The Monetary Entanglement between CBDC and Central Bank Policies

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ABSTRACT

Using a two-period equilibrium model, we show that the effects of introducing a Central Bank Digital Currency (CBDC) depend on the ongoing monetary policy. We derive neutrality conditions without direct pass-through policies and find that they do not always hold with quantitative easing, as bank lending shrinks if demand for CBDC is above a certain threshold. Moreover, we find that commercial banks optimally liquidate excess reserves in the system to accommodate households' demand for CBDC. This leads to the replacement of banks with households on the liability side of the central bank balance sheet, making quantitative tightening difficult to implement.

Keywords: CBDC, central banking, monetary policy, quantitative easing, neutrality.

JEL classification: E42, E52, E58, G21, G28.

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1 Introduction

Most major central banks are considering introducing a retail central bank digital currency (CBDC), i.e., a digital payment instrument, denominated in the national unit of account, that is a direct liability of the central bank (BIS, 2020). Advocates of CBDC projects argue that they would strengthen monetary sovereignty, enrich monetary policy toolkits, and foster financial innovation and inclusion.[1] Nonetheless, the introduction of a CBDC would lead central banks into uncharted territory as they would directly compete with banks for deposits, raising concerns about financial stability as well as privacy issues (Armelius et al., 2020). The burgeoning literature on the topic focuses on several aspects, such as disintermediation risk, deposit competition, and optimal design (see, e.g., Fernández-Villaverde et al., 2021b; Agur et al., 2022).

However, the interaction between a CBDC and monetary policy remains an open question (see, e.g., BOE, 2020; ECB, 2020). This is particularly relevant, as the balance sheets of central banks reached record levels after the global financial crisis and expanded even further due to COVID-19 relief programs. Despite the ongoing tightening cycle, the system is still characterized by large amounts of excess reserves.[2] In this context, we address the following questions: Does ongoing monetary policy matter for the introduction of a CBDC? What role does the central bank balance sheet play?

We answer these questions using a simple general equilibrium model that allows the joint study of different central bank policies. We find that the equilibrium impact of a CBDC depends on the ongoing monetary policy. Under quantitative easing, the economy reaches different equilibrium allocations than under standard monetary policy. Furthermore, under a standard policy, the introduction of a CBDC can be neutral even without a direct pass-through policy. On the contrary, under quantitative easing, neutrality is impossible when the demand for CBDC is larger than the amount of excess reserves in the system. We also show that commercial banks optimally liquidate their excess reserves to accommodate households' demand for CBDC. Such a mechanism leads to households replacing banks as counterparts on the liability side of the central bank's balance sheet. As retail deposits are typically highly inelastic (Chiu and Hill, 2018), we argue that quantitative tightening might become more difficult.

We obtain these results by extending the model proposed by Magill, Quinzii, and Rochet (2020) with an interest-bearing CBDC backed by central bank assets (consistently with

[1] G7 Finance Ministers and Central Bank Governors' Communiqué, Art. 17, June 5th 2021, www.g7uk.org/g7-finance-ministers-and-central-bank-governors-communique.

[2] See the "Excess Reserves of Depository Institutions" in the United States. Link https://fred.stlouisfed.org/series/EXCSRESNS.

ECB, 2020).[3] The model is a two-period general equilibrium that features an economy with a private and a public sector. The private sector consists of three agents (i.e., households, cash pools, and investors) and a representative commercial bank. The public sector consists of a central bank and a fiscal authority, which are treated as a unique entity, namely the government. Agents are utility maximizers, whereas the representative commercial bank maximizes the shareholders' expected profit, subject to central bank regulation (i.e., liquidity and capital requirements). In this framework, the central bank implements two different monetary policies. The first is standard monetary policy, where the central bank holds government bonds, their interest rate is kept above the one on reserves, and liquidity requirements are binding. The second is quantitative easing policy, where the central bank holds risky securities, the interest rates on treasuries and reserves are equal, and there are excess reserves in the system.

We find that the impact of the introduction of a CBDC mainly depends on the amount of bank deposits converted into CBDC as well as on the ongoing monetary policy. When depositors decide to convert one unit of bank deposits into one of CBDC, commercial banks must transfer one unit of resources to the central bank to settle the transaction. We show that, when converting bank deposits into CBDC deposits, the commercial bank optimally decides to reduce its excess reserves. When the demand for CBDC is lower than the amount of excess reserves, the size of the central bank's balance sheet remains the same, as one unit of reserves is simply converted to CBDC and transferred from the commercial bank's reserve account to the households' CBDC account. When there are no excess reserves, the commercial bank is forced to transfer other assets to the central bank, leading to an increase in the size of the central bank's balance sheet.

Under standard policy, the liquidity requirement is binding, and there are no excess reserves. When households convert part of their savings into CBDC deposits, the central bank needs to buy additional government bonds to back the new CBDC liabilities. This crowds out cash pools' demand for treasuries, which in turn have to invest in bank debt, compensating the commercial bank for the loss in deposits. If CBDC and bank deposits have the same liquidity properties, this indirect pass-through policy renders the introduction of a CBDC neutral, consistently with the equivalence theorem of Brunnermeier and Niepelt (2019).[4] Together, these conditions on standard policy and liquidity equivalence make the theorem hold even in the absence of a direct pass-through policy.

[3]Even if it were possible for a central bank to issue an unbacked CBDC, it would result in a decline in central bank equity and would be akin to helicopter money, which is not considered a viable option by policymakers (BIS, 2020).

[4]The theorem states that neutrality can be obtained only through liquidity and span-neutral open-market operations with compensating transfers and a corresponding central bank pass-through policy.

Under quantitative easing, the equilibrium outcomes depend on the amount of excess reserves in the system. As long as the amount of CBDC deposits does not exceed the amount of excess reserves, the introduction of a CBDC leads to a reduction in both deposits and reserves without consequences for bank lending. When we impose liquidity equivalence between reserves, CBDC, and bank deposits, the introduction of a CBDC is neutral without a direct pass-through policy. However, when households' demand for CBDC exceeds the amount of excess reserves, the commercial bank can swap reserves for CBDC deposits until the liquidity requirement is binding. After this point, the central bank needs to buy additional risky securities to back the demand for CBDC deposits. The commercial bank loses deposits, which represent a cheap funding source, and in turn, receives more costly equity injections, leading to a contraction in lending. In this scenario, CBDC neutrality can be only obtained by allowing for a direct pass-through policy (as in Brunnermeier and Niepelt, 2019).

Furthermore, when we introduce a CBDC under quantitative easing, quantitative tightening might be impaired. Substituting banks with households on the liability side of the central bank balance sheet might have significant consequences.[5] Banks are always willing to swap back risky assets for reserves with the central bank via open market operations. However, this is not the case for households whose deposits tend to be extremely inelastic (Chiu and Hill, 2018). Therefore, the central bank might not be able to smoothly tighten its balance sheet as if it were dealing only with the banking sector.

Although our model encompasses salient real-world features, including liquidity and capital requirements, explicit and implicit deposit guarantees, and excess reserves, it has limitations. First, the state of the economy is exogenous and taken as given by the actors. Second, monetary policies, including the introduction of a CBDC, are exogenous. Third, all interest rates in the model are real rates, and thus, there is no inflation from one period to the next. Providing an exhaustive theoretical account of the general equilibrium effects of introducing a CBDC is beyond the scope of this paper.

Finally, our results directly inform the debate about CBDCs in two ways. First, our findings suggest that the decision to issue a CBDC should consider the ongoing monetary policy. More specifically, central banks should complete the tightening cycle before issuing a CBDC to avoid the concerns highlighted in this paper. Second, if a central bank nevertheless decides to launch a CBDC before the end of the tightening cycle, it should make sure that the amount of CBDC in circulation is lower than the amount of excess reserves, e.g., via restrictions on holdings.

[5] Back-of-the-envelope calculations suggest that excess reserves could accommodate the conversion of at most 25% of US deposits. Source: St. Louis Fred

Related literature. Our paper contributes to the burgeoning literature that studies the introduction of a CBDC, its design and the implications for the banking sector.[6][7] To the best of our knowledge, we are the first to focus on the interaction between a CBDC and ongoing monetary policies.

We contribute to the literature related to the disintermediation risk of the banking sector due to the introduction of a CBDC. For instance, Whited et al. (2022) estimate a structural model that highlights how a CBDC would reduce banks' deposit funding. Fernández-Villaverde et al. (2021a) and Fernández-Villaverde et al. (2021b) focus on these issues by using a modified version of the model by Diamond and Dybvig (1983), where a central bank engages in large-scale intermediation by competing with private financial intermediaries for deposits and investing in long term projects. They find that the set of allocations achieved with private financial intermediation is also achieved with a CBDC and that, during a run, the central bank is more stable than the commercial banking sector. For this reason, they conclude that the central bank would arise as a deposit monopolist. Brunnermeier and Niepelt (2019) and Niepelt (2020) provide conditions under which swapping private money with public money (e.g., CBDC) is indifferent for equilibrium allocations. In their setting, the central bank collects retail deposits and lends them to commercial banks to compensate for missing funding, de facto eliminating any disintermediation effect. Chiu et al. (2023) focus on banks' market power and show that when banks have no market power, issuing a CBDC would crowd out private banking. However, when banks have deposit market power, a CBDC with a reasonable interest rate would encourage banks to pay higher interests or offer better services to keep their customers (see also Andolfatto, 2021). We contribute to this strand of literature by showing that the general equilibrium effects that might render a CBDC neutral for the banking sector depend on the excess reserves in the system and on the baseline monetary policy.

