alpha}_i) = g(\alpha_i) + \frac{1}{\sqrt{T}}g^{(1)}(\alpha_i)\eta_{i,T} + \frac{1}{2T}g^{(2)}(\bar{\alpha}_i)\eta_{i,T}^2, \qquad \text{(A2)}$$

where $\bar{\alpha}_i$ is between $\hat{\alpha}_i$ and α_i for all i. Thus, we get

$$\begin{aligned}&\sqrt{T}\left(\frac{1}{n}\sum_i g(\hat{\alpha}_i)\mathbf{1}_i^{\chi} - E\left[g(\alpha_i)\right]\right) && \text{(A3)} \\ &= \sqrt{T}\left(\frac{1}{n}\sum_i g(\alpha_i) - E\left[g(\alpha_i)\right]\right) - \sqrt{T}\frac{1}{n}\sum_i g(\alpha_i)\left(1-\mathbf{1}_i^{\chi}\right) && \text{(A4)} \\ &+ \frac{1}{n}\sum_i \mathbf{1}_i^{\chi}g^{(1)}(\alpha_i)\tau_{i,T}E_1'\hat{Q}_{x,i}^{-1}\left(\frac{1}{\sqrt{T}}\sum_t I_{i,t}x_t\varepsilon_{i,t}^*\right) && \text{(A5)} \\ &+ \frac{1}{n}\sum_i \mathbf{1}_i^{\chi}g^{(1)}(\alpha_i)\tau_{i,T}E_1'\hat{Q}_{x,i}^{-1}\left(\frac{1}{\sqrt{T}}\sum_t I_{i,t}x_t u_{O,t}'\right)b_{i,O}^* && \text{(A6)} \\ &+ \frac{1}{2\sqrt{T}n}\sum_i \mathbf{1}_i^{\chi}g^{(2)}(\bar{\alpha}_i)\eta_{i,T}^2 =: I_1 + I_2 + I_3 + I_4 + I_5. && \text{(A7)}\end{aligned}$$

We control the five terms separately, the leading term being the fourth one and the others being asymptotically negligible, *i.e.*, of probability order $o_p(1)$.

i) Proof that $I_1 = o_p(1)$. By Assumptions A.1 and A.6, and the standard CLT, we have $I_1 = O_p(\sqrt{T/n})$. By using $T/n = o(1)$, it follows $I_1 = o_p(1)$.

ii) Proof that $I_2 = o_p(1)$. We have $E[|I_2|] \leq \sqrt{T} E[|g(\alpha_i)|(1-\mathbf{1}_i^\chi)] \leq \sqrt{T} E[|g(\alpha_i)|^2]^{1/2} P[\mathbf{1}_i^\chi = 0]^{1/2}$, by the Cauchy-Schwarz inequality. By Lemma 7 in Gagliardini, Ossola, and Scaillet (2016), $P[\mathbf{1}_i^\chi = 0] = O(T^{-\bar{b}})$, for any $\bar{b} > 0$. From Assumption A.6, $E[|I_2|] = o(1)$.

iii) Proof that $I_3 = o_p(1)$. We have

$$E[I_3^2 | f_{\underline{T}}, \gamma_i^*, I_{i,\underline{T}}, i = 1, ..., n] \tag{A8}$$

$$= \frac{1}{n^2 T} \sum_{i,j} \sum_{t,s} \mathbf{1}_i^\chi \mathbf{1}_j^\chi g^{(1)}(\alpha_i) g^{(1)}(\alpha_j) \tau_{i,T} \tau_{j,T} E_1' \hat{Q}_{x,i}^{-1} I_{i,t} x_t E_1' \hat{Q}_{x,j}^{-1} I_{i,s} x_s E[\varepsilon_{i,t}^* \varepsilon_{j,s}^* | f_{\underline{T}}, \gamma_i^*, \gamma_j^*]. \tag{A9}$$

By using $\mathbf{1}_i^\chi \tau_{i,T} \leq \chi_{2,T}$ and $\mathbf{1}_i^\chi \|\hat{Q}_{x,i}^{-1}\| \leq C\chi_{1,T}$ for a generic constant C (see Gagliardini, Ossola, and Scaillet (2016), proof of Lemma 3), we get

$$E[I_3^2 | f_{\underline{T}}, \gamma_i^*, I_{i,\underline{T}}, i = 1, ..., n] \leq \frac{C \chi_{1,T}^2 \chi_{2,T}^2}{n^2 T} \sum_{i,j} \sum_{t,s} |g^{(1)}(\alpha_i)| \, |g^{(1)}(\alpha_j)| \, \|x_t\| \|x_s\| \, |E[\varepsilon_{i,t}^* \varepsilon_{j,s}^* | f_{\underline{T}}, \gamma_i^*, \gamma_j^*]|. \tag{A10}$$

By the Cauchy-Schwarz inequality, we get

$$E[I_3^2] \leq C \chi_{1,T}^2 \chi_{2,T}^2 E[|g^{(1)}(\alpha_i)|^8]^{1/4} E[\|x_t\|^8]^{1/4} \frac{1}{n^2 T} \sum_{i,j} \sum_{t,s} E\left[|E[\varepsilon_{i,t}^* \varepsilon_{j,s}^* | f_{\underline{T}}, \gamma_i^*, \gamma_j^*]|^2\right]^{1/2}. \tag{A11}$$

From Assumptions A.3-A.6, we get $E[I_3^2] = o(1)$.

iv) Proof that $I_4 \to_d N\left(0, V_g\right)$. We have

$$I_4 = \frac{1}{n} \sum_i g^{(1)}(\alpha_i) \tau_i E_1' Q_x^{-1} \left(\frac{1}{\sqrt{T}} \sum_t I_{i,t} x_t u_{O,t}' \right) b_{i,O}^* + o_p(1) \tag{A12}$$

$$= E_1' Q_x^{-1} \frac{1}{\sqrt{T}} \sum_t x_t u_{O,t}' \left(\frac{1}{n} \sum_i I_{i,t} \tau_i g^{(1)}(\alpha_i) b_{i,O}^* \right) + o_p(1), \tag{A13}$$

where $\tau_i = E[I_{i,t}|\gamma_i^*]^{-1}$ by Assumption A.2. By Assumptions A.1 and A.2, the cross-sectional average $\frac{1}{n} \sum_i I_{i,t} \tau_i g^{(1)}(\alpha_i) b_{i,O}^*$ converges in probability to the expectation $E[I_{i,t} \tau_i g^{(1)}(\alpha_i) b_{i,O}^*]$. Moreover, we have the chain of equalities: $E[I_{i,t} \tau_i g^{(1)}(\alpha_i) b_{i,O}^*] = E[E[I_{i,t}|\gamma_i^*] \tau_i g^{(1)}(\alpha_i) b_{i,O}^*] =$

$E[g^{(1)}(\alpha_i)b^*_{i,O}]$. This expectation is finite by Assumptions A.1 and A.6. Thus, we get

$$I_4 = E_1' Q_x^{-1} \frac{1}{\sqrt{T}} \sum_t x_t u_{O,t}' E[g^{(1)}(\alpha_i)b^*_{i,O}] + o_p(1). \quad \text{(A14)}$$

Now, we use $x_t u_{O,t}' E[g^{(1)}(\alpha_i)b^*_{i,O}] = \left(E[g^{(1)}(\alpha_i)b^*_{i,O}]' \otimes I_{d+1}\right)(u_{O,t} \otimes x_t)$, where d denotes the number of factors included in the model. Thus, we get

$$\left(E[g^{(1)}(\alpha_i)b^*_{i,O}]' \otimes E_1' Q_x^{-1}\right) \frac{1}{\sqrt{T}} \sum_t u_{O,t} \otimes x_t \to_d N(0, V_g), \quad \text{(A15)}$$

by Assumption A.3, and the result follows.

v) Proof that $I_5 = o_p(1)$. We have

$$\frac{1}{n} \sum_i \mathbf{1}_i^{\chi} g^{(2)}(\bar{\alpha}_i) \eta_{i,T}^2 = E[g^{(2)}(\alpha_i)\eta_{i,T}^2] + o_p(1), \quad \text{(A16)}$$

from Assumptions A.1-A.6, and the result follows.

For the proportion and quantile estimators, we proceed in a similar manner even if the indicator $g(\alpha) = \mathbf{1}\{\alpha \leq a\}$ is not differentiable. To understand intuitively the asymptotic variance of the proportion estimator, we can consider that the derivative of the indicator function is minus the Dirac function $g^{(1)}(\alpha) = -\delta(\alpha - a)$ in the sense of distribution theory. Thus, we have $E[g^{(1)}(\alpha_i)b^*_{i,O}] = -\int \delta(\alpha - a) m(\alpha) d\alpha = -m(a)$, where $m(a) = E[b^*_{i,O}|\alpha_i = a]\phi_{ac}(a)$. By plugging this expression in Equation (A15), we obtain the asymptotic distribution of the proportion estimator. The asymptotic distribution of the quantile estimator is derived from that of the cdf estimator by means of the Bahadur (1966) representation $\hat{Q}(u) - Q(u) = -\frac{1}{\phi_{ac}(Q(u))}\left(\hat{P}(Q(u)) - u\right) + o_p(1)$.

Next, we turn to the proof of Proposition 2, which examines the difference in characteristics between models. Whereas our empirical analysis focuses on the comparison between various models and the CAPM, the results in Proposition 2 are general and apply to any pair of models k and l (which can be nested or non-nested). We can view Proposition 2 as a corollary of Proposition 1. For each characteristic (moments, proportion, quantile), we simply need to work with $g(\hat{\alpha}_i^k) - g(\hat{\alpha}_i^l)$ substituted for $g(\hat{\alpha}_i)$, and apply the delta method to obtain the results.

I.C. Proof of Proposition 3

In this section, we provide the theoretical arguments to show that the asymptotic variance estimators are consistent. We focus on the estimation of the asymptotic variance of Proposition 1, namely

$$V_g = \left(E[g^{(1)}(\alpha_i)b_{i,O}^*]' \otimes E_1'Q_x^{-1}\right)\Omega_{ux}\left(E[g^{(1)}(\alpha_i)b_{i,O}^*] \otimes Q_x^{-1}E_1\right), \tag{A17}$$

using the notations of appendix I.A. Because the arguments are similar for the consistency of the estimators of the asymptotic variances in Proposition 2, we omit their lengthy developments.