Furthermore, we contribute to the literature on CBDC design. The choice of CBDC design has sizeable real effects on the economy in terms of technological innovation, users' privacy, and the bank's ability to intermediate. A comprehensive BIS report by Auer and Böhme (2020) studies the differences between three main architectural choices: account- vs token-based system, one- or two-tier distribution, and whether to adopt a decentralized ledger technology (see also Armelius et al., 2020). Agur et al. (2022) studies the relation

[6] Notably, Barrdear and Kumhof (2016) are among the first to study CBDCs, by focusing on their potential as additional monetary policy tools to stabilize the business cycle. For a more extensive review of the literature, please refer to (Ahnert et al., 2022; Chapman et al., 2023; Carapella and Flemming, 2020).

[7] Note that there is still little empirical research on CBDC as only a few CBDC projects are in advanced stages, and data is not yet available for research (e.g., see Auer and Böhme, 2020; Kosse and Mattei, 2022).

between preferences over anonymity and security by developing a theoretical model where depositors can choose between cash, CBDC, and bank deposits. They conclude that the optimal CBDC design trades off bank intermediation against the social value of maintaining diverse payment instruments. By contrast, Keister and Sanches (2021) study CBDC optimal design in a setting with financially constrained banks and with a liquidity premium on bank deposits. They highlight an important policy trade-off: while a digital currency tends to promote efficiency, it may also crowd out bank deposits, raise banks' funding costs, and decrease investment. They also find that despite these effects, introducing a CBDC often increases welfare. Our approach is rather agnostic in term of design as our model allows for different kinds of CBDC designs (i.e., token-based, account-based, interest-bearing and so forth). Nevertheless, we contribute to this literature providing evidence that the design should depend on the ongoing monetary policy and that limiting the scope of the CBDC in circulation would minimize the risk of negatively impacting the economy.

The rest of the paper is organised as follows. Section 2 describes the model setup. Section 3 reviews the possible mechanisms to issue new CBDC deposits and introduce them in the economy. Sections 4 presents the equilibrium conditions. Section 5 discusses the implications of introducing a CBDC under different policy regimes, and the neutrality conditions. Finally, Section 6 concludes.

2 Model

For our analysis, we extend the model developed by Magill, Quinzii, and Rochet (2020) by adding a one-tier interest-bearing CBDC. The model has two periods and an economy with a private and a public sector. The private sector consists of agents and a representative commercial bank, whereas the public sector consists of a central bank (CB) and a fiscal authority, which are treated as a single actor, the government.

Agents are households, investors, and institutional cash pools. Households and cash pools are infinitely risk-averse and only lend to banks if they are sure of having their funds returned. Deposits are explicitly insured (e.g., DGS in the Eurozone or FDIC in the US). In addition to the deposit interest rate, households benefit from the payment services provided by the banks. Cash pools invest indifferently in public and bank debt and consider the latter to be implicitly insured by the government. This belief was essentially confirmed in 2008 when the government bailed out most failing financial institutions or provided relief by purchasing assets through the central bank. Because of the public insurance on the bank liabilities, there is no possibility of bank runs. On the other hand, investors are the only agents willing to accept risk and therefore invest in bank equity.

Banks have a unique technology that allows them to invest in risky ventures and perform maturity transformation. They channel funds from savers to entrepreneurs and allow savers to transfer funds from one period to the next. We do not explicitly model entrepreneurs' decision-making. We assume that banks invest in productive ventures without explicitly modelling the bank's screening process. The government regulates banks, bails them out of bankruptcy when needed, issues debt to fund its spending, and collects taxes from investors to repay its debt.

In this setting, we include a CBDC, by which households have the option to deposit their funds at the central bank. CBDC deposits pay an interest and provide payment services.

2.1 Households

The representative household is infinitely risk averse and receives an endowment $w_{h,0}$ at time 0 and no endowment in period 1. Households can place their funds either in a commercial bank (as a standard bank deposit) or in the central bank (as a CBDC deposit) to transfer them to time 1 for consumption. They also benefit from the payment services provided by the bank and the central bank.[8]

The agent's utility derives from the consumption stream x_h, which consists of $x_{h,0}$ at time 0 and the random consumption $\tilde{x}_{h,1}$ at time 1. The total utility is given by:

$$u_h(x_{h,0}) + \min \tilde{x}_{h,1} + \rho \min \tilde{x}_{h,1}, \tag{1}$$

where u_h is a concave increasing function, $\min \tilde{x}_{h,1}$ represents the household's infinite risk aversion, and ρ captures the convenience yield obtained from the transaction services at time 1. We assume that the convenience yield is linear. If R^h denotes the deposit interest paid by banks, a bank deposit h generates a consumption $R^h h$ at time 1. Similarly, if R^d denotes the deposit interest paid by the central bank, d worth of CBDC deposit generates a consumption $R^d d$ in period 1. The total consumption is therefore $x_h = (w_{h,0} - h - d, R^h h + R^d d)$ and the household utility is $u_h(w_{h,0} - h - d) + (1 + \rho_h)R^h h + (1 + \rho_d)R^d d$, where ρ_h and ρ_d are the convenience yields from bank and central bank services respectively.

If in time 0 the utility function of households u_h satisfies the Inada conditions $\frac{\partial u_h(x_{h,0})}{\partial h} \to \infty$ as $x_{h,0} \to 0$ and $\frac{\partial u_h(x_{h,0})}{\partial d} \to \infty$ as $x_{h,0} \to 0$, then the solutions to the maximization

[8] Note that in the main version of the model we do not include cash. However, when we allow households to hold it, the implications of the model do not change; only the magnitudes of the effects are different.

problem are characterized by the following first-order conditions:

$$\frac{\partial u_h(w_{h,0} - h - d)}{\partial h} = (1 + \rho_h)R^h, \tag{2}$$

$$\frac{\partial u_h(w_{h,0} - h - d)}{\partial d} = (1 + \rho_d)R^d. \tag{3}$$

2.2 Cash Pools

The cash pool agents represent the wholesale money market, which includes money market funds, wealth managers, and the like. Just like households, cash pools are infinitely risk averse and invest only in safe and liquid assets. The representative cash pool has an endowment $w_{c,0}$ only at time 0, and it has a utility function $u_c(x_{c,0}) + \min \tilde{x}_{c,1}$, where u_c is an increasing concave function that captures the opportunity cost of the cash pool funds.

During the 2008 financial crisis, the actions by the central bank and the treasury prevented runs and confirmed the perception that bank liabilities are implicitly insured by the government. Since cash pools invest only in safe assets, they choose between government bonds and bank liabilities, which have to be interpreted as short-term debt, either loans or bonds.[9] When treasuries are not enough to satisfy the demand of cash pools, part of their savings is therefore absorbed by the bank (c_b). The representative cash pool chooses how much to invest (c) in order to maximize $u_c(w_{c,0} - c) + R^c c$, where R^c is the interest received by the bank or the government.

If in time 0 the utility function of cash pools u_c satisfies the Inada conditions $\frac{\partial u_c(x_{c,0})}{\partial c} \to \infty$ as $x_{c,0} \to 0$, then the solution to their maximization problem is characterized by the first-order condition:

$$\frac{\partial u_c(w_{c,0} - c)}{\partial c} = R^c. \tag{4}$$

2.3 Investors

Investors play two roles in the model. They are long-term investors who take risks, and they act as taxpayers.[10] Investors receive an endowment in both periods $w_i = (w_{i,0}, w_{i,1})$ and are risk neutral. Their utility function is $u_i(x_{i,0}) + \mathbb{E}(\tilde{x}_{i,1})$, where u_i is an increasing concave function that satisfies the Inada conditions. Investors can place their funds in safe assets (either government bonds or bank debt that we denote by c_i), and bank equity (that we denote by e). If they invest in safe assets, they receive the same return R^c as cash pools.

[9] Potentially, they could invest also in bank and CBDC deposits. Since cash pools do not benefit from the payment services, these options are not attractive enough.

[10] We better describe taxes in Section 2.5, where we describe the government.

The payoff of bank equity is $V(y)$ per unit of equity, where y is the realization of the random payoff $\tilde{y}$ per unit of investment in risky projects. The investor problem is to choose (c_i, e) to maximize

$$u_i(w_{i,0} - c_i - e) + \mathbb{E}(w_{i,1} - t(y) + V(y)e + R^c c_i), \tag{5}$$

where $t(y)$ is a lump-sum tax due to the government at time 1. Given the expected return on equity $R^E = \mathbb{E}\left[V(\tilde{y})\right]$, we exclude the case where $R^c > R^E$ for which $c_i > 0$ and $e = 0$, since banks must have positive equity in equilibrium. We assume that when $R^E = R^c$, investors choose to invest only in equity. Finally, when $R^E > R^c$, investors prefer to invest only in equity and $c_i = 0$.

Therefore, the first-order condition that characterizes the solution of the investor maximization problem is:

$$\frac{\partial u_i(w_{i,0} - e)}{\partial e} = R^E. \tag{6}$$

2.4 Commercial Bank

The banking sector is modeled with a representative commercial bank that can either store funds in reserves (M) at the central bank or invest (K) in a productive risky technology. To finance its assets, the bank collects deposits from households (h), obtains financing from cash pools (c_b), and issues equity (E). Hence, it holds that $M + K = h + c_b + E$.

The commercial bank is the only actor in the model that can perform risk and maturity transformation: it borrows short safe deposits and lends long risky loans to entrepreneurs. It offers bank deposits with a series of complimentary services and faces a unitary cost μ_h at time 1, which represents the cost of maintenance of the infrastructure, managing of accounts, and so forth. In light of what occurred in the aftermath of the 2008 crisis, our model encompasses two kinds of insurances. The first one is explicit and refers to the households, featuring the deposit guarantee schemes of major economies. The second one is implicit and applies only to cash pools, who believe that, in case of crisis, the government would bail out the banking sector following the too-big-to-fail argument.[11]

The central bank pays an interest rate R^M on reserves, while the risky technology delivers $\tilde{y}$ at time 1. The distribution of returns is characterized by the density function $f(y)$ on $\mathbb{R}_{\geqslant 0}$,

[11] It is worth mentioning that cash pools receive $\underline{y}K$ as collateral from the bank. Thus, in case of default, they are only interested in the fact that the government would repay them the difference between what they lent out and the collateral value.

and it is different from zero for $\tilde{y} > \underline{y} > 0$.[12] Our model also incorporates current banking regulations with liquidity and capital requirements. The bank is forced to store at least δ of its deposits in reserves to satisfy the liquidity requirement and finance at least $\bar{\alpha}$ of the risky projects with equity for the capital requirement.

The representative bank optimally chooses the items of its balance sheet $(M,\ K,\ h,\ c_b,\ E)$ taking as given the interest rates in the economy $(R^M,\ R^h,\ R^c,\ R^E)$, and it maximizes the shareholders' expected profit:

$$\max_{h,\, c_b,\, E,\, M,\, K} \int_{\hat{y}}^{\infty} [R^M M + yK - (1+\mu_h)R^h h - R^c c_b] f(y) dy - R^E E, \tag{7}$$

subject to

$$h + c_b + E = K + M, \quad \text{(balance sheet constraint)} \tag{8}$$

$$M \geqslant \delta h, \quad \text{(liquidity requirement)} \tag{9}$$

$$E \geqslant \bar{\alpha} K, \quad \text{(capital requirement)} \tag{10}$$

where $\hat{y}$ is the minimum return on the risky technology that allows the bank to repay its creditors, i.e., $R^M M + \hat{y} K = (1+\mu_h)R^h + R^c c_b$. The bank is solvent for $y > \hat{y}$.

2.5 Government

We consider the fiscal authority and the central bank as a single entity (i.e., the government) that conducts guarantee, prudential, interest rate, and balance sheet policies. Similarly, to the commercial bank, the central bank offers CBDC deposits facing a unitary cost μ_d at time 1. To finance its expenditures G, the government issues bonds $(B = G)$ at time 0, on which it pays an interest R^B at time 1.[13] The central bank can influence this interest rate via open market operations, namely repos and reverse-repos with cash pools.[14] The interest rate takes different values according to the monetary policy regime. At time 1, the government levies taxes on the investors to service its bonds. We make the strong assumption that prices are fully rigid as it allows us to work with a real variable model.

As mentioned before, the government provides explicit and implicit insurance to households and cash pools to avoid bank runs, and it sets the liquidity (δ) and capital $(\bar{\alpha})$ requirements.

[12] As in Magill et al. (2020), we assume that all shocks are perfectly correlated and, due to the law of large numbers, we can treat $\tilde{y}$ as an aggregate shock for the economy.

[13] In this model, the amount of government expenditures is not affected by the introduction of a CBDC.

[14] We consider only two periods, so we interpret B as very short-term bonds.

The central bank manages the funds coming from reserves (M) and CBDC deposits (d) by deciding the compositions of its assets. Hence, it either invests in government bonds (B^{CB}) or in risky securities (E^{CB}), which in our model are represented by the bank's equity. We define a baseline *standard policy* where the central bank holds government bonds against its reserves and a *quantitative easing (QE) policy* setting where the reserves are backed by risky assets (i.e., bank equity, which is the only risky asset in the model). It is worth noting that purchasing distressed assets from the banking sector is economically equivalent to recapitalize banks by injecting equity.

In standard policy, the liquidity requirement is always binding ($M = \delta h$), and the interest rate on government bonds is larger than the one on reserves, $R^B > R^M$. In a QE setting, the amount of reserves exceeds the liquidity requirement ($M \geqslant \delta h$), and the banking sector holds excess reserves ($M - \delta h$) at the central bank. In our model, the amount of excess reserves can be considered as exogenous to the banking sector, as it is solely due to the asset purchase programs of the central bank. Finally, under QE, there is a low interest rate environment with the interest rate on reserves equal to the one on government bonds, $R^B = R^M$.

Finally, when the commercial bank is solvent ($y > \hat{y}$), the tax is equal to the difference between the bondholders' repayment and the net seignorage revenue (θ). The seignorage is defined as the profit made by the government, depending on the composition of its assets. In case the bank goes bankrupt ($y \leqslant \hat{y}$), the tax also includes the repayment of bank's guaranteed liabilities (household's deposits and cash pools' funds) after the liquidation of the assets. Thus, we define the bankruptcy costs as $\phi = (1+\mu_h)R^h h + R^c c_b - (yK + R^M M)$. Taxes are given by:

$$t = R^B B - \theta + \phi \mathbb{1}_{y \leqslant \hat{y}}. \tag{11}$$

3 CBDC Introduction Mechanism

3.1 Institutional Settings

In standard times, the central bank conducts a conventional monetary policy, regulating the commercial banks and setting the short-term interest rates to stimulate or slow down the economy. However, in times of crisis, lowering the interest rates might not be enough. In these cases, the central bank could implement an unconventional monetary policy, called quantitative easing (QE). When conducting quantitative easing policies, the central bank creates new reserves and uses them to purchase assets. Normally, the asset purchases programmes focus on longer-term securities or distressed assets, with the purpose of manipulating the longer maturities of the yield curve. Such policies aim to support the financial system and ease the pressure on governments and banks.

The result is an increase in the central bank's balance sheet size and an abundance of reserves in the banking system (Joyce et al., 2012). As banks are subject to liquidity requirements, the abundance of reserves should help to boost lending. However, in the US, the launch of quantitative easing programs in 2008 has led to a significant amount of excess reserves, i.e., reserves above liquidity requirements. Figure 1 shows the evolution of the FED's balance sheet size and the amount of excess reserves in the system between 2006 and 2021. The strong link between quantitative easing and excess reserves is clearly visible.

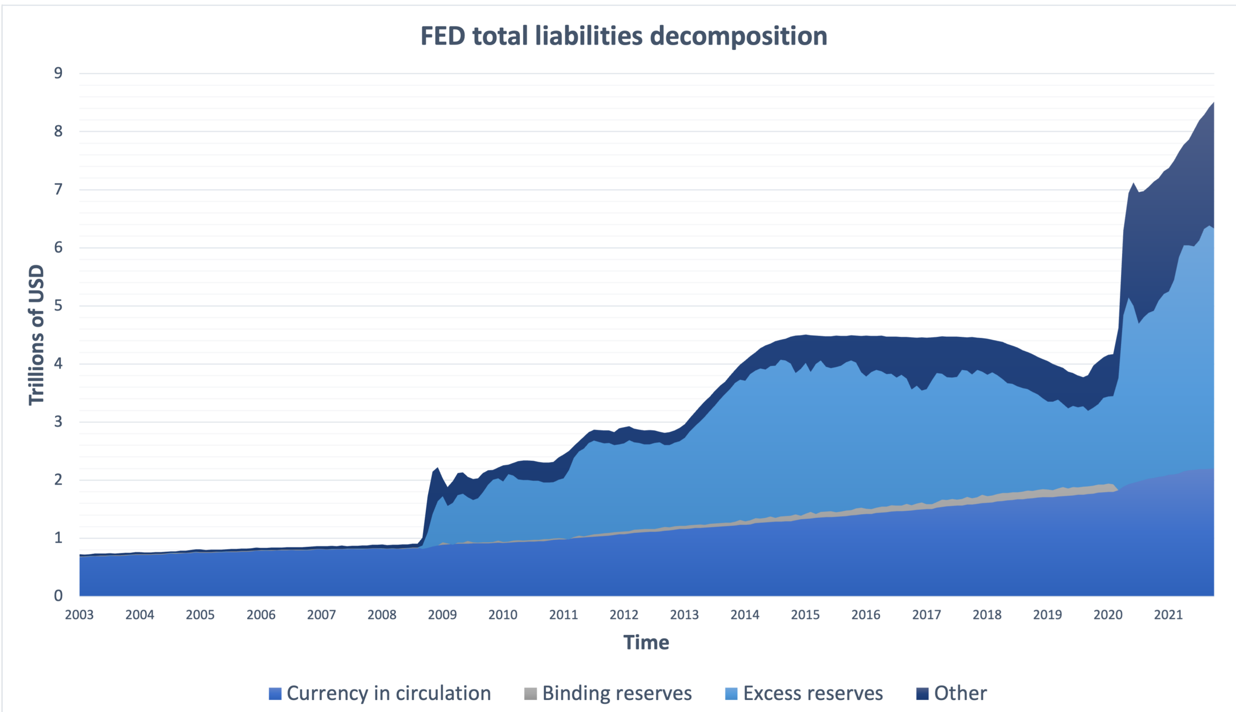

Figure 1. FED's total liabilities decomposition. Source: FRED, Federal Reserve Bank of St. Louis, December 2021.

Other central banks that implemented quantitative easing over the years show a similar pattern. Figure 2 exhibits the liabilities decomposition for the Bank of England (BoE) and the ECB, with an increase of excess reserves after each asset purchase round.

Quantitative tightening (QT) is the reversion of quantitative easing policies to go back to a standard regime. When central banks want to tighten, they sell assets to the market and cancel outstanding reserves in exchange, effectively decreasing the size of their balance sheets. Reducing the balance sheet implies reducing both assets and liabilities at the same time.