The asymptotic variance V_g depends on the omitted factors and their loadings. We can still estimate it without knowing them through the pseudo-residuals defined as $\hat{\varepsilon}_{i,t} = r_{i,t} - \hat{\gamma}_i'x_t$, where $\hat{\gamma}_i = (\hat{Q}_{x,i})^{-1}\frac{1}{T_i}\sum_t I_{i,t}x_t r_{i,t}$ is the vector of coefficients of the time-series regression in Equation (4) of the paper. We build

$$\hat{V}_g = \frac{1}{n^2T}\sum_i\sum_j\sum_t \mathbf{1}_i^\chi \tau_{i,T} I_{i,t}\mathbf{1}_j^\chi \tau_{j,T} I_{j,t}\hat{a}_{i,t}\hat{a}_{j,t}, \tag{A18}$$

where $\hat{a}_{i,t} = E_1'\hat{Q}_x^{-1}g^{(1)}(\hat{\alpha}_i)\hat{\varepsilon}_{i,t}x_t$, $i = 1, ..., n$.[1] To simplify the presentation, we assume a scalar omitted factor, and we treat vector x_t as a scalar in some terms.[2] The pseudo-residuals are

$$\hat{\varepsilon}_{i,t} = r_{i,t} - \hat{\gamma}_i'x_t = \varepsilon_{i,t}^* + b_{i,O}^*u_{O,t} - (\hat{\gamma}_i - \gamma_i)'x_t. \tag{A19}$$

Then, we have $\hat{V}_g = E_1'\hat{Q}_x^{-1}I_6\hat{Q}_x^{-1}E_1$, where

$$I_6 := \frac{1}{n^2T}\sum_i\sum_j\sum_t \mathbf{1}_i^\chi \tau_{i,T} I_{i,t}\mathbf{1}_j^\chi \tau_{j,T} I_{j,t} g^{(1)}(\hat{\alpha}_i)\hat{\varepsilon}_{i,t} g^{(1)}(\hat{\alpha}_j)\hat{\varepsilon}_{j,t}x_t x_t'. \tag{A20}$$

By using Equation (A19) of the pseudo-residuals, we can decompose I_6 into six terms, the leading

[1] In the empirical analysis of the paper, we replace n with n_χ to obtain conservative estimators of the variance. This replacement has no effect on the asymptotic properties of the variance estimator derived in this section.

[2] For expository purpose, we only develop the case where both the error terms and the factors are independent across time. When the error terms and/or the factors are correlated across time, we need to modify the estimator by including weighted cross-terms at different dates (Newey and West, 1987).

term being the second one and the other five ones being asymptotically negligible

$$I_6 = \frac{1}{n^2T}\sum_i\sum_j\sum_t \mathbf{1}_i^{\chi}\tau_{i,T}I_{i,t}\mathbf{1}_j^{\chi}\tau_{j,T}I_{j,t}g^{(1)}(\hat{\alpha}_i)\varepsilon^*_{i,t}g^{(1)}(\hat{\alpha}_j)\varepsilon^*_{j,t}x_tx_t' \tag{A21}$$

$$+\frac{1}{T}\sum_t\left(\frac{1}{n}\sum_i \mathbf{1}_i^{\chi}\tau_{i,T}I_{i,t}g^{(1)}(\hat{\alpha}_i)b^*_{i,O}\right)^2 u^2_{O,t}x_tx_t' \tag{A22}$$

$$+\frac{1}{T}\sum_t\left(\frac{1}{n}\sum_i \mathbf{1}_i^{\chi}\tau_{i,T}I_{i,t}g^{(1)}(\hat{\alpha}_i)(\hat{\gamma}_i-\gamma_i)\right)^2 x_t^4 \tag{A23}$$

$$+\frac{2}{n^2T}\sum_i\sum_j\sum_t \mathbf{1}_i^{\chi}\tau_{i,T}I_{i,t}\mathbf{1}_j^{\chi}\tau_{j,T}I_{j,t}g^{(1)}(\hat{\alpha}_i)\varepsilon^*_{i,t}g^{(1)}(\hat{\alpha}_j)b^*_{j,O}u_{O,t}x_tx_t' \tag{A24}$$

$$-\frac{2}{n^2T}\sum_i\sum_j\sum_t \mathbf{1}_i^{\chi}\tau_{i,T}I_{i,t}\mathbf{1}_j^{\chi}\tau_{j,T}I_{j,t}g^{(1)}(\hat{\alpha}_i)\varepsilon^*_{i,t}g^{(1)}(\hat{\alpha}_j)(\hat{\gamma}_j-\gamma_j)x_t^3 \tag{A25}$$

$$-\frac{2}{T}\sum_t\left(\frac{1}{n}\sum_i \mathbf{1}_i^{\chi}\tau_{i,T}I_{i,t}g^{(1)}(\hat{\alpha}_i)b^*_{i,O}\right)\left(\frac{1}{n}\sum_i \mathbf{1}_i^{\chi}\tau_{i,T}I_{i,t}g^{(1)}(\hat{\alpha}_i)(\hat{\gamma}_i-\gamma_i)\right)u_{O,t}x_t^3 \tag{A26}$$

$$=: I_{61}+I_{62}+I_{63}+I_{64}+I_{65}+I_{66}\,. \tag{A27}$$

We control the six terms separately.

i) Proof that $I_{61} = o_p(1)$. We have

$$I_{61} = \frac{1}{n^2}\sum_i\sum_j \tau_i\tau_j g^{(1)}(\alpha_i)g^{(1)}(\alpha_j)\frac{1}{T}\sum_t I_{i,t}I_{j,t}\varepsilon^*_{i,t}\varepsilon^*_{j,t}x_t^2 + o_p(1) \tag{A28}$$

$$= \frac{1}{n^2}\sum_i\sum_j g^{(1)}(\alpha_i)g^{(1)}(\alpha_j)\frac{1}{T}\sum_t E[\varepsilon^*_{i,t}\varepsilon^*_{j,t}x_t^2|\gamma_i^*,\gamma_j^*] + o_p(1) =: I_{611} + o_p(1)\,. \tag{A29}$$

From the Cauchy-Schwarz inequality and the law of iterated expectations, we have $E[\varepsilon^*_{i,t}\varepsilon^*_{j,t}x_t^2|\gamma_i^*,\gamma_j^*] = E[E[\varepsilon^*_{i,t}\varepsilon^*_{j,t}|f_{\underline{T}},\gamma_i^*,\gamma_j^*]x_t^2|\gamma_i^*,\gamma_j^*] \le E[E[\varepsilon^*_{i,t}\varepsilon^*_{j,t}|f_{\underline{T}},\gamma_i^*,\gamma_j^*]^2|\gamma_i^*,\gamma_j^*]^{1/2}E[\|x_t\|^4]^{1/2}$. Thus, we get:

$$|I_{611}| \le E[\|x_t\|^4]^{1/2}\frac{1}{n^2T}\sum_i\sum_j\sum_t |g^{(1)}(\alpha_i)||g^{(1)}(\alpha_j)|E[E[\varepsilon^*_{i,t}\varepsilon^*_{j,t}|f_{\underline{T}},\gamma_i^*,\gamma_j^*]^2|\gamma_i^*,\gamma_j^*]^{1/2}. \tag{A30}$$

By applying again the Cauchy-Schwarz inequality, we get

$$E[|I_{611}|] \le E[\|x_t\|^4]^{1/2}E[|g^{(1)}(\alpha_i)|^4]^{1/2}\frac{1}{n^2T}\sum_i\sum_j\sum_t E[E[\varepsilon^*_{i,t}\varepsilon^*_{j,t}|f_{\underline{T}},\gamma_i^*,\gamma_j^*]^2]^{1/2}. \tag{A31}$$

From Assumptions A.3, A.4, and A.6, we get $E[|I_{611}|] = o(1)$ and thus $I_{611} = o_p(1)$.

ii) Proof that $I_{62} = E[g^{(1)}(\alpha_i)b^*_{i,O}]^2 E[u^2_{O,t}x_t x'_t] + o_p(1)$. We have

$$I_{62} = \frac{1}{T}\sum_t \left(\frac{1}{n}\sum_i \tau_i I_{i,t} g^{(1)}(\alpha_i) b^*_{i,O}\right)^2 u^2_{O,t}x_t x'_t + o_p(1) \tag{A32}$$

$$= \frac{1}{T}\sum_t E[\tau_i I_{i,t} g^{(1)}(\alpha_i) b^*_{i,O}]^2 u^2_{O,t}x_t x'_t + o_p(1), \tag{A33}$$

from Assumptions A.1 and A.2. Now, we have $E[\tau_i I_{i,t} g^{(1)}(\alpha_i) b^*_{i,O}] = E[g^{(1)}(\alpha_i) b^*_{i,O}]$, and this expectation is finite by Assumptions A.1 and A.6. Further, $\frac{1}{T}\sum_t u^2_{O,t}x_t x'_t = E[u^2_{O,t}x_t x'_t] + o_p(1)$, and the conclusion follows.

iii) Proof that $I_{63} = o_p(1)$. We have

$$I_{63} = \frac{1}{T}\sum_t \left(\frac{1}{n}\sum_i \tau_i I_{i,t} g^{(1)}(\alpha_i)(\hat{\gamma}_i - \gamma_i)\right)^2 x^4_t + o_p(1) \tag{A34}$$

$$= \frac{1}{T}\sum_t \left(\frac{1}{n}\sum_i \tau_i I_{i,t} g^{(1)}(\alpha_i) Q^{-1}_{x,i}\frac{1}{T}\sum_s I_{i,s}x_s\varepsilon_{i,s}\right)^2 x^4_t + o_p(1), \tag{A35}$$

since $\sup_i \mathbf{1}^{\chi}_i \|\hat{Q}^{-1}_{x,i} - Q^{-1}_{x,i}\| = O_p(T^{-c})$ for some $c > 0$ under Assumption A.5 (see Gagliardini, Ossola, and Scaillet, 2016, proof of Lemma 3 (iii), Equation (38)). Now, from Assumptions A.1 and A.2, we have $\frac{1}{n}\sum_i \tau_i I_{i,t} g^{(1)}(\alpha_i) Q^{-1}_{x,i}\frac{1}{T}\sum_s I_{i,s}x_s\varepsilon_{i,s} = E[\tau_i I_{i,t} g^{(1)}(\alpha_i) Q^{-1}_{x,i}\frac{1}{T}\sum_s I_{i,s}x_s\varepsilon_{i,s}] + o_p(1)$. By applying the law of iterated expectations, the conclusion comes from $E\left[\tau_i I_{i,t} g^{(1)}(\alpha_i) Q^{-1}_{x,i}\frac{1}{T}\sum_s I_{i,s}x_s\varepsilon_{i,s}\right] = 0$, since $E[\varepsilon_{i,s}|x_{\underline{T}}, I_{i,\underline{T}}, \gamma^*_i] = 0$.

iv) Proof that $I_{64} = o_p(1)$. We have

$$I_{64} = \frac{2}{n^2}\sum_i\sum_j \tau_i\tau_j g^{(1)}(\alpha_i) g^{(1)}(\alpha_j) b^*_{j,O}\frac{1}{T}\sum_t I_{i,t}I_{j,t}\varepsilon^*_{i,t}u_{O,t}x_t x'_t + o_p(1) \tag{A36}$$

$$= \frac{2}{n^2}\sum_i\sum_j g^{(1)}(\alpha_i) g^{(1)}(\alpha_j) b^*_{j,O}\frac{1}{T}\sum_t E[\varepsilon^*_{i,t}u_{O,t}x_t x'_t|\gamma^*_i, \gamma^*_j] + o_p(1). \tag{A37}$$