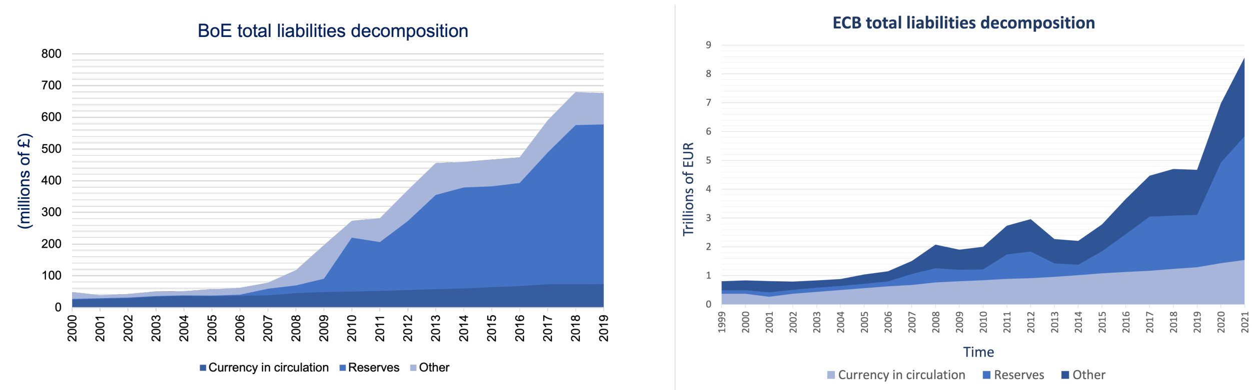

Figure 2. (a) Bank of England's total liabilities decomposition. Source: Bank of England. (b) ECB's total liabilities decomposition. Source: ECB.

3.2 Transferring Money into a CBDC Deposit

With the introduction of a CBDC, households will want to transfer part of their savings from bank deposits into CBDC deposits. By definition, a CBDC is a direct liability of the central bank, like cash (banknotes). From an accounting perspective, it is reasonable to assume that transferring money into a CBDC deposit will work similarly to withdrawing cash from an ATM. In both cases, households exchange a liability of the commercial bank (private money) for a liability of the central bank (public money). The commercial bank transfers resources to the central bank to accommodate the household's demand for public money, either cash or CBDC.

When the liquidity requirement is not binding ($M > \delta h$), the commercial bank easily exchanges part of its excess reserves for the central bank liabilities. After the swap, the commercial bank reduces the household's deposit account and delivers the banknotes or the CBDC. The operation is neutral for the size of the central bank's balance sheet, as one type of liabilities (excess reserves) is transformed into another (CBDC deposits). On the other hand, when the liquidity requirement is binding ($M = \delta h$), the commercial bank needs to keep reserves on its balance sheet and cannot swap them for cash or CBDC. In this case, it is forced to liquidate the other assets in favour of the central bank, leading to an increase in the size of the central bank's balance sheet.

Figure 3 provides a graphical representation of the mechanism described. As long as there are enough excess reserves in the system, the transfer is neutral for the size of the central bank's balance and the central bank's liabilities only change in type. Once excess reserves are exhausted, and the liquidity requirement is binding, the commercial bank liquidates assets in favor of the central bank, that in turn can create new liabilities in the form of CBDC. This operation increases the size of the central bank's balance sheet.

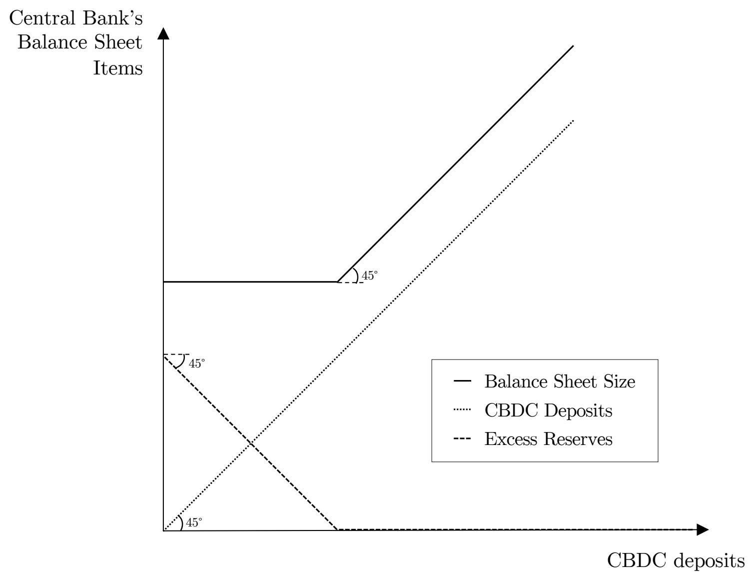

Figure 3. Relationships between CBDC deposits, excess reserves, and central bank's balance sheet size. If the liquidity requirement is not binding, the commercial bank swaps excess reserves for CBDC deposits. In this case, the size of the central bank does not change as one type of liability is simply transformed into another. Once the liquidity requirement is met, the commercial bank liquidates assets in favor of the central bank, increasing its size.

THEOREM 1. *Let the representative commercial bank be a profit maximizer as described in Section 2. The optimal choice for accommodating the households' demand for CBDC is to exchange reserves rather than liquidating other assets, unless the liquidity requirement is binding.*

Proof. When the commercial bank transfer the households' savings into CBDC deposits, it can stop paying the interest on the lost deposits and their cost of maintenance. When it accommodates the households' demand by exchanging reserves for CBDC deposits, it loses the interest on the swapped reserves. The difference in the expected profit is

$$\Delta\pi' = d\left[(1+\mu_h)R^h - R^r\right]. \tag{12}$$

On the other hand, when the commercial bank liquidates other assets than reserves in favor of the central bank, its expected profits change accordingly:

$$\Delta\pi'' = d\left[(1+\mu_h)R^h - \int_{\hat{y}}^{\infty} y f(y) dy\right]. \tag{13}$$

It must hold that $R^r < \int_{\hat{y}}^{\infty} y f(y) dy$ as an incentive for the commercial bank to invest in risky projects, implying $\Delta\pi' > \Delta\pi''$. Since the commercial bank is a profit maximizer, whenever it is possible, it optimally chooses to reduce its excess reserves to accommodate the demand for CBDC. □

The commercial bank can reduce its reserves only until the liquidity requirement is binding. After that point, the commercial bank has no choice but to liquidate its assets in favor of the central bank. We define $\bar{d}$ as the maximum demand for CBDC deposits for which the commercial bank can swap excess reserves. This amount is such that the liquidity requirement is binding, $M - \bar{d} = \delta(h - \bar{d})$, i.e., the maximum amount for which the reduction in reserves fully compensates the reduction in deposits.

COROLLARY 1. *The commercial bank can swap a maximum of $\bar{d}$ reserves into CBDC deposits, where $\bar{d}$ is defined such that the liquidity requirement becomes binding:*

$$\bar{d} = \frac{M - \delta h}{1 - \delta}. \tag{14}$$

If the demand for CBDC deposits exceeds the threshold ($d > \bar{d}$), then the commercial bank swaps as many reserves as possibile. Only when it runs out of excess reserves, i.e., the liquidity requirement is binding, the commercial bank then liquidates assets in favor of the central bank. We define $\tilde{d} = d - \bar{d}$ as the demand of CBDC that that the commercial bank accommodates by liquidating assets. In this case, since the liquidity requirement is binding, the bank compensate the loss in deposits by partly reducing its reserves by an additional $\delta\tilde{d}$, on top of the $\bar{d}$ optimally used.

3.3 Central Bank's Balance Sheet

When there is an abundance of excess reserves, the commercial bank optimally swaps them for CBDC deposits, without altering the size of the central bank's balance sheet. The composition of the central bank's liabilities change, but the asset side of its balance sheet is left unaltered.

This is not the case when the central bank issues new liabilities in the form of CBDC deposits, as CBDCs must always be backed by assets (ECB, 2020). The central bank could acquire either government bonds or risky securities to hold against the CBDC deposits. In theory, holding risky securities against households' deposits could be justified by the fact that there might not be sufficient safe assets (i.e., government bonds) to fully absorb the overall demand. However, backing the issuance of new liabilities with the purchase of risky securities corresponds to a new quantitative easing round, and it should be a measure for times of crisis.

Nevertheless, if the commercial bank converted its excess reserves into CBDC deposits, it would be much harder for the central bank to revert QE programs. The central bank would go from having a limited number of financial institutions as counterparts to having a large number of small households. Households would use a CBDC for payments and savings and would probably be much less elastic than financial institutions. It is reasonable to assume that the CBDC deposits' elasticity would be similar to the bank deposits' one, which tends to be low (Chiu and Hill, 2018). Quantitative tightening means selling assets on the one side and canceling liabilities on the other. An inelastic liability side would render quantitative easing policies semi-permanent.

REMARK 1. *The adoption of a CBDC under quantitative easing might render this policy quasi-permanent, as it will be even more difficult to revert.*

4 Equilibrium

In this Section, we study how introducing a CBDC under different monetary policy scenarios changes the respective equilibrium allocations. We first outline assumptions to ensure that banks fund themselves with households' deposits and wholesale funding at equilibrium. Then we define the equilibria in different monetary policy regimes. Finally, we briefly discuss the Pareto-optimality of our equilibrium allocations.

4.1 Assumptions

ASSUMPTION 1. *Investors.*

(a) Investors are better off investing in bank equity: $\frac{\partial u_i(w_{i,0})}{\partial w_{i,0}} < \mathbb{E}[\tilde{y}]$.
(b) Investors have enough endowment at time 1 to pay the tax: $w_{i,1} > (w_{h,0}(1+\mu_h) + w_{c,0})\,\mathbb{E}[\tilde{y}]$.

The first part of the assumption guarantees that investors do not prefer to consume all their endowment at time 0 but always want to invest in the technology. It is worth noting that it also ensures that bank equity is never zero, especially under standard monetary policy. The second part of the assumption guarantees that investors have enough resources to repay households and cash pools, as investors pay the tax to the government (including the cost of bankruptcy). The condition considers even the limit case in which at date 0 households store all their endowment in deposits, and cash pools invest all their endowment in wholesale funds.

ASSUMPTION 2. *Cash pools.*

(a) Cash pools want to buy both government bonds and bank debt: $\frac{\partial u_c(w_{c,0}-B)}{\partial w_{c,0}} < R^B \leqslant \mathbb{E}[\tilde{y}]$.

This assumption entails that at equilibrium there is a shortage of government bonds. Cash pools want to invest an amount bigger than the amount of government bonds in the economy. For this reason, cash pools resolve to wholesale funding at the commercial bank.

ASSUMPTION 3. *Households.*

(a) Households prefer bank deposits to government bonds: $\rho_h > \mu_h$, $0 \leqslant \delta \leqslant \frac{\rho_h - \mu_h}{1+\rho_h}$.
(b) Households would want treasuries if they had no other choice: $\frac{\partial u_h(w_{h,0})}{\partial w_{h,0}} < \frac{\partial u_c(w_{c,0}-B)}{\partial w_{c,0}}$.

This assumption guarantees positive bank deposits at equilibrium. The first part ensures that households get a greater utility from the saving technology and payment services offered by bank deposits, rather than investing in government bonds. The second part states that, in an economy without bank and CBDC deposits, households would prefer treasuries rather than consume all their endowment in time 0.

4.2 Equilibrium definition

In our model, there exist corner and interior solutions. We have corner solutions when households decide to hold either CBDC or bank deposits. Trivially, if households do not hold CBDC, the economy is unaffected. Conversely, the equilibrium with no bank deposits is highly unrealistic, as central banks are keen on avoiding it (e.g., Armelius et al., 2018). Therefore, we focus on the interior solution, which is characterized by households holding both CBDC and bank deposits. This happens when CBDC and bank deposits are imperfect substitutes. A good example is that the average PayPal user also has a bank account and keeps only a small sum in their PayPal account.[15] For any interior solution, the following proposition must hold.

PROPOSITION 1. *If the utility function of households u_h satisfies the Inada conditions, then positive holdings in bank and CBDC deposits, $(h, d) > 0$, are guaranteed if and only if*

$$(1+\rho_h)\, R^h = (1+\rho_d)\, R^d. \tag{15}$$

Proof. Using Leibniz's notation, $\frac{\partial u_h}{\partial h} = \frac{\partial u_h}{\partial x_{h,0}} \frac{\partial x_{h,0}}{\partial h}$ and $\frac{\partial u_h}{\partial d} = \frac{\partial u_h}{\partial x_{h,0}} \frac{\partial x_{h,0}}{\partial d}$. In this model, it holds that $\frac{\partial x_{h,0}}{\partial h} = \frac{\partial x_{h,0}}{\partial d}$ and, therefore, that $\frac{\partial u_h}{\partial h} = \frac{\partial u_h}{\partial d}$. Applying this result to (2) and (3), it follows (15). □

[15] Source: Demos, T. June 1st 2016, *PayPal Isn't a Bank, But It May Be the New Face of Banking*, The Wall Street Journal.

In other words, there is an interior solution if the unitary utilities, considering interest rates and convenience yields, for bank and CBDC deposits are the same. This condition guarantees that, at equilibrium, households holds both, even if interest rates are set to zero.

In addition, we consider an economy with scarcity of safe assets (i.e., government bonds), which are not enough to satisfy the demand of cash pools. Therefore, it must hold that

$$R^c = R^B \tag{16}$$

to make bank debt attractive enough to cash pools. It is worth noting that, this way, the central bank can influence the cost of bank funding by setting R^B.

Moreover, the commercial bank keeps both bank deposits and wholesale funds as a source of funding in terms of debt. Intuitively, when the commercial bank wants to invest its debt and lend money to entrepreneurs, it consider 1 unit of bank deposits equivalent to $(1-\delta)$ unit of wholesale funding, because of the liquidity requirement in equation (9). Therefore, they must also have the same opportunity costs[16]: $(1+\mu_h)R^h - \delta R^r = (1-\delta)R^B$. We get that

$$R^h = \frac{(1-\delta)R^B + \delta R^r}{1+\mu_h}. \tag{17}$$

We define the investable debt of the bank as all the debt that can be invested in the risky technology, excluding reserves:

$$D = h + c_b - M. \tag{18}$$

Replacing equation (18) into the bank's balance sheet constraint (8), we obtain:

$$K = E + D. \tag{19}$$

Let's define $\alpha \geq \bar{\alpha}$ as $\alpha = E/K$. From equation (19), we have that $D = (1-\alpha)K$. If we substitute it into the first order conditions, we find the bank's maximization problem is reduced to the choice of $(\alpha,\ E)$ that maximizes $E\left(\frac{1}{\alpha}\int_{(1-\alpha)R^B}^{\infty}\left[y-(1-\alpha)R^B\right]f(y)dy - R^E\right)$, where $\hat{y} = (1-\alpha)R^B$ comes from the bankruptcy definition at equilibrium. This problem has solution if and only if the capital requirement (10) is binding:

$$E = \bar{\alpha}K, \tag{20}$$

and the zero profit condition is satisfied:

$$R^E = \frac{1}{\bar{\alpha}}\int_{(1-\bar{\alpha})R^B}^{\infty}\left[y-(1-\bar{\alpha})R^B\right]f(y)dy. \tag{21}$$

[16]This condition comes from the first order conditions of the commercial bank's maximization problem.

Finally, from equations (19) and (20), we find that $D = (1-\bar{\alpha})K$, from which we derive

$$E = \frac{\bar{\alpha}}{1-\bar{\alpha}} D. \tag{22}$$

We now define the equilibrium conditions under the two monetary policy regimes. For simplicity, we consider that when the central bank chooses the type of assets to back the issuance of CBDCs, it carries on with the ongoing monetary policy. Therefore, it chooses government bonds under standard policy and risky securities in QE. We always use the same structure for the equilibria definitions. Conditions *(i)* are the common ones we discussed above. Condition *(ii)* specifies the agents' optimal choices. Condition *(iii)* refers to whether the liquidity requirement is binding or not. Condition *(iv)* derives from the dynamics of the money market, in which cash pools invest in (short-term) government bonds, and they lend the remaining part to the bank. Finally, condition *(v)* imposes market clearing for bank equity.

DEFINITION 1. ***Equilibrium under standard policy.***
Given the central bank standard monetary policy $(R^B,\ R^r,\ \delta,\ \bar{\alpha})$*, with interest rate policy* $R^B > R^r$ *and balance sheet policy* $(B^{CB},\ E^{CB}) = (M+d,\ 0)$*, the banking equilibrium consists of rates of return* $(R^h,\ R^d,\ R^c,\ R^E)$ *and choices* $(h,\ d,\ c,\ e,\ E,\ D,\ M,\ K)$ *such that:*

(i) *Conditions (15), (16), (17), (18), (19), (22), (21) hold;*
(ii) (h,d) *is optimal for households, given* (R^h, R^d)*;* c *is optimal for cash pools, given* R^c*;* e *is optimal for investors, given* (R^B, R^E)*;*
(iii) $M = \delta h$*;*
(iv) $c_b = c - (B - M - d)$*;*
(v) $e = E$*.*

DEFINITION 2. ***Equilibrium under quantitative easing.***
If the demand for CBDC deposits is such that $d > \bar{d}$*, given the central bank quantitative easing policy* $(R^B,\ R^r,\ \delta,\ \bar{\alpha})$*, with interest rate policy* $R^B = R^r$ *and balance sheet policy* $(B^{CB},\ E^{CB}) = (0,\ M + \tilde{d})$*, then the banking equilibrium consists of rates of return* $(R^h,\ R^d,\ R^c,\ R^E)$ *and choices* $(h,\ d,\ c,\ e,\ E,\ D,\ M,\ K)$ *such that:*

(i) *Conditions (15), (16), (17), (18), (19), (22), (21) hold;*
(ii) (h,d) *is optimal for households, given* (R^h, R^d)*;* c *is optimal for cash pools, given* R^c*;* e *is optimal for investors, given* (R^B, R^E)*;*
(iii) $M \geqslant \delta h$*;*
(iv) $c_b = c - B$*;*

(v) $e + M + \tilde{d} = E$.

Figure 4 depicts the balance sheets at equilibrium at time 1.

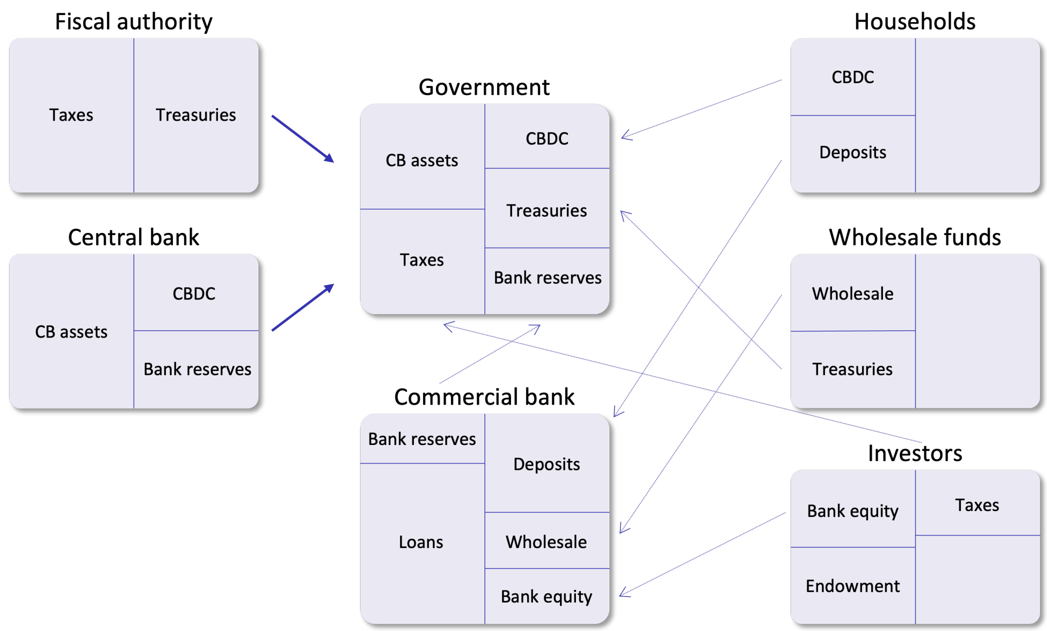

Figure 4. Actors' balance sheets and relationships at time 1.