The result follows from the law of iterated expectations, and Assumption A.1 implying $E[\varepsilon^*_{i,t}|f_t, \gamma^*_i, \gamma^*_j] = E[\varepsilon^*_{i,t}|f_t] = 0$.

v) Proof that $I_{65} = o_p(1)$. We have

$$I_{65} = -\frac{2}{n^2}\sum_i\sum_j \tau_i\tau_j g^{(1)}(\alpha_i)g^{(1)}(\alpha_j)(\hat{\gamma}_j-\gamma_j)\frac{1}{T}\sum_t I_{i,t}I_{j,t}\varepsilon^*_{i,t}x_t^3 + o_p(1) \tag{A38}$$

$$= -\frac{2}{n^2}\sum_i\sum_j g^{(1)}(\alpha_i)g^{(1)}(\alpha_j)(\hat{\gamma}_j-\gamma_j)\frac{1}{T}\sum_t E[\varepsilon^*_{i,t}x_t^3|\gamma_i^*,\gamma_j^*] + o_p(1). \tag{A39}$$

The result follows from the law of iterated expectations, and Assumption A.1 implying $E[\varepsilon^*_{i,t}|f_t,\gamma_i^*,\gamma_j^*] = E[\varepsilon^*_{i,t}|f_t] = 0$.

vi) Proof that $I_{66} = o_p(1)$. We have

$$I_{66} = -\frac{2}{T}\sum_t\left(\frac{1}{n}\sum_i \tau_i I_{i,t}g^{(1)}(\alpha_i)b^*_{i,O}\right)\left(\frac{1}{n}\sum_i \tau_i I_{i,t}g^{(1)}(\alpha_i)(\hat{\gamma}_i-\gamma_i)\right)u_{O,t}x_t^3 + o_p(1) \tag{A40}$$

$$= -\frac{2}{T}\sum_t E[g^{(1)}(\alpha_i)b^*_{i,O}]E\left[\tau_i I_{i,t}g^{(1)}(\alpha_i)Q_{x,i}^{-1}\frac{1}{T}\sum_s I_{i,s}x_s\varepsilon_{i,s}\right]u_{O,t}x_t^3 + o_p(1). \tag{A41}$$

The conclusion comes from $E\left[\tau_i I_{i,t}g^{(1)}(\alpha_i)Q_{x,i}^{-1}\frac{1}{T}\sum_s I_{i,s}x_s\varepsilon_{i,s}\right] = 0$, by applying the law of iterated expectations and $E[\varepsilon_{i,s}|x_{\underline{T}}, I_{i,\underline{T}}, \gamma_i^*] = 0$.

Finally, by using $\hat{Q}_x^{-1} = Q_x^{-1} + o_p(1)$ and $E_1'Q_x^{-1}E[g^{(1)}(\alpha_i)b^*_{i,O}]^2E[u_{O,t}^2x_tx_t']Q_x^{-1}E_1 = V_g$, we deduce that $\hat{V}_g = V_g + o_p(1)$.

I.D. List of Terms for the Asymptotic Variance Estimators

To obtain the variance estimator for each characteristic in Proposition 1, we simply need to plug the correct $\hat{a}_{i,t}$ in Equation (A18). To mitigate the impact of outliers, we also winsorize the observed values $\hat{a}_{i,t}$ at 99%.

For the mean M_1, we have

$$\hat{a}_{i,t} = E_1'\hat{Q}_{x,i}^{-1}\hat{\varepsilon}_{i,t}x_t. \tag{A42}$$

For the standard deviation, we need to apply the delta method. Let us denote the derivative of the standard deviation M_2, w.r.t. $E[\alpha_i^j]$ by $\nabla_j M_2$. We get $\nabla_2 M_2 = (2M_2)^{-1}$, $\nabla_1 M_2 = -M_1/M_2$. For the second moment $E[\alpha_i^2]$, we have $\hat{a}_{i,t} = 2\hat{\alpha}_i E_1'\hat{Q}_x^{-1}\hat{\varepsilon}_{i,t}x_t$. We can build an estimate of the variance of the standard deviation from a weighted sum of the contributions corresponding to the moments of orders 2 and 1:

$$\hat{a}_{i,t} = \left(\widehat{\nabla_2 M_2} \times 2\hat{\alpha}_i + \widehat{\nabla_1 M_2}\right) E_1' \hat{Q}_{x,i}^{-1} \hat{\varepsilon}_{i,t} x_t, \tag{A43}$$

where $\widehat{\nabla_2 M_2}$ is a plug-in estimate of the derivative of M_2 w.r.t. $E[\alpha_i^2]$.

For the proportion $P(a)$ at point a, we can approximate the Dirac function by a smooth bump, namely a kernel function K, and take a vanishing bandwidth h. Hence, we can use

$$\hat{a}_{i,t} = -h^{-1} K((\hat{\alpha}_i - a)/h) E_1' \hat{Q}_{x,i}^{-1} \hat{\varepsilon}_{i,t} x_t, \tag{A44}$$

where K is a kernel function such that $K \geq 0$, $\int K(u)du = 1$, $\int uK(u)du = 0$, and $\int u^2 K(u)du < \infty$. In practice, we use a Gaussian kernel corresponding to the Gaussian density, and take the Silverman rule of thumb for the bandwidth selection, namely $h = 1.06 \hat{M}_2 n_\chi^{-1/5}$.

For the quantile $Q(u)$ of level u, we can rely on the Bahadur (1966) representation, and use

$$\hat{a}_{i,t} = -h^{-1} K((\hat{\alpha}_i - \hat{Q}(u))/h) \hat{\phi}_{ac}(\hat{Q}(u))^{-1} E_1' \hat{Q}_{x,i}^{-1} \hat{\varepsilon}_{i,t} x_t. \tag{A45}$$

Extending the above analysis to the characteristic differences in Proposition 2 is straightforward. For each estimated difference, we simply plug the appropriate $\hat{a}_{i,t}$ redefined as $\hat{a}_{i,t} = \hat{a}_{i,t}^k - \hat{a}_{i,t}^l$, where we obtain $\hat{a}_{i,t}^k$ and $\hat{a}_{i,t}^l$ for models k and l from the previous expressions in Equations (A42)-(A45).

I.E. Application of the Methodology to the Beta Component

Whereas our description of the methodology focuses on the distribution of the alpha component, we can apply the same arguments to the distribution of the beta component. For each fund, we simply need to replace the estimated alpha component $\hat{ac}_i^k$ with the estimated beta component $\hat{bc}_i^k = \hat{\mu}_i - \hat{ac}_i^k$, where $\hat{\mu}_i = \frac{1}{T_i} \sum_t I_{i,t} r_{i,t}$. Using this information, we can then compute the different characteristics of the cross-sectional distribution (mean, standard deviation, proportion, and quantile). We omit the detailed technical derivation of the asymptotic properties for the beta component since it parallels closely the lines and arguments used in the previous subsections for the alpha component. We can also adapt the regularity assumptions (Section I.A) to the case of the beta component in a straightforward manner. The same remarks apply to the estimate $\hat{bc}_{i,j}^{k}$ of the contribution associated with each factor j included in model k analysed the next section.

To compute the asymptotic variance terms for each distribution characteristic, we proceed as follows. Since the residuals $\hat{\varepsilon}_{i,t}^{k}$ are centered (their time series average is zero), we get the identity $0 = \frac{1}{T_i}\sum_t I_{i,t}r_{i,t} - \hat{ac}_i^k - \hat{bc}_i^k$, and thus $\hat{bc}_i^k = \frac{1}{T_i}\sum_t I_{i,t}r_{i,t} - \hat{ac}_i^k$. Hence, we can use this expression to get $\hat{bc}_i^k - bc_i^k = (\frac{1}{T_i}\sum_t I_{i,t}r_{i,t} - E[r_{it}]) - (\hat{ac}_i^k - ac_i^k)$. We then compute the term $\hat{a}_{i,t}$ to estimate the asymptotic variance of each estimated characteristic (as per Equation (A18)).

We then use the following estimated quantities $\hat{a}_{i,t}$ to build an estimate of the asymptotic variance based on the the average return and the pseudo-residuals $\hat{\varepsilon}_{i,t}^{k}$ inferred from the least-squares regression:

$$\hat{a}_{i,t} = g^{(1)}(\hat{bc}_i^k)r_{i,t} - E_1'(\hat{Q}_x^k)^{-1}g^{(1)}(\hat{bc}_i^k)\hat{\varepsilon}_{i,t}^k x_t^k \,. \tag{A46}$$

I.F. Application of the Methodology to the Factor Contribution

We can further apply our methodology to the cross-sectional distribution of the contribution associated with each factor j included in model k. For each fund, we simply need to replace the estimated alpha component $\hat{ac}_i^k$ with the estimated factor contribution $\hat{bc}_{i,j}^k = \hat{b}_{i,I,j}^k\hat{\lambda}_{I,j}^k$, where $\hat{\lambda}_{I,j}$ is the empirical average of $f_{I,t,j}^k$. Using this information, we can then compute the different characteristics of the cross-sectional distribution (mean, standard deviation, proportion, and quantile).

To compute the asymptotic variance terms, we apply the delta method to obtain $(\hat{b}_{i,I,j}^k - b_{i,I,j}^k)\lambda_{I,j}^k + b_{i,I,j}^k(\hat{\lambda}_{I,j}^k - \lambda_{I,j}^k)$. We then compute the term $\hat{a}_{i,t}$ to estimate the asymptotic variance of each estimated characteristic (as per Equation (A18)). This term depends on the residual $\hat{\varepsilon}_{i,t}^k$ obtained from the regression of the fund return on the factors included in model k. Formally, we have

$$\hat{a}_{i,t} = \hat{\lambda}_{I,j}^k E_{j+1}'(\hat{Q}_x^k)^{-1}g^{(1)}(\hat{bc}_{i,j}^k)\hat{\varepsilon}_{i,t}^k x_t^k + \hat{b}_{i,I,j}^k g^{(1)}(\hat{bc}_{i,j}^k) f_{I,t,j}^k \,, \tag{A47}$$

where E_{j+1} is a vector with one in the $j+1$ entry and zeros elsewhere.

II. Monte Carlo Analysis

II.A. Setup

We now conduct a Monte-Carlo analysis to evaluate the finite-sample properties of the estimated characteristics of the alpha distribution when the model is misspecified. We consider a hypothetical population of n funds with T return observations (n=1,000, 2,500, 5,000, 7,500, and 10,000; T=50, 100, 250, 500, and 1,000). Building on our example in Section II.C.3 of the paper, we model the fund excess return as

$$r_{i,t} = \alpha_i^* + b_{i,m}^* r_{m,t} + b_{i,1}^* f_{1,t} + b_{i,2}^* f_{2,t} + b_{i,3}^* f_{3,t} + \varepsilon_{i,t}^* , \tag{A48}$$

where $r_{m,t}$ is the market excess return, $f_{1,t}$, $f_{2,t}$, and $f_{3,t}$ denote the excess returns of three uncorrelated factors that track alternative strategies, and $\varepsilon_{i,t}^*$ is the fund residual. For each fund, the true alpha α_i^* is drawn from a normal $N(\mu_\alpha^*, \sigma_\alpha^{*2})$, $b_{i,m}^*$ from a normal $N(\mu_b^*, \sigma_b^{*2})$, and $b_{i,j}^*$ from a normal $N(\mu_{b_j}^*, \sigma_b^{*2})$, where $\mu_{b_j}^*$ is positive to capture the exposure of hedge funds to alternative strategies. We further assume that the first factor is a more important driver of hedge fund returns by setting $\mu_{b_1}^* = \mu_b^*$ and $\mu_{b_2}^* = \mu_{b_3}^* = \mu_b^*/3$.