4.3 Pareto Optimal Allocations

The maximization of social welfare determines the optimal allocations of resources at time 0 and the optimal weight of each agent. The Pareto problem can be written as:

$$\max_{x_{h,0},\, x^h_{h,1},\, x^d_{h,1},\, x_{c,0},\, x_{c,1},\, x_{i,0},\, \{x_{i,1}(y)\}_{y\in Y},\, K} \quad \beta_h\Big[u_h(x_{h,0}) + (1+\rho_h)x^h_{h,1} + (1+\rho_d)x^d_{h,1}\Big] + \tag{23}$$
$$+\ \beta_c\Big[u_c(x_{c,0}) + x_{c,1}\Big] +$$
$$+\ \beta_i\Big[u_i(x_{i,0}) + \textstyle\int_0^\infty x_{i,1}(y)f(y)dy\Big]$$

subject to

$$x_{h,0} + x_{c,0} + x_{i,0} + K + G = w_{h,0} + w_{c,0} + w_{i,0}, \tag{24}$$

$$(1+\mu_h)x^h_{h,1} + (1+\mu_d)x^d_{h,1} + x_{c,1} + x_{i,1}(y) = w_{i,1} + Ky, \tag{25}$$

$$x_{h,1} = x^h_{h,1} + x^d_{h,1}, \tag{26}$$

where $(\beta_h,\ \beta_c,\ \beta_i) > 0$ are the relative weights of the agents, and equations (24) and (25) represent the resource constraints at time 0 and 1, respectively. Substituting $x_{i,1}(y)$[17] in the maximization problem and computing the first order conditions with respect to $x_{h,1}^h$, $x_{h,1}^d$, and $x_{c,1}$, we find that a solution exists only if

$$\beta_c = \beta_i = \frac{1+\rho_h}{1+\mu_h}\beta_h = \frac{1+\rho_d}{1+\mu_d}\beta_h. \tag{27}$$

Interestingly, equation (27) shows that, at Pareto optimum, the ratio between the benefits and the costs of CBDC and bank deposits have to be the same, i.e., $\frac{1+\rho_h}{1+\mu_h} = \frac{1+\rho_d}{1+\mu_d}$.

The necessary and sufficient conditions for a Pareto optimal equilibrium can be summarized by

$$\frac{1+\mu_h}{1+\rho_h}\frac{\partial u_h(x_{h,0})}{\partial x_{h,0}} = \frac{\partial u_c(x_{c,0})}{\partial x_{c,0}} = \frac{\partial u_i(x_{i,0})}{\partial x_{i,0}} = \mathbb{E}\left[\tilde{y}\right], \tag{28}$$

and the resource constraints (24) and (25).[18]

Furthermore, the implicit contributions $(d^*,\ h^*,\ c^*,\ e^*)$ of all the agents are given by:

$$\begin{aligned}
\frac{\partial u_h(w_{h,0} - h^* - d^*)}{\partial h^*} &= \frac{1+\rho_h}{1+\mu_h}\mathbb{E}\left[\tilde{y}\right], \\
\frac{\partial u_h(w_{h,0} - h^* - d^*)}{\partial d^*} &= \frac{1+\rho_d}{1+\mu_d}\mathbb{E}\left[\tilde{y}\right], \\
\frac{\partial u_c(w_{c,0} - c^*)}{\partial c^*} &= \mathbb{E}\left[\tilde{y}\right], \\
\frac{\partial u_i(w_{i,0} - e^*)}{\partial e^*} &= \mathbb{E}\left[\tilde{y}\right].
\end{aligned}$$

PROPOSITION 2. *In any Pareto optimal allocation, the implicit rates of return are:*

$$(1+\mu_h)\,R^h = (1+\mu_d)\,R^d = R^c = R^E = \mathbb{E}\left[\tilde{y}\right]. \tag{29}$$

Proof. It follows from the combination of the Pareto optimal allocations and the first order conditions of the single agents' maximization. □

5 Results

This section is dividend in two parts. In the first, we perform a counterfactual exercise by analyzing the first-order effects of introducing a CBDC. Because of the introduction of a CBDC, households will transfer part of their bank deposits to the central bank to convert

[17] We derive the equation for $x_{i,1}(y)$ from the resource constraint (25).

[18] Equation (28) derives from the first order conditions with respect to $x_{h,0}$, $x_{c,0}$, $x_{i,0}$, and K.

them into CBDC. Hence, the main mechanism driving the results in Sections 5.1 and 5.2 is the reduction in bank deposits (as in Klein et al., 2020; Kumhof and Noone, 2018).[19] Proofs are in Appendix B. In the second, we derive the conditions for the CBDC introduction is neutral under each monetary policy regime.

5.1 Introduction of a CBDC under Standard Policy

The introduction of a CBDC under standard monetary policy leads to a decline in deposits by the amount of households' savings placed in CBDC (d). Since in equilibrium the liquidity constraint is binding, the bank reserves held at the central bank decline by δd, and the size of the commercial bank's balance sheet (S) shrinks. Furthermore, since net liabilities shrink and equity remains unchanged, the commercial bank's leverage declines.[20] The central bank's treasury holding increases by d and declines by δd, as the reduction in bank deposits is followed by a decrease in central bank reserves (M) by the commercial bank. This additional demand for government bonds, $(1-\delta)d$, from the central bank to back CBDC deposits crowds out cash pools that cannot buy as many treasuries as they desire. Consequently, cash pools compensate by investing $(1 - \delta)d$ more in bank debt. The amount of investable funds D for the bank does not change, as the decrease in deposits, $-d$, is fully compensated by the reduction in reserves, $-\delta d$, and the increase in cash pool funding, $(1 - \delta)d$. In other words, the expansion of the central bank balance sheet generates a general equilibrium effect for which wholesale funding substitutes deposits on the commercial bank balance sheet. The result of this general equilibrium effect is that the bank does not change the amount invested in risky loans (K). Bankruptcy costs (ϕ) are unchanged.

The effect on the government sector depends on the cost of issuing CBDC deposits, namely interest rate (R^d) and management cost $(1+\mu_d)$. The impact on seignorage revenues is determined by the difference between the cost of deposits for the central bank, $(1+\mu_d)R^d$, and the commercial bank, $(1 + \mu_h)R^h$. When the cost of deposits for the central bank is higher than for the commercial bank (i.e., $(1 + \mu_d)R^d > (1 + \mu_h)R^h$), seignorage revenues decrease, and taxes increase. Vice versa when $(1 + \mu_d)R^d < (1 + \mu_h)R^h$.

5.2 Introduction of a CBDC under Quantitative Easing

Under quantitative easing, there is an abundance of excess reserves and the reserve requirement is not binding. As shown in section 3.2, when the demand for CBDC remains

[19] Note that, for simplicity, our model does not include cash, however, our findings do not change when we account for it.

[20] We define leverage as bank liabilities divided by the size of the balance sheet, i.e., $(h + c_b)/(h + c_b + E)$.

under the threshold $\bar{d}$, the commercial bank optimally chooses to swap reserves for CBDC deposits. In this scenario, the size of the commercial bank decreases but everything else remains equal. The reduction in bank deposits is fully compensated by the reduction in excess reserves and loans are not affected. Furthermore, the size of the central bank's balance sheet does not change, as there are no additional asset purchases. Nevertheless, the composition of the central bank balance sheet changes, as commercial bank reserves are converted into CBDC deposits. The government asks the investors to pay higher or lower taxes depending on the relative costs of reserves and CBDC deposits that the central bank needs to sustain. If the cost for CBDC deposits is higher than the one for reserves, than taxes increase, and viceversa.

When the demand for CBDC exceeds the amount of excess reserves, i.e., $d > \bar{d}$, the commercial bank swaps reserves for CBDC deposits until the liquidity requirement is binding. At that point, the reduction in deposits cannot be fully compensated by the reduction of reserves anymore. The central bank needs to issue new liabilities to satisfy the demand for CBDC deposits and holds risky securities against them. Therefore, the commercial bank loses deposits, which are a cheap source of funding, and receives more costly equity injections. The result is a reduction in lending, due to the replacement of a cheap source of funding by a more expensive one.

As in Magill et al. (2020), for $\bar{\alpha} > \alpha_c$ the bank has enough capital to absorb the losses even when $\tilde{y}$ is $\underline{y}$, its lowest possible realization. With such a macroprudential policy, there are no bankruptcies, and the equilibrium is Pareto optimal. The central bank holds riskier assets on its balance sheet, with higher expected seignorage revenues. Seignorage volatility increases as the central bank holds more risky assets on its balance sheet. Consequently, taxes are lower in expectation but more volatile. When $\bar{\alpha} < \alpha_c$, the impact on the government sector depends on the relative levels of R^B, R^h, and $V(y)$. In this case, the impact on seignorage is ambiguous.