To construct the return time-series for each iteration, we need to draw values for the factors and the fund residuals. We draw the market return $r_{m,t}$ from a normal $N(\lambda_m, \sigma_m^2)$, and the returns of the each alternative factor $f_{j,t}$ ($j = 1, 2, 3$) from a normal $N(\lambda_j, \sigma_j^2)$, where we set $\lambda_j = \lambda_m$ and $\sigma_j^2 = \sigma_m^2$ for simplicity. Finally, we draw $\varepsilon_{i,t}^*$ from a normal $N(0, \sigma_\varepsilon^{*2})$.

We use our monthly dataset to calibrate the parameters of the model. We set λ_m and σ_m^2 equal to the empirical average and variance of the equity market. We set μ_b^* and σ_b^* equal to the cross-sectional average and volatility of the fund market betas. Finally, we calibrate μ_α^*, σ_α^*, and σ_ε^{*2} using the values reported for mutual funds by Barras, Gagliardini, and Scaillet (2022).[3] This calibration yields the following values on a monthly basis: $\lambda = 0.63\%$, $\sigma_m = 4.36\%$, $\mu_b^* = 0.3$, $\sigma_b^* = 0.4$, $\sigma_\varepsilon^* = 1.67\%$, $\mu_\alpha^* = 0\%$, and $\sigma_\alpha^* = 0.13\%$.

In our simulations, we evaluate hedge fund performance using the CAPM. Given the above assumptions, the CAPM is misspecified because it does not include the three alternative factors (we have $f_{I,t} = r_{m,t}$ and $f_{O,t} = (f_{1,t}, f_{2,t}, f_{3,t})'$). We conduct a total of $S = 1{,}000$ simulation iterations. For each iteration s ($s = 1, ..., S$), we follow the following steps. First, we draw values for $\alpha_i^*(s), b_{i,m}^*(s), b_{i,1}^*(s), b_{i,2}^*(s)$, and $b_{i,3}^*(s)$ for each fund i ($i = 1, ..., n$). Second, we draw values

[3] The rationale for calibrating the values under the correct model using mutual fund data is that the issue of misspecification is far less severe than for hedge funds. We find that choosing alternative values does not change the finite-sample properties of the estimators.

for the factors

$$f_t(s) = (r_{m,t}(s), f_{1,t}(s), f_{2,t}(s), f_{3,t}(s))' \,, \tag{A49}$$

for $t = 1, ..., T$ and the fund residuals $\varepsilon^*_{i,t}(s)$ for $i = 1, ..., n$ and $t = 1, ..., T$. Third, we construct the return time-series of each fund as

$$r_{i,t}(s) = \alpha^*_i(s) + b^*_{i,m}(s) r_{m,t}(s) + b^*_{i,1}(s) f_{1,t}(s) + b^*_{i,2}(s) f_{2,t}(s) + b^*_{i,3}(s) f_{3,t}(s) + \varepsilon^*_{i,t}(s) \,. \tag{A50}$$

Fourth, we estimate the CAPM alphas for each fund by regressing its return on the market:

$$\hat{\alpha}_i(s) = E_1'(\hat{Q}_{x,i}(s))^{-1} \frac{1}{T} \sum_t x_t(s) r_{i,t}(s) \,, \tag{A51}$$

where E_1 is a vector with one in the first position, $x_t(s) = (1, r_{m,t}(s))'$, and $\hat{Q}_{x,i}(s) = \frac{1}{T}\sum_t x_t(s)x_t(s)'$. Finally, we apply our approach to compute the distribution characteristics of the CAPM alpha distribution using as inputs the estimated alphas across funds $\hat{\alpha}_i(s)$ ($i = 1, ..., n$). We compute (i) the cross-sectional mean and standard deviation, $\hat{M}_1(s)$ and $\hat{M}_2(s)$, (ii) the proportion of funds with negative alphas $\hat{P}(0)(s)$, and (iii) the quantiles at 10% and 90%, $\hat{Q}(0.1)(s)$ and $\hat{Q}(0.9)(s)$.[4]

For each estimated characteristic $\hat{C} \in \{\hat{M}_1, \hat{M}_2, \hat{P}(0), \hat{Q}(0.1), \hat{Q}(0.9)\}$, we compute the mean squared error (MSE) as

$$\mathit{MSE}(\hat{C}) = \mathit{bs}^2(\hat{C}) + \sigma^2(\hat{C}) \,, \tag{A52}$$

where $\mathit{bs}(\hat{C})$ and $\sigma^2(\hat{C})$ denote the bias and variance of the estimator $\hat{C}$. These terms are given by

$$\mathit{bs}(\hat{C}) = \frac{1}{S}\sum_s \hat{C}(s) - C \,, \tag{A53}$$

$$\sigma^2(\hat{C}) = \frac{1}{S}\sum_s \left(\hat{C}(s) - \frac{1}{S}\sum_s \hat{C}(s)\right)^2 \,, \tag{A54}$$

where the population value C for each characteristic can be easily computed because the CAPM alpha distribution is normally distributed.

[4]We do not examine the estimated proportion of positive alpha funds whose properties are identical to $\hat{P}(0)(s)$.

II.B. Main Results

In Table AI, we report the MSE, bias, and standard deviation of the five estimated characteristics for the different combinations of T and n. We express the MSE in squared percent per month (multiplied by 100). We express the bias and standard deviation in percent per year for the mean, standard deviation, and quantiles, and in percent for the proportion of negative-alpha funds.

The simulation results are in line with the theoretical analysis in Proposition 2. First, the convergence rate of each estimator depends on T and not on n. As shown in the rightmost columns, the standard deviation decreases when the sample period increases. In contrast, increasing the population size does not produce more precise estimators because the omitted factors $f_{1,t}$, $f_{2,t}$, and $f_{3,t}$ have an impact on the estimated alphas of all funds simultaneously.

Second, the bias of each estimator vanishes relatively quickly as we increase the sample sizes n and T. As a result, it is smaller in magnitude than the standard deviation. To illustrate, we consider the proportion estimator under the scenario where $n = 5{,}000$ and $T = 100$, which provides a conservative analysis of our actual sample after trimming (*i.e.*, we have $n_\chi = \sum_{i=1}^n \mathbf{1}_i^\chi = 5{,}231$ and $T_\chi = \frac{1}{n_\chi} \sum_{i=1}^n \mathbf{1}_i^\chi T_i = 125$). Whereas the bias of the estimated proportion equals 2.9%, its standard deviation is around two times larger (5.7%).

Consistent with these results, we find that the MSE of the estimators (i) decreases with the number of observations T, and (ii) is primarily driven by the standard deviation, and not the bias. This analysis departs significantly from the well-specified case examined by Barras, Gagliardini, and Scaillet (2022). In their Monte-Carlo simulations reported in the appendix, we see that the standard deviation of the estimators decreases with the number of funds n. In addition, the bias dominates the standard deviation and thus requires an error-in-variable bias adjustment procedure.

Please insert Table AI here

III. Data Description

III.A. Construction of the Hedge Funds Dataset

We use monthly net-of-fee returns of individual funds (including dead funds) across four data providers (Barclayhedge, HFR, Morningstar, and TASS). The initial sample shown in Panel A of Table AII contains 65,142 funds that classify themselves across four investment categories: equity (long-short and market neutral), macro (global macro and CTA/managed futures), arbitrage (relative value and event-driven), and other (multi-strategy and funds of funds). To map the specific investment styles used by each database into one of the four categories above, we apply the mapping proposed by Joenväärä et al. (2021).[5] We convert the fund returns into USD using the exchange rates at the end of the month retrieved from Bloomberg and remove monthly returns lower than -90% and above 300%.

We apply a set of filters to the initial population. For each database, we include the fund if it: (i) has more than 12 observations (in order to compute return correlations), (ii) reports continuously to the database, (iii) exhibits less than three consecutive zero returns, (iv) has a non-zero return volatility, and (v) reports in USD, EUR, GBP, or JPY. As shown in Panel B, these filters reduce the total size of the population to 40,169 funds.

Next, we remove the duplicates for each database. We use the fund manager ID to cluster funds based on a string matching approach based on the Jaro-Winkler distance (see Joenväärä, Kosowski, and Tolonen, 2016). Within each of these clusters, we identify funds with pairwise return correlations above 0.99, and keep one fund using the following priority rule: (i) maximum number of observations, (ii) largest average size, (iii) USD as reporting currency, and (iv) onshore.[6] Panel C shows that removing the duplicates reduces the total number of funds to 30,734.

Finally, we remove the duplicates across all four databases. To this end, we compute the pairwise correlations across all funds in the aggregated dataset to identify groups of funds with correlations above 0.99. For each group, we then keep one fund using the following priority rule: (i) maximum number of observations, (ii) largest average size, (iii) USD as reporting currency, and (iv) onshore. As shown in Panel D, the final sample size includes a total of 21,293 funds.

Please insert Tables AII here

III.B. Data Sources for the Factors

In this section, we provide additional information on the factors included in the standard models. We download the market, size, value, momentum, investment, and profitability factors from Ken French's website. For the bond factors, we use the FRED database. The term factor is defined as

[5] We also use an earlier version of their paper (Joenväärä, Kosowski, and Tolonen, 2016) to obtain the mapping for long-short and market neutral funds.

[6] TASS does not provide information about the fund manager ID. To remove the duplicates in this database, we therefore conduct a correlation analysis on the entire population to detect funds with correlations above 0.99.

the monthly change in the 10-year treasury constant maturity yield, and the default factor is defined as the monthly change in the Moody's Baa yield less the 10-year Treasury constant maturity yield. These two series capture changes in yields and thus provide an approximation of the return of the term and default strategies (using the duration formula). Data on the bond, currency, and commodity straddles are obtained from David Hsieh's website.

Turning to the description of the alternative factors, we obtain the time series of the traded liquidity factor from Lubos Pastor's website. We obtain the return of the BAB strategy from the website of AQR. For the variance factor, we do not directly observe quotes of traded variance swaps on the S&P 500. Therefore, we use the FRED database to compute the difference between the monthly sum of the daily squared S&P 500 returns and the squared VIX (at the start of the month), divided by the squared VIX.[7] In the presence of jumps, our computation provides an approximation of the return of variance swaps (Martin, 2017). This approximation is quite accurate given that our summary statistics are in line with those reported by Dew-Becker et al. (2017) using actual swap quotes.[8] We download the return of the time-series momentum strategy from the website of AQR. For carry, we download the return time-series of the carry factors for equity, bonds (level and slope), currency, and commodity from Ralph Koijen's website. We then compute the average return of these five strategies (scaled by their volatility) to obtain the carry factor.

IV. Additional Results

IV.A. Misspecification Diagnostic Criterion

We now provide additional information on the misspecification diagnostic criterion proposed by Gagliardini, Ossola, and Scaillet (2019). This criterion, which is computed for each model k is defined as

$$GOS = \mu_1^k(\hat{V}) - g(n_\chi, T)\,. \qquad \text{(A55)}$$

The first term μ_1^k is the largest eigenvalue of the matrix $\hat{V} = \frac{1}{n_\chi T}\sum_i \mathbf{1}_i^\chi \bar{\varepsilon}_i^k \bar{\varepsilon}_i^{k'}$, where $\bar{\varepsilon}_i^k$ is of size T and gathers the values $\bar{\varepsilon}_{i,t}^k / \sqrt{\frac{1}{T}\sum_t (\bar{\varepsilon}_{i,t}^k)^2}$ with $\bar{\varepsilon}_{i,t} = I_{i,t}\hat{\varepsilon}_{i,t}^k$. The second term $g(n_\chi, T)$ is the penalization equal to $\frac{n_\chi + T}{n_\chi T}\ln(\frac{n_\chi T}{n_\chi + T})$. As n and T converge to infinity, GOS is positive with

[7]To compute the annualized statistics in Table II, we further divide the variance return by 10 to obtain similar magnitude as the other factors.

[8]They find that the average monthly excess return of the one-month swap is equal to -25.7% over the period 1995-2013 (see their Table II). We find a monthly average of -31.7% over the period 1994-2020.

probability one if the model is misspecified and omits a strong factor. We find that all the models are misspecified because the GOS is always positive, both in the entire population and in each investment category (equity, macro, and arbitrage).

IV.B. Model Comparisons

IV.B.1. Impact of Data Filters

In this section, we examine how different data filers impact the comparisons of models. We begin our analysis by changing the minimum number of observations. Our initial sample is free of survivorship bias because it includes both living and dead funds. However, our fund selection rule requires that each fund has a minimum number of return observations T_{min} to estimate its alpha. Our results could therefore be subject to survivorship bias if negative-alpha funds disappear early (*i.e.*, the reported alphas could be too high). At the same time, choosing a small T_{min} increases the severity of the reverse survivorship bias (Linnainmaa, 2013), which arises because some positive-alpha funds may perform unexpectedly poorly and disappear early (*i.e.*, the reported alphas could be too low). To examine these issues, we repeat our CAPM-based comparison using two alternative thresholds for T_{min} equal to 36 and 84.

Next, we use different filters to construct the hedge fund database. We apply the backfill bias correction proposed by Joenväärä et al. (2021), which eliminates all the return observations before the fund listing date. Whereas this alternative procedure provides a more stringent control of the backfill bias, it potentially discards important information about the fund performance by eliminating a large number of observations—in some cases, more than five years of data (see Aggarwal and Jorion, 2010; Fung and Hsieh, 2009, for a discussion). We also apply the five filters proposed by Straumann (2009) and applied by Almeida, Ardison, and Garcia (2020) to remove errors in reported hedge fund returns. These filters are based on the number of returns equal to zero, the proportion of unique values, the repetition of identical values, the occurrence of identical sequences of returns, and the presence of rounding errors. Applying the filters of Straumann (2009) leads to a reduction in the number of selected funds in the three main categories (equity, macro, arbitrage) from 15,567 to 13,877.

For each of these changes, we formally compare the alpha distribution of each proposed model with that of the CAPM. The results in Table AIII show that the CAPM-based comparisons remain robust to all these changes. Whereas the standard and machine learning models are similar to the CAPM, the JKKT and CP models produce sharp differences. These results are consistent with intuition—changing the data filters affects all models uniformly. It therefore leaves their differences unchanged.

Please insert Table AIII here

IV.B.2. Analysis of Investment Categories

We next perform the CAPM-based comparison for each investment category separately (equity, macro, arbitrage). Consistent with our main results, Table AIV reveals almost no statistically significant differences in characteristics between the standard and machine learning models and the CAPM. Whereas both the JKKT and CP models shift the alpha distribution towards zero, the magnitude of this shift is different. The CP model produces a statistically significant reduction in average alphas for all three categories. In contrast, the reduction obtained with the JKKT model is not statistically significant at conventional levels for equity and arbitrage funds.

Please insert Table AIV here

IV.C. Impact of Factor Trading Costs

In our baseline comparisons, we do not include the costs of trading the five alternative factors. To address this issue, we approximate these costs using estimates from previous studies. The costs of trading illiquidity, carry, and TS momentum are modest because these strategies are rebalanced annually or implemented in futures markets. For illiquidity, we use a value of 4.5 bps equal to the average cost estimate for size and value (Novy-Marx and Velikov, 2016). For carry and TS momentum, we choose a value of 9.7 bps, which is equal to the average estimated costs of rolling futures positions (Bollerslev et al., 2018). In contrast, the costs of trading the BAB and variance factors are significantly higher. For BAB, we take the estimate of Novy-Marx and Velikov (2022) equal to 60 bps. For variance, we use a value of 75 bps, which corresponds to the costs of trading variance swaps (Dew-Becker et al., 2017).

Consistent with intuition, we find in Table AV that accounting for trading costs increases the alpha components. However, this increase is generally modest—the average alphas under the JKKT and CP models are equal to 1.3% and 1.0% per year (versus 1.0% and 0.4% without trading costs). As a result, these two models still produce alpha distributions that depart significantly from the CAPM.

Please insert Table AV here

IV.D. Return Decomposition for Multi-Strategy Funds and Funds of Funds

We estimate the distributions of the alpha and beta components for two additional categories—multi-strategy funds and funds of funds. Consistent with our baseline results, Table AVI reveals that the JKKT and CP model produces a sizable reduction in the alpha component and a sizable increase in the beta component. In both categories, the decrease in fund alphas is particularly strong under the CP model. For multi-strategy funds, the average alpha is 0.1% and only 50.7% of the funds deliver positive alphas. Among funds of funds, the performance is even lower (-2.5% for the average alpha and 23.1% for the proportion of positive-alpha funds). Whereas it is well known that the alpha of these funds is hampered by their additional fees (*e.g.*, Agarwal, Mullally, and Naik, 2015), we find that the underperformance is worse than previously documented.

Please insert Table AVI here

IV.E. Alternative Strategies Within Investment Categories

We deepen the analysis of the economic importance of the alternative factors by splitting each investment category into two subcategories. For the equity category, we have long-short and market neutral funds. For the macro category, we have macro and CTA/managed futures funds. For the arbitrage category, we have relative value and event-driven funds. For each subcategory, we apply our approach to estimate the distribution of contributions associated with each factor.

Consistent with our baseline results, Table AVII provides substantial evidence that hedge funds follow alternative strategies to boost their returns (*e.g.*, Carhart et al., 2014). Across the six categories and the five alternative factors, the proportion of funds with positive betas is above 50% in all but seven cases. We also confirm that carry is an important driver of hedge fund returns—it is among the three most relevant alternative factors for each subcategory.

The variation in factor loadings across subcategories is largely in line with economic intuition. CTA funds, which are known to exploit market trends, load extensively on TS momentum—its average contribution to the beta component reaches 3.7% per year. The BAB factor is particularly important among market neutral funds as it allows them to take advantage of leverage flexibility, while maintaining a neutral exposure to the market and various industries (see Pedersen, 2015, ch. 9). Long-short equity funds are exposed to the variance factor, possibly because it reduces the effectiveness of their hedging strategies (Buraschi, Kosowski, and Trojani, 2014). This is also the case for relative value funds, which commonly use option-based strategies (Duarte, Longstaff, and Yu, 2006), and for event-driven funds, which take short put positions when they engage in merger arbitrage (Mitchell and Pulvino, 2001).

Please insert Table AVII here

IV.F. Characteristics of the Worst and Best Funds

In this section, we examine the characteristics of the best and worst funds under the CP model. To this end, we compute the alpha component of all funds in the population and sort them into quantiles (5%, 10%, 25%, 75%, 90%, 95%). We then measure the average characteristics of each group along several dimensions including (i) the investment style composition, (ii) the beta component due to each factor included in the CP model, (iii) measures of management incentives, and (iv) measures of managerial discretion.

We find in Table AVIII that the worst funds load heavily on the factors. For instance, the average beta components of variance and TS momentum are equal to 8.8% and 4.7% per year in the bottom quantile (versus 0.67% and -5.25% for the top quantile). The best funds follow equity strategies more extensively and macro strategies less extensively than the worst funds. Finally, we find that the vast majority of the best funds use high watermark provisions and impose lockup and notice periods more frequently than the worst funds.

Please insert Table AVIII here

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TABLE AI. Finite-Sample Properties of the Estimated Characteristics of the Alpha Distribution
This table reports the Mean Squared Error (MSE), bias, and standard deviation of the different characteristic estimators under the CAPM for different combinations for the numbers of funds n and return observations T. In the simulations, the average fund returns are explained by four factors (the market and three alternative factors 1, 2, and 3). The CAPM is misspecified because it omits factors 1, 2, and 3. We examine a total of five characteristics, which are the mean, standard deviation, proportions of funds with negative alphas, and quantiles at 10% and 90%. The bias and standard deviation are expressed in percent per year for the mean, standard deviation, and quantiles, and in percent for the proportion of negative-alpha funds.