PROPOSITION 3. *A different monetary policy in place when introducing a CBDC determines different equilibrium allocations and, consequently, a different impact of the CBDC on the economy.*

Proof. The result comes from the derivation shown in Appendix B and the analysis in Sections 5.1 and 5.2. □

Our analysis shows that the effects of introducing a CBDC depend on the ongoing monetary policy. Specifically, the equilibrium depends on interest rates and on the composition of the central bank balance sheet. Such a relationship highlights an unappreciated problem. Currently, the central bank balance sheet has been a function of monetary policy and financial stability. Issuing a CBDC would add an additional layer of complexity by permanently locking assets on central bank balance sheet, in so inevitably interacting with ongoing monetary policies.

While our model is static and does not capture transition dynamics, in Appendix A we model a hybrid scenario where the central bank conducts QE policy, but it backs the CBDC with government bonds (as in standard policy). In such a scenario, the central bank holds risky assets on its balance sheet but, once it absorbes excess reserves, it decides to accommodate the inflows of CBDC deposits by purchasing treasuries, as it does not want to pursue further QE expansions. In such a scenario, we find that introducing a CBDC increases the speed of transition towards standard policy by pushing the sales of risky assets as households convert bank deposits into CBDC.

5.3 Neutrality

Brunnermeier and Niepelt (2019) derive the conditions under which the introduction of a CBDC does not change the equilibrium allocations in the economy. Their equivalence theorem states that neutrality can be obtained only through liquidity and span-neutral open-market operations with compensating transfers and a corresponding central bank pass-through policy. In this Section, we derive the neutrality conditions in our model.

In our framework, CBDC and bank deposits have the same liquidity properties when they are perfect substitutes, namely when the convenience yields and the maintenance costs are the same ($\rho_h = \rho_d$, $\mu_h = \mu_d$). The span-neutrality condition is always guaranteed.

Under standard policy, the introduction of a CBDC increases the amount of liabilities on the central bank's balance sheet. Since the central bank holds treasuries against CBDC deposits, it decreases the amount of safe assets available to cash pools, thus pushing them toward bank debt. This mechanism fully compensates commercial banks for the reduction in bank deposits. For this reason, the bank's lending to the economy is not affected by the introduction of a CBDC. Moreover, since the convenience yields and the maintenance costs are the same, also seigniorage and taxes do not change.

Under QE, the central bank holds risky securities on its balance sheet. This means that additional purchases do not crowd out cash pools, and the same compensation mechanism of the standard policy scenario does not occur. Nevertheless, as long as the demand for CBDC remains below the threshold $\bar{d}$, the reduction in bank deposits is fully compensated by the reduction in reserves without affecting bank lending. The swap between reserves and bank deposits is neutral when they have the same liquidity. Hence, the introduction of a CBDC does not affect the seigniorage and taxes without the need for central bank intervention.

However, if the demand for CBDC is higher than the amount of excess reserves ($d > \bar{d}$), the commercial bank has to fully liquidate assets to accommodate it, with lending decreasing mechanically. In such a scenario, neutrality is impossible unless the central bank conducts a direct pass-through policy through open-market operations, as in Brunnermeier and Niepelt (2019).

THEOREM 2. *When bank and CBDC deposits have the same liquidity properties, the introduction of a CBDC is neutral to the economy, without direct central bank pass-though policy, if one of the following conditions holds:*

1. *The central bank conducts standard policy;*
2. *The central bank conducts quantitative easing, the demand for CBDC is below the threshold $\bar{d}$, and bank deposits have the same liquidity properties of reserves.*

Proof. Under standard policy, $\Delta_K^{sB} = 0$ and $\Delta_t^{sB} = \left[(1+\mu_d)R^d - (1+\mu_h)R^h\right]h$ as explained in Appendix B. If $\rho_h = \rho_d$ and $\mu_h = \mu_d$, then $(1+\mu_d)R^d = (1+\mu_h)R^h$ because of equation (15), and $\Delta_t^{sB} = 0$. Under quantitative easing, if the demand for CBDC deposits is lower than the amount of excess reserves, the commercial bank can swap excess reserves for CBDC deposits and $\Delta_K^q = 0$. Since the central bank transforms one type of liabilities into another, the impact on taxes is $\Delta_t^q = \left[(1+\mu_d)R^d - R^r\right]h$. If bank deposits have the same liquidity properties of reserves, then $(1+\mu_d)R^d = R^r$, and $\Delta_t^q = 0$. □

6 Conclusions

When central banks issue a CBDC, the equilibrium effects on the economy depend on the ongoing monetary policy. In this paper, we use a simple two-period model to compare two illustrative cases, the first where the central bank pursues standard monetary policy and the second where it implements quantitative easing. Our paper sheds light on the key equilibrium mechanisms that affect the banking and government sectors and derives neutrality implications without a direct pass-through policy.

First, we find that the economic effects do indeed differ depending on the interaction with the ongoing monetary policy. For instance, introducing a CBDC under standard policy does not affect lending to the economy, but it can reduce it under QE. This fact can be regarded as a warning that the debate over CBDC cannot be held in a vacuum, as it will interact with the other central bank policies.

Second, the impact of introducing a CBDC while the central bank is conducting QE depends on the amount of excess reserves in the system. Banks optimally transfer excess reserves to households when creating new CBDC deposits. Therefore, a CBDC has no impact on the banking sector as long as the demand for CBDC does not exceed excess reserves. Above this threshold, introducing a CBDC is problematic as banks lose a cheap source of funding, which is not replaced. Furthermore, it is worth noting that substituting banks with households on the liability side of the central bank's balance sheet is not without consequences. Households tend to be inelastic, so it would be difficult for the central bank to reduce the size of its balance sheet when reverting QE policies. In this sense, introducing a CBDC might render QE quasi-permanent.

These findings are relevant for policymakers in charge of designing future digital currencies. CBDCs have the potential to radically change monetary policy transmission, and central banks should have a comprehensive approach that considers the interaction with current monetary policies.

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A Equilibrium Under Quantitative Easing with CBDC Backed by Government Bonds

DEFINITION 3. *If the demand for CBDC deposits is such that $d > \bar{d}$, given the central bank quantitative easing policy $(R^B,\ R^r,\ \delta,\ \bar{\alpha})$, with interest rate policy $R^B = R^r$ and balance sheet policy $(B^{CB},\ E^{CB}) = (d,\ M - \bar{d})$, the banking equilibrium consists of rates of return $(R^h,\ R^d,\ R^c,\ R^E)$ and choices $(h,\ d,\ c,\ e,\ E,\ D,\ M,\ K)$ such that:*

(a) *Conditions (15), (16), (17), (18), (19), (22), (21) hold;*
(b) *(h, d) is optimal for households, given (R^d, R^h); c is optimal for cash pools, given R^c; e is optimal for investors, given (R^B, R^E);*
(c) *$M \geqslant \delta h$;*
(d) *$c_b = c - (B - d)$;*
(e) *$e + M - \bar{d} = E$.*

Under QE policy, there is a positive amount of excess reserves in the system due to the asset-purchase programs. Thus, the liquidity requirement is not binding. When the central bank decides to hold treasuries against CBDC deposits, the amount of risky investments in the economy (K) increases but not the size of the bank (S). This happens because the decrease in deposits is fully offset by increased cash pools' funding as there are $\tilde{d}$ less bonds available in the economy. At the same time, the reduction in deposits allows the commercial bank to further decrease its reserves, increasing the amount of investable debt. Thus, the commercial bank has more funding to allocate in risky loans. The difference with the standard policy setting is that, in this scenario, bank reserves are not backed by treasuries but by bank equity. Therefore, a reduction in bank reserves has no impact on the treasury market, and it does not allow cash pools to purchase more treasuries. We find that the bank's leverage decreases and that the larger investable debt increases bankruptcy costs.

In this scenario, since some bank reserves are swapped into CBDC deposits, and some are simply reduced, the central bank asset side is less risky. Therefore, the introduction of CBDCs backed by treasuries, under QE policy, reduces the seigniorage volatility. Consequently, the economy benefits from more stable taxes.

B CBDC Equilibrium Effects - Proofs

The superscripts s and q denote the standard policy and the QE policy scenarios, respectively, without the CBDC. In this section, we always consider the QE policy when the amount of CBDC deposits exceeds the amount of excess reserves in the economy ($d > \bar{d}$) and the liquidity requirement is binding. With the introduction of a CBDC, a B superscript indicates when the central bank decides to hold government bonds against CBDC deposits and a E superscript when the CBDC is backed by bank equity (risky securities). The Δ_x^{sB} is defined as the difference between the generic variable x in the case of standard policy with CBDC backed by treasuries and the same variable in a scenario with the same policy but no CBDC: $\Delta_x^{sB} = x^{sB} - x^s$. Similarly, the differences $\Delta_x^{sE} = x^{sE} - x^s$, $\Delta_x^{qB} = x^{qB} - x^q$, and $\Delta_x^{qE} = x^{qE} - x^q$ illustrate the variation with the respective baseline scenarios.

B.1 Agents' optimal choices

We assume that the monetary policy interest rates (R^r, R^B), the amount of government bonds in the economy (B), and the convenience yield of deposits (ρ_h) do not change with the introduction of a CBDC. If we also assume that the initial endowments of the agents do not change, it implies that the optimal amounts of savings for depositors and cash pools remain the same with the introduction of a CBDC.

B.2 Bank deposits and reserves

In scenarios without the CBDC, bank deposits are the same: $h^s = h^q$. With the introduction of a CBDC, we always have that part of the depositors' savings goes to the central bank and, therefore, bank deposits decrease:

$$h^{sB} = h^{sE} = h^s - d,$$
$$h^{qB} = h^{qE} = h^q - d,$$

with $\Delta_h^{sB} = \Delta_h^{sE} = \Delta_h^{qB} = \Delta_h^{qE} = -d < 0$.