Mean (True Value 3.75%)

	MSE (x100)					Bias (Annualized)					Standard Deviation (Annualized)				
n\T	50	100	250	500	1000	50	100	250	500	1000	50	100	250	500	1000
1000	3.93	2.12	0.95	0.43	0.25	0.13	-0.01	0.02	-0.02	0.02	2.38	1.75	1.17	0.79	0.60
2500	4.11	2.19	0.87	0.45	0.22	-0.01	-0.02	-0.04	0.02	0.04	2.43	1.78	1.12	0.80	0.56
5000	4.24	2.03	0.94	0.45	0.24	-0.01	-0.02	0.03	0.04	0.04	2.47	1.71	1.16	0.80	0.59
7500	4.73	2.12	0.94	0.45	0.23	0.02	0.06	0.02	0.02	0.04	2.61	1.75	1.16	0.81	0.58
10000	4.39	2.31	0.81	0.42	0.23	-0.13	-0.05	-0.04	0.03	0.01	2.51	1.82	1.08	0.77	0.57

Volatility (True Value 7.35%)

	MSE (x100)					Bias (Annualized)					Standard Deviation (Annualized)				
n\T	50	100	250	500	1000	50	100	250	500	1000	50	100	250	500	1000
1000	5.26	2.03	0.77	0.34	0.18	2.00	1.00	0.45	0.19	0.11	1.89	1.39	0.95	0.67	0.50
2500	4.97	2.08	0.67	0.34	0.18	1.90	0.97	0.38	0.23	0.14	1.88	1.44	0.91	0.66	0.48
5000	5.19	2.08	0.79	0.34	0.17	1.96	1.01	0.45	0.22	0.12	1.90	1.41	0.97	0.66	0.49
7500	5.42	2.17	0.77	0.34	0.18	1.96	1.08	0.44	0.20	0.14	1.99	1.40	0.96	0.67	0.48
10000	4.95	2.11	0.64	0.33	0.17	1.83	0.98	0.39	0.22	0.12	1.94	1.44	0.87	0.66	0.48

Probability < 0 (True Value 30.5%)

	MSE (x100)					Bias (Annualized)					Standard Deviation (Annualized)				
n\T	50	100	250	500	1000	50	100	250	500	1000	50	100	250	500	1000
1000	0.79	0.45	0.18	0.09	0.05	4.38	2.90	1.31	0.70	0.23	7.75	6.04	4.04	2.85	2.26
2500	0.92	0.46	0.17	0.08	0.04	4.93	2.94	1.40	0.63	0.24	8.22	6.07	3.86	2.69	1.93
5000	0.89	0.42	0.17	0.07	0.04	5.01	2.95	1.28	0.53	0.22	8.03	5.76	3.90	2.63	1.93
7500	1.00	0.45	0.17	0.07	0.04	5.07	2.80	1.32	0.58	0.25	8.60	6.07	3.91	2.65	1.91
10000	0.99	0.50	0.16	0.06	0.04	5.35	3.14	1.42	0.58	0.34	8.36	6.32	3.75	2.47	1.86

10th Percentile (True Value -5.67%)

	MSE (x100)					Bias (Annualized)					Standard Deviation (Annualized)				
n\T	50	100	250	500	1000	50	100	250	500	1000	50	100	250	500	1000
1000	6.43	2.26	0.61	0.30	0.20	-2.42	-1.28	-0.53	-0.25	-0.11	1.85	1.27	0.77	0.60	0.52
2500	6.50	2.14	0.59	0.22	0.13	-2.45	-1.26	-0.53	-0.26	-0.15	1.84	1.22	0.75	0.50	0.40
5000	6.82	2.13	0.56	0.20	0.10	-2.53	-1.31	-0.55	-0.25	-0.11	1.86	1.16	0.71	0.47	0.36
7500	6.67	2.22	0.54	0.19	0.10	-2.49	-1.33	-0.54	-0.24	-0.14	1.85	1.20	0.70	0.46	0.36
10000	6.70	2.17	0.51	0.18	0.09	-2.47	-1.31	-0.54	-0.25	-0.14	1.88	1.19	0.67	0.45	0.33

90th Percentile (True Value -13.16%)

	MSE (x100)					Bias (Annualized)					Standard Deviation (Annualized)				
n\T	50	100	250	500	1000	50	100	250	500	1000	50	100	250	500	1000
1000	18.60	8.70	3.89	1.76	0.98	2.67	1.25	0.57	0.21	0.15	4.43	3.31	2.30	1.58	1.18
2500	18.12	9.18	3.38	1.81	0.93	2.44	1.21	0.44	0.31	0.20	4.49	3.43	2.16	1.59	1.14
5000	18.72	8.77	3.94	1.83	0.96	2.50	1.26	0.61	0.32	0.19	4.55	3.32	2.30	1.59	1.16
7500	20.60	9.10	3.87	1.84	0.96	2.53	1.43	0.58	0.28	0.21	4.82	3.33	2.29	1.60	1.16
10000	18.32	9.35	3.20	1.75	0.94	2.22	1.21	0.46	0.31	0.16	4.63	3.46	2.10	1.56	1.15

TABLE AII. Construction of the Hedge Fund Dataset

This table summarizes the different steps for forming the consolidated hedge fund dataset. Panel A shows the total number of funds in each database. Panel B provides the same information after imposing the filters on each database. Panel C provides the same information after removing the duplicates within each database. Panel D provides the same information after removing the duplicates across all databases.

	BarclayHedge	HFR	Morningstar	TASS	Aggregate
Panel A: Raw Databases					
All Funds	22,315	15,275	6,120	21,432	65,142
Equity					
Long/Short	4,401	3,639	1,467	4,886	14,393
Market Neutral	687	931	227	873	2,718
Macro					
Global Macro	1,834	2,499	235	1,228	5,796
CTA/Managed Futures	3,944	750	260	3,071	8,025
Arbitrage					
Relative Value	3,431	2,674	1,536	904	8,545
Event Driven	1,104	1,204	192	948	3,448
Other					
Fund of Funds	5,813	3,578	383	6,908	16,682
Multi-Strategy	1,101	0	1,820	2,614	5,535
Panel B: Filtered Databases					
	BarclayHedge	HFR	Morningstar	TASS	Aggregate
All Funds	17,815	13,456	1,669	7,229	40,169
Equity					
Long/Short	3,622	3,168	427	1,874	9,091
Market Neutral	561	795	61	352	1,769
Macro					
Global Macro	1,380	2,183	106	360	4,029
CTA/Managed Futures	3,048	664	156	1,426	5,294
Arbitrage					
Relative Value	2,748	2,356	462	370	5,936
Event Driven	966	1,050	98	413	2,527
Other					
Fund of Funds	4,619	3,240	172	2,008	10,039
Multi-Strategy	871	0	187	426	1,484
Panel C: Filtered and Duplicate-Free Databases					
	BarclayHedge	HFR	Morningstar	TASS	Aggregate
All Funds	12,381	11,229	1,116	6,008	30,734
Equity					
Long/Short	2,505	2,688	329	1,576	7,098
Market Neutral	379	686	39	297	1,401
Macro					
Global Macro	1,002	1,870	68	308	3,248
CTA/Managed Futures	2,550	602	100	1,297	4,549
Arbitrage					
Relative Value	1,773	1,984	246	300	4,303
Event Driven	663	870	71	350	1,954
Other					
Fund of Funds	2,938	2,529	142	1,525	7,134
Multi-Strategy	571	0	121	355	1,047
Panel D: Filtered, Duplication-Free, and Merged Databases					
	BarclayHedge	HFR	Morningstar	TASS	Aggregate
All Funds	9,070	7,730	638	3,855	21,293
Equity					
Long/Short	1,736	1,936	177	914	4,763
Market Neutral	273	492	26	162	953
Macro					
Global Macro	811	1,163	34	180	2,188
CTA/Managed Futures	1,991	390	43	1,007	3,431
Arbitrage					
Relative Value	1,306	1,384	161	159	3,010
Event Driven	407	606	32	177	1,222
Other					
Fund of Funds	2,136	1,759	81	1,030	5,006
Multi-Strategy	410	0	84	226	720

TABLE AIII. Model Comparisons Relative to the CAPM – Impact of Data Filters

The table measures the degree of misspecification of the standard models (Carhart, Five-Factor, Fung-Hsieh, AMP), the two machine learning models (KNS1, KNS2), and the two alternative models (JKKT and CP) using different data filters. Panel A examines whether the distribution of the alpha components of each model departs from the one obtained with the CAPM after imposing a minimum number of 36 observations. Lack of differences signals that the model is no better than the CAPM at capturing hedge fund strategies. We report the differences in the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated differences. ***, **, * indicate that the null hypothesis of equal characteristics is rejected at the 1%, 5%, and 10% levels. Panels B to D repeat the analysis after imposing (i) a minimum of 84 observations,(ii) a more stringent backfill bias procedure, and (iii) filters to eliminate reporting errors.

	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
Panel A: Minimum Number of 36 Observations						
	Mean	Std Dev.	Negative	Positive	10%	90%
Carhart	-0.35 (0.44)	-0.33 (0.31)	2.00 (2.46)	-2.00 (2.46)	0.00 (0.41)	-0.46 (0.32)
Five-Factor	-0.17 (0.50)	0.04 (0.33)	1.95 (2.89)	-1.95 (2.89)	-0.15 (0.50)	-0.21 (0.39)
Fung-Hsieh	0.18 (0.63)	-0.12 (0.35)	-0.38 (3.78)	0.38 (3.78)	0.38 (0.57)	0.06 (0.47)
AMP	-0.26 (0.45)	-0.01 (0.35)	1.95 (2.41)	-1.95 (2.41)	-0.24 (0.38)	-0.22 (0.43)
KNS1	0.06 (0.42)	0.69** (0.29)	0.89 (1.84)	-0.89 (1.84)	-0.41 (0.29)	0.56 (0.37)
KNS2	0.55 (0.53)	0.51* (0.30)	-1.67 (2.68)	1.67 (2.68)	0.32 (0.42)	0.98** (0.46)
JKKT	-1.77** (0.69)	0.26 (0.49)	12.51*** (3.87)	-12.51*** (3.87)	-2.25*** (0.67)	-1.75*** (0.67)
CP	-2.42*** (0.78)	2.47*** (0.49)	16.86*** (4.06)	-16.86*** (4.06)	-4.34*** (0.74)	-0.89 (0.73)
Panel B: Minimum Number of 84 Observations						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Carhart	-0.32 (0.36)	-0.17 (0.28)	1.81 (2.43)	-1.81 (2.43)	-0.15 (0.39)	-0.72** (0.35)
Five-Factor	-0.28 (0.42)	0.10 (0.31)	2.37 (2.94)	-2.37 (2.94)	-0.15 (0.51)	-0.34 (0.42)
Fung-Hsieh	0.08 (0.55)	-0.10 (0.34)	0.22 (3.70)	-0.22 (3.70)	0.11 (0.59)	-0.19 (0.49)
AMP	-0.40 (0.41)	0.03 (0.33)	2.62 (2.41)	-2.62 (2.41)	-0.28 (0.38)	-0.44 (0.52)
KNS1	-0.17 (0.34)	0.18 (0.26)	2.09 (1.70)	-2.09 (1.70)	-0.39 (0.30)	0.02 (0.39)
KNS2	0.61 (0.47)	0.07 (0.29)	-2.90 (2.54)	2.90 (2.54)	0.48 (0.40)	0.54 (0.51)
JKKT	-2.08*** (0.64)	0.39 (0.48)	15.91*** (4.00)	-15.91*** (4.00)	-2.72*** (0.67)	-2.24*** (0.70)
CP	-2.64*** (0.73)	1.89*** (0.46)	22.03*** (4.38)	-22.03*** (4.38)	-4.20*** (0.74)	-1.71** (0.81)
Panel C: Stringent Backfill Bias Procedure						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Carhart	-0.29 (0.42)	-0.16 (0.35)	2.18 (2.70)	-2.18 (2.70)	0.13 (0.48)	-0.76** (0.34)
Five-Factor	-0.14 (0.48)	-0.02 (0.38)	2.01 (3.25)	-2.01 (3.25)	0.09 (0.58)	-0.26 (0.37)
Fung-Hsieh	0.10 (0.62)	-0.26 (0.40)	-0.56 (4.14)	0.56 (4.14)	0.45 (0.65)	-0.32 (0.50)
AMP	-0.26 (0.44)	0.07 (0.41)	2.07 (2.53)	-2.07 (2.53)	-0.03 (0.45)	-0.46 (0.47)
KNS1	0.03 (0.39)	0.41 (0.32)	1.21 (1.84)	-1.21 (1.84)	-0.21 (0.34)	0.21 (0.36)
KNS2	0.61 (0.51)	0.26 (0.34)	-2.87 (2.64)	2.87 (2.64)	0.74 (0.49)	0.56 (0.49)
JKKT	-1.91*** (0.69)	0.37 (0.54)	14.05*** (4.19)	-14.05*** (4.19)	-2.17*** (0.72)	-2.13*** (0.71)
CP	-2.54*** (0.75)	2.27*** (0.51)	19.21*** (4.26)	-19.21*** (4.26)	-3.99*** (0.77)	-1.52** (0.75)
Panel D: Filters for Removing Reporting Errors						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Carhart	-0.30 (0.39)	-0.21 (0.31)	2.30 (2.50)	-2.30 (2.50)	-0.13 (0.40)	-0.58* (0.33)
Five-Factor	-0.23 (0.46)	-0.03 (0.34)	2.54 (2.97)	-2.54 (2.97)	-0.17 (0.52)	-0.32 (0.39)
Fung-Hsieh	0.06 (0.60)	-0.16 (0.35)	0.59 (3.85)	-0.59 (3.85)	0.20 (0.58)	-0.15 (0.50)
AMP	-0.30 (0.42)	0.06 (0.38)	2.45 (2.52)	-2.45 (2.52)	-0.30 (0.36)	-0.25 (0.48)
KNS1	-0.04 (0.39)	0.38 (0.29)	1.36 (1.86)	-1.36 (1.86)	-0.37 (0.30)	0.18 (0.38)
KNS2	0.60 (0.50)	0.14 (0.33)	-2.38 (2.69)	2.38 (2.69)	0.47 (0.44)	0.61 (0.48)
JKKT	-1.94*** (0.68)	0.29 (0.51)	14.77*** (4.06)	-14.77*** (4.06)	-2.46*** (0.68)	-2.03*** (0.72)
CP	-2.56*** (0.76)	1.84*** (0.49)	20.39*** (4.41)	-20.39*** (4.41)	-4.36*** (0.73)	-1.39* (0.78)