The amount of bank reserves in standard policy is given by $M^{sB} = M^{sE} = \delta(h^s - d) = M^s - \delta d$, because the liquidity requirement is binding. Under QE policy, the commercial bank swaps $\bar{d}$ excess reserves into CBDC deposits. After this point, the liquidity requirement is binding, and at each further unit of bank deposits reduction corresponds δ units of reserves reduction. We have that $M^{qB} = M^{qE} = M^q - \bar{d} - \delta\tilde{d}$, where $\tilde{d} = d - \bar{d}$. We obtain $\Delta_M^{sB} = \Delta_M^{sE} = -\delta d < 0$, and $\Delta_M^{qB} = \Delta_M^{qE} = -d + (1-\delta)\tilde{d} < 0$.

B.3 Wholesale funding

The wholesale funding is given by the cash pool demand of savings, minus all the available government bonds in the economy. The amount of treasuries available for cash pools is given by the amount of bonds issued by the government minus the ones bought by the central bank. In standard policy $c_b^s = c - (B - M^s)$, while under QE policy the central bank does not hold any bond and $c_b^q = c - B$.

With the introduction of a CBDC backed by treasuries in standard policy, the cash pool funding becomes $c_b^{sB} = c^s - (B^s - M^{sB} - d)$, which translate in an increase of $\Delta_{c_b}^{sB} = c_b^{sB} - c_b^s = (1-\delta)d > 0$. When the CBDC deposits are backed by equity, the mechanism is similar to before, i.e., $c_b^{sE} = c^s - (B^s - M^{sE})$, which corresponds to a decline of $\Delta_{c_b}^{sE} = c_b^{sE} - c_b^s = -\delta d < 0$, given by the decrease in the reserves. Under QE policy, the bank's wholesale funding when the central bank holds bonds against CBDC deposits is $c_b^{qB} = c^q - (B^q - d)$, with an increase of $\Delta_{c_b}^{QB} = c_b^{qB} - c_b^q = \tilde{d} > 0$. The funding does not change if the central bank decides to hold only equity: $c_b^{qE} = c^q - B^q$, with $\Delta_{c_b}^{qE} = c_b^{qE} - c_b^q = 0$.

B.4 Investable debt, bank equity and risky investment

As in equation (18), we define the investable debt of the bank as all the debt fundings that can be invested in the risky technology, excluding the reserves. In all scenarios, the investable debt is determined by:

$$D = h + c_b - M.$$

Under standard policy with CBDC backed by treasuries, there is no difference with the baseline: $\Delta_D^{sB} = 0$. However, if the central bank decides to allocate these funds in bank equity, then the investable debt declines by $\Delta_D^{sE} = -d < 0$. On the other hand, under quantitative easing policy, the CBDC investment in the safe asset translates in an increase in the debt that the banks can use to fund the risky technology, $\Delta_D^{qB} = d - (1-\delta)\tilde{d} > 0$, while an investment in bank equity decreases it, $\Delta_D^{qE} = -(1-\delta)\tilde{d} < 0$.

Let's define $\gamma = \frac{\bar{\alpha}}{1-\bar{\alpha}}$ for simplicity in the notation. At equilibrium, as in equation (22), the amount of bank equity is fixed at $E = \gamma D$, and, because of condition (19), the risky investment is always given by $K = (1+\gamma)D$. For both equity and risky investment, the results are the same as for the investment debt, but scaled by γ and $1+\gamma$, respectively.

B.5 Commercial bank size

We measure the bank size as the sum of all its liabilities or all its assets:

$$S = h + c_b + E = M + K.$$

The introduction of a CBDC in standard policy always leads to a decline in the bank size. In fact, $\Delta_S^{sB} = -\delta d < 0$ and $\Delta_S^{sE} = -(1+\delta+\gamma)d < 0$. Instead, in a QE policy setting, we have that $\Delta_S^{qB} = \gamma[d - (1-\delta)\tilde{d}] > 0$ and $\Delta_S^{qE} = -d - \gamma(1-\delta)\tilde{d} < 0$.

B.6 Bankruptcy costs

Let $\hat{y}$ be the minimum return on the risky technology that allows the bank to repay its creditors. It follows that $\hat{y}$ is such that $K\hat{y} + MR^r = hR^h(1+\mu_h) + c_bR^c$, and the bank is solvent for $y > \hat{y}$. The bankruptcy costs are then given by:

$$\phi = hR^h(1+\mu_h) + c_bR^c - MR^r - Ky,$$

when $y \leqslant \hat{y}$. At equilibrium, it holds that $R^c = R^B$ as in (16), $R^h(1+\mu_h) = (1-\delta)R^B + \delta R^r$ for condition (17), and $D = h + c_b - M = \frac{K}{(1+\gamma)}$ as defined in section B.4. This implies that $\phi = DR^B - Ky$ and $\hat{y} = \frac{R^B}{1+\gamma}$. Hence, the bankruptcy costs can be written as:

$$\phi = \left[R^B - (1+\gamma)y\right]D.$$

For this reason, all the results are the same as for the investable debt D, but scaled by $\big[R^B - (1+\gamma)y\big]$, that is always positive in bankruptcy because $y \leqslant \hat{y}$.

B.7 Seignorage

The seignorage is defined as the profit made by the government. In standard policy, this profit is given by $\theta^s = \big(R^B - R^r\big)M^s$, while under quantitative easing policy we have $\theta^q = \big(V(y) - R^B\big)M^q$. With the introduction of a CBDC, there is an additional term that depends on what the central bank decides to hold against the new funds. If CBDC deposits are backed by bonds, then the seignorage has an additional profit of $\big(R^B - (1+\mu_d)R^d\big)$ per unit of CBDC. Instead, if it they are backed by bank equity, then the additional profit per unit of CBDC becomes $\big(V(y) - (1+\mu_d)R^d\big)$.

Therefore, with the introduction of the CBDC in the standard policy we have that $\theta^{sB} = \big(R^B - R^r\big)M^{sB} + \big(R^B - (1+\mu_d)R^d\big)d$, and $\theta^{sE} = \big(R^B - R^r\big)M^{sE} + \big(V(y) - (1+\mu_d)R^d\big)d$, with a difference from the baseline of $\Delta_\theta^{sB} = \big[(1+\mu_h)R^h - (1+\mu_d)R^d\big]d$, and $\Delta_\theta^{sE} = -\big(R^B - R^r\big)\delta d + \big(V(y) - (1+\mu_d)R^d\big)d$, respectively. Similarly, under quantitative easing policy the seignorage is computed as $\theta^{qB} = \big(V(y) - R^B\big)M^{qB} + \big(R^B - (1+\mu_d)R^d\big)h$ in the scenario with a CBDC backed by safe assets, and as $\theta^{qE} = \big(V(y) - R^B\big)M^{qE} + \big(V(y) - (1+\mu_d)R^d\big)d$ for equity held against the CBDC. The differences with the baseline scenario are respectively $\Delta_\theta^{qB} = \big(R^B - (1+\mu_d)R^d\big)d - \big(V(y) - R^B\big)\big(d - (1-\delta)\tilde{d}\big)$, and $\Delta_\theta^{qE} = \big(R^B - (1+\mu_d)R^d\big)d + \big(V(y) - R^B\big)(1-\delta)\tilde{d}$.

In the quantitative easing policy, Pareto-optimum can be achieved. As $\mathbb{E}\big[V(y)\big] = R^E$ by definition, $R^c = R^B$ at the banking equilibrium, and $R^E = R^c = (1+\mu_d)R^d$ at Pareto-optimum, it follows that:

$$\mathbb{E}\big[\Delta_\theta^{qB}\big] = \mathbb{E}\big[\Delta_\theta^{qE}\big] = 0.$$

It is worth noting that whenever the central bank decides to invest in bank equity, the seignorage is no more deterministic because it depends on the realization of the payoff of the risky technology. Therefore, the only scenarios in which the seigniorage volatility is null are standard policy without CBDC and with CBDC backed by bonds: $\sigma_\theta^s = \sigma_\theta^{sB} = 0$. If the central bank decides to hold equity against CBDC deposits, we have that $\sigma_\theta^{sE} = d\,\sigma_{V(y)}$, where $\sigma_{V(y)}$ is the volatility of the equity payoff. Under quantitative easing policy, the seigniorage is always volatile and, specifically, we have that $\sigma_\theta^q = M^q\,\sigma_{V(y)}$. Introducing a CBDC has opposite effects to the seigniorage volatility depending on where the central bank decides to invest the funds. If the CBDC deposits are backed by treasuries, then $\sigma_\theta^{qB} = M^{qB}\,\sigma_{V(y)}$, reducing the volatility: $\Delta_{\sigma_\theta}^{qB} = -(d - (1-\delta)\tilde{d})\,\sigma_{V(y)} < 0$. On the other hand, holding bank equity increases the volatility of the seigniorage, as $\sigma_\theta^{qE} = M^{qE}\,\sigma_{V(y)}$, and $\Delta^{qE}_{\sigma_\theta} = (1-\delta)\tilde{d}\,\sigma_{V(y)} > 0$.

B.8 Taxes

Taxes are defined in Section 2.5:

$$t(y) = \begin{cases} R^B B - \theta, & \text{if } y > \hat{y} \\ R^B B - \theta + \phi, & \text{if } y \leqslant \hat{y} \end{cases} \; = \; R^B B - \theta + \phi\,\mathbb{1}_{y \leqslant \hat{y}}.$$

For this reason, all the differences in all scenarios can be determined as $\Delta_t = \Delta_\phi\,\mathbb{1}_{y\leqslant\hat{y}} - \Delta_\theta$.