TABLE AIV. Model Comparisons Relative to the CAPM – Investment Categories

The table measures the degree of misspecification of the standard models (Carhart, Five-Factor, Fung-Hsieh, AMP), the two machine learning models (KNS1, KNS2), and the two alternative models (JKKT and CP) across investment styles. Panel A examines whether the distribution of the alpha components of each model departs from the one obtained with the CAPM among equity funds. Lack of differences signals that the model is no better than the CAPM at capturing hedge fund strategies. We report the differences in the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated differences. ***, **, * indicate that the null hypothesis of equal characteristics is rejected at the 1%, 5%, and 10% levels. Panels B and C repeat the analysis for macro and arbitrage funds.

	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Panel A: Equity Funds						
Carhart	-0.25 (0.62)	-0.39 (0.36)	3.10 (3.12)	-3.10 (3.12)	0.21 (0.50)	-1.04* (0.61)
Five-Factor	0.09 (0.68)	0.07 (0.37)	2.45 (3.80)	-2.45 (3.80)	0.27 (0.61)	-0.22 (0.76)
Fung-Hsieh	0.08 (0.74)	-0.30 (0.33)	0.75 (4.20)	-0.75 (4.20)	0.56 (0.65)	-0.16 (0.69)
AMP	-0.09 (0.48)	0.34 (0.41)	4.35* (2.43)	-4.35* (2.43)	-0.18 (0.43)	-0.37 (0.58)
KNS1	-0.16 (0.46)	0.27 (0.31)	2.60 (2.39)	-2.60 (2.39)	-0.52 (0.39)	-0.15 (0.46)
KNS2	0.95 (0.64)	0.35 (0.38)	-2.50 (3.44)	2.50 (3.44)	0.83 (0.51)	1.01 (0.62)
JKKT	-1.28 (0.79)	-0.11 (0.43)	12.74*** (4.81)	-12.74*** (4.81)	-1.22* (0.70)	-1.77** (0.81)
CP	-1.83* (0.94)	1.99*** (0.56)	17.14*** (5.15)	-17.14*** (5.15)	-3.32*** (0.85)	-1.01 (1.11)
Panel B: Macro Funds						
	Mean	Std Dev.	Negative	Positive	10%	90%
Carhart	-0.80 (0.55)	-0.01 (0.39)	6.59*** (2.44)	-6.59*** (2.44)	-0.51 (0.44)	-0.91 (0.69)
Five-Factor	-0.93 (0.59)	0.13 (0.43)	7.83** (3.53)	-7.83** (3.53)	-0.64 (0.56)	-1.09 (0.79)
Fung-Hsieh	-0.32 (1.04)	-0.02 (0.52)	4.60 (3.81)	-4.60 (3.81)	-0.16 (0.61)	-0.20 (1.49)
AMP	-1.04 (0.82)	0.04 (0.60)	5.22 (4.54)	-5.22 (4.54)	-0.67 (0.48)	-1.15 (1.19)
KNS1	0.26 (0.77)	0.86* (0.45)	2.49 (2.88)	-2.49 (2.88)	-0.07 (0.39)	0.69 (0.94)
KNS2	0.02 (0.78)	0.21 (0.42)	1.93 (3.19)	-1.93 (3.19)	0.25 (0.48)	0.23 (1.07)
JKKT	-3.85*** (1.25)	1.10 (1.06)	25.79*** (6.24)	-25.79*** (6.24)	-4.98*** (1.22)	-3.71** (1.87)
CP	-4.10*** (1.37)	3.00*** (1.08)	26.10*** (6.04)	-26.10*** (6.04)	-6.64*** (1.44)	-2.15 (1.83)
Panel C: Arbitrage Funds						
	Mean	Std Dev.	Negative	Positive	10%	90%
Carhart	0.10 (0.57)	-0.12 (0.31)	-3.02 (5.07)	3.02 (5.07)	0.37 (0.71)	-0.27 (0.27)
Five-Factor	0.15 (0.68)	-0.13 (0.34)	-3.02 (5.91)	3.02 (5.91)	0.44 (0.83)	-0.29 (0.39)
Fung-Hsieh	0.46 (1.01)	-0.28 (0.36)	-3.82 (8.21)	3.82 (8.21)	0.72 (1.13)	0.06 (0.53)
AMP	0.12 (0.47)	-0.29 (0.29)	-2.47 (4.11)	2.47 (4.11)	0.55 (0.70)	-0.19 (0.26)
KNS1	-0.11 (0.47)	-0.01 (0.22)	-0.56 (3.61)	0.56 (3.61)	-0.11 (0.55)	-0.15 (0.26)
KNS2	0.82 (0.52)	0.05 (0.24)	-6.79 (4.24)	6.79 (4.24)	0.82 (0.59)	0.58* (0.32)
JKKT	-0.83 (0.79)	0.09 (0.29)	6.29 (5.67)	-6.29 (5.67)	-0.88 (0.86)	-1.07** (0.48)
CP	-1.94** (0.86)	1.20*** (0.31)	16.90*** (6.05)	-16.90*** (6.05)	-2.70*** (0.92)	-1.05* (0.58)

TABLE AV. Impact of Factor Trading Costs

This table measures the impact of the costs of trading the alternative factors (illiquidity, BAB, variance, carry, TS momentum). Panel A examines whether the distribution of the alpha components of each alternative model (JKKT and CP) departs from the one obtained with the CAPM. Lack of differences signals that the model is no better than the CAPM at capturing alternative hedge fund strategies. We report the differences in the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated differences. ***, **, * indicate that the null hypothesis of equal characteristics is rejected at the 1%, 5%, and 10% levels. Panels B and C report the characteristics of the cross-sectional distribution of the alpha and beta components among models. We report the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics.

			Panel A: Comparison With the CAPM			
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
JKKT	-1.59** (0.70)	0.23 (0.53)	11.11** (4.44)	-11.11** (4.44)	-1.84*** (0.70)	-1.86*** (0.67)
CP	-1.96** (0.77)	1.79*** (0.52)	14.57*** (4.58)	-14.57*** (4.58)	-3.26*** (0.77)	-1.13 (0.75)
			Panel B: Distribution of the Alpha Components			
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
CAPM	2.93 (0.94)	7.01 (0.48)	27.15 (5.49)	72.85 (5.49)	-3.95 (0.67)	10.06 (0.67)
JKKT	1.34 (0.72)	7.24 (0.29)	38.25 (4.58)	61.75 (4.58)	-5.79 (0.58)	8.19 (0.40)
CP	0.97 (0.87)	8.80 (0.33)	41.71 (5.26)	58.29 (5.26)	-7.21 (0.74)	8.93 (0.51)
			Panel C: Distribution of the Beta Components			
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
CAPM	2.62 (0.83)	4.37 (0.67)	22.54 (9.57)	77.46 (9.57)	-0.71 (0.61)	8.01 (0.73)
JKKT	4.22 (0.83)	5.71 (0.41)	14.83 (2.88)	85.17 (2.88)	-0.58 (0.31)	10.78 (0.69)
CP	4.59 (0.94)	7.20 (0.47)	16.44 (2.71)	83.56 (2.71)	-1.18 (0.36)	11.91 (1.00)

TABLE AVI. Decomposition of Average Fund Returns – Multi-Strategy and Fund of Funds
Panel A shows the decomposition of average fund returns under the CAPM and the two alternative models (JKKT and CP) across investment multi-strategy funds. We report the characteristics of the cross-sectional distribution of the alpha and beta components for equity funds. We report the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics. Panel B repeats the analysis for funds of funds.

Panel A: Multi-Strategy						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Distribution of the Alpha Components						
CAPM	2.48 (1.09)	6.68 (0.54)	32.23 (7.02)	67.77 (7.02)	-4.29 (1.40)	9.15 (1.00)
JKKT	0.49 (0.83)	7.41 (0.53)	46.45 (4.67)	53.55 (4.67)	-6.83 (1.23)	8.01 (0.80)
CP	-0.11 (0.87)	7.79 (0.60)	49.29 (4.83)	50.71 (4.83)	-8.06 (1.22)	7.66 (0.80)
Distribution of the Beta Components						
CAPM	1.90 (0.81)	3.44 (0.44)	19.43 (13.61)	80.57 (13.61)	-0.37 (0.76)	4.74 (1.14)
JKKT	3.89 (0.74)	4.65 (0.40)	10.43 (4.04)	89.57 (4.04)	0.00 (0.51)	8.13 (0.97)
CP	4.49 (0.80)	4.68 (0.52)	8.53 (3.46)	91.47 (3.46)	0.19 (0.55)	9.65 (1.09)

Panel B: Fund of Funds						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Distribution of the Alpha Components						
CAPM	1.22 (1.39)	4.16 (0.26)	31.82 (16.99)	68.18 (16.99)	-2.76 (1.23)	4.93 (1.11)
JKKT	-1.28 (1.03)	4.42 (0.17)	66.08 (12.46)	33.92 (12.46)	-5.39 (0.95)	2.74 (0.76)
CP	-2.46 (1.08)	5.41 (0.28)	76.88 (7.96)	23.12 (7.96)	-7.18 (1.18)	2.29 (0.70)
Distribution of the Beta Components						
CAPM	1.75 (0.99)	2.56 (0.34)	14.15 (22.75)	85.85 (22.75)	-0.14 (1.01)	4.37 (0.80)
JKKT	4.25 (0.94)	3.40 (0.18)	3.24 (1.28)	96.76 (1.28)	1.35 (0.67)	7.67 (0.84)
CP	5.42 (1.04)	4.49 (0.26)	3.52 (1.04)	96.48 (1.04)	1.66 (0.68)	9.49 (1.14)

TABLE AVII. Decomposition of the Beta Components – Investment Subcategories

This table shows the decomposition of the beta components obtained with the CP model. Panel A reports the characteristics of the cross-sectional distribution of the beta components associated with each factor (market, size, illiquidity, BAB, variance, carry, TS momentum) among long-short equity funds. For each fund, the beta component associated with a given factor is defined as the product between the fund beta and the factor premium. We report the annualized mean and standard deviation, the proportions of funds with negative and positive contributions, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics. Panel B to F repeat the analysis for market neutral, global macro, CTA/managed futures, relative value, and event-driven funds.

	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Panel A: Long-Short						
Market	4.10 (2.30)	4.62 (1.14)	11.51 (8.94)	88.49 (8.94)	-0.10 (0.87)	9.64 (3.33)
Size	0.47 (0.63)	1.39 (0.43)	30.94 (12.81)	69.06 (12.81)	-0.35 (0.14)	1.77 (1.15)
Illiquidity	0.15 (0.11)	1.56 (0.37)	43.93 (2.68)	56.07 (2.68)	-0.90 (0.26)	1.37 (0.51)
Betting Against Beta	0.22 (0.19)	2.79 (0.83)	39.32 (3.83)	60.68 (3.83)	-1.92 (0.62)	2.63 (1.07)
Variance	0.77 (0.28)	5.25 (0.73)	36.92 (2.36)	63.08 (2.36)	-3.14 (0.52)	4.64 (0.89)
Carry	0.34 (0.18)	2.94 (0.49)	45.24 (2.87)	54.76 (2.87)	-2.23 (0.44)	2.84 (0.54)
Time-Series Momentum	0.51 (0.17)	2.99 (0.58)	44.10 (2.11)	55.90 (2.11)	-1.66 (0.47)	3.19 (0.77)
Panel B: Market Neutral Funds						
	Mean	Std Dev.	Negative	Positive	10%	90%
Market	0.63 (0.34)	2.06 (0.55)	36.18 (4.04)	63.82 (4.04)	-0.73 (0.06)	2.21 (0.81)
Size	0.05 (0.11)	0.49 (0.21)	45.12 (6.31)	54.88 (6.31)	-0.35 (NaN)	0.53 (0.24)
Illiquidity	0.06 (0.05)	0.74 (0.27)	45.53 (3.44)	54.47 (3.44)	-0.67 (0.29)	0.73 (0.33)
Betting Against Beta	0.48 (0.31)	1.42 (0.53)	32.52 (5.10)	67.48 (5.10)	-0.53 (0.06)	1.96 (0.80)
Variance	-0.08 (0.18)	3.32 (0.49)	50.41 (2.31)	49.59 (2.31)	-3.04 (0.57)	2.16 (0.33)
Carry	0.35 (0.14)	1.52 (0.31)	41.46 (2.33)	58.54 (2.33)	-1.36 (0.28)	1.96 (0.47)
Time-Series Momentum	0.58 (0.21)	1.24 (0.35)	27.64 (3.30)	72.36 (3.30)	-0.61 (0.13)	2.13 (0.58)
Panel C: Macro Funds						
	Mean	Std Dev.	Negative	Positive	10%	90%
Market	1.97 (0.99)	3.87 (1.24)	27.58 (5.24)	72.42 (5.24)	-0.97 (0.13)	6.79 (2.31)
Size	0.08 (0.04)	0.72 (0.22)	49.85 (5.26)	50.15 (5.26)	-0.44 (0.22)	0.54 (0.20)
Illiquidity	-0.01 (0.07)	0.85 (0.28)	50.29 (3.22)	49.71 (3.22)	-0.71 (0.34)	0.71 (0.29)
Betting Against Beta	0.29 (0.21)	1.89 (0.63)	38.20 (4.36)	61.80 (4.36)	-1.19 (0.40)	1.91 (0.78)
Variance	0.56 (0.27)	5.68 (0.73)	44.10 (2.60)	55.90 (2.60)	-3.83 (0.70)	4.14 (0.69)
Carry	0.38 (0.23)	2.54 (0.54)	39.23 (2.89)	60.77 (2.89)	-2.24 (0.51)	3.20 (0.67)
Time-Series Momentum	2.29 (0.82)	4.50 (1.08)	26.55 (2.87)	73.45 (2.87)	-0.67 (0.08)	7.85 (1.91)
Panel D: CTA/Managed Futures Funds						
	Mean	Std Dev.	Negative	Positive	10%	90%
Market	0.60 (0.37)	3.03 (0.92)	40.60 (3.95)	59.40 (3.95)	-1.77 (0.54)	3.22 (1.30)
Size	0.07 (0.14)	0.91 (0.19)	51.13 (11.18)	48.87 (11.18)	-0.42 (0.26)	0.73 (0.18)
Illiquidity	-0.13 (0.12)	1.49 (0.36)	56.82 (3.54)	43.18 (3.54)	-1.06 (0.37)	0.79 (0.27)
Betting Against Beta	0.14 (0.18)	2.06 (0.66)	42.75 (4.81)	57.25 (4.81)	-1.35 (0.45)	1.81 (0.73)
Variance	-0.02 (0.45)	7.13 (0.98)	53.28 (3.46)	46.72 (3.46)	-5.30 (1.00)	4.72 (0.71)
Carry	0.42 (0.30)	2.95 (0.60)	43.82 (4.08)	56.18 (4.08)	-2.51 (0.65)	3.26 (0.66)
Time-Series Momentum	3.70 (1.32)	6.56 (1.34)	22.88 (3.73)	77.12 (3.73)	-0.79 (0.21)	11.31 (2.84)
Panel E: Relative Value Funds						
	Mean	Std Dev.	Negative	Positive	10%	90%
Market	1.49 (0.76)	2.41 (0.72)	19.94 (5.58)	80.06 (5.58)	-0.28 (0.15)	3.97 (1.34)
Size	0.10 (0.11)	0.49 (0.15)	36.93 (4.35)	63.07 (4.35)	-0.12 (0.01)	0.42 (0.29)
Illiquidity	0.07 (0.12)	0.76 (0.18)	34.72 (7.32)	65.28 (7.32)	-0.33 (0.06)	0.59 (0.35)
Betting Against Beta	0.58 (0.39)	1.69 (0.64)	27.61 (4.65)	72.39 (4.65)	-0.71 (0.15)	2.19 (1.01)
Variance	1.32 (0.40)	3.13 (0.58)	23.45 (2.30)	76.55 (2.30)	-0.93 (0.10)	4.35 (1.00)
Carry	0.73 (0.25)	1.76 (0.33)	22.81 (3.99)	77.19 (3.99)	-0.48 (0.06)	2.53 (0.58)
Time-Series Momentum	-0.27 (0.20)	1.48 (0.37)	61.59 (4.68)	38.41 (4.68)	-1.52 (0.60)	0.82 (0.15)
Panel F: Event Driven Funds						
	Mean	Std Dev.	Negative	Positive	10%	90%
Market	2.05 (1.17)	2.68 (0.77)	10.97 (8.55)	89.03 (8.55)	-0.02 (0.45)	5.02 (1.86)
Size	0.28 (0.39)	0.76 (0.18)	24.54 (17.15)	75.46 (17.15)	-0.09 (0.14)	0.77 (0.59)
Illiquidity	0.07 (0.12)	1.06 (0.19)	41.26 (6.95)	58.74 (6.95)	-0.48 (0.04)	0.64 (0.25)
Betting Against Beta	0.67 (0.40)	1.88 (0.49)	20.82 (5.90)	79.18 (5.90)	-0.56 (0.09)	2.03 (0.80)
Variance	1.78 (0.48)	3.71 (0.50)	18.77 (3.32)	81.23 (3.32)	-0.51 (0.17)	4.87 (0.97)
Carry	0.20 (0.17)	2.14 (0.34)	43.49 (4.29)	56.51 (4.29)	-1.75 (0.33)	1.80 (0.38)
Time-Series Momentum	-0.34 (0.19)	1.64 (0.36)	63.20 (3.71)	36.80 (3.71)	-1.87 (0.57)	0.92 (0.18)

TABLE AVIII. Characteristics of the Worst and Best Funds Under the CP Model
This table examines the properties of the worst and best performing funds identified by the alpha under the CP model (5%, 10%, 25%, 75%, 90%, and 95%). Panel A reports the average investment style composition (equity, macro, and arbitrage) for each quantile. Panel B reports the average beta component due to each factor included in the CP model (market, size, illiquidity, BAB, variance, carry, TS momentum). Panel C reports the averages of three measures of managerial incentives (management and performance fees, proportion of funds with high-mark provisions). Panel D reports the averages of two measures of managerial discretion (lockup and notice periods).

Panel A: Investment Style

	Quantile of the Alpha Distribution					
	5th	10th	25th	75th	90th	95th
Arbitrage (%)	16.86	18.36	22.88	29.84	22.37	19.16
Equity (%)	33.33	35.76	38.94	38.41	38.24	42.91
Macro (%)	49.81	45.89	38.18	31.75	39.39	37.93

Panel B: Beta Component Due to each Factor

	Quantile of the Alpha Distribution					
	5th	10th	25th	75th	90th	95th
Market	3.27	3.47	3.33	1.62	1.81	2.22
Size	0.32	0.31	0.28	0.22	0.20	0.25
Illiquidity	0.39	0.38	0.22	-0.17	-0.26	-0.41
Betting Against Beta	1.11	0.90	0.67	-0.09	-0.46	-0.78
Variance	8.78	5.59	3.35	-1.56	-3.41	-5.25
Carry	2.38	1.92	1.34	-0.33	-0.55	-0.81
Time-Series Momentum	4.73	3.61	2.27	0.62	0.70	0.67

Panel C: Measures of Management Incentives

	Quantile of the Alpha Distribution					
	5th	10th	25th	75th	90th	95th
Management Fees (% per year)	1.58	1.52	1.42	1.51	1.54	1.56
Performance Fees (% per year)	18.68	17.44	15.68	19.29	19.87	19.86
High Water Mark (%)	58.24	60.69	62.82	78.46	76.82	76.25

Panel D: Measures of Managerial Discretion

	Quantile of the Alpha Distribution					
	5th	10th	25th	75th	90th	95th
Lockup (months)	2.47	2.62	2.65	4.17	4.41	4.61
Notice (months)	0.83	0.83	0.83	1.27	1.14	1.12

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